1994

MATHEMATICAL METHODS TRIAL CAT 2

Based on the Victorian Certificate of Education Mathematics Study Design.

CHEMISTRY ASSOCIATES
P.O. BOX 2227
KEW, VIC., 3101
AUSTRALIA
TEL:(03) 9817 5374 **FAX: (03) 9817 4334**

email: chemas@vicnet.net.au

Internet: http://www.vicnet.net.au/~chemas/education.htm

Victorian Certificate of Education
Mathematics 1994

MATHEMATICAL METHODS
1994 TRIAL CAT 2
Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

Part I

MULTIPLE-CHOICE QUESTION BOOKLET

This task has two parts: part I (multiple-choice questions) and part II (short answer questions). Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, continue immediately to the other part.
A detachable formula sheet for use in both parts is included with this booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

Directions to students

Materials
Question booklet of 13 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.
An approved calculator may be used.

The task
Detach the formula sheet from this booklet during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for part I.
All questions should be answered on the answer sheet for multiple-choice questions provided. Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

Specific instructions for Part I

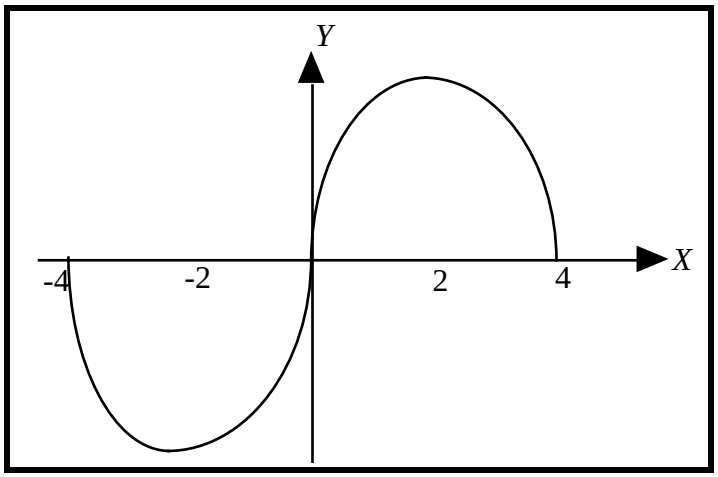
Part I consists of 33 questions.

Answer **all** questions in this section on the multiple-choice answer sheet provided.

A correct answer scores 1, an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers and you should attempt every question.

For each of the following, put a line through the correct response on the answer sheet provided.

Question 1



The graph shown above has a negative gradient for

A. $\{x : -4 < x < 0\}$

B. $\{x : -2 < x < 2\}$

C. $\{x : -4 < x < -2\}$ $\{x : 2 < x < 4\}$

D. $\{x : x \leq -2\}$ $\{x : x \geq 2\}$

E. $\{x : x = -2\}$ $\{x : x = 2\}$

Question 2

The graph of $y = 2 \log_2 x$ could have

A. a Domain where $x \in R$.

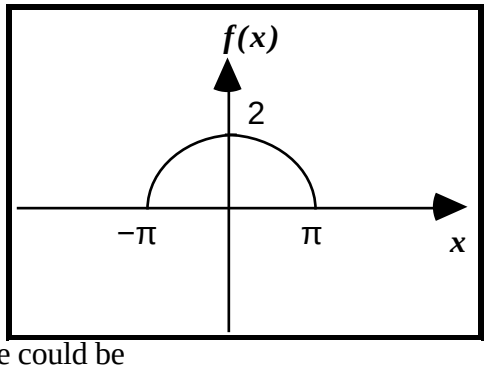
B. contain the point (0,1).

C. an asymptote with equation $y = 2$.

D. an asymptote with equation $x = 0$.

E. an inverse with equation $y = 2^x$.

Question 3



The equation of the graph above could be

A. 2 cos$x$

B. cos$x$ + 1

C. 2 sin (x - π)

D. sin$x$ + 1

E. 2 sin$x$

Question 4

If sin 2x = 1 , 0 ≤ x ≤ 2π, then the number of solutions is

A. 0

B. 1

C. 2

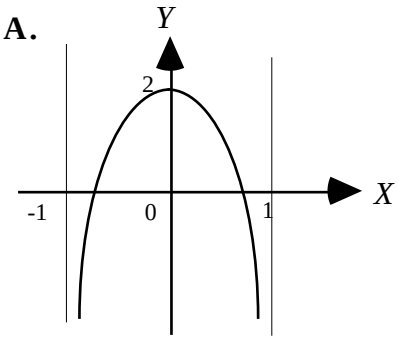
D. 3

E. 4

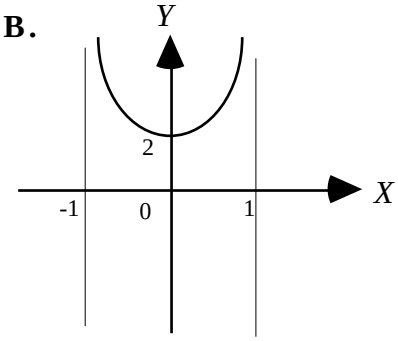
Question 5

Which one of the following best represents the graph with equation $y = \dfrac{2}{x^2 + 1}$?

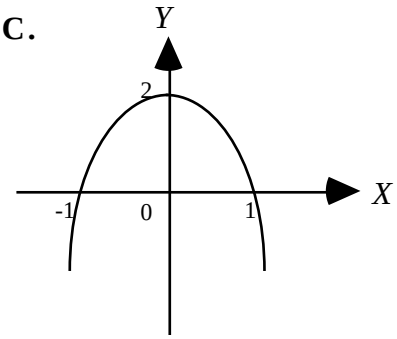
A.



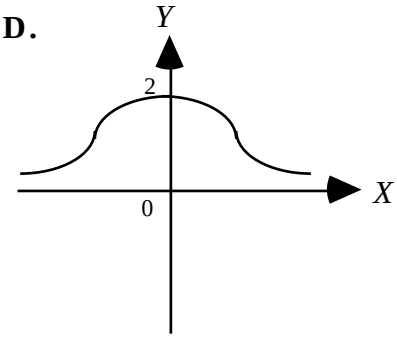
B.



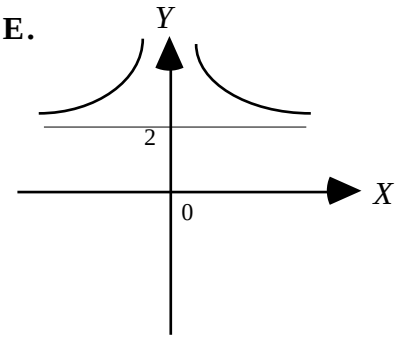
C.



D.



E.



Question 6

If $\cos x = -\frac{1}{2}$ and $0 < x < \pi$, then x equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

E. $\frac{3\pi}{4}$

Question 7

Which one of the following statements is always true?

A. $\tan\theta \leq 1$ for all values of θ.

B. $\sin\theta = -\cos\theta$ for all values of θ.

C. $\sin\theta$ is positive for all values of θ.

D. $\cos\theta$ is negative for $\frac{3\pi}{2} < \theta < 2\pi$.

E. $\tan\theta$ is positive for $\pi < \theta < \frac{3\pi}{2}$

Question 8

The graph of $3 + \cos 2x$ has a minimum value of

A. -5

B. -3

C. 0

D. 1

E. 2

Question 9

The largest possible domain for the function $f(x) = \log (2x - 3)$ is

A. R

B. R^+

C. $R^+ \quad \{0\}$

D. $x > 1\frac{1}{2}$

E. $x \geq 1\frac{1}{2}$

Question 10

The graph of $2 \cos 3(x - \frac{\pi}{4}) + 1$ has

A. an amplitude of 3.

B. an amplitude of 1.

C. a period of $\frac{2\pi}{3}$.

D. a period of $\frac{\pi}{4}$.

E. a period of $\frac{\pi}{2}$.

Question 11

The derivative of $\frac{1}{\sqrt{2x^3}}$ is equal to

A. $\frac{1}{3\sqrt{x}}$

B. $\frac{3}{4\sqrt{x}}$

C. $\frac{-3}{4x^5}$

D. $\frac{-3}{2\sqrt{2x}}$

E. $\frac{-3}{2\sqrt{2x^5}}$

Question 12

A graph has the rule $y = e^x + e$. The angle the tangent to this graph, at the point $x = 0$, makes with the positive direction of the X axis, is closest to

A. 15^o

B. 30^o

C. 45^o

D. 60^o

E. 70^o

Question 13

If $xy - y - x = 0$, then $\frac{dy}{dx}$ is equal to

A. $\frac{y - 1}{x - 1}$

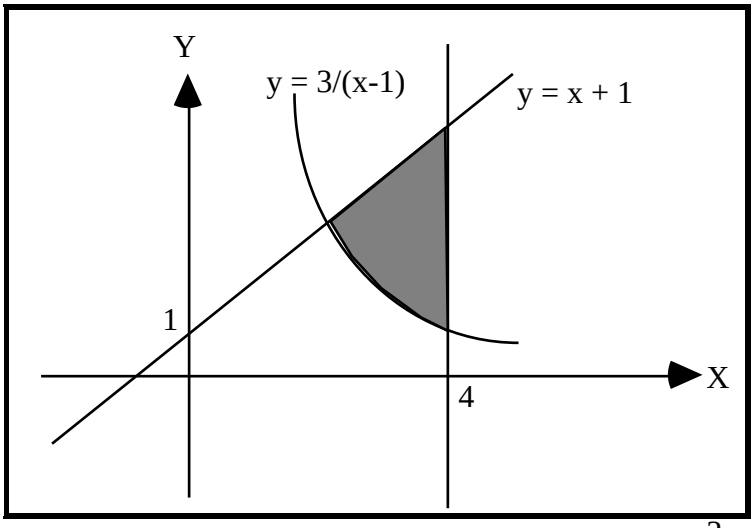
B. $\frac{1 - y}{x - 1}$

C. $\frac{y - 1}{x + 1}$

D. $y - x$

E. 0

Question 14



In the diagram, the shaded area shown is the region between the curve $y = \frac{3}{x - 1}$, the line $y = x + 1$ and the line $x = 4$. The area can be found from

A. $\left[\frac{x^2}{2} + x - 3\log_e(x - 1)\right]_2^4$

B. $\left[\frac{x^2}{2} + x - 3\log_e(x - 1)\right]_3^4$

C. $\left[1 - 3\log_e(x - 1)\right]_3^4$

D. $\left[\frac{x^2}{2} + x + \frac{3}{(x - 1)^2}\right]_2^4$

E. $\left[1 - \frac{3}{(x - 1)^2}\right]_2^4$

Question 15

If $\int_{-2}^{2} c(x - 5)\,dx = 1$ where c is a constant, then c equals

A. $\frac{1}{5}$

B. $\frac{1}{10}$

C. $-\frac{1}{10}$

D. $-\frac{1}{16}$

E. $-\frac{1}{20}$

Question 16

If $2 + \log_{10} 3x = \log_{10} y$, then

A. $y = \frac{3x}{1024}$

B. $y = \frac{3x}{100}$

C. $y = 6x$

D. $y = 300x$

E. $y = 3072x$

Question 17

The graph of $y = (1 - x)^2 (7 + 2x)$ cuts the X axis when x equals

A. $1 , \frac{2}{7}$

B. $1 , -\frac{2}{7}$

C. $1 , 3\frac{1}{2}$

D. $1 , -3\frac{1}{2}$

E. $\pm 1 , -3\frac{1}{2}$

Question 18

If $f(x)$ equals $1 + x + \cos x$, then $f(\frac{\pi}{2})$ is closest to

A. -1

B. 0

C. 1

D. 3

E. 91

Question 19

The graph of $2 \sin 2x - \sqrt{3}$, $0 \leq x \leq 2\pi$, cuts the X axis when x equals

A. $\frac{\pi}{3}, \frac{2\pi}{3}$ only.

B. $\frac{\pi}{6}, \frac{\pi}{3}$ only.

C. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ only.

D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$ only.

E. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ only.

Question 20

The derivative of $y = x^2 \sin x$ with respect to x, is best expressed as

A. $\sin 2x^2 + \cos x^3$

B. $2x \sin x + x^2 \cos x$

C. $2x \cos x$

D. $-2x \cos x$

E. $2\cos x^2$

Question 21

$\int_0^2 \frac{2x}{x^2 + 2}\, dx$ is closest to

A. 0.5

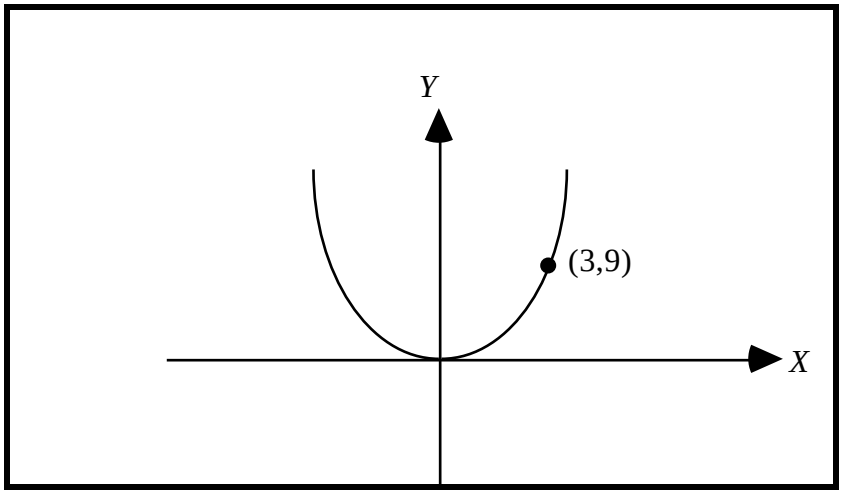
B. 0.7

C. 0.8

D. 1.1

E. 1.8

Question 22



The gradient of the curve $y = x^2$ at the point (3 , 9) is

A. 0

B. 6

C. 9

D. 18

E. 81

Question 23

If $f(x) = (2x + 5)^4$, then $f'(x)$ equals

A. $4(2x + 5)$

B. $4(2x + 5)^3$

C. $4(2x + 5)^4$

D. $8(2x + 5)^3$

E. $8(2x + 5)^4$

Question 24

$\int_0^1 (2x - 1)^5 \, dx$ is equal to

A. 5

B. 1

C. $\frac{1}{6}$

D. $\frac{1}{12}$

E. 0

Question 25

$\int (e^x - e^{-x})^2 \, dx$ is equal to

A. $-2x + C$

B. $e^{2x} - 2x + C$

C. $\frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + C$

D. $\frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x} - 2x + C$

E. $\frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} - 2x + C$

Question 26

If $x^2 + 2x = 0$, then x equals

A. -2

B. 2

C. 0

D. 2 or 0

E. -2 or 0

Question 27

If $2^{2(x-1)} = 8$, then x equals

A. 0.4

B. 2.0

C. 2.5

D. 3.5

E. 5.0

Question 28

The value of x for which $x^2 + 8x + 9$ is a minimum is

A. -9

B. -7

C. -4

D. 4

E. 9

Question 29

The gradient of the graph $y = x^2 - 2x + 3$ is

A. always positive.

B. positive for all values of x greater than -2.

C. positive for all values of x greater than -1.

D. positive for all values of x greater than 1.

E. always negative.

Question 30

Which **one** of the following is true?

A. An estimator is a specific observed value of a statistic.

B. An estimator is a sample statistic used to estimate a population parameter.

C. An estimator is a sample parameter used to estimate a population statistic.

D. An estimator is the set of values that lie within 2 standard errors of the mean with 95% confidence.

E. An estimator is the set of values that lie within 3 standard errors of the mean with 95% confidence.

Question 31

The standard deviation of a sample proportion where the probability of a success, $p = 0.3$, and the sample size, $n = 100$, is

A. 0.0021

B. 0.0030

C. 0.0458

D. 0.0548

E. 0.0837

The following table should be used to answer **Question 32**.
Cumulative Normal Distribution

$$F(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-0.5t^2}\, dt$$

a	$F(a)$	a	$F(a)$	a	$F(a)$
0.0	0.500				
0.1	0.540	1.1	0.864	2.1	0.982
0.2	0.579	1.2	0.885	2.2	0.986
0.3	0.618	1.3	0.903	2.3	0.989
0.4	0.655	1.4	0.919	2.4	0.992
0.5	0.692	1.5	0.933	2.5	0.994
0.6	0.726	1.6	0.945	2.6	0.995
0.7	0.758	1.7	0.955	2.7	0.996
0.8	0.788	1.8	0.964	2.8	0.997
0.9	0.816	1.9	0.971	2.9	0.998
1.0	0.841	2.0	0.977	3.0	0.999

Question 32
If the random variable X is normally distributed with mean 1.0 and standard deviation 1.5, then the $\Pr(X > 4)$ is closest to

A. 0.01

B. 0.02

C. 0.05

D. 0.08

E. 0.09

Question 33
The best sized sample to take so that the standard error will be less than or equal to 0.01 if $p = 0.3$ is

A. 21.

B. 110.

C. 210.

D. 2000.

E. 2100.

END PART I GO ON IMMEDIATELY TO PART II

STUDENT NUMBER								LETTER
figures								
words								

Victorian Certificate of Education
Mathematics 1994

MATHEMATICAL METHODS
1994 TRIAL CAT 2
Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

Part II

QUESTION AND ANSWER BOOKLET

This task has two parts: part I (multiple-choice questions) and part II (short answer questions). Part I consists of a separate question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, continue immediately to the other part.
A detachable formula sheet for use in both parts is included in the part I question booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of this question and answer booklet (part II) and hand them in.

Directions to students

Materials
Question and answer booklet of 5 pages including one blank page for rough working.
Working space is provided throughout the booklet.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the part I booklet during reading time.
Ensure that you write your **student number** in the space provide on the cover of this booklet.
The marks allotted to each question are indicated at the end of the question.
There is a total of 17 marks available for part II.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of this question and answer booklet (part II) and hand them in.

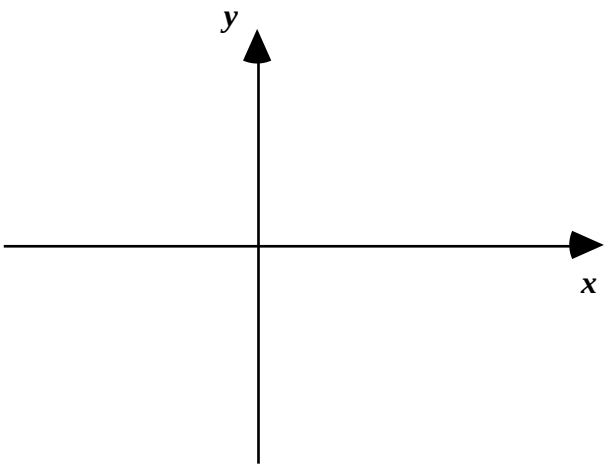
Specific instructions for Part II

Part II consists of 6 questions.

Answer **all** questions in this booklet.

Question 34

(a) Sketch the graph of $y = \log_e (3x + 2)$ showing the intercepts with both axes.



(b) Give the equation of the vertical asymptote.

_____

_____

_____

_____

_____

(4 marks)

Question 35

Find all values of θ between 0 and 2π for which $\cos 2\theta = \frac{1}{2}$.

(3 marks)

Question 36

Solve for x where

$$2e^{2x} - e^{x} = 1$$

(2 marks)

Question 37

Find the derivative of

$$y = e^{\sin 3x}$$

(2 marks)

Question 38

Find the area under the curve

$$y = (2x + 1)^3 \text{ for } 0 \leq x \leq 1$$

(3 marks)

Question 39

Cumulative Normal Distribution

$$F(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-0.5t^2}\, dt$$

a	$F(a)$	a	$F(a)$	a	$F(a)$
0.0	0.500				
0.1	0.540	1.1	0.864	2.1	0.982
0.2	0.579	1.2	0.885	2.2	0.986
0.3	0.618	1.3	0.903	2.3	0.989
0.4	0.655	1.4	0.919	2.4	0.992
0.5	0.692	1.5	0.933	2.5	0.994
0.6	0.726	1.6	0.945	2.6	0.995
0.7	0.758	1.7	0.955	2.7	0.996
0.8	0.788	1.8	0.964	2.8	0.997
0.9	0.816	1.9	0.971	2.9	0.998
1.0	0.841	2.0	0.977	3.0	0.999

A certain brand of milk is sold in litre cartons. The actual amount of milk (in litres) in each carton is a normally distributed random variable with mean 0.995 and standard deviation 0.05. Find the proportion of milk cartons containing more than 1 litre.

(3 marks)

End Of Questions 1994 Mathematical Methods Trial Cat 2

CHEMISTRY ASSOCIATES
PO BOX 2227
KEW
VICTORIA 3101
AUSTRALIA
TEL: (03) 9817 5374 FAX: (03) 9817 4334

email: chemas@vicnet.net.au
Internet: http://www.vicnet.net.au/~chemas/education.htm

ROUGH WORKING

SECTION A CORE

1	**C**	**12**	**C**	**23**	**D**
2	**D**	**13**	**B**	**24**	**E**
3	**B**	**14**	**A**	**25**	**E**
4	**C**	**15**	**E**	**26**	**E**
5	**D**	**16**	**D**	**27**	**C**
6	**D**	**17**	**D**	**28**	**C**
7	**E**	**18**	**D**	**29**	**D**
8	**D**	**19**	**E**	**30**	**B**
9	**D**	**20**	**B**	**31**	**C**
10	**C**	**21**	**D**	**32**	**B**
11	**E**	**22**	**B**	**33**	**E**

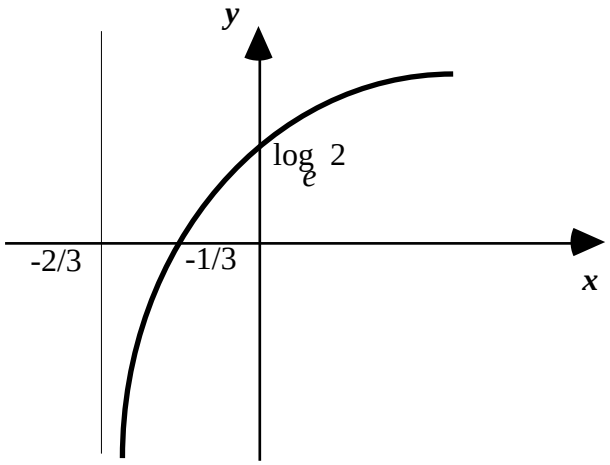
PART II

Question 34

(a) When $3x + 2 = 1$, $y = 0$. Hence, $3x = -1$ and $x = -\frac{1}{3}$

When $x = 0$, $y = \log_e 2$

There is an asymptote at $x = -\frac{2}{3}$.



Question 35

$\cos 2\theta = \frac{1}{2}$, $0 \le \theta \le 2\pi$, $0 \le 2\theta \le 4\pi$

Therefore,

$2\theta = \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$, $2\pi + \frac{\pi}{3}$, $4\pi - \frac{\pi}{3}$.

Hence, $2\theta = \frac{\pi}{3}$, $\frac{5\pi}{3}$, $\frac{7\pi}{3}$, $\frac{11\pi}{3}$

and $\theta = \frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$ **ANS**

Question 36

$2(e^x)^2 - e^x - 1 = 0$

$(2e^x + 1)(e^x - 1) = 0$

$2e^x + 1 = 0$ or $e^x - 1 = 0$

$2e^x = -1$ or $e^x = 1$

$e^x = -\frac{1}{2}$ or $e^x = 1$.

But $e^x > 0$ for all values of x.

Therefore, $e^x = 1$ and $x = 0$ **ANS**

Question 37

Let $\sin 3x = u$

$\frac{du}{dx} = 3\cos 3x$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= e^u \times 3\cos 3x$

$= 3e^u \cos 3x$

$= 3e^{\sin 3x} \cos 3x$ **ANS**

Question 38

$A = \int_0^1 (2x + 1)^3 \, dx$

$= \frac{1}{8}(2x + 1)^4 \Big]_0^1$

$= \frac{1}{8}(3)^4 - \frac{1}{8}(1)^4$

$= \frac{81}{8} - \frac{1}{8}$

$= \frac{80}{8}$

$= 10$ square units. **ANS**

Question 39

$\sigma = 0.05$

$Z = \frac{X - \mu}{\sigma}$

$= \frac{1 - 0.995}{0.05}$

$= \frac{0.005}{0.05}$

$= 0.1$

$\Pr(X > 1) = \Pr(Z > 0.1)$

$= 1 - \Pr(Z < 0.1)$

$= 1 - 0.540$

$= 0.46.$

Therefore, 46 % of cartons contain more than 1 litre. **ANS**

End Of Suggested Solutions 1994 Mathematical Methods Trial Cat 2

CHEMISTRY ASSOCIATES
PO BOX 2227
KEW
VICTORIA 3101
AUSTRALIA
TEL: (03) 9817 5374 FAX: (03) 9817 4334

email: chemas@vicnet.net.au
Internet: http://www.vicnet.net.au/~chemas/education.htm