The Mathematical Association of Victoria

Trial Exam 2024

GENERAL MATHEMATICS

WRITTEN EXAMINATION 2

SOLUTIONS

Data analysis

Question 1

a. 77.8% 1 mark

There are 14 lifespans that are less than 7 years.

$\frac{14}{18} = 77.8\%$

b. Upper fence = $6 + 1.5 \times (6 - 4) = 9$, $10 > 9$ so it is an outlier. 2 marks

The data is ordered so the median and quartiles can be easily by hand.

These values are also shown in the boxplot in part (c).

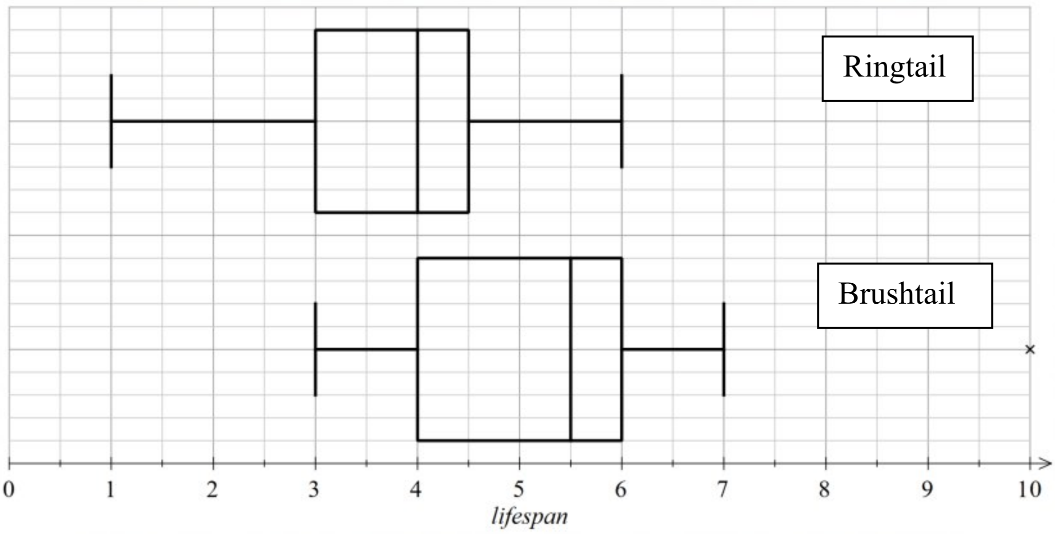
3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 10

Median = 5.5

$Q_1 = 4$ $Q_3 = 6$

Using rule for the upper fence, $Q_3 + 1.5(IQR) = 6+1.5(2) = 9$, so 9 is the upper fence. A1
$10 > 9$, so 10 is an outlier. A1

c. The graph as added below: 1 mark



d. Yes, the boxplots do support the contention that *lifespan* is associated with *tail type*, Brushtail or Ringtail because the median for the *Brushtail* possum (5.5 years) is higher than the median for the *Ringtail* possum (4 years).
or
Yes, the boxplots do support the contention that *lifespan* is associated with *tail type*, Brushtail or Ringtail because the IQR for the *Brushtail* possum (2 years) is greater than the IQR for the *Ringtail* possum (1.5 years).

Statement that association exists due to difference in medians or IQR A1

Correctly stating values for medians or IQRs A1

Question 2

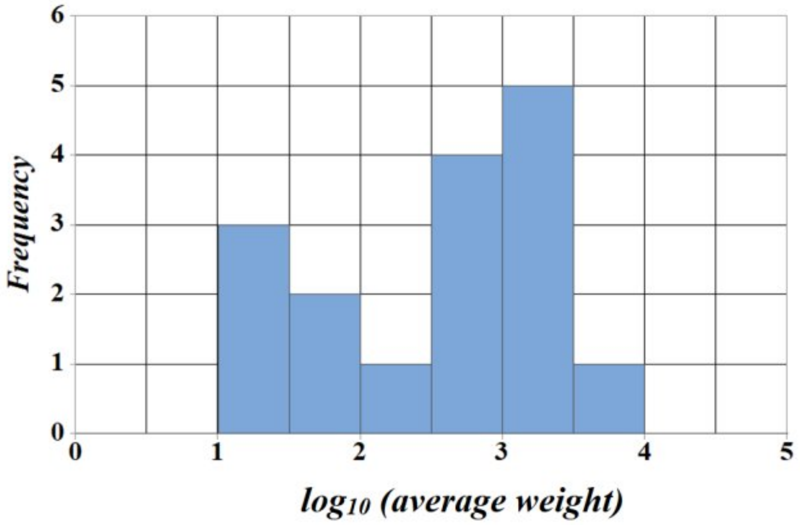
a. Table completed as below: 1 mark

Daintree River Ringtail	950	2.98
Honey	10	1.00

Daintree River Ringtail: $\log_{10}(950) = 2.98$

Honey: $10^1 = 10$

b. The graph as shown below: 1 mark



A frequency table of the *log_{10} (average weight)* with intervals of 0.5 is

log_{10} (average weight	frequency
1.0 - < 1.5	3
1.5 - < 2.0	2
2.0 - < 2.5	1
2.5 - < 3.0	4
3.0 - < 3.5	5
3.5 - < 4.0	1
Total	16

This can then be used to construct the histogram.

Question 3

a. One 1 mark

The only ordinal variable is *conservation status.* The numbers are just code for the categories: least concern, near threatened and critically endangered which are ordered conservation levels from lowest to highest.

b. As r = −0.775, there is a negative association between *body length* and *litter size.* 1 mark

Pearson's correlation coefficient for *body length* and *litter size* is r = −0.775 which indicates a strong negative correlation. This supports the researcher's belief that 'smaller possum species, based on *body length,* tend to have a larger *litter size'*.

The value of Pearson's can be calculated using CAS as shown below:

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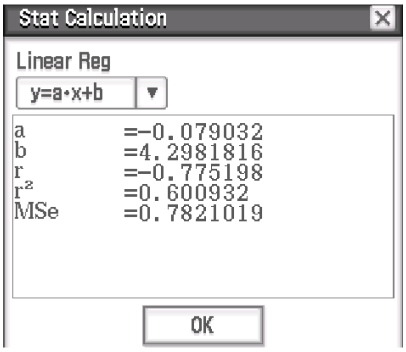
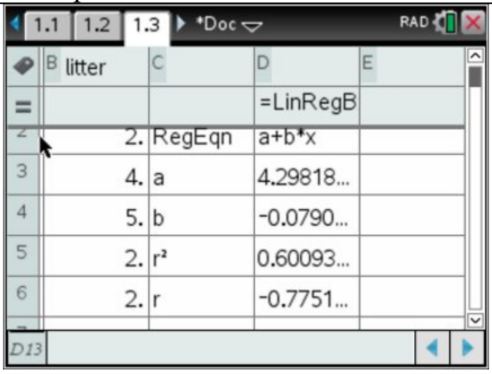
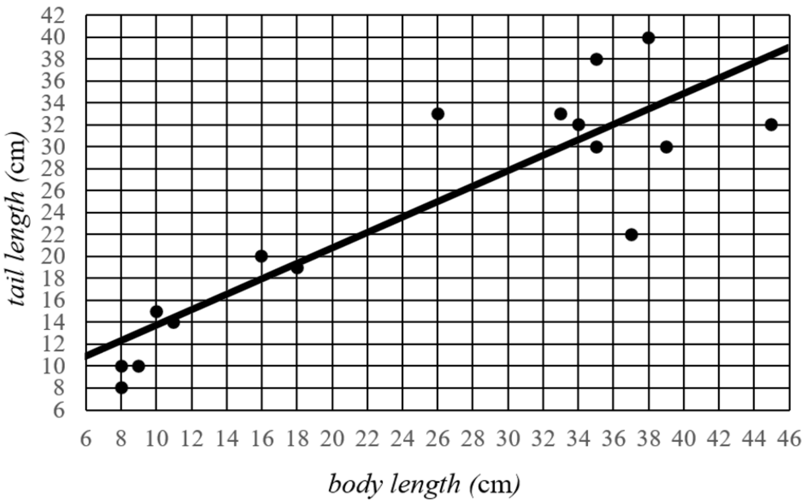
	

c. Line as shown below passing through (6, 10.6) and (46, 38.6): 1 mark



Find two points that lie on the line, by substituting two different body length values into the given equation. The line must be drawn for the entire axes given therefore choose the two end points.

When body length is 6: *tail length* = 6.4 + 0.70 × *6* = 10.6 giving the point (6, 10.6).

When body length is 46: *tail length* = 6.4 + 0.70 × 4*6* = 38.6 giving the point (46, 38.6).

d. **i.** $r^2 = 0.777$ 1 mark

The value of the coefficient of determination can be calculated using the square of the given r value:
$r^2 = 0.8816^2 = 0.777218...$

ii. 77.7% of the variation in *tail length* can be explained by the variation in *body length*. 1 mark

The value of $r^2 = 0.777$ is converted to a percentage $0.777 \times 100 = 77.7\%$.

This value is then inserted into the standard statement shown.

e. −10.3 2 marks

The predicted *tail length* of a rock haunting ringtail possum with a *body length* of 37cm is calculated as shown: *tail length* = 6.4 + 0.70 × 37 = 32.3. A1
The residual is the difference between the actual and predicted tail length:

residual = actual – predicted
= 22 – 32.3
= −10.3 A1

Question 4

		habitat			
		Bushland	Forest	Rocky	Total
conservation status	Least concern	4	5	1	10
	Near threatened	0	**3**	1	4
	Critically endangered	0	1	1	2
	Total	4	9	3	**16**

a. 3 1 mark

The number of species whose *habitat* is forest and have the *conservation status* of near threatened is 3. This can be seen by completing the column to sum to 9 or the row to sum to 4.

b. One of the statements below: 2 marks

The percentages support the environmental researcher's belief that the possums' *conservation status* is associated with their *habitat* because there is a difference in the percentages.
A1

For example, 100% of possum species in bushland habitat have the conservation status of least concern, compared with only 56% of the possums in forest habitat and 33% of the possums in the rocky habitat. A1

or

The percentages support the environmental researcher's belief that the possums' *conservation status* is associated with their *habitat* because there is a difference in the percentages.
A1

For example, 0% of possum species in bushland habitat have the conservation status of near threatened, compared with 33% of the possums in forest habitat and 33% of the possums in the rocky habitat. A1

or

The percentages support the environmental researcher's belief that the possums' *conservation status* is associated with their *habitat* because there is a difference in the percentages.
A1

For example, 0% of possum species in bushland habitat have the conservation status of critically endangered, compared with only 11% of the possums in forest habitat and 33% of the possums in the rocky habitat. A1

The percentages for each cell in the table is shown below:

		habitat		
		Bushland	Forest	Rocky
conservation status	Least concern	100%	56%	33%
	Near threatened	0%	**33%**	33%
	Critically endangered	0%	11%	33%
	Total	100%	100%	99%

Question 5

a. −21.5 1 mark

Use the statistics application of the calculator to find *log (body length)* and then find the least squares regression line.

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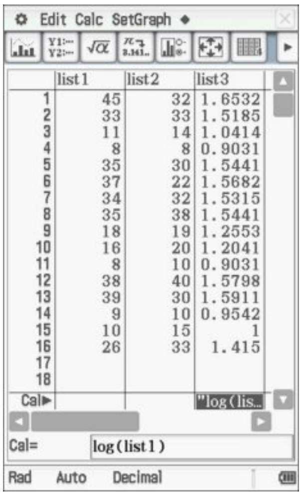	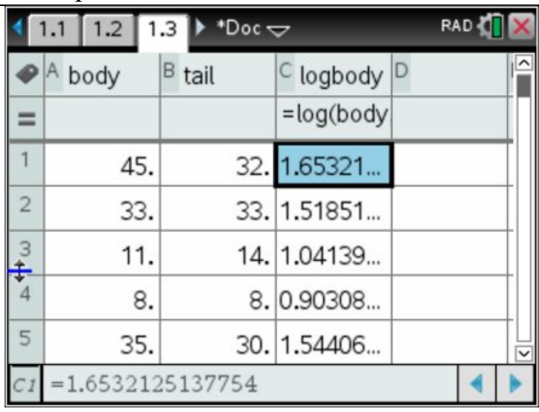

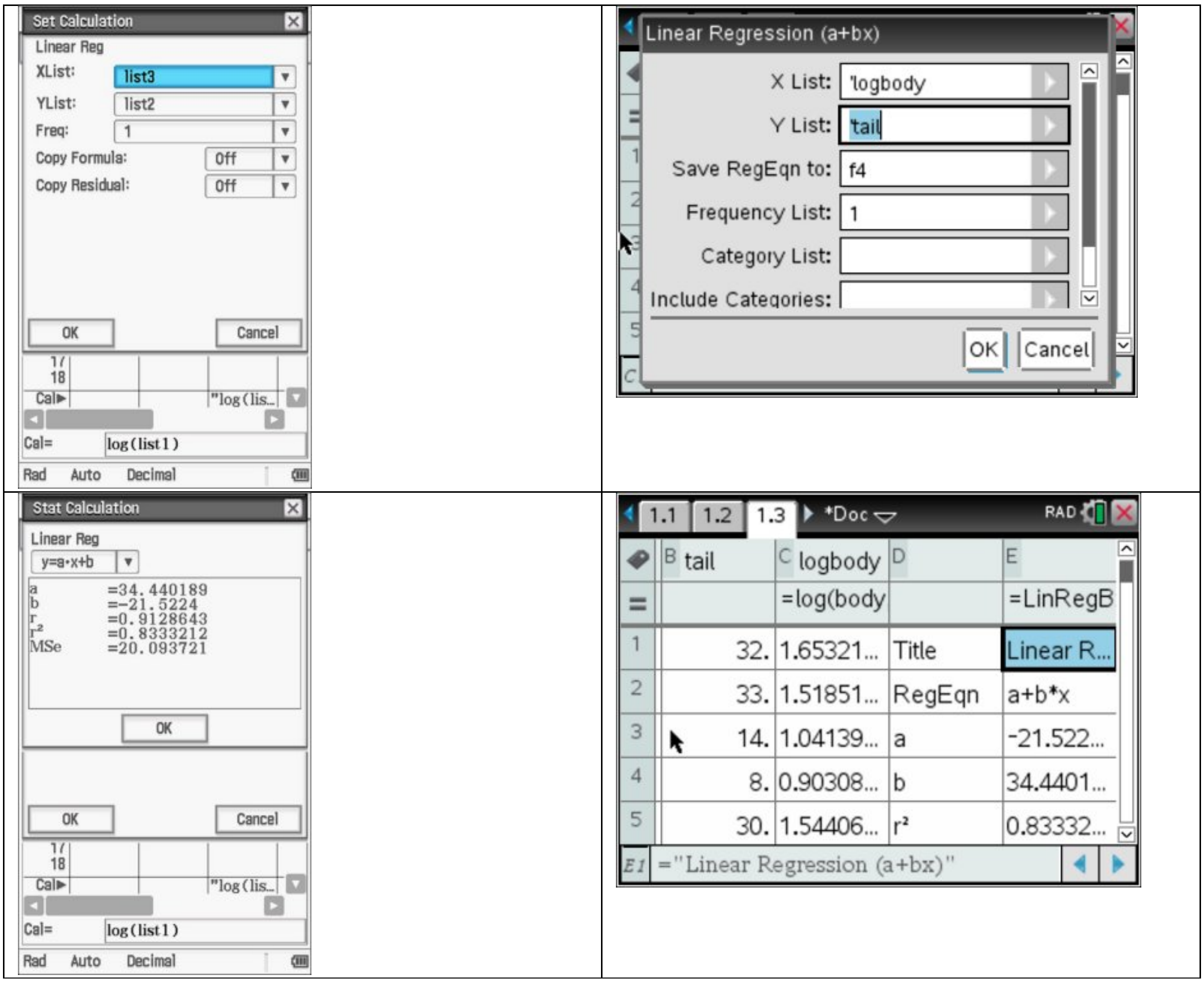

It can be seen that the slope is −21.5224, which is −21.5 correct to three significant figures.

tail length = [-21.5] + 34.4 × $\log_{10}$(*body length*)

b. On average, the *tail length* increases by 34.4 cm for every increase of one unit in the $\log_{10}$(*body length*). 1 mark

The slope of the least squares line represents the increase in the response variable (*tail length*) for every increase of one unit in the explanatory variable (*body length*).

c. *tail length* = -21.5 + 34.4 × $\log_{10}(35)$ = 31.6159…. 1 mark

The substitution into the least squares line must be shown along with either the rounded or unrounded value that rounds to 31.6, correct to one decimal place.

d. The predicted answer in part **c.** is more reliable than the given predicted value because the first answer is interpolation and the second is extrapolation. 1 mark

The calculated answer is considered reliable because it is interpolation with a high r^2 value. The given answer is less reliable because it is extrapolation, even though the it is using the same equation with a high r^2 value.

Question 6

a. An increasing trend with random fluctuations. 1 mark

b. 49.6 1 mark

The five-mean smoothed population at 2011 can be calculated as follows:

year	*population*	
2009	22	
2010	40	
2011	54	$\frac{(22+40+54+48+84)}{5} = 49.6$
2012	48	
2013	84	

Recursion and financial modelling

Question 7

a. \$11 825 1 mark

The rule $C_n = 12800 - 0.0975n$ can be used where n = 10 000:

$$C_{10000} = 12800 - 10\,000 \times 0.0975 = \$11825$$

b. 69 744 (coffees) 1 mark

The rule $C_n = 12800 - 0.0975n$ can be used where n = 6000:

$$12\,800 - 0.0975n = \$6000$$
$$-0.0975n = -6800$$
$$n = 6800 \div 0.0975 = 69\,743.58974...$$

The value of 69 743.58974… must be rounded up to ensure that the value falls below \$6000, so 69 744 coffees is the required answer.

c. Unit Cost depreciation 1 mark

The depreciation is based on the usage (number of coffees made), where each coffee reduces the value of the machine by \$0.0975 or 9.75 cents. This is therefore unit cost depreciation.

Question 8

a. $7.2 \div 12 = 0.6$ 1 mark

This is a "show that" calculation where the working of 7.2% divided by 12 months is required.

b. 600.00 174.94 425.06 28 732.42 (in that order) 1 mark

The payment continues to be \$600.00.

The monthly interest is calculated as 0.6% of the previous balance of \$29 157.74 as follows:

$$\text{Interest} = \frac{0.6}{100} \times 29\,157.48 = \$174.94.$$

The principal reduction is calculated by subtracting the interest from the \$600 payment as follows:

Principal reduction = $600 - 174.94 = \$425.06$

The balance is calculated by subtracting the principal reduction from the previous balance as follows:

Balance $= 25157.48 - 425.06 = \$28732.42$

All values are expected to be calculated correct to the nearest cent.

c. $R_0 = 30000,\ R_{n+1} = 1.006R_n - 600$ 1 mark

The loan has a starting value of \$30 000, so $R_0 = 30000$.

The interest rate is 0.6% per month, so the multiplier is $(1 + \frac{0.6}{100}) = 1.006$.

The amount paid each month is \$600, so an amount of 600 must be subtracted.

d. \$3592.89 2 marks

This question requires a number of steps.

Step 1: Calculating the balance owed after two years.

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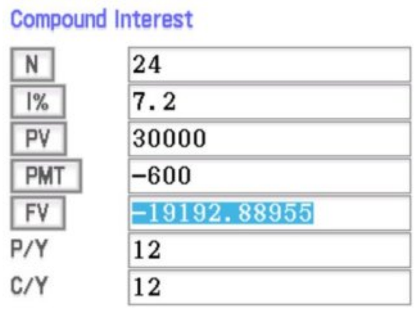
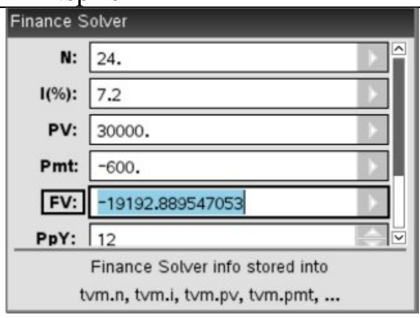
	

Step 2: Calculating the amount paid off after two years

Balance after two years = \$19 192.89, so 30 000 – 19 192.89 = \$10 807.11 paid off. A1

Step 3: Calculating the amount paid on the loan after two years.

Total of 24 × 600 = \$14 400 paid.

Step 4: Calculating the Interest

Interest = 14 400 – 10 807.11 = \$3592.89. A1

Question 9

a. Depreciation amount reduces each year (as the depreciation is calculated on a reducing balance). 1 mark

b. 23% 1 mark

Any subsequent two values can be used to determine the rate, but the initial value of $70 000 and the value after one year of $53 900 are used in the calculation below:

$$\frac{53900}{70000} = 0.77$$

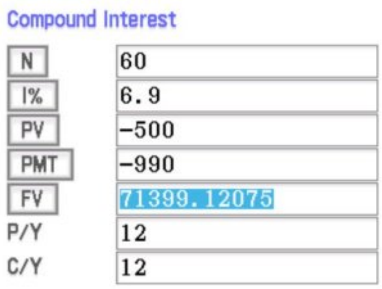
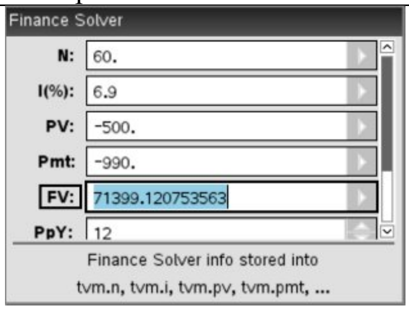
$$1 - \frac{r}{100} = 0.77$$

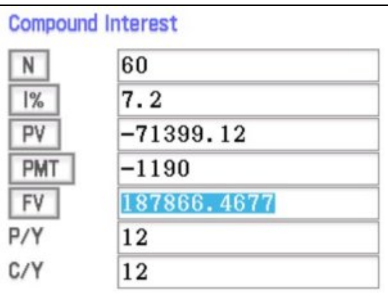
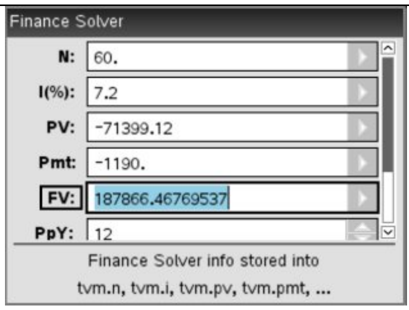
$$r = 23\%$$

Question 10

30.1% 2 marks

A number of steps were required to determine this answer:

<table>
<tr><td></td><td>Casio ClassPad</td><td>TI-Nspire</td></tr>
<tr><td>The balance after the first five years must be determined.
After five years there is $71 399.12 in the account.</td><td>
</td><td>
</td></tr>
<tr><td>The balance after the second five years must be determined. The previous balance of $71 399. 12 is now the present value.
After another five years there is $187 866.47 in the account.

(1 mark for final balance after 10 years)</td><td>
</td><td>
</td></tr>
<tr><td colspan="3">The amount of interest included in the balance is calculated as follows:
$Interest = 187866.47 - (500 + 60 \times 990 + 60 \times 1190) = \$56566.47$</td></tr>
<tr><td colspan="3">The percentage of interest included in the final balance is calculated as follows:
$percentage = \frac{56566.47}{187866.47} \times 100 = 30.1099... \approx 30.1\%$</td></tr>
</table>

Matrices

Question 11

a. There are two breeding ponds that breeding pond C connects to directly by pipes.

1 mark

The initial information indicates that the matrix represents connections from the rows to the columns. There is a "1" at each location from row 3 (from C) to each of B and D.

b. $x = 2$ and $y = 1$ 1 mark

The CAS calculator can be used to calculate the square of matrix *P*:

$$P^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 1 & \boxed{2} & 1 \\ \boxed{1} & 1 & 1 & 2 \end{bmatrix}$$

It can be seen that $x = 2$ and $y = 1$.

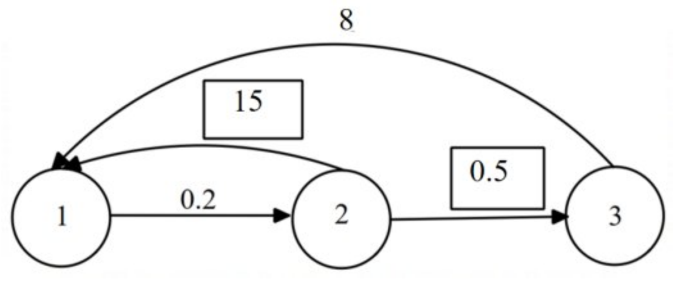
c. There is one way to get from breeding pond C to breeding pond B via another pond, CDB.

1 mark

Matrix P^2 represents the two step connections, or the connection between breeding ponds via another pond. This element shows the connection from C to B via another pond and, looking at the diagram, it can be seen that the other pond is D.

Question 12

a. 15 and 0.5 (from left to right, as shown in the diagram below). 1 mark



b. 3.2% 1 mark

The following calculation should be completed:

$$S_1 = \begin{bmatrix} 0 & 15 & 8 \\ 0.2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 90 \\ 0 \\ 3 \end{bmatrix}$$

There are a total of 90 + 0 + 3 frogs, where 3 are in the 6-9 age group: $\frac{3}{93} \times 100 = 3.2258... \approx 3.2\%$.

c. $0 \times 24 + 0.5 \times 18 + 8 \times 0 = 9.$ 1 mark

To find the expected number of frogs after three three-year periods:

$$S_3 = \begin{bmatrix} 0 & 15 & 8 \\ 0.2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \times \begin{bmatrix} 24 \\ 18 \\ 0 \end{bmatrix} = \begin{bmatrix} 270 \\ 4.8 \\ 9 \end{bmatrix}$$

The specific element required in row 3 is calculated by multiplying row 3 of matrix L by S_2:

$0 \times 24 + 0.5 \times 18 + 8 \times 0 = 9.$

d. 85% 1 mark

Find two consecutive matrices S_k and S_{k+1} for a high value of k, for example:

$$S_{50} = \begin{bmatrix} 6.388 \times 10^{14} \\ 6.902 \times 10^{13} \\ 1.861 \times 10^{13} \end{bmatrix} \text{ and } S_{51} = \begin{bmatrix} 1.184 \times 10^{15} \\ 1.278 \times 10^{14} \\ 3.451 \times 10^{13} \end{bmatrix}$$

$\frac{1.184 \times 10^{15}}{6.388 \times 10^{14}} = 1.85$ $\frac{1.278 \times 10^{14}}{6.902 \times 10^{13}} = 1.85$ $\frac{3.451 \times 10^{13}}{1.861 \times 10^{13}} = 1.85$

This is an 85% increase for each three-year time period.

Question 13

a. 3 1 mark

The number mature frogs initially is 20 and the mature frogs who are still mature frogs after one year is given by $G_{22} = 0.15$.

$0.15 \times 20 = 3$ frogs.

b. 5 1 mark

Using a 1×2 summing matrix and the matrices supplied:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix}^3 \times \begin{bmatrix} 120 \\ 20 \end{bmatrix} = \begin{bmatrix} 4.9125 \end{bmatrix}$$

The expected total number of frogs after three years, correct to the nearest whole number is five.

c. 75 1 mark

After seven calculations the number of adult and mature frogs settles at 75 adult frogs.

$$S_7 = \begin{bmatrix} 74.999 \\ 42.641 \end{bmatrix} \text{ and } S_8 = \begin{bmatrix} 74.999 \\ 42.645 \end{bmatrix}$$

d. i. 86 1 mark

After two years there are 72 adult and 32.5 mature frogs predicted:

$$C_1 = \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 60 \\ 10 \end{bmatrix} = \begin{bmatrix} 60 \\ 10 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix} \times \begin{bmatrix} 60 \\ 10 \end{bmatrix} + \begin{bmatrix} 60 \\ 10 \end{bmatrix} = \begin{bmatrix} 72 \\ 32.5 \end{bmatrix}$$

If the same number of frogs is added for another year there will be 74.4 adult frogs:

$$C_3 = \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix} \times \begin{bmatrix} 72 \\ 32.5 \end{bmatrix} + \begin{bmatrix} 60 \\ 10 \end{bmatrix} = \begin{bmatrix} 74.4 \\ 40.075 \end{bmatrix}$$

So, an additional 26 frogs are added to the 60 already added so 86 adult frogs will be added to make 100.4 > 100 adult frogs.

ii. 56 1 mark

The new matrix after three years is:

$$C_3 = \begin{bmatrix} 0.20 & 0 \\ 0.35 & 0.15 \end{bmatrix} \times \begin{bmatrix} 72 \\ 32.5 \end{bmatrix} + \begin{bmatrix} 86 \\ 10 \end{bmatrix} = \begin{bmatrix} 100.4 \\ 40.075 \end{bmatrix}$$

Continuing with the relation $C_{n+1} = GC_n + B$ where $C_3 = \begin{bmatrix} 100.4 \\ 40.075 \end{bmatrix}$ and $B = \begin{bmatrix} 86 \\ 10 \end{bmatrix}$, it can be seen that the number of mature frogs approaches 56.02941176… or 56 mature frogs.

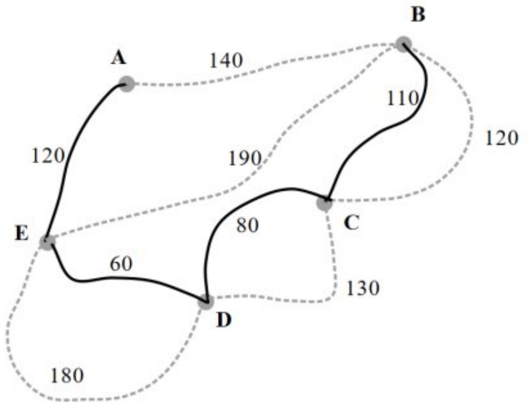
Networks and decision mathematics

Question 14

a. 250 (metres) 1 mark

The shortest path can be found using Djikstra's algorithm or by inspection. By inspection the shortest path is CBA with a length of 110 + 140 = 250 m.

b. The tracks shown on the diagram below: 1 mark



The minimum length of bunting used to connect the pergolas would form a minimal spanning tree that can be determined using Prim's algorithm.

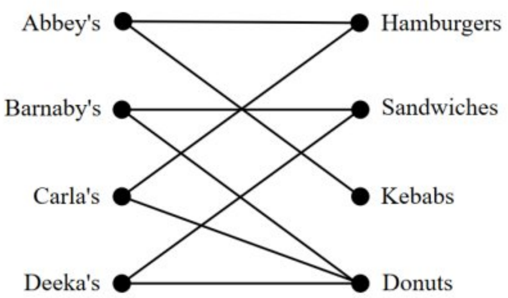
c. Eulerian circuit or Euler circuit 1 mark

Vera will start and finish at pergola A. As she wants to return to the starting point the route she will take is a circuit.

She will walk along every track at least once, taking the minimum possible distance. Because the degree of every vertex in the graph is even, an Eulerian circuit is possible and this would give the minimum distance because she will not repeat any edges.

Question 15

a. The completed bipartite graph shown below: 1 mark



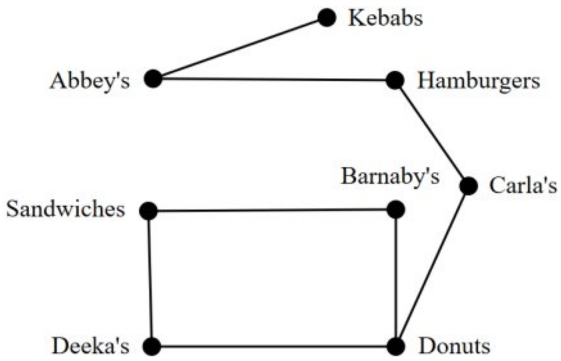
b. Two 1 mark

From the bipartite graph it can be seen that Abbey's must do Kebabs, as only this caterer supplies kebabs. As Abbey's is already allocated to Kebabs, that only leaves Carla's to do Hamburgers.

Barnaby's and Deeka's can both do either Sandwiches or Donuts, so there are two different possible allocations.

c. Yes. It can be redrawn without edges crossing (only meeting at vertices) 1 mark

A graph is planar if it can be redrawn without edges crossing. One possible version of this graph is drawn below without edges crossing:



Question 16

A table of the earliest and latest starting times and float times for each activity for this question is shown below:

Activity	EST	LST	Float time
A	0	0	0
B	0	2	2
C	7	9	2
D	5	6	1
E	5	5	0
F	5	8	3
G	10	13	3
H	10	14	4
I	13	14	1
J	13	13	0
K	14	17	3
L	11	12	1
M	15	15	0
N	11	13	2

a. I, J, L and N 1 mark

Activities L and N follow on directly from the end of activity D, whereas I and J are linked to D via the dummy from the end of D to the start of I and J.

b. 14 days 1 mark

The earliest starting time for activity L is 11 days and the completion time for L is 3 days, so an earliest finishing time of 11 + 3 = 14 days.

c. 20 days 1 mark

The critical path or longest path through the network is AEJM with a time of 5 + 8 + 2 + 5 = 20 days. This corresponds to the activities with zero float time. The critical path represents the shortest completion time for the whole project.

d. H 1 mark

Activity H has an EST of 10 and an LST of 14 days, so a float time of 4 days. This is the longest float time of any activity.

e. One day 1 mark

The critical path of AEJM would be reduced to a time of 15 days if E was reduced by five days. The second longest path of AEI would also be reduced from 19 days to 14 days. Path ADLM also has a length of 19 days, and as E is not on this path, the time would not be reduced and this would be the new critical path with a time of 19 days, one less than the previous time.

f. 4 days 1 mark

A crashing table is shown below where every path through the network is included. The critical path(s) at each stage are highlighted:

Path	Time	A X 1	A X 1	M X 1	E X 1 & C X 1
BCJM	17	17	17	16	15
BCI	16	16	16	16	15
BCLM	18	18	18	17	16
BCN	17	17	17	17	16
AFGK	17	16	15	15	15
AFHK	16	15	14	14	14
AEI	19	18	17	17	16
AEJM	20	19	18	17	16
ADLM	19	18	17	16	16
ADJM	18	17	16	16	16
ADI	17	16	15	15	15
ADN	18	17	16	16	16

Initially AEJM is the longest path. A is reduced as it also appears on the two next longest paths. This occurs twice.

After A is reduced by two days, AEJM and BCLM are both critical at 18 days. They share activity M, so it is reduced next by one, resulting in four critical paths of 17 days, BCLM, BCN, AEI and AEJM.

There are no activities in common between these four paths, but two share C and two share E, so each of E and C are reduced by one each, resulting in seven critical paths, shown above with a length of 16 days.

This represents a reduction of four days from the original critical time of 20 days.