The Mathematical Association of Victoria

Trial Examination 2024

GENERAL MATHEMATICS

Written Examination 1

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of mars*
40	40	40
		Total 40

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- One bound reference (which may be annotated).
- One approved technology with numerical, graphical, symbolic, financial and statistical functionality and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 27 pages.
- Formula sheet.
- Answer sheet for multiple choice questions.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

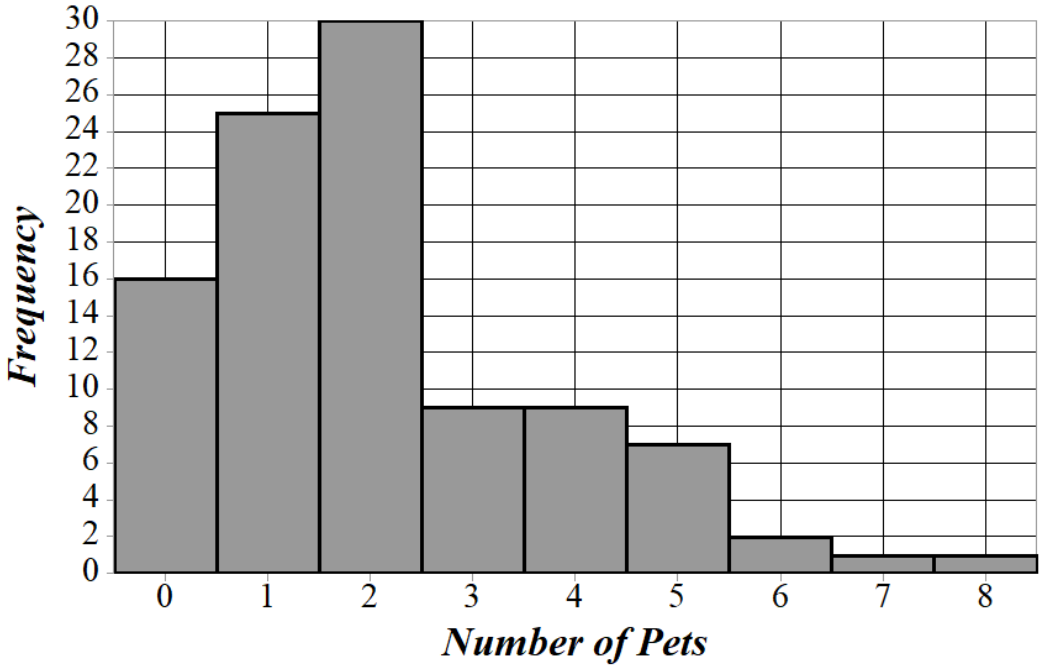
No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

Data analysis

Use the following information to answer questions 1, 2 and 3:

The graph below shows the responses given by 100 people who were asked "How many pets do you have at home?"



Question 1

The shape of the distribution is best described as

A. symmetric with one or more outliers.

B. positively skewed with one or more outliers.

C. negatively skewed with one or more outliers.

D. bimodal with one or more outliers.

Question 2

Which of the following statements is true about the distribution of the *Number of Pets*?

A. The mean, median and mode are all the same in this distribution.

B. The median and mode are the same in this distribution, but the mean is a larger value.

C. The median and mode are the same in this distribution, but the mean is a smaller value.

D. All of the values for the median, mean and mode are different in this distribution.

Question 3

Which of the following statements is true about the outlier(s) in the distribution of the *Number of Pets*?

A. The outliers are the two people who had 7 and 8 pets, because both values are greater than 6.

B. The outliers are the two people who had 7 and 8 pets, because both values are greater than 6.5.

C. The outliers are the four people who had 6, 7 and 8 pets respectively, because all of these values are greater than 5.

D. The outlier is the person who had 8 pets, because this value is greater than 7.

Question 4

In a set of univariate data values, which of the following statistical measures will **always** be equivalent to at least one of the original data values?

A. Mean

B. Median

C. Mode

D. Q_1

Question 5

A family gathering of three generations is being held. The generations consist of two grandparents with a mean age of 75, six of their children with a mean age of 42 and 28 grandchildren with a mean age of 15.

The mean age of all people attending the family gathering would be closest to

A. 44 years.

B. 42 years.

C. 36 years.

D. 23 years.

Question 6

A speed camera is set up on the side of the road, where the speed limit is 60 km/h.

12 800 cars passed the speed camera and had their speeds recorded.

The distribution of speeds of passing cars was normally distributed, with a mean speed of 57 km/h.

320 speed fines were issued, all for cars travelling faster than 60 km/h.

The standard deviation of speeds, in km/h, recorded was

A. 3

B. 2

C. 1.5

D. 1

Question 7

At a basketball draft camp, the heights of all players were measured and found to be normally distributed.

Sam's height was recorded as 183 cm and he had a standardised score of -1.28.

Akram's height was recorded as 192 cm and he had a standardised score of 2.32.

The mean and standard deviation respectively, in cm, of players at the draft camp were

A. 186.2 and 8.65

B. 186.2 and 2.5

C. 187.5 and 8.65

D. 187.5 and 2.5

Question 8

A researcher has been collecting and recording the *length* and *sex* of the endangered Lord Howe stick insects in a particular location on Lord Howe Island.

He recorded the *length* of all *male* and *female* specimens collected, in cm, correct to one decimal place, before releasing the insects again.

The results of his collection for one week are shown in the back-to-back stem and leaf plot below:

male (n = 25)						*female* (n = 32)									
				2	9										
			6	1	10										
8	8	6	4	3	11										
		7	6	5	12	0	4	4	7	9					
	6	4	1	1	13	3	3	9							
		9	9	7	14	1	2	2	4	5	7	7	8	9	9
9	9	8	2	2	15	2	5	6							
			1	0	16	5	6			Key 14|1 = 14.1 cm					
					17	4	6	6	8						
					18	6	7	7	9	9					

Which of the following statements is true?

A. The *length* of the stick insects is associated with their *sex*, because the range of female *lengths* is larger at 6.9 cm compared to the range of male *lengths* at 6.8 cm.

B. The *length* of the stick insects is associated with their *sex*, because the interquartile range of female *lengths* is smaller at 3.5 cm compared to the interquartile range of male *lengths* at 3.7 cm.

C. The *length* of the stick insects is associated with their *sex*, because the female median *length* is longer at 14.85 cm compared to the male median *length* of 13.1 cm.

D. The *length* of the stick insects is associated with their *sex*, because the female median *length* is longer at 14.85 cm compared to the male median *length* of 13.4 cm.

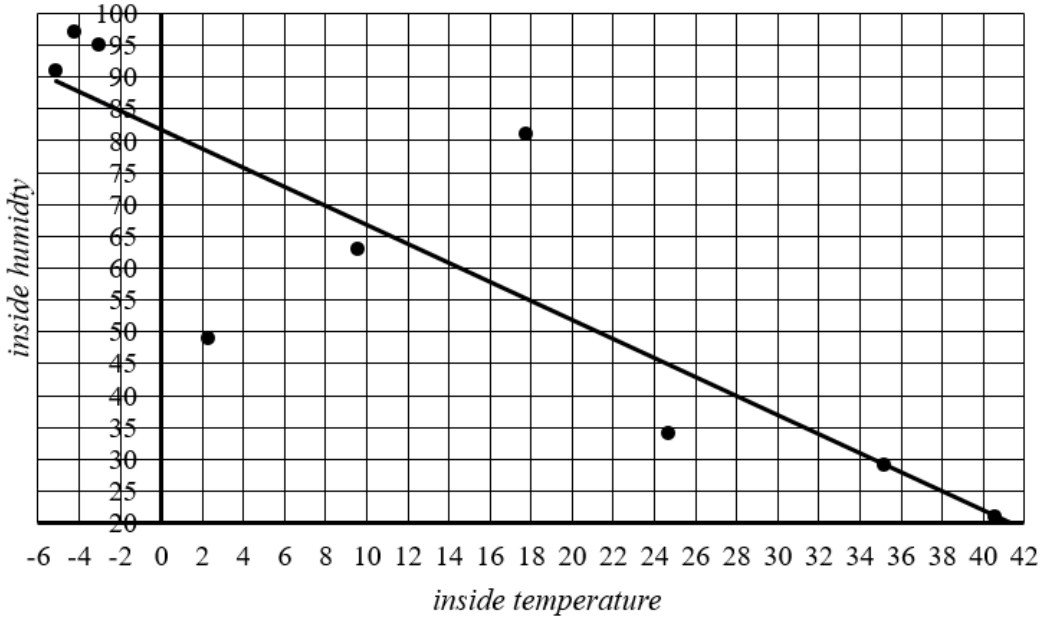
Question 9

A manufacturer specialises in alpine tents designed to protect campers in very cold conditions.

The tent is pitched in the snow where the temperature outside is $-6°$ Celsius.

A heater is started inside the tent and the *inside temperature* (in C°) and *inside humidity* (in %) are recorded over a period of 90 minutes.

A scatterplot of the measurements for *inside temperature* and *inside humidity* is shown below, along with the least squares line for the association:



The equation of the least squares line shown is closest to

A. $inside\,humidity = 92.1 - 1.50 \times inside\,temperature$

B. $inside\,humidity = 85.0 - 1.04 \times inside\,temperature$

C. $inside\,humidity = 81.8 - 1.50 \times inside\,temperature$

D. $inside\,temperature = 81.8 - 1.50 \times inside\,humidity$

Question 10

The data below shows the *median house price* (in thousands of dollars) and the *population* (in thousands) in Melbourne, recorded every second year from 1990 to 2020:

Year	*median house price* (in $1000's)	*population* (in 1000's)
1990	131	3154
1992	125	3217
1994	130	3260
1996	131	3305
1998	155	3333
2000	191	3361
2002	258	3423
2004	321	3522
2006	360	3624
2008	385	3775
2010	520	3932
2012	508	4114
2014	553	4322
2016	675	4541
2018	735	4771
2020	780	4859

If a least squares regression line is fitted to this data so that the *median house price* can be predicted from the *population,* the predicted *median house price* in 2024, when the *population* is 5 316 000 would be closest to

A. $820 000

B. $956 000

C. $978 000

D. $1 595 000

Question 11

In a particular inland lake, there is a positive association between the maximum daily temperature and the amount of algae present in the lake. The coefficient of determination is 0.49.

From this information it can be concluded that

A. an increase in temperature will cause the amount of algae to increase.

B. when the temperature is higher the amount of algae tends to increase.

C. a decrease in temperature will cause the amount of algae to decrease.

D. when the temperature is lower the amount of algae tends to increase.

Question 12

A mathematics teacher has been analysing her students' test scores. She asked each student to record the *time spent studying*, in minutes, the night before a test and she then recorded the *test score* that they achieved as a percentage.

The mean *time spent studying* was 50 minutes and the mean *test score* was 72%.

She has found that for every increase of one minute of *time spent studying*, the student's *test score* increased by 0.8 marks, on average.

She also found that 36% of the variation in the students' *test score* could be explained by the variation in the *time spent studying*.

Which of the following statements would also be true?

A. A student who did not study the night before the test would be predicted to get a *test score* of 32%.

B. A student whose *time spent studying* was 75 minutes would be predicted to get a *test score* of 100%.

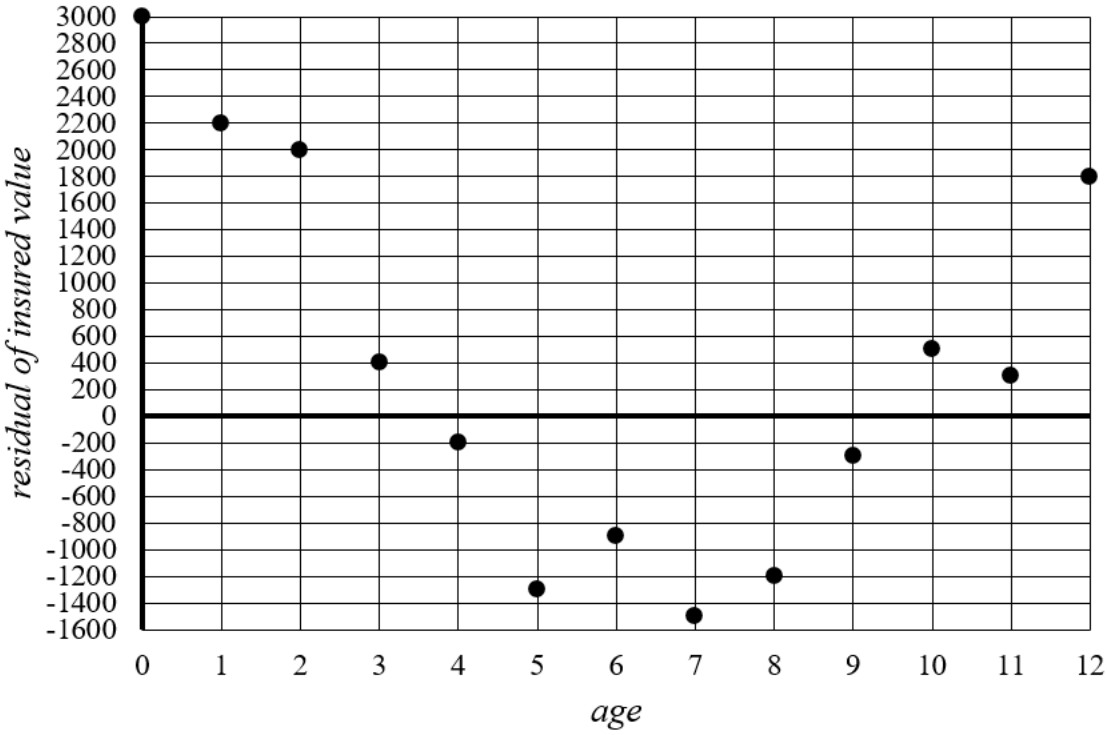
C. There is a weak positive linear association between *time spent studying* and *test score*.

D. Students whose *time spent studying* was higher than 50 minutes would all get a *test score* above 72%.

Question 13

There is a **negative** association between the *age* of a car and its *insured value* in dollars.

The residual plot when the linear association for this association is explored is shown below:



Based on this residual plot

A. the linear association is appropriate to describe the association between *age* and *insured value.*

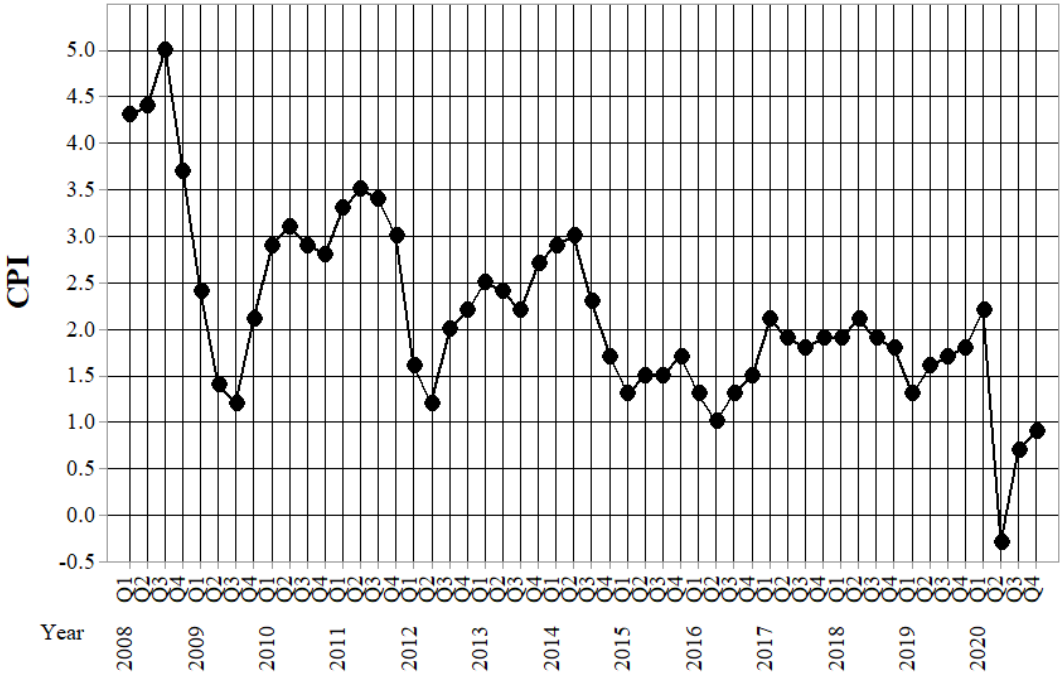
B. to linearise the association it could be appropriate to use an age^2 transformation.

C. to linearise the association it could be appropriate to use an $insured\ value^2$ transformation.

D. to linearise the association it could be appropriate to use a log(*age*) transformation.

Question 14

The time series below shows the *CPI* (Consumer Price Index) recorded at the end of every quarter from the start of 2008 to the end of 2020.



The time series is best described as

A. seasonal with random fluctuations.

B. seasonal with random fluctuations and a decreasing trend.

C. cyclic with random fluctuations.

D. cyclic with random fluctuations and a decreasing trend.

Question 15

A small cafe records its quarterly *profit* over the years 2021, 2022 and 2023 and the yearly *profit* (the sum of the quarterly *profit* for each year).

The *profit*, in dollars recorded every quarter, correct to the nearest $100, along with the total profit for each year are shown in the table below:

	Quarter 1	**Quarter 2**	**Quarter 3**	**Quarter 4**	**Yearly**
2021	12 500	8700	7600	15 500	44 300
2022	13 400	9500	7700	16 800	47 400
2023	13 300	9800	9000	17 200	49 300

Based on this information, on average, the *profit* during the first quarter is

A. approximately 13% above the yearly average.

B. approximately 12% above yearly average.

C. approximately 11% above the yearly average.

D. approximately 8% above the yearly average.

Question 16

The sales of surfboards at a particular shop is seasonal.

To correct the sales of surfboards in June for seasonality, the actual number of surfboards, is increased by 60%**.**

The seasonal index for June is

A. 1.6

B. 0.6

C. 0.4

D. 0.625

Recursion and financial modelling

Question 17

Consider the recurrence relation shown below:

$$A_0 = 6,\ A_{n+1} = 1.5A_n - 2$$

The value of A_2 in the sequence generated by this recurrence relation is given by

A. $A_2 = 1.5 \times 1 - 2$

B. $A_2 = 1.5 \times 2 - 2$

C. $A_2 = 1.5 \times 6 - 2$

D. $A_2 = 1.5 \times 7 - 2$

Question 18

$6000 is invested in an account earning 4% per annum simple interest.
The money is withdrawn after *n* months, when the investment has a value of $6180.
The value of *n* is

A. 1

B. 9

C. 12

D. 26

Question 19

A tractor is purchased for $44 200.
The value of the tractor is depreciated using a flat rate method of depreciation.
After 9 years, the value of the tractor is $2431.
The depreciation rate, per annum, is closest to

A. 5.5%

B. 10.5%

C. 18.2%

D. 27.6%

Question 20

An amount of $15 000 is invested in an account earning interest at the rate of 4.8% per annum, compounding quarterly.

A recurrence relation in terms of R_0, R_{n+1} and R_n, that gives the balance in dollars, after n quarters is given by

A. $R_0 = 15000, \quad R_{n+1} = 1.004R_n$

B. $R_0 = 15000, \quad R_{n+1} = 1.012R_n$

C. $R_0 = 15000, \quad R_{n+1} = 1.016R_n$

D. $R_0 = 15000, \quad R_{n+1} = 1.048R_n$

Question 21

Patricia has a perpetuity investment that is invested at a rate of 8.4% per annum and pays her $3780 per month. The value of the investment is

A. $317 520

B. $381 024

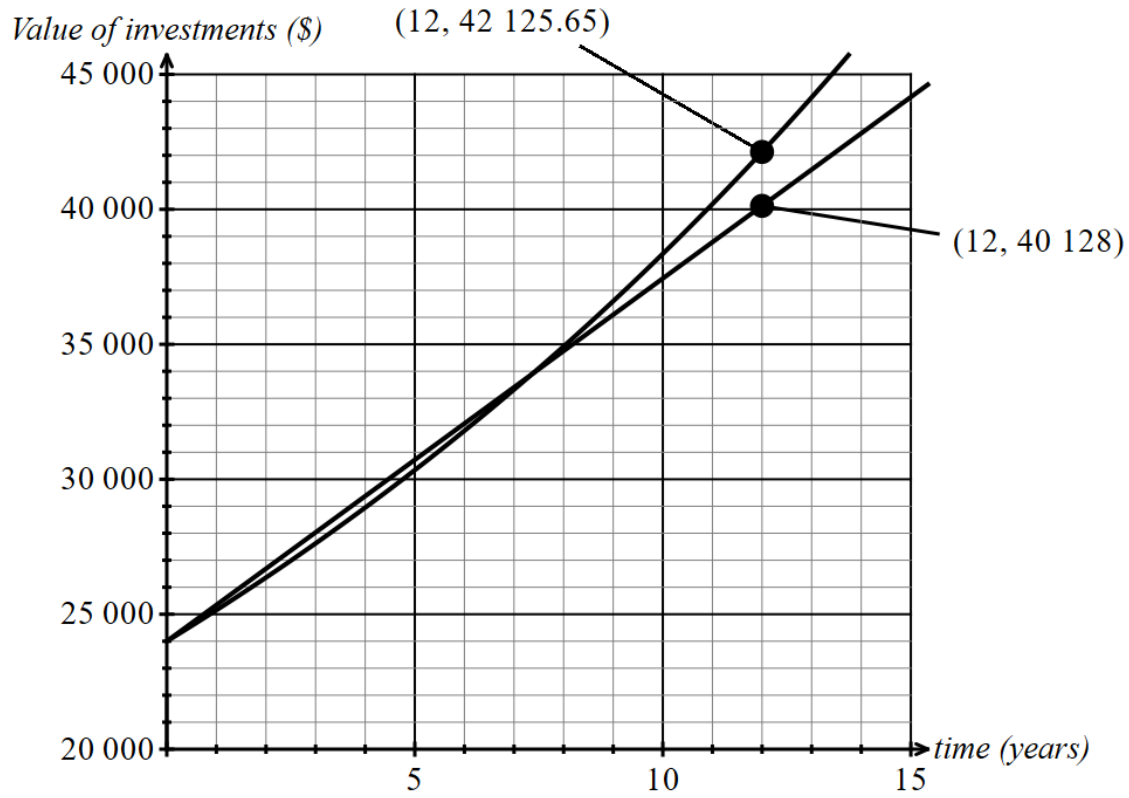
C. $450 000

D. $540 000

Question 22

The graph below compares two investments:

- Investment A: $24 000 invested in an account earning simple interest where interest is calculated and added annually.
- Investment B: $24 000 invested in an account earning compound interest where interest is calculated and added annually.



Which one of the following statements is **not** true?

A. The value of the Investment A is less than the value of Investment B at six years.

B. The interest rate per annum for Investment B is less than the interest rate per annum for Investment A.

C. At five years the value of Investment B is less than the value of Investment A.

D. The value of Investment A at 12 years is $40 128.

Question 23

$25 000 is invested in an account with interest compounding monthly.
After one year, the balance of the account was $26 505.
The difference between the rate of interest per annum for the investment and the effective annual rate of interest is closest to

A. 0.15%

B. 0.16%

C. 0.17%

D. 0.18%

Question 24

Ten years ago, Min took out a reducing balance loan of $543 000 to buy a new house.

The loan had a fixed interest rate of 4.8% per annum compounding monthly and the monthly repayments were $3500.

After five years, Min made a one-off payment. The interest rate was fixed for another five years at a rate of 3.6% compounding monthly. She continued with the same monthly repayment of $3500.

The balance of her loan today is $ 270 767.70.

The value of the one-off payment was closest to

A. $27 545

B. $34 303

C. $35 000

D. $65 748

Matrices

Question 25

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is an example of

A. an identity matrix

B. a permutation matrix

C. a symmetric matrix

D. a triangular matrix

Question 26

Matrix P is a permutation matrix and matrix Q is a column matrix where

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } Q_0 = \begin{bmatrix} m \\ a \\ t \\ e \\ s \end{bmatrix}$$

When Q_0 is multiplied repeatedly by P such that $Q_n = P^n Q_0$ the minimum number of permutations, n, until the product is back to $\begin{bmatrix} m \\ a \\ t \\ e \\ s \end{bmatrix}$ is

A. 2

B. 3

C. 5

D. 6

Question 27

Consider the matrix, M where $M = \begin{bmatrix} -1 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$.

The element in row i and column j of matrix M is m_{ij}.

The elements in matrix M are determined by the rule

A. $m_{ij} = i - j - 1$

B. $m_{ij} = j - 2i$

C. $m_{ij} = i + j - 3$

D. $m_{ij} = i - 2j$

Question 28

X is a 5 × 6 matrix, Y is a 5 × 5 matrix and Z is a 4 × 5 matrix.

Which one of the following matrix expressions is not defined?

A. $(ZYZ^T)^{-1}$

B. Y^2XZ^T

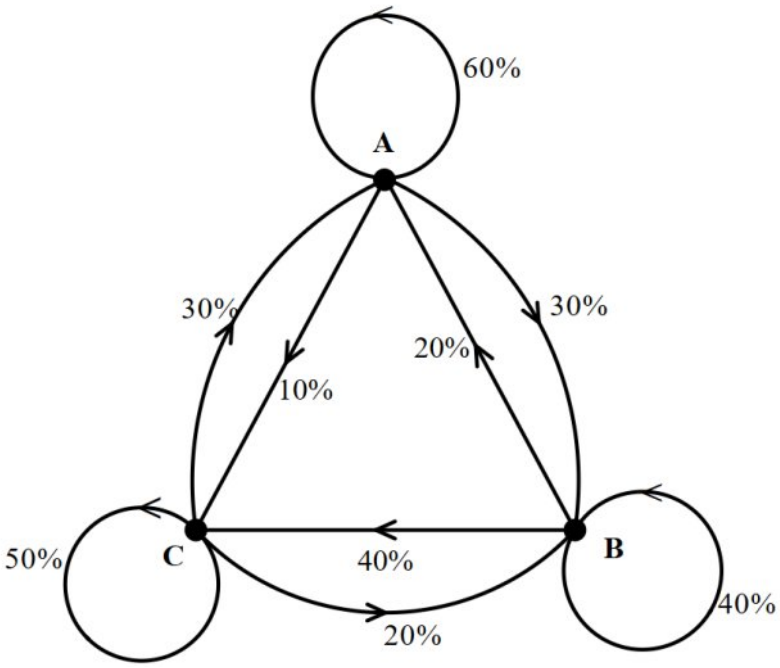
C. $X^TY^{-1}Z^T$

D. ZYX

Question 29

A large town has three different bus locations, A, B and C, where they store the buses overnight.

The transition diagram below shows the change in the location of the buses from night to night.



On Monday night the number of buses at each location is given in the matrix

$$L = \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}.$$

The number of buses expected to be at location B on Tuesday night is

A. 10

B. 12

C. 14

D. 18

Question 30

The matrix below shows the result of each match between four players, *A, B, C* and *D*, in a chess competition. Each player competed against each other player once, until there was a final winner and loser.

The results of the competition are shown in the matrix below, where a "1" represents a win, so the "1" in row 1, column 2 represents that A defeated B.

$$\begin{array}{cc} & \begin{array}{cccc} loser & & & \\ A & B & C & D \end{array} \\ \begin{array}{c} winner \\ \\ \\ \\ \end{array} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Using one- and two-step dominances, the player who ranked second was

A. *A*

B. *B*

C. *C*

D. *D*

Question 31

A study of the population of kangaroos in a region has shown that the kangaroos regularly move between three locations, X, Y and Z every week.

Let K_n be the state matrix that shows the population of kangaroos in each location n weeks after the study began.

The expected population of kangaroos in each location can be determined by the matrix recurrence relation $K_{n+1} = T \times K_n - D$ where

$$T = \begin{matrix} \\ X \\ Y \\ Z \end{matrix} \begin{matrix} \begin{matrix} \textit{this week} \\ X \quad Y \quad Z \end{matrix} \\ \begin{bmatrix} 0.4 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \end{matrix} \quad \textit{next week} \quad \text{and } D = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \begin{matrix} X \\ Y \\ Z \end{matrix}.$$

Two months after the study began $K_2 = \begin{bmatrix} 36 \\ 58 \\ 41 \end{bmatrix} \begin{matrix} X \\ Y \\ Z \end{matrix}$.

The number of kangaroos that were initially located at X is closest to

A. 15

B. 27

C. 40

D. 95

Question 32

A college canteen only serves burgers on a Wednesday. The students can choose between a chicken burger (C) and a beef burger (B). The proportion of students who choose each type of burger changes from week to week, as shown in the transition matrix

$$T = \begin{array}{c} \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cc} \textit{this week} & \\ B & C \end{array} \\ \begin{bmatrix} 0.65 & 0.20 \\ 0.35 & 0.80 \end{bmatrix} \end{array} \begin{array}{l} \\ B \\ C \end{array} \quad \textit{next week}$$

The matrix recurrence relation $S_0 = \text{initial state matrix}$, $S_{n+1} = TS_n$ is used to model this situation.

In the long run, the fraction of students who will choose a beef burger is

A. $\frac{1}{5}$

B. $\frac{4}{11}$

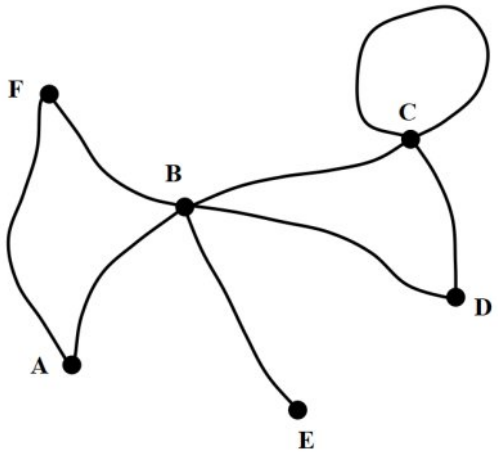
C. $\frac{13}{20}$

D. $\frac{17}{40}$

Networks and decision mathematics

Question 33

Consider the following graph:



How many of the following five statements are true?

- The graph is planar.
- The graph has an Eulerian trail.
- The graph contains a Hamiltonian cycle.
- The degree of vertex C is four.
- The edge DC is a bridge.

A. 2

B. 3

C. 4

D. 5

Question 34

A connected, planar graph has seven vertices.

Which of the following statements **could not** be true?

A. There must be a minimum of six edges.

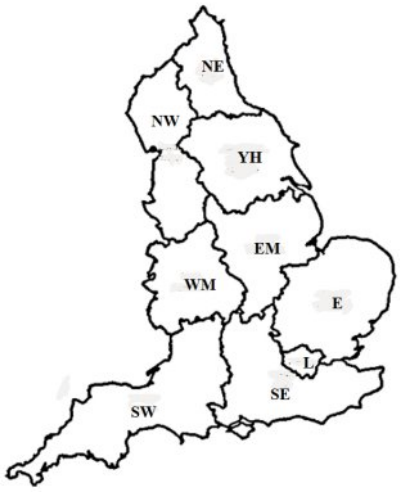
B. There must be five more edges than the number of faces.

C. The graph could be complete.

D. The degree of each vertex must be at least one.

Question 35

A map of the regions of England is shown below along with a key to vertex labels on a network:

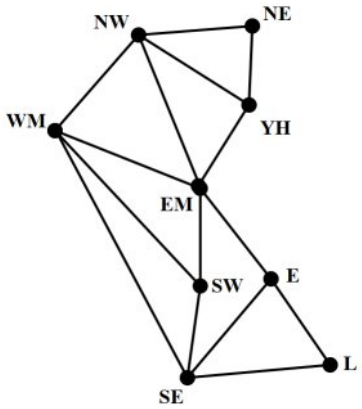


Key to regions on network:
North East = NE
North West = NW
Yorkshire and the Humber = YH
West Midlands = WM
East Midlands = EM
South West = SW
South East = SE
East of England = E
Greater London = L

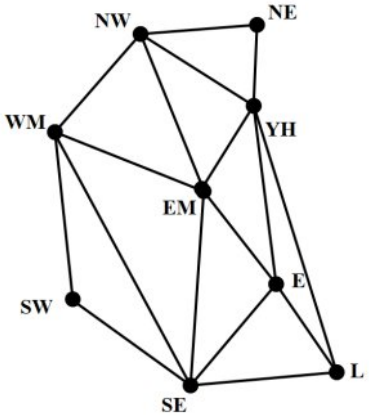
A network has been constructed where vertices represent each of the nine regions and the edges represent borders between the regions.

Which of the following correctly represents the borders between the nine regions?

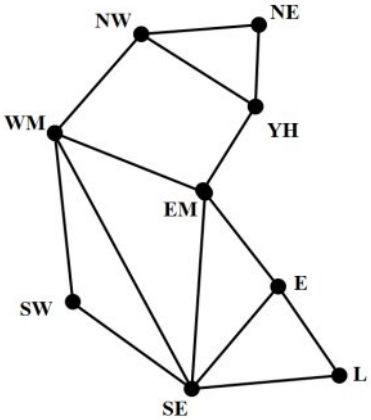
A.



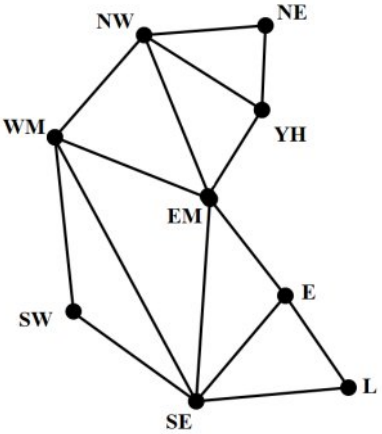
B.



C.



D.



Question 36

Consider the adjacency matrix for a network below:

		to				
		A	B	C	D	E
from	A	0	1	1	1	1
	B	0	0	0	0	1
	C	0	1	0	0	1
	D	0	0	1	0	1
	E	0	0	0	0	0

Which of the following statements is **not** true?

A. The network is a directed network.

B. Vertex E would be considered a sink.

C. Vertex D is reachable from vertex B.

D. Vertex A is unreachable from any other vertex.

Question 37

A school wants to take four groups on four different excursions on the one day. They contact four bus companies to get quotes for each trip. They will allocate one excursion to each of the four bus companies using the cheapest overall cost.

The costs, in dollars, quoted by each of the companies (E, F, G and H) for each of excursions 1, 2, 3 and 4 are shown in the table below:

	Excursion 1	**Excursion 2**	**Excursion 3**	**Excursion 4**
Company E	\$80	\$130	\$60	\$210
Company F	\$90	\$110	\$70	\$230
Company G	\$80	\$120	\$70	\$230
Company H	\$100	\$120	\$70	\$230

The school determines the minimum total cost and allocates the buses, but at the last minute, company E states that they can no longer take Excursion 4. Company E is still able to take any of the other three excursions.

The school determines the new minimum cost if each company is allocated one excursion and Company E does not take Excursion 4.

Which of the following statements is true?

A. There is no change in total cost under the changed circumstances.

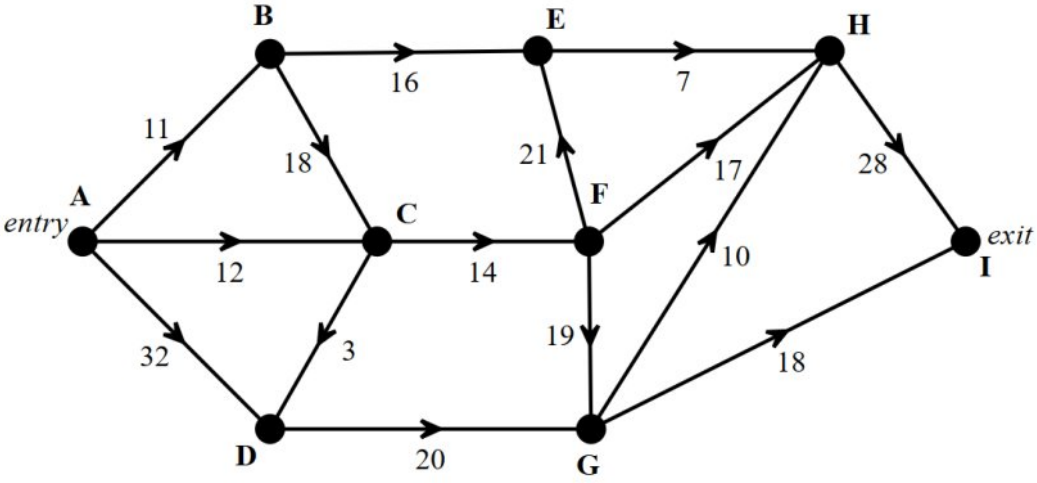
B. There is an increase of \$10 in total cost under the changed circumstances.

C. There is an increase of \$20 in total cost under the changed circumstances.

D. There is an increase of \$30 in total cost under the changed circumstances.

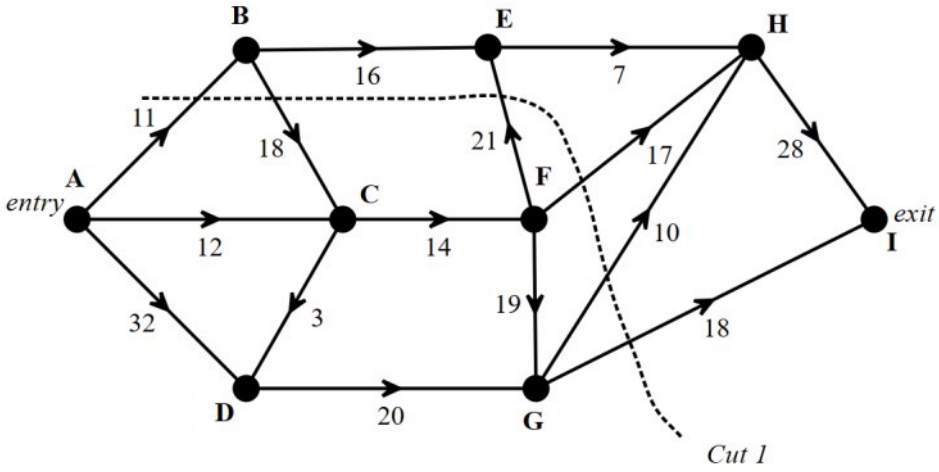
Use the following information to answer Questions 38 and 39.

The network below shows the one-way paths between the entrance, A, and the exit, I, of a multi-story car park.
The vertices represent the intersections of the one-way paths.
The number on each edge is the maximum number of cars that can travel along that path per minute.



Question 38

Cut 1 is shown on another copy of the network below:



The capacity of *Cut 1* is

A. 95

B. 77

C. 56

D. 46

Question 39

The car park management must increase flow through the car park.

Which of the following actions, when done individually, would increase the flow through the car park by the greatest amount?

A. Increasing flow from D to G by 20, so that the new capacity is 40.

B. Increasing flow from C to F by 20, so that the new capacity is 34.

C. Increasing flow from E to H by 20, so that the new capacity is 27.

D. Reversing the direction of flow from F to E, so it now flows from E to F.

Question 40

A particular building project has eight activities that must be completed.
These activities, along with their durations in hours, their earliest start time (EST) and their immediate predecessor(s) are shown in the table below.

Activity	Immediate predecessor(s)	Duration (in hours)	Earliest Start Time
A	-	7	0
B	A	4	7
C	A	5	7
D	A	10	7
E	B, C	6	12
F	B	8	11
G	B	11	11
H	D, E, F	5	19

The overall project must be reduced in time.

Which of the following statements is true?

A. If activity F can be reduced by three hours, the overall project completion time will be reduced by two hours.

B. If activity D can be reduced by two hours, the overall project completion time will be reduced by one hour.

C. If activity H can be reduced by three hours, the overall project completion time will be reduced by two hours.

D. If activity B can be reduced by two hours, the overall project completion time will be reduced by two hours.

END OF QUESTION BOOK