The Mathematical Association of Victoria

Trial Exam 2024

GENERAL MATHEMATICS

WRITTEN EXAMINATION 1

SOLUTIONS

Q1	**B**	**Q21**	**D**
Q2	**B**	**Q22**	**A**
Q3	**A**	**Q23**	**B**
Q4	**C**	**Q24**	**C**
Q5	**D**	**Q25**	**C**
Q6	**C**	**Q26**	**D**
Q7	**B**	**Q27**	**B**
Q8	**C**	**Q28**	**B**
Q9	**C**	**Q29**	**C**
Q10	**C**	**Q30**	**C**
Q11	**B**	**Q31**	**A**
Q12	**A**	**Q32**	**B**
Q13	**D**	**Q33**	**B**
Q14	**D**	**Q34**	**D**
Q15	**C**	**Q35**	**C**
Q16	**D**	**Q36**	**B**
Q17	**D**	**Q37**	**B**
Q18	**B**	**Q38**	**B**
Q19	**B**	**Q39**	**A**
Q20	**B**	**Q40**	**C**

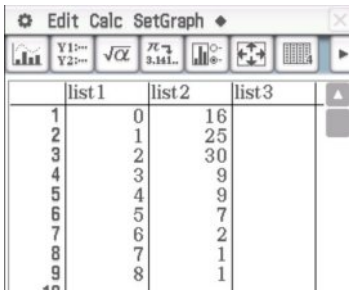
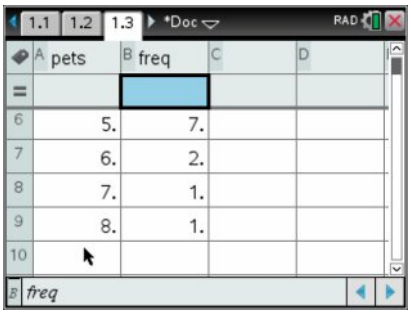
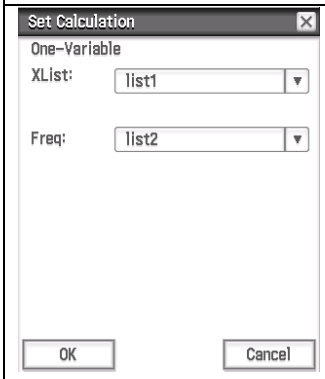
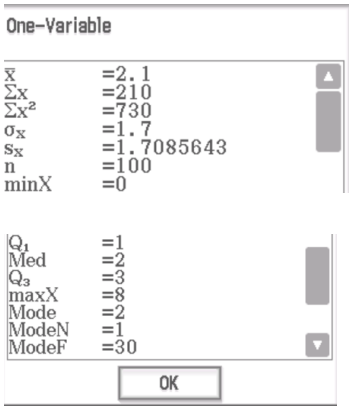
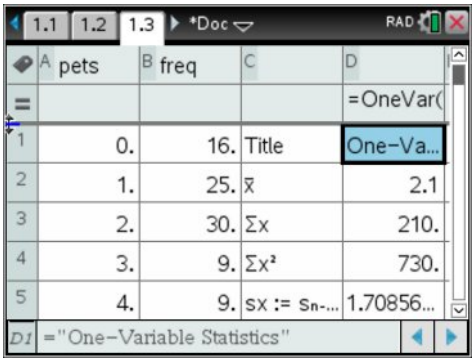
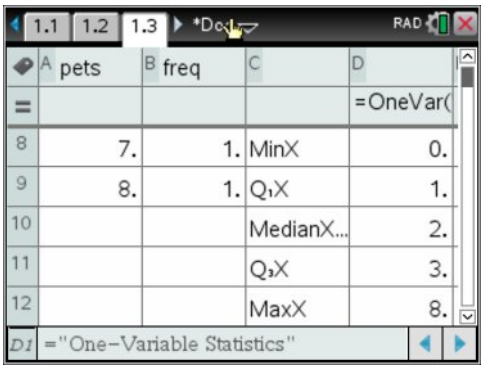
Data analysis

Question 1 Answer B

The shape of the distribution of the *Number of Pets* is best described as positively skewed with one or more outliers. The majority of the data is clustered in the lower region of 1, 2 or 3 pets with a tail in the data out to 8 pets. All options suggested the possibility of the same number of outliers, so it was only shape that was relevant.

Question 2 Answer B

The median and mode are the same in this distribution, but the mean is a larger value. The data can be entered as a frequency table into the CAS as shown below:

Casio ClassPad	TI-Nspire
	
 	 

It can be seen that the mean is 2.1, while both the median and mode are 2, so the median and mode are the same, but the mean is larger.

Question 3 **Answer A**

From the previous question it can be seen that $Q_1 = 1$ and $Q_3 = 3$.

The interquartile range is $Q_3 - Q_1 = 3 - 1 = 2.$

The upper fence is $Q_3 + 1.5 \times IQR = 3 + 1.5 \times 2 = 6.$

The values 7 and 8 are both outliers because they are greater than the upper fence value of 6.

Question 4 **Answer C**

The mode will always be at least one of the original data values as it is the most common value.

The mean is calculated from the sum of all values divided by the number of data values and will not always be a data value.

The median and Q_1 are values obtained by their position in an ordered data set. If the number of data values is even, the median is the mean of the two middle values and if the two middle values are different, the median will not be an original data value. The value of Q_1 is the median of the bottom half of the data set, and if the bottom half contains an even number of values the same situation will apply and the value will not be an original value.

All of these statistical values could be an original data value, but only the mode is always an original data value.

Question 5 **Answer D**

The mean is calculated by dividing the sum of all values by the number of data values.

The following calculation is required:

$$\bar{x} = \frac{2 \times 75 + 6 \times 42 + 28 \times 15}{2 + 6 + 28} = 22.8\dot{3} \approx 23$$

Question 6 **Answer C**

There were 320 cars travelling faster than 60 km/h, so $\frac{320}{12800} \times 100 = 2.5\%$

of the cars were travelling faster than 60 km/h.

Because the speeds are normally distributed, this means that 60 km/h is two standard deviations above the mean:

$60 = 57 + 2 \times s_x$

$s_x = 1.5$

Question 7 **Answer B**

Two simultaneous equations can be set up from the information provided:

$$-1.28 = \frac{183 - \overline{x}}{s_x}$$

$$2.32 = \frac{192 - \overline{x}}{s_x}$$

The CAS calculator can be used to solve these equations:

Casio ClassPad	TI-Nspire
$\left\{ -1.28 = \frac{183-a}{b},\ 2.32 = \frac{192-a}{b} \right\} \Big\vert_{a,b}$ $\{a = 186.2, b = 2.5\}$	$\text{solve}\left(\left\{ -1.28 = \frac{183-x}{y},\ 2.32 = \frac{192-x}{y} \right\}, \{x, y\} \right)$ $x = 186.2$ and $y = 2.5$

Therefore, the mean is 186.2 cm and the standard deviation is 2.5 cm.

Question 8 **Answer C**

The following statistics can be determined for each of the *male* and *female* samples:

	Min	Q_1	Med	Q_3	Max	IQR	Range
male	9.2	11.7	13.1	15.2	16.1	3.5	6.9
female	12.0	14.0	14.85	17.5	18.9	3.5	6.9

Considering each of the options given:

A	The *length* of the stick insects is associated with their *sex*, because the range of female *lengths* is larger at 6.9 cm compared to the range of male *lengths* at 6.8 cm.	False. The range for both male and female samples is the same at 6.9 cm.
B	The *length* of the stick insects is associated with their *sex*, because the interquartile range of female *lengths* is smaller at 3.5 cm compared to the interquartile range of male *lengths* at 3.7 cm.	False. The interquartile range for both samples is 3.5 cm.
C	The *length* of the stick insects is associated with their *sex*, because the female median *length* is longer at 14.85 cm compared to the male median *length* of 13.1 cm.	True. The medians are quoted correctly and different, so there is an association.
D	The *length* of the stick insects is associated with their *sex*, because the female median *length* is longer at 14.85 cm compared to the male median *length* of 13.4 cm.	False. The medians are different, but the male median is quoted incorrectly.

Question 9 **Answer C**

The explanatory variable is *inside temperature* and the response variable is *inside humidity*, so the equation is of the form:

$inside\,humidity = a + b \times inside\,temperature$.

The line passes close to the points $(-2, 85)$ and $(38, 25)$. Using these two point the slope can be determined as follows:

$$b = \frac{25 - 85}{38 - -2} = -1.50$$

Using a slope of -1.50 and the point (38, 25), the intercept can be calculated as follows:

$$25 = a - 1.50 \times 38$$
$$a = 82$$

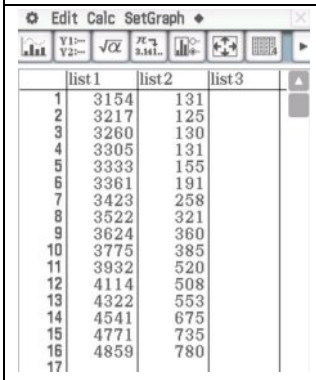
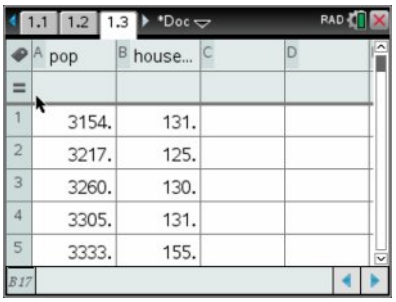
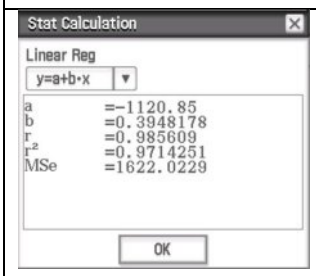
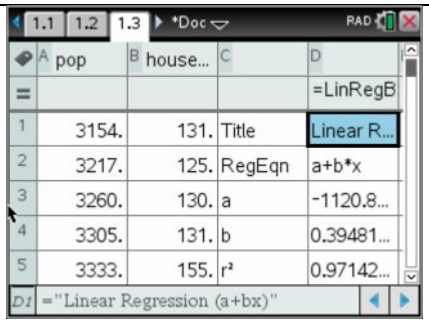
Therefore, the equation of the least squares line is closest to:

$inside\,humidity = 81.8 - 1.50 \times inside\,temperature$.

Question 10 **Answer C**

The least squares line is fitted to this data so that the *median house price* can be predicted from the *population,* so the population is the explanatory variable and the median house price is the response variable.

The least squares line can be calculated on CAS as shown below:

Casio ClassPad	TI-Nspire
	
	

From the CAS screen it can be seen that the equation of the least squares line is:

$$median\ house\ price\,(in\,\$1000's) = -1120.85 + 0.3948 \times population\,(in\,1000's).$$

Using this equation, when the *population* is 5 316 000:

$$median\ house\ price\,(in\,\$1000's) = -1120.85 + 0.3948 \times 5316 = 977.9... \approx 978.$$

Therefore, the predicted *median house price* is closest to $978 000.

Question 11 **Answer B**

There is a positive association between the maximum daily temperature and the amount of algae present in the lake. This means that as the temperature increases, the amount of algae will also tend to increase. This corresponds with option B. Option A can be discounted because it implies that there is a causal relationship between maximum daily temperature and the amount of algae present in the lake. A correlation coefficient cannot be used to establish causality.

The coefficient of determination is given as 0.49, but this value is not necessary to determine the correct response.

Question 12 **Answer A**

A number of different pieces of information are given:

"The teacher found that for every increase of one minute of *time spent studying*, the student's *test score* increased by 0.8 marks, on average."

From this statement it can be determined that the least squares line is of the form:

$test\ score = a + 0.8 \times time\ spent\ studying$ where a is the vertical intercept.

"The mean *time spent studying* was 50 minutes and the mean *test score* was 72%."

From this statement the value of the vertical intercept can be determined:

$a = \overline{y} - b \times \overline{x}$

$a = 72 - 0.8 \times 50 = 32$

It can now be determined that the least squares line is: $test\ score = 32 + 0.8 \times time\ spent\ studying$.

"She also found that 36% of the variation in the students' *test score* could be explained by the variation in the *time spent studying*."

From this statement and the positive value of the slope it can be determined that the value of the correlation coefficient is:

$\sqrt{0.36} = 0.6$.

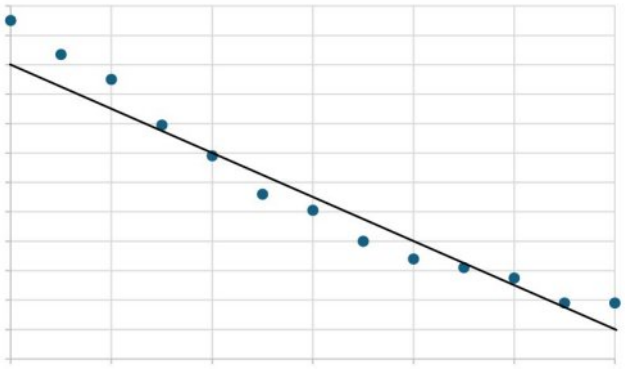
Considering each of the options:

A	A student who did not study the night before the test would be predicted to get a *test score* of 32%.	True. The vertical intercept is 32, so when *time spent studying* is zero, the *test score* is predicted to be 32%.
B	A student whose *time spent studying* was 75 minutes would be predicted to get a *test score* of 100%.	False. Using the least squares line $test\ score = 32 + 0.8 \times 75 = 92\%$.
C	There is a weak positive linear association between *time spent studying* and *test score*.	False. As $r^2 = 0.36$, the value of r is 0.6, so there is a moderate positive linear association between *time spent studying* and *test score*.
D	Students whose *time spent studying* was higher than 50 minutes would all get a *test score* above 72%.	False. Students whose *time spent studying* was higher than 50 minutes are predicted to get a *test score* above 72%, but there is no absolute guarantee.

Question 13 **Answer D**

There is a **negative** association between the *age* of a car and its *insured value* in dollars and the residual shows a clear pattern, so option A can be excluded.

Because there is a negative association, the line and the position of the original points curving around that line, can be estimated to be close to that shown below:



It can be seen from the circle of transformations that appropriate transformations would be a reciprocal transformation of either axis or a logarithmic transformation of either axis. An alternative approach is to consider the process of compressing either axis.

The only appropriate transformation given as an option is a log(*age*) transformation.

Question 14 **Answer D**

The *CPI* (Consumer Price Index) can be seen to decrease over the period of time from the start of 2008 to the end of 2020. This means that options A and C can be excluded as they do not refer to a trend being present. There are also random fluctuations.

The *CPI* is recorded quarterly, but there are no consistent peak and trough positions each year, therefore the data cannot be described as seasonal. The *CPI* does, however, rise and fall during the time period, over periods of greater than one year, so the pattern is considered to be cyclic.

Question 15 **Answer C**

The seasonal average (SA) for each year must be calculated as follows:

2021	2022	2023
$SA = 44300/4 = 11075$	$SA = 47400/4 = 11850$	$SA = 49300/4 = 12325$

The seasonal proportion (SP)can now be calculated for each year for Quarter 1:

2021:

$$SP = 12500/11075$$
$$= 1.128668...$$

2022:

$$SP = 13400/11850$$
$$= 1.130801...$$

2023:

$$SP = 13300/12325$$
$$= 1.079107...$$

The seasonal index (SI) can now be calculated for Quarter 1:

$$SI = (1.128668 + 1.130801 + 1.079107)/3 = 1.11285...$$

This represents that Quarter 1 profits are approximately 11% above the yearly average.

Question 16 **Answer D**

To correct the sales of surfboards in June for seasonality or deseasonalise, the actual number of surfboards, is increased by 60%**.** This means that the actual number of surfboards must be multiplied by

$$(1 + \frac{60}{100}) = 1.6\,.$$

Deseasonalising is division by the seasonal index or multiplying by the reciprocal of the seasonal index, so:

$$\frac{1}{SI} = 1.6$$
$$SI = \frac{1}{1.6} = 0.625$$

Recursion and financial modelling

Question 17 Answer D

First find the value of A_1 by substituting A_0 into the recurrence relation:

$A_1 = 1.5 \times 6 - 2$

$A_1 = 7$

Substitute A_1 into the recurrence relation to get A_2:

$A_2 = 1.5 \times 7 - 2$

Question 18 Answer B

The interest per year is $0.04 \times 6000 = \$240.$

Using the given information:

$$6000 + 240n = 6180$$
$$n = \frac{6180 - 6000}{240} = 0.75$$

$$n = 0.75 \times 12 = 9\ months$$

$n = 0.75$ years or 9 months. (Answer required in months).

Question 19 Answer B

The total depreciation is $44200 - 2431 = 41769.$

The depreciation per year is $\frac{41769}{9} = 4641.$

The depreciation rate, per annum, is $\frac{4641}{44200} \times 100 = 10.5\%.$

Question 20 Answer B

The initial amount is \$15 000, so $R_0 = 15000$.

The interest rate per annum is 4.8%, so the quarterly interest rate is $\frac{4.8}{4} = 1.2\%.$

Using the quarterly interest rate, the multiplier is $1 + \frac{1.2}{100} = 1.012$ so the recurrence relation is

$R_0 = 15000, \quad R_{n+1} = 1.012R_n$.

Question 21 **Answer D**

A perpetuity investment is an investment where the principal remains constant, that is the payments are equal to the interest.

The monthly interest rate is $\frac{8.4}{12} = 0.7$ and 0.7% is 0.007 as a decimal.:

This means that the principal P can be found using:

$P \times 0.007 = 3780$

$P = \frac{3780}{0.007} = \$540\,000$

Question 22 **Answer A**

The linear graph is the simple interest investment, Investment A, and after 12 years the value of the investment \$40 128.

The interest added each year is $\frac{40128 - 24\,000}{12} = \$1344.$

The simple interest rate for Investment A is $\frac{1344}{24\,000} \times 100 = 5.6\%.$

The compound interest investment, that is Investment B, has a value of \$42 125.65 after 12 years, so:

$24\,000 \times R^{12} = 42125.65$

$R = 1.048$

$R = (1 + \frac{r}{100}) = 1.048$

$r = 4.8\%.$

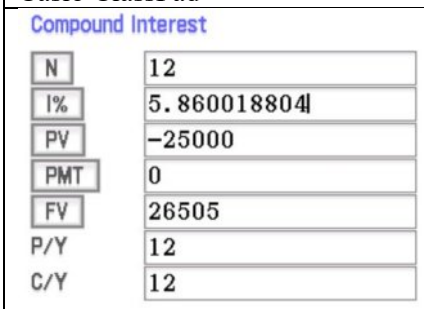
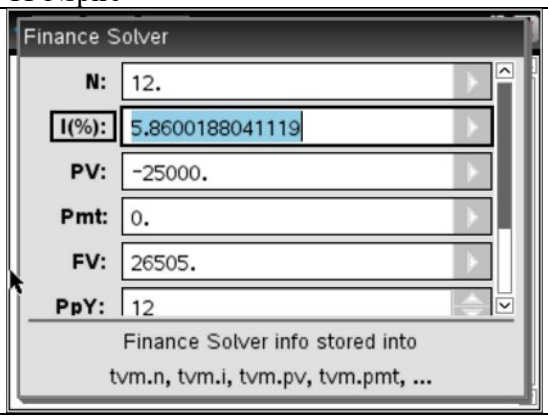
Considering each of the options:

A	The value of Investment A at six years is 24 000 + 6 ×1344 = \$32 064. The value of Investment B at six years is 24 000 × 1.048^6 = \$31 796.47. The value of Investment A value of Investment B at six years. This can also be seen on the graph as the linear graph is higher when the time is 6 years. Statement A is **not** true.
B	The simple interest rate for Investment A is 5.6% per annum. The compound interest rate for Investment A is 4.8% per annum. Interest rate for Investment B < Interest rate of Investment A. Statement B is true.

C	The value of Investment A at five years is 24 000 + 5 ×1344 = 30720. The value of Investment B at five years is 24 000 × 1.048^5 = \$30340.15. At five years, value of Investment B < value of Investment A. Statement C is true. This can also be seen on the graph.
D	From coordinates of the labelled point on the linear graph we can see that the value of Investment A at 12 years is \$40 128. Statement D is true.

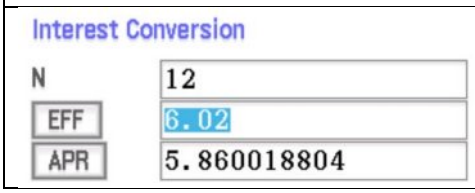
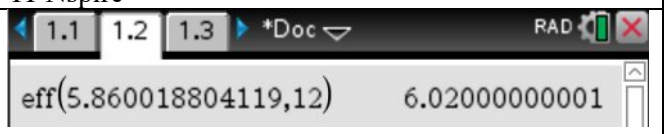
Question 23 **Answer B**

The nominal interest rate for the account can be calculated using CAS:

Casio ClassPad	TI-Nspire
	

The nominal rate is therefore 5.86% correct to two decimal places.

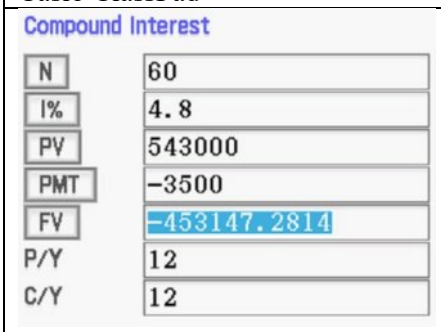
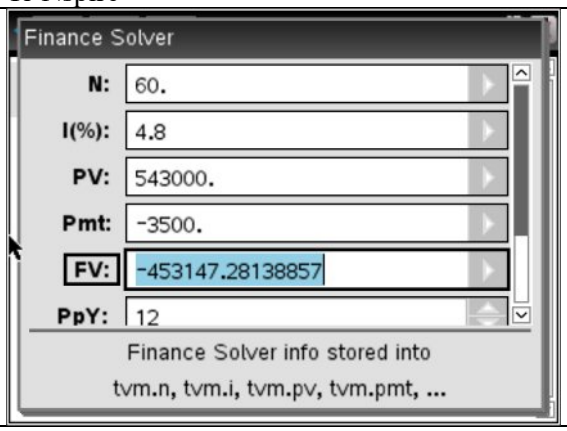
The effective interest rate can be found using CAS:

Casio ClassPad	TI-Nspire
	

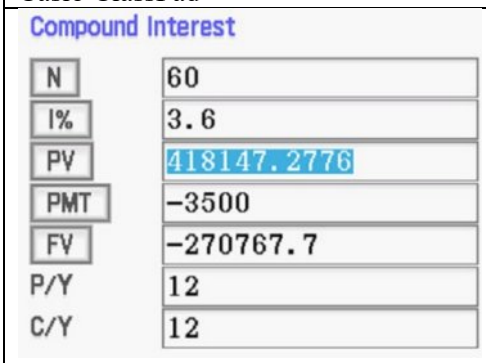
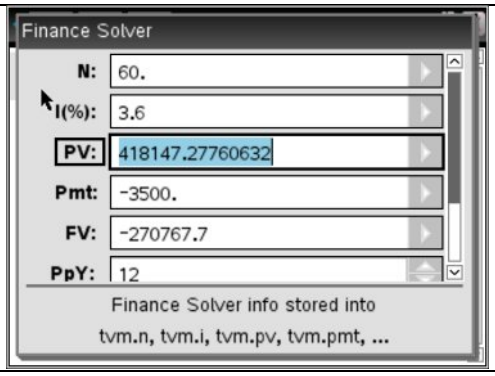
The difference between the effective and nominal interest rate is therefore $6.02 - 5.86 = 0.16\%$.

Question 24 **Answer C**

Step 1: Find the balance of the loan after five years using the financial application of the calculator.

Casio ClassPad	Ti-Nspire
	

Step 2: Work backwards from today's balance of $270 767.70 with the new interest rate using CAS to find the starting value of the new loan five years ago.

Casio ClassPad	
	

The difference between the value after five years and the starting balance for the next five years is the value of the one-off payment: 453147.28 – 418147.28 = $35 000.

Matrices

Question 25 Answer C

Considering each of the options:

A	An identity matrix only has 1's on the leading diagonal and the other elements are 0's.	False – there are 1's in places other than the leading diagonal.
B	A permutation matrix has only one 1 in each row and column, and 0's elsewhere.	False – there are two 1's in the second row and the third row. There are two 1's in second column and the third column.
C	A symmetic matrix has a line of symmetry	True $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$
D	A triangular matrix only has numbers other than zero in the top or bottom of the matrix.	False - A triangular matrix is a square matrix in which all elements below or above the leading diagonal are zeros.

Question 26 Answer D

The top left section of the matrix gives a permutation of the *{m, a, t}*

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

from {m, a, t} to {t, m, a} to {a, t, m} and back to {m, a, t}

The bottom right section of the matrix gives a permutation of the {e, s}

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

from {e, s} to {s, e} and back to {e, s}

It will take 2 × 3 = 6 permutations until all the letters are back to the original

Question 27 **Answer B**

The element in row i and column j of matrix M is m_{ij}.

A	$m_{ij} = i - j - 1$ $\begin{bmatrix} 1-1-1 & 1-2-1 & 1-3-1 \\ 2-1-1 & 2-2-1 & 2-3-1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 \\ 0 & -1 & -2 \end{bmatrix}$	False
B	$m_{ij} = j - 2i$ $\begin{bmatrix} 1-2(1) & 2-2(1) & 3-2(1) \\ 1-2(2) & 2-2(2) & 3-2(2) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$	True – all the elements are correct.
C	$m_{ij} = i + j - 3$ $\begin{bmatrix} 1+1-3 & 1+2-3 & 1+3-3 \\ 2+1-3 & 2+2-3 & 2+3-3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$	False
D	$m_{ij} = i - 2j$ $\begin{bmatrix} 1-2(1) & 1-2(2) & 1-2(3) \\ 2-2(1) & 2-2(2) & 2-2(3) \end{bmatrix} = \begin{bmatrix} -1 & -3 & -5 \\ 0 & -2 & -4 \end{bmatrix}$	False

Question 28 **Answer B**

X is a 5 × 6 matrix, Y is a 5 × 5 matrix and Z is a 4 × 5 matrix.

Matrix multiplication is only defined when the number of columns in the first matrix is equal to the number of rows in the second matrix.

Z^T is the transpose of Z and is a 5 × 4 matrix and X^T is the transpose of X and is a 6 × 5 matrix.

Only square matrices have powers and the product has the same order as the original.

Considering each of the options:

A	(4 × 5) × (5 × 5) × (5 × 4) means that ZYZ^T is defined and is a 4 × 4 matrix and has an inverse.	True
B	Y^2XZ^T is not defined because the number of columns in X is not equal to the number of rows in Z^T and (5 × 5) × (5 × 6) × (5 × 4)	False
C	(6 × 5) × (5 × 5) × (5 × 4) means that $X^TY^{-1}Z^T$ is defined and is a 6 × 4 matrix.	True
D	(4 × 5) × (5 × 5) × (5 × 6) means that ZYX *is* defined and is a 4 × 6 matrix.	True

Question 29 **Answer C**

The transition matrix for the system is:

$$\begin{matrix} & \textit{this night} & & \\ & A & B & C \\ \end{matrix}$$
$$\begin{bmatrix} 0.6 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \quad \textit{next night}$$

To find the number of buses at each location on Tuesday:

$$\begin{bmatrix} 0.6 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 20 \\ 14 \\ 16 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

The number of buses expected to be at location *B* on Tuesday night is 14.

Question 30 **Answer C**

Using the one – step dominance, find the sum of the element in each row.

Player	Score
A	1
B	2
C	2
D	1

Player B and Player C have the same winning total. To decide the overall winner and second place, consider the values for B and C by squaring the matrix to give the two – step dominances:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Find sum of the element in these rows for the square matrix.

Player	Score
B	3
C	2

If we consider the one and two – step dominances, Player B has a greater two-step dominance than Player C. The player who came second was C.

Question 31 **Answer A**

To find K_1, add D to K_2 and then multiply by the inverse of the transition matrix:

$$K_2 = TK_1 - D$$
$$K_2 + D = TK_1$$
$$T^{-1} \times \left(K_2 + D\right) = K_1$$
$$K_1 = \begin{bmatrix} 0.4 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}^{-1} \times \left(\begin{bmatrix} 36 \\ 58 \\ 41 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} 40 \\ 70 \\ 40 \end{bmatrix}$$

$$K_1 = TK_0 - D$$
$$K_1 + D = TK_0$$
$$T^{-1} \times \left(K_1 + D\right) = K_0$$
$$K_0 = \begin{bmatrix} 0.4 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}^{-1} \times \left(\begin{bmatrix} 40 \\ 70 \\ 40 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} 15 \\ 120 \\ 30 \end{bmatrix}$$

The number of kangaroos that were initially located at X is 15.

Question 32 **Answer B**

In the long run the proportion of students who choose a beef burger will stay the same from one transition to the next. Let x be the proportion of students who choose a beef burger:

$$\begin{bmatrix} 0.65 & 0.20 \\ 0.35 & 0.80 \end{bmatrix} \times \begin{bmatrix} x \\ 1-x \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$$

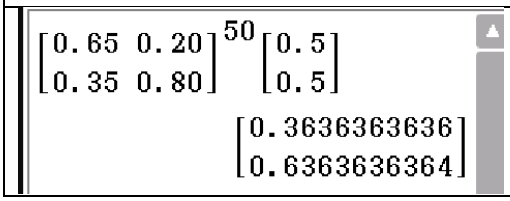
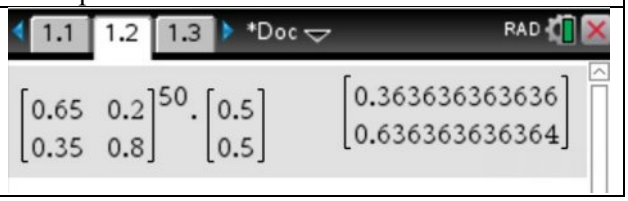
$$\begin{bmatrix} 0.65x + 0.2(1-x) \\ 0.35x + 0.8(1-x) \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$$

Equating the matrices, we get two equations that are actually the same equation:
$0.65x + 0.2(1-x) = x$ and $0.35x + 0.8(1-x) = 1-x$.

Solving either of these will give $x = \dfrac{4}{11}$ (or 0.3636363636…):

$$0.65x + 0.2(1-x) = x$$
$$0.65x + 0.2 - 0.2x = x$$
$$0.2 = x - 0.65x + 0.2x = 0.55x$$
$$x = \frac{0.2}{0.55} = \frac{4}{11}$$

The decimal approximation could also be found using after a large of transitions with an initial where the elements sum to one. For example:

Casio ClassPad	TI-Nspire
	

Networks and decision mathematics

Question 33 Answer B

Considering all of the five statements:

The graph is planar.	True. The graph is drawn so that no edges cross and edges only meet at vertices.
The graph has an Eulerian trail.	True. All vertices are of even degree except vertices B and E, so an Eulerian trail exists.
The graph contains a Hamiltonian cycle.	False. It is impossible to go through all vertices exactly once and return to the start without repeating a vertex.
The degree of vertex C is four.	True. There are four edge ends attached to vertex C.
The edge DC is a bridge.	False. If the edge DC is removed the graph remains connected.

There are, therefore, three true statements.

Question 34 **Answer C**

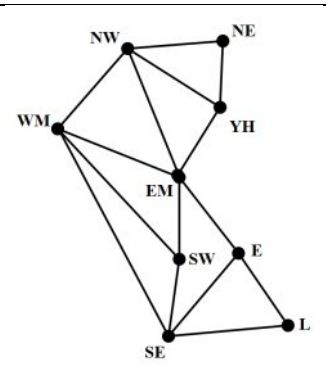
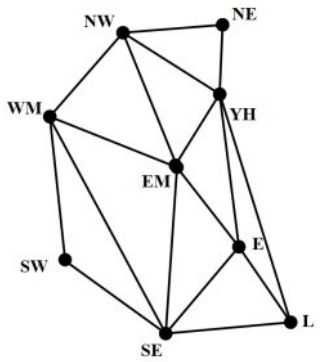
Considering each of the four statements:

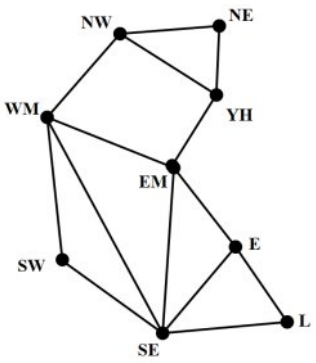
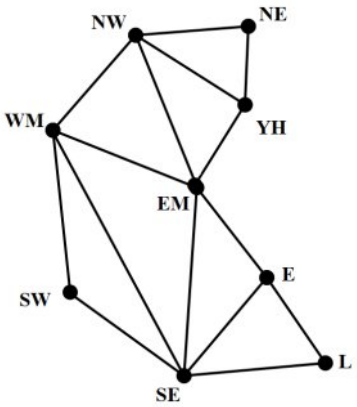
There must be a minimum of six edges.	True. For a connected graph with seven vertices, there must be at least six edges.
There must be five more edges than the number of faces.	True. Using Euler's rule: $V + F = E + 2$ $7 + F = E + 2$ $F = E + 2 - 7 = E - 5$ So there are five more edges than faces.
The graph could be complete.	False. If the graph is complete, it could not be drawn without edges crossing.
The degree of each vertex must be at least one.	True. The graph is connected, so there are no isolated vertices. The degree of every vertex must be at least one.

The false statement is required and the required response is therefore option C.

Question 35 **Answer D**

Each of the options is explored below:

A		This option incorrectly shows a border between EM and SW.
B		This option incorrectly shows borders between YH and both E and L.

C		This option ignores the border between NW and EM.
D		This option correctly represents the borders between the regions and is, therefore, the required response.

Question 36 **Answer C**

Each of the options is explored below:

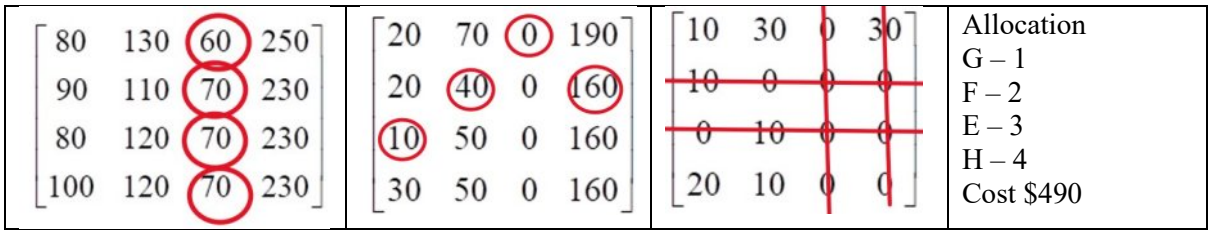
A	The network is a directed network.	True. The adjacency matrix is asymmetric so the network must be directed.
B	Vertex E would be considered a sink.	True. There is a row of zeros from E, but some connections to E, so E is a sink.
C	Vertex D is reachable from vertex B.	False. B can only go to E, but E is a sink and so D cannot reach B.
D	Vertex A is unreachable from any other vertex.	True. No other vertex goes to A as there is a column of zeros to A.

Question 37 **Answer B**

When the initial allocation was made the following working was completed using the Hungarian Algorithm:

$\begin{bmatrix} 80 & 130 & \textcircled{60} & 210 \\ 90 & 110 & \textcircled{70} & 230 \\ 80 & 120 & \textcircled{70} & 230 \\ 100 & 120 & \textcircled{70} & 230 \end{bmatrix}$	$\begin{bmatrix} 20 & 70 & \textcircled{0} & \textcircled{150} \\ 20 & \textcircled{40} & 0 & 160 \\ \textcircled{10} & 50 & 0 & 160 \\ 30 & 50 & 0 & 160 \end{bmatrix}$	$\begin{bmatrix} \cancel{10} & \cancel{30} & \cancel{0} & \cancel{0} \\ \cancel{10} & \cancel{0} & \cancel{0} & \cancel{10} \\ \cancel{0} & \cancel{10} & \cancel{0} & \cancel{10} \\ 20 & 10 & 0 & 10 \end{bmatrix}$	Allocation G – 1 F – 2 H – 3 E – 4 Cost \$480

E was selected to take Excursion 4, but now cannot, so increasing the value of E taking Excursion 4 to \$250 (more than any other cost) will ensure that it is not selected to do this task. The new working is shown below:



There is, therefore, an increase of \$10 from \$480 to \$490 under the changed circumstances.

Question 38 **Answer B**

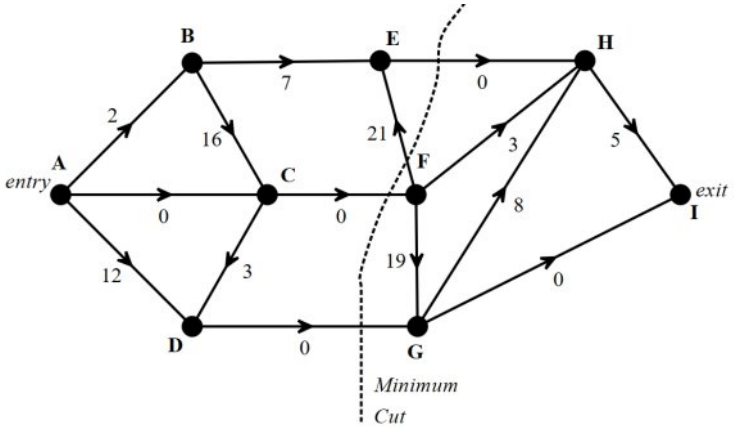
The capacity of *Cut 1* is 11 + 0 + 21 + 17 + 10 + 18 = 77 cars.

The capacity of the cut is the sum of all of the capacities of the edges the cut crosses, so long as the edge crosses the cut from the entry side to the exit side.

The edge *BC* is not counted in the capacity of *Cut 1* because it passes through *Cut 1* from exit side to entry side.

Question 39 **Answer A**

The network is shown with minimum cut below. The capacities of the edges have been reduced by taking 7 cars through ABEHI, 12 cars through ACFHI, 18 cars along ADGI, 2 cars through ADGHI and 2 cars through ABCFHI, a total of 41 cars.



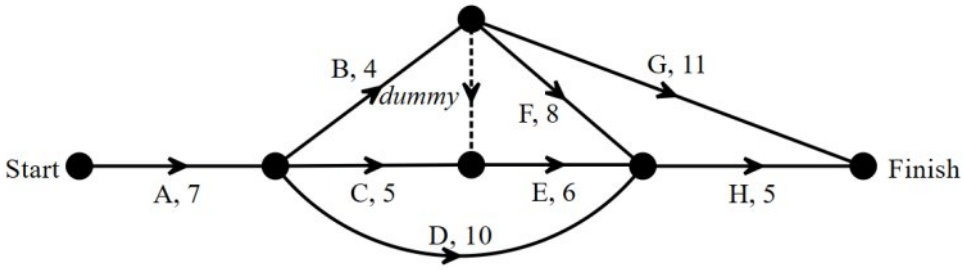
Exploring each of the options:

A	Increasing flow from D to G by 20, so that the new capacity is 40.	This would result in an increase of flow of 5 cars, through ADGHI.
B	Increasing flow from C to F by 20, so that the new capacity is 34.	This would result in an increase of flow of 2 cars, through ABCFHI.
C	Increasing flow from E to H by 20, so that the new capacity is 27.	This would result in an increase of flow of 2 cars, through ABEHI.
D	Reversing the direction of flow from F to E, so it now flows from E to F.	There would be no increase in flow.

Therefore, the greatest increase in flow is given in option A.

Question 40 **Answer C**

The activity network can be constructed from the table as shown below:



The activities, along with their earliest start time (EST) and latest start time (LST) are shown in the table below.

Activity	Earliest Start Time	Latest Start Time
A	0	0
B	7	7
C	7	8
D	7	9
E	12	13
F	11	11
G	11	13
H	19	19

The critical path is ABFH with a critical time of 24 hours. There are a total of five paths through the network ABG – 22 hours, ABFH – 24 hours, ABEH – 22 hours, ACEH – 23 hours and ADH – 22 hours.

Exploring each option:

A	If activity F can be reduced by three hours, the overall project completion time will be reduced by two hours.	False. Reducing F by 3 hours will reduce path ABFH only leaving ACEH as the critical path, a reduction of 1 hour.
B	If activity D can be reduced by two hours, the overall project completion time will be reduced by one hour.	False. Activity D is not on the critical path and so reducing this activity will not affect the completion time.
C	If activity H can be reduced by three hours, the overall project completion time will be reduced by two hours.	True. Reducing H by 3 hours will reduce ABFH to 21, ABEH to 19, ACEH to 20 and ADH to 19, leaving ABG as critical at 22 hours, a saving of 2 hours.
D	If activity B can be reduced by two hours, the overall project completion time will be reduced by two hours.	False. Reducing B by 2 hours will reduce ABG to 20, ABFH to 22 and ABEH to 20 hours. ACEH is the critical path at 23 hours, a reduction of 1 hour.