

# General Mathematics

## Written Examination 1

### 2024 Insight Year 12 Trial Exam Paper

#### Worked Solutions

This book presents:

- answers
- explanatory notes
- tips.

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**Answers to multiple-choice questions**

Question	Answer
1	D
2	B
3	B
4	D
5	D
6	A
7	A
8	D
9	B
10	C
11	B
12	D
13	D
14	B
15	C
16	A
17	C
18	C
19	B
20	C

Question	Answer
21	D
22	D
23	B
24	A
25	C
26	B
27	C
28	D
29	B
30	D
31	D
32	A
33	B
34	C
35	B
36	B
37	A
38	D
39	A
40	B

## Data analysis

### Question 1

**Answer: D**

#### Explanatory notes

The median is sitting to the right of the boxplot, indicating that the data is negatively skewed. There is also an outlier present.

### Question 2

**Answer: B**

#### Explanatory notes

25% of the data is below 200:  $0.25 \times 24 \text{ hours} = 6 \text{ hours}$ .

After removing the outlier of 90, as it is outside the range required (160–200), there are 5 hours remaining.

### Question 3

**Answer: B**

#### Explanatory notes

Option B is correct, as it can represent the two sets of data recorded for each type of vehicle (one boxplot for number of trucks, one for the number of vans).

Option A is incorrect because histograms cannot represent the categorical variable of this data (i.e. the type of vehicle (truck, van)); they can only represent one set of numerical data. Parallel histograms would be required to represent both types of vehicles.

Option C is incorrect because they represent two categorical variables.

Option D is incorrect because scatterplots represent two numerical variables, so it cannot differentiate between the two types of vehicles.

**Question 4****Answer: D****Explanatory notes**

$$\log_{10}(320) = 2.5$$

$$\log_{10}(10\,000) = 4$$

There are  $8 + 7 + 5 = 20$  stores between the intervals of 2.5 and 4.

**Question 5****Answer: D****Explanatory notes**

*Height* is the response variable in this scenario, with *age* as the explanatory variable. This means the transformation is applied to *height*. So after the reciprocal transformation, the

response variable would be  $\frac{1}{\text{height}}$ .

**Question 6****Answer: A****Explanatory notes**

$$\text{Average} = \frac{12 + 14 + 15 + 19 + 20 + 24 + 21}{7} = 17.86$$

$$\text{SI} = \frac{\text{actual}}{\text{average}} = \frac{15}{17.86} = 0.84$$

**Question 7****Answer: A****Explanatory notes**

Options B and D are incorrect because the slope is positive but the equations show a negative slope.

Option C is also incorrect, as the  $y$ -intercept in the equation (i.e. 1.8) represents when  $x$  is 10, instead of when  $x$  is zero.

Alternative method:

Identify two points on the line (e.g. (20, 10) and (32, 21)) and use them to calculate an approximate slope and intercept. The equation for option A has the values closest to these.

A x	B y		
		a	-8.3333...
20	10		
32	21	b	0.91666...

**Tip**

- Remember that the  $y$ -intercept is the value of the response variable when the explanatory variable is zero, so some options can be eliminated if the value given in the equation occurs when  $x \neq 0$ .

**Question 8****Answer: D****Explanatory notes**

Options A, B and C are incorrect as an association or correlation is not the same as causation: one variable may not *cause* something to happen to the other.

Option A also refers to a positive association instead of a negative association, so is therefore incorrect.

**Tip**

- A negative association is one where an increase in one variable is associated with a decrease in the other: as the explanatory variable increases, the response variable tends to decrease.

**Question 9****Answer: B****Explanatory notes**

To determine the mean of the 30 scores:

$$\frac{574 + 25 + 27 + 19 + 24 + 21}{30} = 23$$

To determine which score results in a  $z$ -score of 1.6:

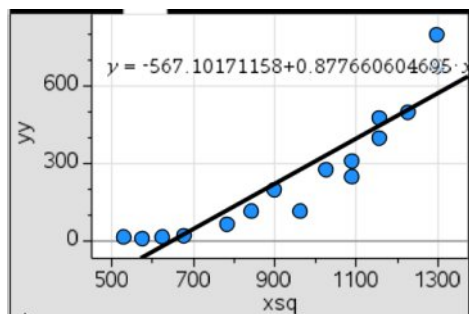
$$1.6 = \frac{x - 23}{1.2}$$

$$x = 24.92$$

Any competitor scoring above 24.92 will move onto the next round, this being Grant and Freyja.

**Question 10****Answer: C****Explanatory notes**Due to the units for the costs being in \$'000, the intercept refers to the initial costs being  $4.5 \times 1000 = \$4500$ , which means that statements I and II are both incorrect.The slope is an increase of  $0.245 \times 1000 = \$245$  for every additional attendee, so statement V is correct. $4.5 + 0.245 \times 5000 = 1229.5$ , which is \$1 229 500; and  $4.5 + 0.245 \times 12\,000 = 2944.5$ , which is \$2 944 500, meaning that statement III is incorrect but statement IV is correct.**Question 11****Answer: B****Explanatory notes**

After the transformation is applied and the least squares line fitted:

The equation needs to be written in terms of the transformed variable,  $x^2$ .

**Question 12****Answer: D****Explanatory notes**

$$\text{solve}\left(\log_{10}(y)=1.8+0.049 \cdot 20,y\right)$$

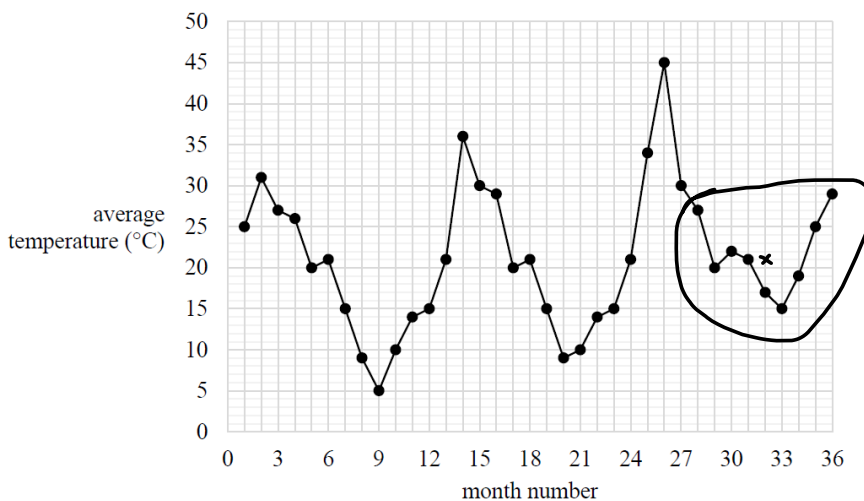
$$y=602.559586074$$

**Question 13****Answer: D****Explanatory notes**

The time series plot has peaks every 12 months (month 2, 14, 26) and increases over the long term.

**Question 14****Answer: B****Explanatory notes**

The smoothed value is shown on the graph below: a value of 21 on month 32.

**Tip**

- *A nine-point smoothing will lose four points on either end when being plotted, so a nine-median smooth value at month 33 cannot be calculated. Seven-point smoothing will lose three points on either end, five-point smoothing will lose two, and three-point smoothing will lose one point on either end.*



**Question 15****Answer: C****Explanatory notes**

The four-point moving mean, with centring, for week 5 requires the averages of weeks 3–6 and weeks 4–7, then the average of those two values.

$$\frac{275+310+345+510}{4} + \frac{310+345+510+580}{4}$$

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$$\frac{\phantom{275+310+345+510} + \phantom{310+345+510+580}}{2}$$
$$398.125$$

**Question 16****Answer: A****Explanatory notes**

It needs to be increased, as the seasonal index of 0.4 indicates that it is 60% below the average sales revenue.

$$\frac{100}{0.4} = 250\%$$

$250\% - 100\% = 150\%$ , so an increase of 150% is required.

## Recursion and financial modelling

### Question 17

**Answer: C**

#### Explanatory notes

Following  $T_0 = 170$ , the sequence shows that 310 is produced at  $T_4$ .

170	170
170+35	205
205+35	240
240+35	275
275+35	310

### Question 18

**Answer: C**

#### Explanatory notes

$$R = 1 + \frac{4.68}{4 \times 100} = 1.0117$$

### Question 19

**Answer: B**

#### Explanatory notes

The effective interest rates and the difference between them is shown below.

$\text{eff}(3.45, 4)$	3.49489157605
$\text{eff}(3.41, 365)$	3.46864223211
$3.4948915760528 - 3.4686422321137$	0.026249343939

**Question 20****Answer: C****Explanatory notes**

The common ratio is found to be 0.88, meaning a rate of depreciation of 12% per annum.

$$\frac{14080}{16000} = 0.88$$

$$\frac{12390.4}{14080} = 0.88$$

Finance Solver	
<b>N:</b>	13.094987746938
<b>I(%):</b>	-12
<b>PV:</b>	-16000
<b>Pmt:</b>	0
<b>FV:</b>	3000
<b>PpY:</b>	1

Using 12%, it is found that it takes 14 time periods to reach \$3000.

Alternative method:

Solving also shows  $n = 13.09$ , which means that it will be worth less than \$3000 after 14 years.

$$\text{solve}(3000 = 16000 \cdot (0.88)^n, n)$$

$$n = 13.0949877469$$

**Tip**

- *Rounding up is required when counting time periods, as the balance will fall short if the time periods have been rounded down.*

**Question 21****Answer: D****Explanatory notes**

$$\frac{\text{Amount of depreciation}}{\text{Number of years}} = \frac{9800 - 4558.40}{3} = \frac{5241.60}{3} = \$1747.20$$

$$\frac{\$1747.20}{52 \times 140} = \$0.24$$

**Question 22****Answer: D****Explanatory notes**

Using the principal reduction for the 117th payment:  $5401.51 + 1889.01 = \$7290.52$

**Question 23****Answer: B****Explanatory notes**

The withdrawal amount needs to be higher than 1599.36, as interest will be added in the last month.

Finding the decimal multiplier and using it to calculate the interest for the 120th month determines that the interest is \$6.80.

$$\frac{22.96}{5401.51} \cdot 1599.36 = 6.79834076027$$

Withdrawal amount = interest + principal reduction

$$\text{Withdrawal amount} = 6.80 + 1599.36$$

$$1599.36 + 6.8 = 1606.16$$

**Question 24****Answer: A****Explanatory notes**

First, calculate the amount Leva requires when she retires (*PV*), for the 20 years (240 months) of monthly withdrawals:

<b>N:</b>	240
<b>I(%):</b>	3.5
<b>PV:</b>	-603490.18931381
<b>Pmt:</b>	3500
<b>FV:</b>	0
<b>PpY:</b>	12

Then use this amount (as *FV*) to determine what additional deposits are needed for the 15 years (180 months) leading up to retirement:

<b>N:</b>	180
<b>I(%):</b>	3.2
<b>PV:</b>	-220000
<b>Pmt:</b>	-1076.0491158775
<b>FV:</b>	603490.189314
<b>PpY:</b>	12

## Matrices

### Question 25

**Answer: C**

#### Explanatory notes

An identity matrix is a square matrix where the elements in the *leading diagonal* are 1 and all other elements are zero.

### Question 26

**Answer: B**

#### Explanatory notes

A summing matrix to add the rows is required. To add the rows of a matrix, a column summing matrix is required. Three values need to be added together, so the summing matrix needs three rows:

$$\begin{bmatrix} 17 & 18 & 32 \\ 10 & 21 & 19 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 67 \\ 50 \end{bmatrix}$$

### Question 27

**Answer: C**

#### Explanatory notes

The row number ( $i$ ) and column number ( $j$ ) are shown for the  $3 \times 2$  matrix:

$$\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \\ 3,1 & 3,2 \end{bmatrix} = \begin{bmatrix} 2(1-1)=0 & 2(1-2)=-2 \\ 2(2-1)=2 & 2(2-2)=0 \\ 2(3-1)=4 & 2(3-2)=2 \end{bmatrix}$$

This can also be created on the CAS calculator by using Menu, 7, 1, A (Construct Matrix):

$$\text{constructMat}(2 \cdot (i-j), i, j, 3, 2) = \begin{bmatrix} 0 & -2 \\ 2 & 0 \\ 4 & 2 \end{bmatrix}$$

The values in the bracket after 'construct matrix' need to be the rule, symbols for the row and column (i.e.  $i$  and  $j$ ), and the values for the order of the matrix. All parts are separated by commas.

**Question 28****Answer: D****Explanatory notes**

The element in  $L_{13}$  represents the reproduction rate for female adults *producing* eggs. It represents that, on average, 3 female eggs will be produced by female adults.

**Question 29****Answer: B****Explanatory notes**

Option A: has a 1 in the leading diagonal, which indicates  $Z$  has played herself.

Option C: both  $W$  and  $X$  are recorded as a defeat against each other.

Option D: has a 1 in the leading diagonal, which indicates  $X$  has played himself.

**Question 30****Answer: D****Explanatory notes**

As matrix  $A$  is a  $1 \times 5$  matrix, the matrix product needs to be  $A \times P$  for it to be defined.

$$[r \ i \ n \ s \ e] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} [r \ e \ s \ i \ n]$$

**Tip**

- *If a permutation matrix is pre-multiplied (e.g.  $P \times A$ ), then the order of the columns of  $A$  is changed. If it is post-multiplied (e.g.  $A \times P$ ), then the order of the rows of  $A$  is changed.*

**Question 31****Answer: D****Explanatory notes**

Using the percentages of the long term, the proportions will be approximately 11% at the oval, 46% at the courtyard, 22% at the basketball court and 21% at the library.

$$\begin{bmatrix} 0.2 & 0.1 & 0.2 & 0 \\ 0.4 & 0.6 & 0.25 & 0.4 \\ 0.1 & 0.1 & 0.45 & 0.3 \\ 0.3 & 0.2 & 0.1 & 0.3 \end{bmatrix}^{100} = \begin{bmatrix} 0.112021857924 & 0.112021857924 & 0.112 & 0.112 \\ 0.459016393443 & 0.459016393443 & 0.459 & 0.459 \\ 0.218579234973 & 0.218579234973 & 0.218 & 0.218 \\ 0.210382513661 & 0.210382513661 & 0.210 & 0.210 \end{bmatrix}$$

**Question 32****Answer: A****Explanatory notes**

Using  $x$  = number at the oval on Tuesday, then the initial state matrix will be:

$$S_{\text{Tue}} = \begin{bmatrix} x \\ 2x \\ 3x \\ 4x \end{bmatrix}$$

Using Solve:

$$0.4 \times x + 0.6 \times 2x + 0.25 \times 3x + 0.4 \times 4x = 79$$

$$x = 20$$

This means there were  $60 + 80 = 140$  students in the library and on the basketball court on Tuesday.

$$S_{\text{Tue}} = \begin{bmatrix} 20 \\ 40 \\ 60 \\ 80 \end{bmatrix}$$

## Networks and decision mathematics

### Question 33

**Answer: B**

#### Explanatory notes

Worker C can only complete task 3 and worker A must complete task 1, which leaves only worker B available to complete task 2.

### Question 34

**Answer: C**

#### Explanatory notes

The degrees can be added to determine the total, or alternatively using

$$\text{Degree sum} = \text{number of edges} \times 2.$$

There are three diagrams with seven edges, therefore a degree sum of 14 (i.e. the first three diagrams).

### Question 35

**Answer: B**

#### Explanatory notes

A complete graph with six vertices needs  $\frac{v \times (v-1)}{2} = \frac{6 \times 5}{2} = 15$  edges.

This graph has nine vertices, so another six edges are needed to be added.

This could also be determined by drawing a complete graph and counting the edges.



**Question 36****Answer: B****Explanatory notes**

Option B is correct because it starts and ends at the same vertex, and goes to every other vertex once only.

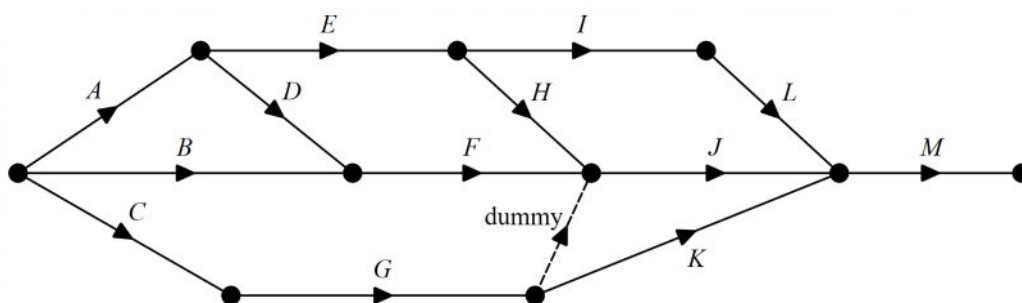
Option A is incorrect because it does not return to  $A$ , so it is not a circuit.

Option C is incorrect because it includes the vertex  $I$ , which should not exist in the network.

Option D is incorrect because it does not include vertex  $E$ .

**Question 37****Answer: A****Explanatory notes**

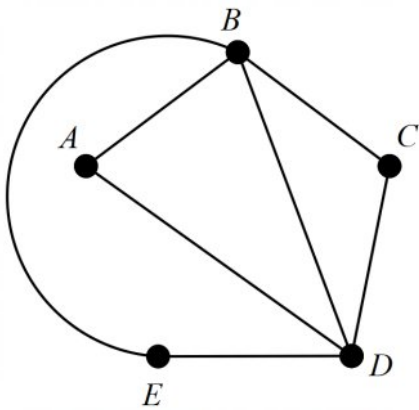
The activity table shows that both activities  $J$  and  $K$  need  $G$  as an immediate predecessor, however  $J$  also needs  $F$  and  $H$  to precede it. This requires a dummy from  $G$  to  $J$  to ensure the precedence structure.

**Tip**

- *Keep in mind that a dummy joins two vertices with a common predecessor, from the one with fewer immediate predecessors to the one that has more.*

**Question 38****Answer: D****Explanatory notes**

A version of the network could look like:



Alternatively, using the matrix:

- Five vertices exist ( $A$  to  $E$ ).
- As it is an undirected graph, the values above and below the leading diagonal are duplicated edges. Counting the values above the leading diagonal (circled) identifies the total number of edges (removing the duplicates).
- Faces can then be calculated using  $v + f - e = 2$ .

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

**Question 39****Answer: A****Explanatory notes**

The maximum amount of water that can flow through pipes  $A-D-E-G-H$  is 9 L/min due to the edge  $E-G$ . Although one part of the path ( $G-H$ ) has a capacity for 20 L/min, the 9 is considered a 'bottleneck' for the water to get through further. Similarly, although 13 L can flow through  $A-D$ , path  $D-E$  restricts it to only 10 L/min, and that again drops to only 9 L/min moving through path  $E-G$ .

**Question 40**

**Answer: B**

**Explanatory notes**

The minimum cut is shown, which results in a maximum flow of  $13 + 13 + 10 = 36$  L/min. One value of 13 on the cut is not included because the flow is in the wrong direction.

