NAME: _____

VCE® General Mathematics

Unit 3&4 Examination 2

Reading time: 10 minutes
Writing time: 90 minutes

QUESTION AND ANSWER BOOK

Section	Number of questions	Number of questions to be answered	Number of marks
A	3	3	24
B	2	2	12
C	3	3	12
D	4	4	12
		Total	60

This exam will be marked **out of 60.**

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are permitted to bring into the examination room: Bound reference notebook, 1 CAS calculator, 1 Scientific Calculator.

Materials supplied
- Question and Answer Book of 17 pages.

Instructions
- Write your **student name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring into the examination room mobile phones and/or any other unauthorised electronic devices.

Section A   **Suggested time: 36 minutes**

Question 1 (8 marks)

The yearly dental bill for 40 random people is shown in the table below as a yearly total in Dollars ($).

37	4771	2001	1932	8045	1255	7559	2625	1224	5732
38	4949	2022	1967	8210	1909	7931	4510	2129	1129
1008	5212	2068	1234	5955	2209	1172	5525	2100	5510
1124	5419	2081	1244	5983	2384	1218	5672	2113	2093

   a. Is the data in the table Discrete Numerical, or Continuous Numerical?   1 mark

   b. Fill the frequency table below.   1 mark

Class Interval	Frequency
0<1000	2
1000<2000	
2000<3000	11
3000<4000	0
4000<5000	3
5000<6000	
6000<7000	0
7000<8000	2
8000<9000	2

   c. Determine the 5-figure summary for the dental cost data to 1 decimal place.

    2 marks

   d. By calculating the upper and lower fences, determine if there are any outliers.

    2 marks

    2

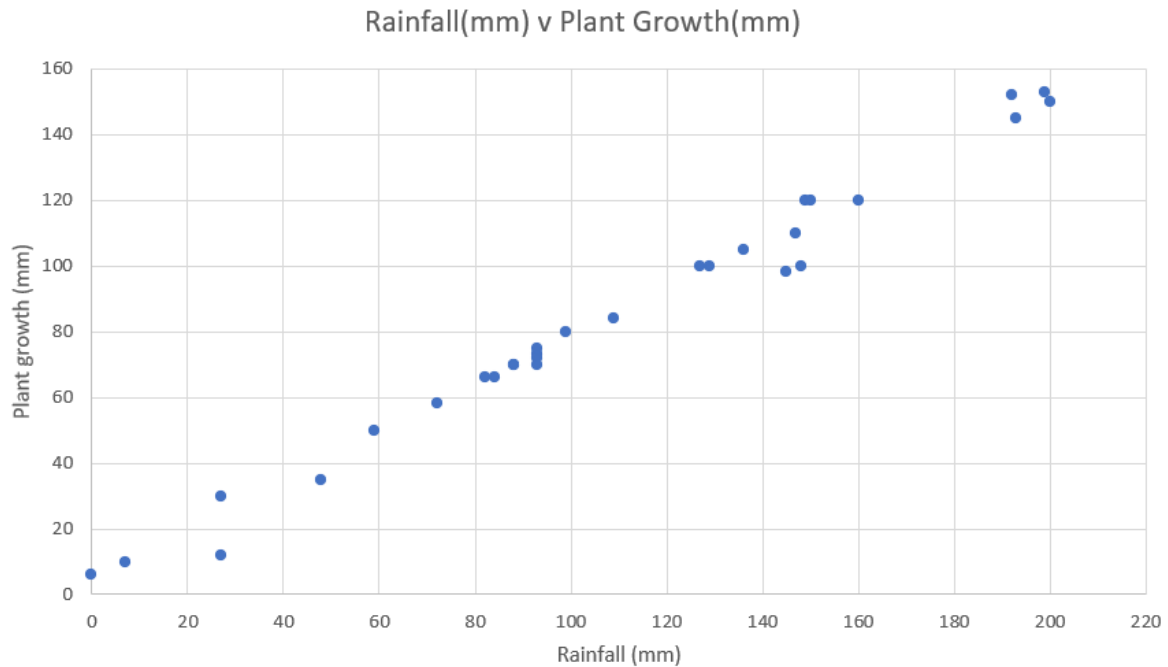**e.** On the grid below create a box plot of the data.   1 mark
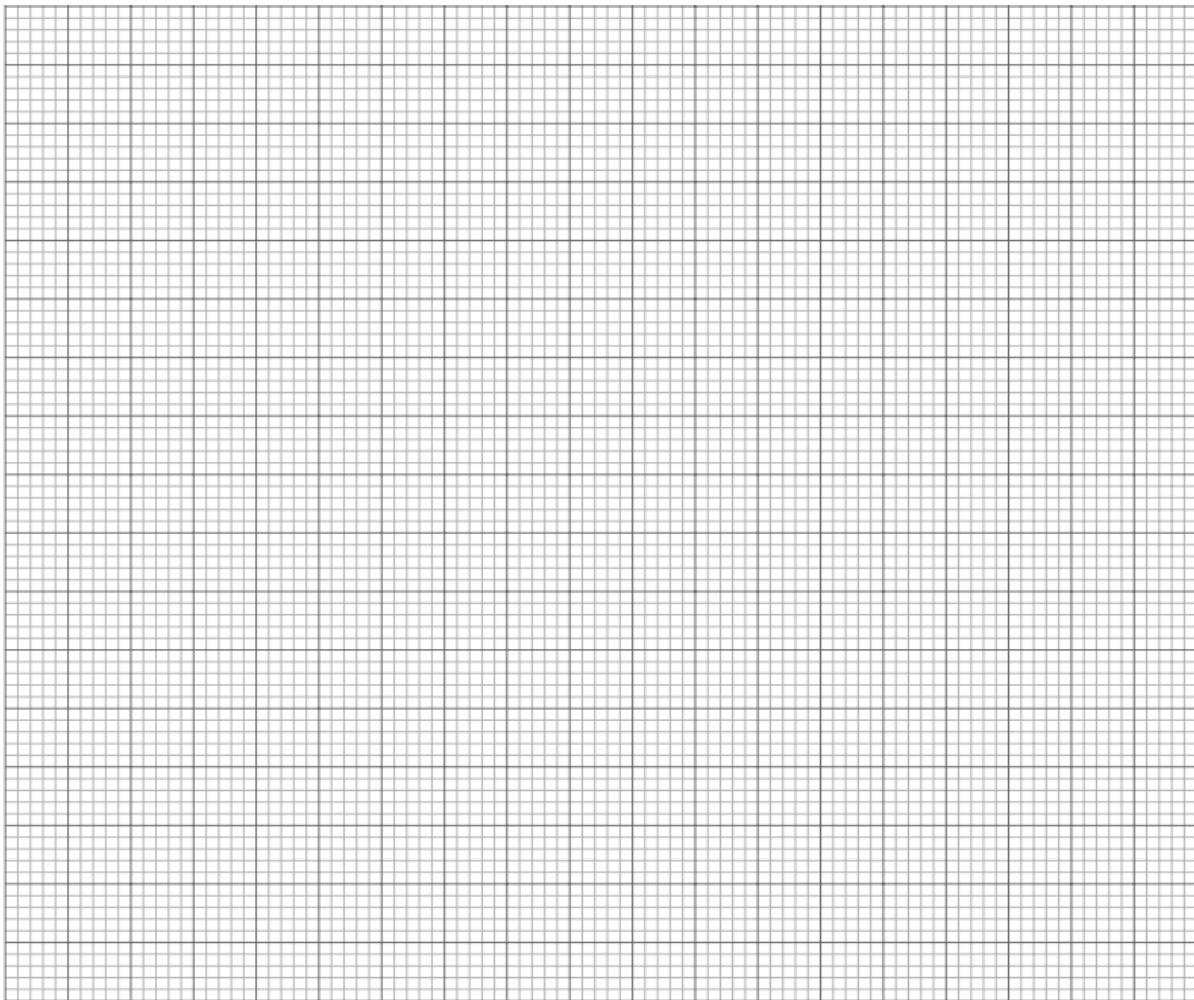
f. Determine the values that 68% of the data is between if we assume the data is normally distributed.   1 mark

Question 2 (6 marks)

The graph below shows the amount of water given to plants, and the amount of growth of the plant over time.

Rainfall(mm) v Plant Growth(mm)

a. Explain which variable is the Explanatory variable.   1 mark

The equation of the relationship is: $Growth = 3.73 + 0.74(Rainfall)$.

b. Explain the meaning of the gradient in terms of the variables involved.   1 mark

c. Plot the equation on the graph on the axes above.   1 mark

d. Use the equation to predict the value of the rainfall when the growth is 145mm.

1 mark

e. Explain the accuracy of the result.   1 mark

f. The actual amount of water the plant received was 193mm. What is the residual from this value?   1 mark

Question 3 (10 marks)

Flights from a selected airport show how many times planes fly a specific route over a 20-day period.

Day	1	2	3	4	5	6	7	8	9	10
Flights	174	180	232	335	297	261	176	176	220	297
Day	11	12	13	14	15	16	17	18	19	20
Flights	335	297	220	174	176	220	300	335	297	186

    a. Find the equation of the least squares regression line for this data set to 4 significant figures.   **2 marks**

    b. It is thought that the airport is large enough to handle growth. Currently, the airport has the capability to reach 380 flights a day for this route. If it takes 100 days for any upgrades to occur at the airport, explain if the airport will be able to maintain its current trajectory, or if it will exceed the 380 flights a day. (Include values in your explanation)   **2 marks**

    c. Calculate the 4-point mean smoothed data with centring for days 8, 9 and 10 below. (Refer to the data supplied above.)   **3 marks**

Day	Flights	4 point moving mean	Centred
			■
8	176		
			■
9	220		
			■
10	297		
			■
11	335		

Over a 2-week period, the following data set was obtained.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
247	193	177	220	297	336	297
281	176	158	260	312	342	330

d. Complete the table below by finding the missing seasonal indices for the 2 weeks.

2 marks

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1.018	0.713		0.925	1.176		1.210

e. The week after this data set was recorded, the Sunday had 336 flights on the route. De-seasonalise to the nearest whole number how many flights this is.   1 mark

Section B   **Suggested time: 18 minutes**

Question 1 (3 marks)

David is trying to save some money. He has currently saved $8300, and is looking to invest it in shares that will grow his money at a flat rate of 3.2% per year.

    a. Determine the Recurrence Relation that models this situation.   1 mark

    b. Show that the value after 2 years is $8831.20.   1 mark

    c. David plans to stop the investment once the value reaches $100000. How long will it take David to reach this target?   1 mark

Question 2 (9 marks)

Sharron is recently retired and has saved $980000. She currently has the money saved in a bank account that follows the recurrence relation below:

$$V_0 = 980000, \ V_{n+1} = 1.0105V_n - 4500$$

Sharron believes that this is a perpetuity that pays her each quarter.

    a. Determine the interest that Sharron's account earns in the first quarter.   1 mark

    b. How much will Sharron receive each quarter from the account?   1 mark

    c. Determine if Sharron is correct about the type of account she has and explain why.

           2 marks

    d. Complete the table below showing the first year of the account.

Quarter	Interest	Payment	Difference	Balance
0	0	0	0	980000
1				
2				
3				
4				

           2 marks

    e. Determine the annual interest rate for this account.   1 mark

    f. What would be the annual interest rate for this account to be a perpetuity?   1 mark

    g. What is the interest rate per quarter for the perpetuity?   1 mark

Section C   Suggested time: 18 minutes

Question 1 (5 marks)

The Matrix movies' initial box office returns are shown in the matrix M below.

$$M = \begin{bmatrix} 27788331 \\ 91774413 \\ 48475154 \\ 10749011 \end{bmatrix} \begin{matrix} \textit{The Matrix} \\ \textit{Reloaded} \\ \textit{Revolutions} \\ \textit{Resurrections} \end{matrix}$$

  a. What percentage of the total returns did Revolutions make?   1 mark

  b. What is the order of the matrix M?   1 mark

  c. To work out the sum of all the returns, M can be multiplied by another matrix S. What is matrix S?   1 mark

  d. After the first weekend, the movies increased in returns. The Matrix increased by 7%, Reloaded by 19%, Revolutions by 8.2% and Resurrections by 1.4%. Show the multiplication needed to produce matrix W below, rounded to the nearest value.

$$W = \begin{bmatrix} 29733514 \\ 109211551 \\ 52450117 \\ 10899497 \end{bmatrix}$$
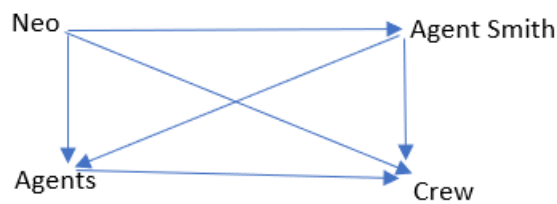
2 marks

Question 2 (4 marks)

Neo has been tracking how many agents he encounters at 4 locations, and displays them in the matrix A below.

$$A = \begin{bmatrix} 37 & 42 \\ 3 & 1000 \end{bmatrix}$$

   a. Show the determinant of matrix A is 36874.   2 marks

   b. Neo has created the diagram below, showing the dominance of Himself, Agents, Agent Smith and the crew of the Nebuchadnezzar. Convert this into a matrix.

1 mark

   c. Calculate the 2-step dominance, and hence the order of dominance of the 4 categories.

1 mark

Question 3 (3 marks)

The people of Zion are split into 4 age groups. The reproduction and survival rate of the 4 groups are shown in the Leslie Matrix below, with 625000 people in each group.

$$\begin{bmatrix} 0 & 1 & 1.5 & 1.2 \\ 0.8 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .25 & 0 \end{bmatrix}$$

    a. What is the birth rate of the people in group 3?   1 mark

    b. By creating a Recurrence Relation, show that after 1 year there are 3281250 people in Zion.   1 mark

    c. Determine the long-term growth rate of the population of Zion by using the 49th and 50th year populations.   1 mark

Section D   **Suggested time: 18 minutes**

Question 1 (3 marks**)**

A group of friends have roads that connect their houses as shown in the matrix below.

$$
\begin{array}{c}
\quad\quad A\ B\ C\ D\ E \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array}
\begin{bmatrix}
0 & 1 & 0 & 2 & 1 \\
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 2 & 0 \\
2 & 1 & 2 & 0 & 1 \\
1 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

    a. Draw the network that the matrix will produce.   1 mark

    b. Confirm that the graph satisfies Euler's Rule.   1 mark

    c. Ben(B) wants to deliver a game to each of his friend's houses. Determine a path he could take that would enable him to return home without repeating any roads.

                                                          1 mark

Question 2 (3 marks)

In playing their new game there are different pathways that evolve, with the time each section takes shown in the graph below. Note: The lengths do not correlate to time i.e., a short path can take longer than a long path.
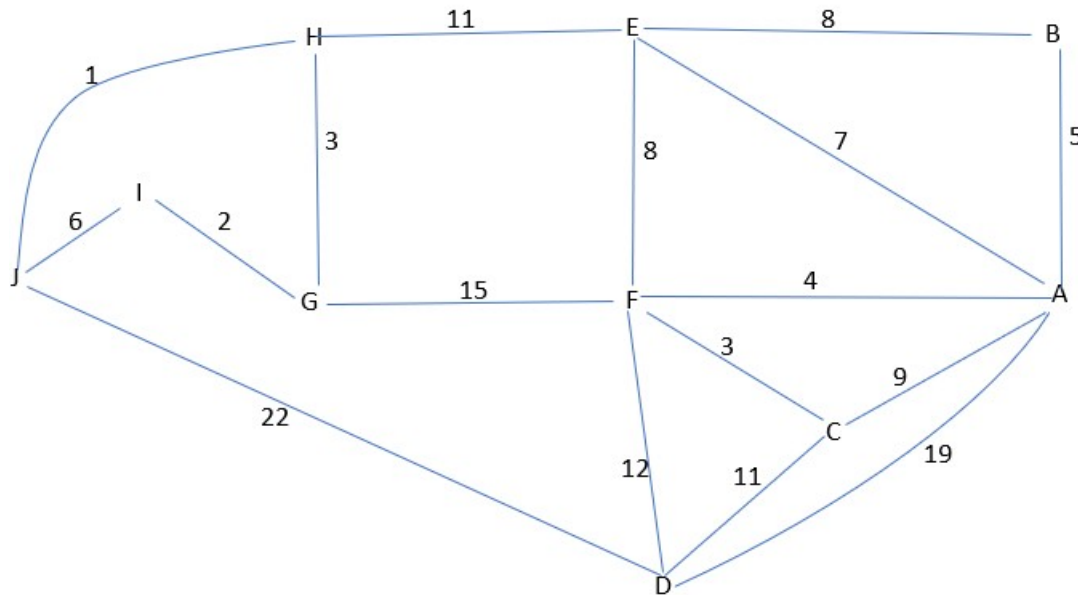
a. The group wants to play for as long as possible. Highlight the maximum spanning tree on the graph above that the game will produce, and determine the length.   2 marks

b. If they were to quickly redo activity B, then go back to I, what is the shortest path they would follow.   1 mark

Question 3 (2 marks)

It takes time to complete specific tasks whilst playing a game. Each friend is better at some tasks than others. As a result, they take different times to complete the tasks.

Below is the table that shows the times for each activity.

	Task 1	Task 2	Task 3	Task 4	Task 5
Adam	12	8	9	10	11
Ben	13	6	12	12	9
Claire	12	10	13	8	20
Demi	8	9	12	9	16
Erik	7	7	15	22	12

By completing a Hungarian algorithm, determine which of the friends should complete each task to finish the game the most quickly.

Question 4 (4 marks)

The game flows by the activity table and network below.

Activity	Predecessor	Duration
A	-	6
B	A	3
C	A	8
D	D	2
E	B	12
F	B, C	4
G	D	6
H	G, F	2
I	E, H	4
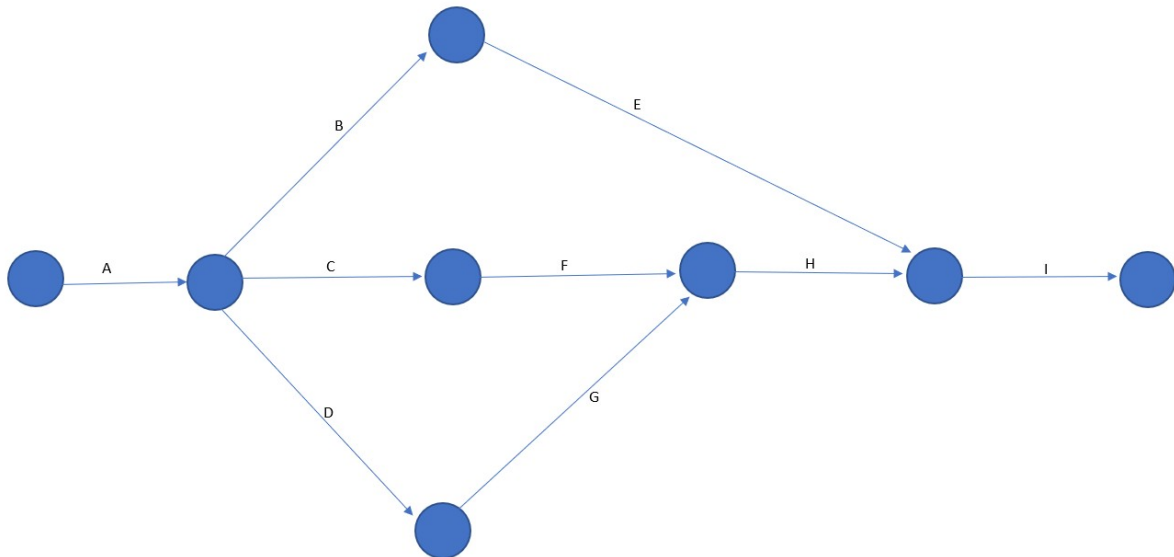
a. Add in the dummy activity in the network diagram.   1 mark

b. Find the LST of activity G.   1 mark

c. Determine the critical path for this activity table.   1 mark

d. Some of the activities are changed as per the table below. Determine the new critical path.   1 mark

Activity	New Duration
E	14
F	7
G	12

VCE® General Mathematics

Unit 3 & 4 Examination 2

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Solution Pathway

Below are sample answers. Please consider the merit of alternative responses.

Section A - 24 marks

Question 1 (8 marks)

a. Discrete Numerical, as the data is in the form of whole numbers.

- **1 mark** for correct answer.

b. The table should be filled out as below.

Class Interval	Frequency
0<1000	2
1000<2000	12
2000<3000	11
3000<4000	0
4000<5000	3
5000<6000	8
6000<7000	0
7000<8000	2
8000<9000	2

- **1 mark** for correct answers.

c. The 5-figure summary is:
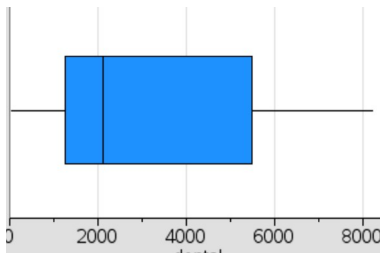
Minimum	Q1	Median	Q3	Maximum
37	1249.5	2106.5	5464.5	8210

- **1 mark** for 3 correct answers.
- **2 marks for** all correct.

d. The upper fence $Q_3 + 1.5 \times IQR = 5464.5 + 1.5(4215) = 11787$
The lower fence $Q_1 - 1.5 \times IQR = 1249.5 - 1.5(4215) = -5073.10$
So there is no Outliers.

- **1 mark** for correct fences.
- **1 mark** for no outliers.

e. Box plot as below

- **1 mark** for correct plot.

f. 68% of the data would be between 917.01 and 5647.95, as the data has a mean of 3282.48 and a standard deviation of 2365.47.

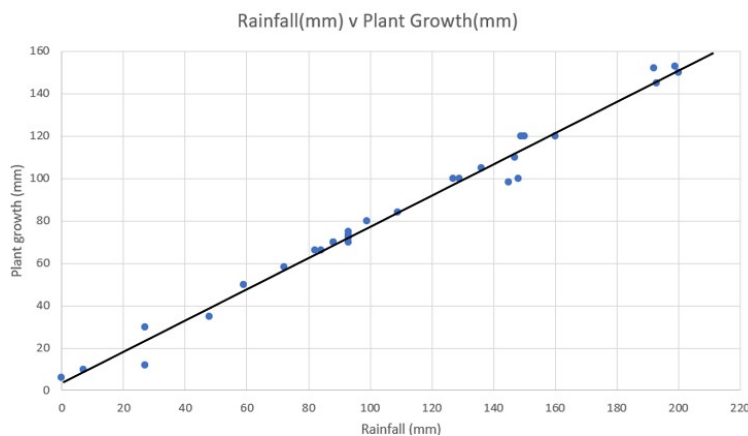- **1 mark** for correct answer.

Question 2 (6 marks)

a. The explanatory variable is the rainfall(mm) as the plants need water to grow.

- **1 mark** for correct answer - must include reason.

b. The gradient is 0.74. This means that for each increase of 1mm of rainfall, the plant growth is predicated to increase by 0.74mm.

- **1 mark** for correct answer.

c. The line should be as below

Line should start at approx.(0,3.73) and cross (200,151.73).

- **1 mark** for correct answer.

d. When the rainfall is 145mm, the growth will be 190.91mm.

$145 = 3.73 + 0.74 \times R$

$141.27 = 0.74 \times R$

$R = 190.91$

- • **1 mark** for correct answer.

e. The accuracy of the result is very accurate as the data is interpolated.

- • **1 mark** for correct answer.

f. The residual is found by actual – predicted. This gives 193-190.91 = 2.09.

- • **1 mark** for correct answer.

Question 3 (10 marks)

a. The equation of the regression line for the data is $flights = 227.3 + 1.632 \times Day$.

- • **1 mark** for correct answer.
- • **1 mark** for correct significant figures.

b. In 100 days, the number of flights would be 390.5 as $227.3 + 1.632(100) = 390.5$. This is not maintainable as it will be more than the 380 flights it can handle.

- • **1 mark** for correct answer.
- • **1 mark** for correct reasoning.

c. The four point smoothed data should be as below:

Day	Flights	4 point moving mean	Centred
		208.25	
8	176		212.75
		217.25	
9	220		237.125
		257	
10	297		272.125
		287.25	
11	335		

- • **1 mark** for each correct centred value (3 total).

d. The missing indices are:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1.018	0.713	0.648	0.925	1.176	1.31	1.210

These values are found as:

Sum of week 1 = 1767

Sum of week 2 = 1859

Weekly average 1 $= \dfrac{1767}{7} = 252.43$

Weekly average 2 $= \dfrac{1859}{7} = 265.57$

For wednesdays: $\dfrac{\left(\dfrac{177}{252.43} + \dfrac{158}{265.57} \right)}{2} = 0.648$

Saturday can be found the same way or 7 − (sum of the other 6 days)

- **1 mark** <u>for each</u> correct answer (2 total).

e. De-seasonalised, 336 flights become $\dfrac{336}{1.210} = 278$

- **1 mark** for correct answer.

Section B – 12 marks

Question 1 (3 marks)

a. The difference is $\dfrac{3.2}{100} \times 8300 = 265.60$

The recurrence relations is then $T_0 = 8300,\ T_{n+1} = T_n + 265.60$.

- **1 mark** for correct answer.

b. By using the recursion relation twice, the result is found as:

$T_1 = 8300 + 265.60 = 8565.60$

$T_2 = 8565.60 + 265.60 = 8831.20$

- **1 mark** for correct answer.

c. At the rate suggested, it will take 346 years, as $\begin{array}{c} 100000 - 8300 = 91700 \\ \dfrac{91700}{265.60} = 345.256 \end{array}$

- **1 mark** for correct answer.

Question 2 (9 marks)

a. The interest that the account earns is 10290 as $0.0105 \times 980000 = 10290$.

- **1 mark** for correct answer.

b. Each quarter Sharron will receive $4500.

- **1 mark** for correct answer.

c. Sharron is not correct, as the account it growing each quarter not staying constant.

- **1 mark** for correct answer.
- **1 mark** for correct reasoning.

d. The table should be completed as below:

Quarter	Interest	Payment	Difference	Balance
0	0	0	0	980000
1	10290	4500	5790	985790
2	10350.80	4500	5850.80	991640.80
3	10412.23	4500	5912.23	997553.03
4	10474.31	4500	5974.31	1003527.34

- **1 mark** for 2 correct rows.
- **2 marks** for all correct.

e. The annual interest rate is $(1.0105 - 1) \, 400 = 4.2\%$

- **1 mark** for correct answer.

f. For the account to be a perpetuity the interest rate would need to be:

$$\frac{4500 \cdot 4}{980000} \times 100 = 1.84\%$$

- **1 mark** for correct answer.

g. The interest rate per quarter would be $\dfrac{1.84}{4} = 0.46\%$
- **1 mark** for correct answer.

Section C – 12 marks

Question 1 (5 marks)

a. The percentage of total returns is $\dfrac{48475154}{178786909} \times 100 = 27.11\%$

-   **1 mark** for correct answer.

b. The order of the matrix is 4x1.

-   **1 mark** for correct answer.

c. To determine the sum of the matrix, it will be pre multiplied by $S = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

-   **1 mark** for correct answer.

d. The multiplication needed to get W, is below.

$$\begin{bmatrix} 1.07 & 0 & 0 & 0 \\ 0 & 1.19 & 0 & 0 \\ 0 & 0 & 1.082 & 0 \\ 0 & 0 & 0 & 1.014 \end{bmatrix} \times \begin{bmatrix} 27788331 \\ 91774413 \\ 48475154 \\ 10749011 \end{bmatrix} = \begin{bmatrix} 29733514 \\ 109211551 \\ 52450117 \\ 10899497 \end{bmatrix}$$

-   **1 mark** for correct multiplication.
-   **1 mark** for correct answer.

Question 2 (4 marks)

a. The determinant of the matrix is found by $\begin{aligned} 37 \times 100 - 42 \times 3 &= 37000 - 126 \\ &= 36874 \end{aligned}$

-   **1 mark** for correct answer.
-   **1 mark** for calculation.

b. The matrix of the diagram is

$$\begin{array}{c} \\ Neo \\ Smith \\ Agents \\ Crew \end{array} \begin{array}{cccc} Neo & Smith & Agents & Crew \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-   **1 mark** for correct answer.

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c. The 2 step dominance is found by squaring the matrix from b) which results in

$$\begin{matrix} Neo \\ Smith \\ Agents \\ Crew \end{matrix} \begin{bmatrix} 6 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

Summing this with the first step results in

Meaning Neo is most dominant, followed by Smith, the Agents and the Crew.

  - **1 mark** for correct answer (could be sum of the steps or the order).

Question 3 (3 marks)

a. The birth rate of the people in group 3 is 1.5 babies per year.

  - **1 mark** for correct answer.

b. The recurrence relation is

$$\begin{bmatrix} 0 & 1 & 1.5 & 1.2 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} \times \begin{bmatrix} 625000 \\ 625000 \\ 625000 \\ 625000 \end{bmatrix} = \begin{bmatrix} 2312500 \\ 500000 \\ 312500 \\ 156250 \end{bmatrix}$$, summing this final matrix results in a total of 3281250.

  - **1 mark** for correct answer.

c. The long term growth rate is found by dividing the values in year 50 by the values in 49. This gives 1.18.

$$T_{50} = \begin{bmatrix} 5073540357.79 \\ 3437450288.94 \\ 1455599006.77 \\ 308188961.35 \end{bmatrix} \text{ and } T_{49} = \begin{bmatrix} 4296812861.18 \\ 2911198013.54 \\ 1222755845.41 \\ 261007146.77 \end{bmatrix}$$

$$\frac{5073540357.79}{4296812861.18} = 1.18$$

$$\frac{3437450288.94}{2911198013.54} = 1.18$$

$$\frac{1455599006.77}{1222755845.41} = 1.18$$
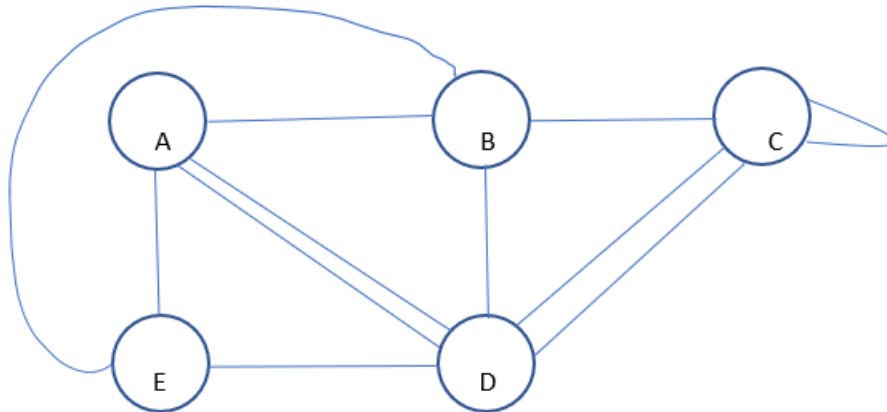
$$\frac{308188961.35}{261007146.77} = 1.18$$

Dividing each corresponding value results in:

  - **1 mark** for correct answer.

Section D - 12 marks

Question 1 (3 marks)

a. The network is:

- **1 mark** for correct answer.

b. Euler's rule is F-E+V=2

This graph gives E=11, F=8, V=5. These values satisfy the equation.
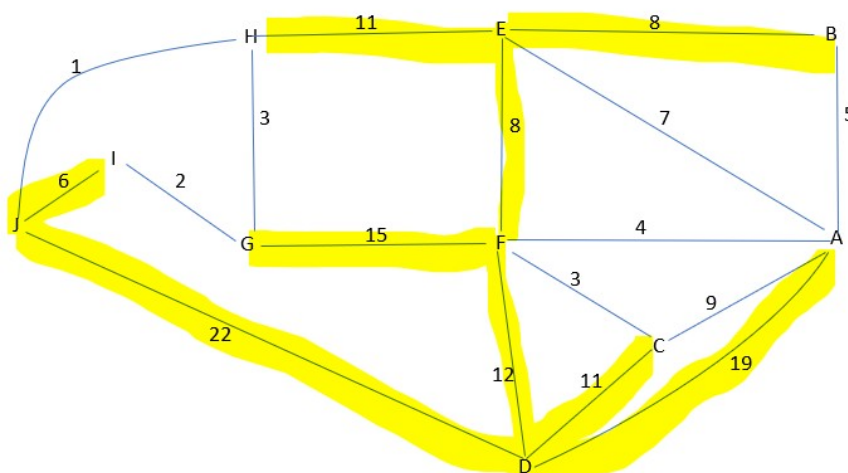
- **1 mark** for correct answer.

c. One of the possible paths is **B-C-D-E-A-B (or in reverse)**

The other path is B-C-D-A-E-B (or in reverse).

- **1 mark** for correct answer.

Question 2 (3 marks)

a. The spanning tree is:

It has a length of 112 min.

- **1 mark** for correct tree.
- **1 mark** for correct length.

b. The shortest path from B to I is **B-E-H-G-I.**

- **1 mark** for correct path.

Question 3 (2 marks)

a. The Hungarian algorithm is shown below:

	Task 1	Task 2	Task 3	Task 4	Task 5
Adam	12	8	9	10	11
Ben	13	6	12	12	9
Claire	12	10	13	8	20
Demi	8	9	12	9	16
Erik	7	7	15	22	12

Row reduction gives:

	Task 1	Task 2	Task 3	Task 4	Task 5
Adam	4	0	1	2	3
Ben	7	0	6	6	3
Claire	4	2	5	0	12
Demi	0	1	4	1	8
Erik	0	0	8	15	5

Column reduction gives:

	Task 1	Task 2	Task 3	Task 4	Task 5
Adam	4	0	0	2	0
Ben	7	0	5	6	0
Claire	4	2	4	0	9
Demi	0	1	3	1	5
Erik	0	0	7	15	2

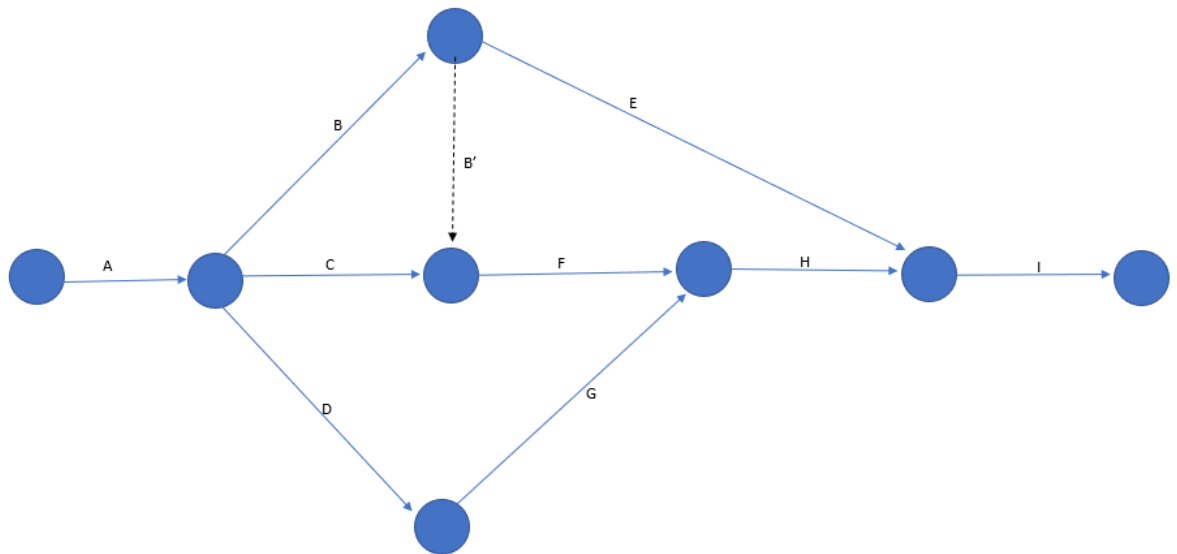This results in 5 lines to cover, so now allocation results in:
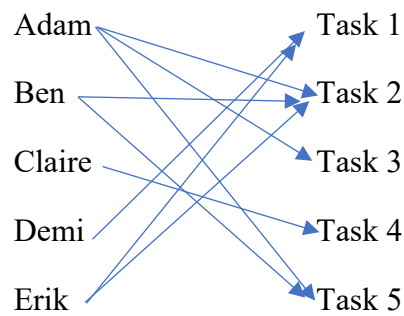
This means that Adam will do task 3, Ben will do task 5, Claire will do task 4, Demi will do task 1, and Erik will do task 2.

- **1 mark** for correct allocation.
- **1 mark** for correct Hungarian.

Question (4 marks)

    a. The dummy activity is from the end of B to the start of F as shown below.

           1 mark for correct dummy activity.

    b. The LST of activity G is 13.

- **1 mark** for correct answer.

    c. The critical path is **A-B-E-I.**

- **1 mark** for correct answer.

    d. The new critical path will be the same as the original critical path as it is still the longest path.

- **1 mark** for correct answer.