THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name……………………………………

GENERAL MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 2

2023

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

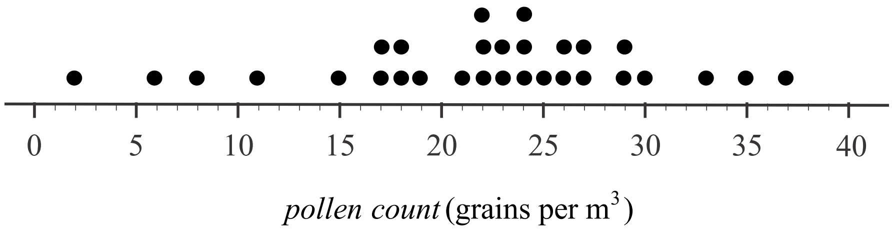
Instructions to students

This exam consists of 15 questions.
All 15 questions should be answered.
There are a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Where a numerical answer is required, students should only round their answer when instructed to do so.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
Formula sheets can be found on pages 20 and 21 of this exam.

Data analysis

Question 1 (5 marks)

The dot plot below displays the distribution of the daily maximum grass *pollen count*, in average number of grass pollen grains per cubic metre of air (grains per m^3), recorded at a particular location for the 30 days in June 2022.



a. Use the dot plot to determine

i. the median. 1 mark

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ii. the value of the first quartile (Q_1). 1 mark

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iii. the percentage of days with a grass pollen count greater than 30. 1 mark

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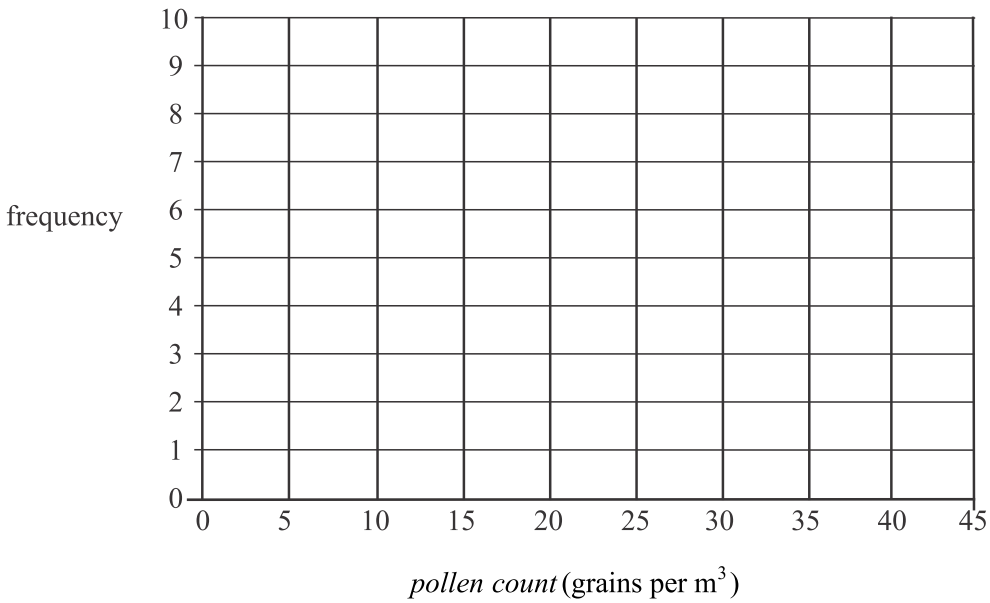
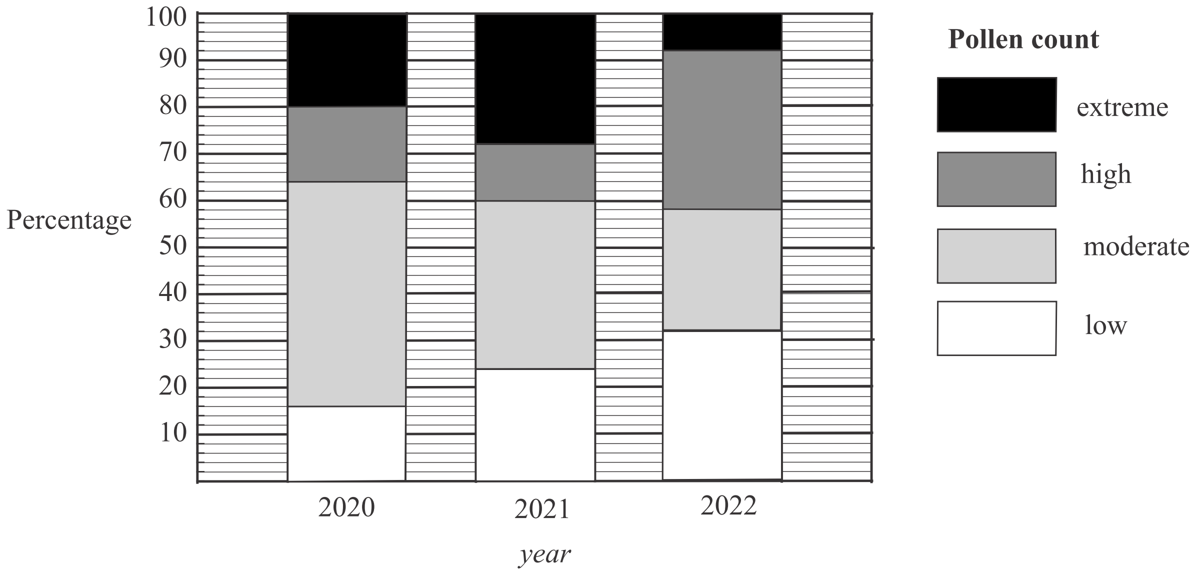
b. Use the grid below to construct a histogram that displays the distribution of the *pollen count*. Use class intervals of five, with the first interval starting at 0. 2 marks



Question 2 (3 marks)

The segmented bar chart below shows the percentage of days during the years 2020, 2021 and 2022 when the pollen count was classified as low, moderate, high or extreme.



a. Warnings are given to the general public on days when the pollen count is classified as high or extreme.
On how many of the 365 days in 2021 were such warnings given to the general public? 1 mark

b. Does the percentaged segmented bar chart support the contention that the pollen count is associated with year? Justify your answer by quoting appropriate percentages. 2 marks

Question 3 (5 marks)

The table below shows data for six variables, collected at a pollen counting location at 12 noon on 11 particular days in December 2021.

day (in December 2021)	***temperature*** (°C)	***wind speed*** (km/h)	***rainfall*** (in previous 24 hours)	***relative humidity*** (%)	***pollen count*** (grains per m^3)
2nd	30	7	yes	81	29
5th	27	11	yes	89	44
8th	26	14	no	78	53
10th	22	10	no	44	47
14th	23	12	no	66	55
17th	36	31	no	15	73
19th	29	16	yes	65	46
20th	28	19	no	36	63
23rd	31	17	yes	55	69
28th	32	21	no	32	78
31st	35	26	no	8	90

a. Write down the number of categorical variables in this table of data. 1 mark

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b. Determine the mean temperature, in degrees Celsius, for this 11 day period. 1 mark

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A least squares line is to be fitted to some of the data in the table with the aim of predicting the *pollen count* from *relative humidity*.

c. **i.** Name the response variable for this least squares line. 1 mark

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ii. Determine the slope of this least squares line. Round your answer to three significant figures. 1 mark

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iii. Determine the value of the correlation coefficient. Round your answer to three significant figures. 1 mark

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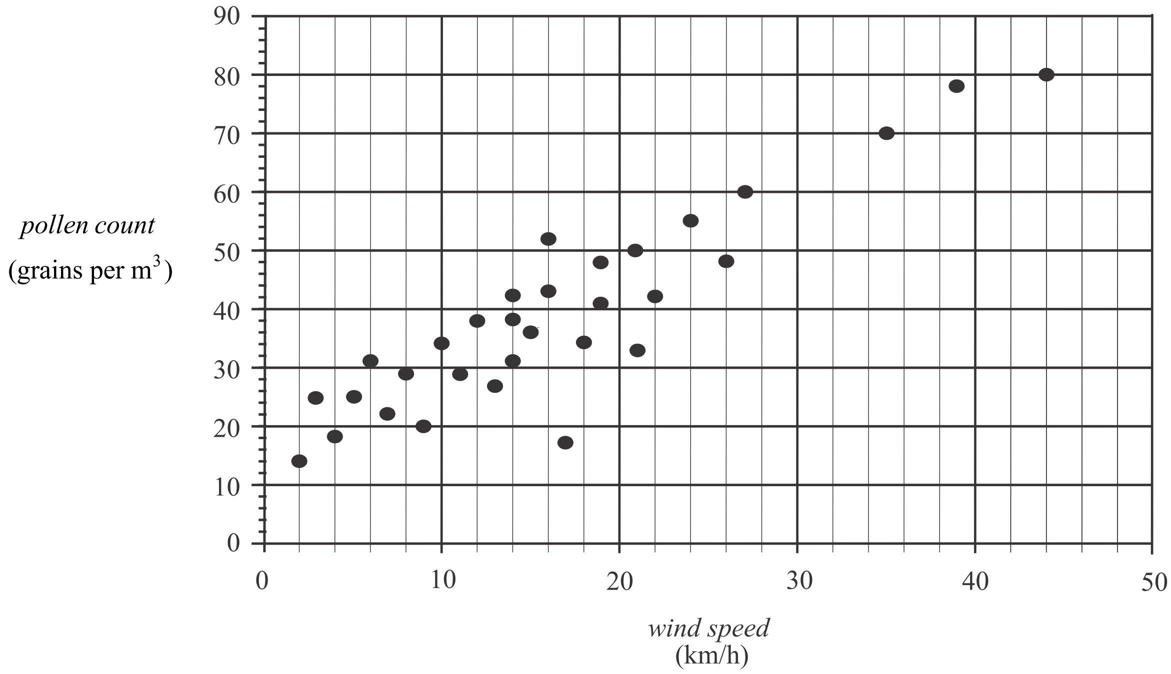
Question 4 (7 marks)

The scatterplot below shows the *pollen count* (grains per m^3) plotted against *wind speed* (km/h) at a regional location at midday for the 31 days of March 2020.



A least squares line is fitted to the scatterplot. The equation of the line is given by

$$pollen\ count = 14.3394 + 1.5058 \times wind\ speed$$

a. Draw the graph of the least squares line on the scatterplot above. 1 mark

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b. Describe the association between *pollen count* and *wind speed* in terms of form and direction. 1 mark

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c. Interpret the intercept of the least squares line in terms of the variables *pollen count* and *wind speed*. 1 mark

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d. Use the given equation of the least squares line to predict the pollen count when the wind speed is 30 km/h. Round your answer to two decimal places. 1 mark

e. The predicted *pollen count* when the *wind speed* is 9 km/h, is 27.8916 grains per m^3. The actual *pollen count* when the *wind speed* is 9 km/h is 20 grains per m^3. Determine the residual value in grains per m^3. 1 mark

f. The mean and standard deviation for the variables *wind speed* and *pollen count* are shown in the table below.

Statistic	***Wind speed (km/h)***	***Pollen count (grains per m^3)***
mean	16.4839	39.1613
standard deviation	10.1386	16.8030

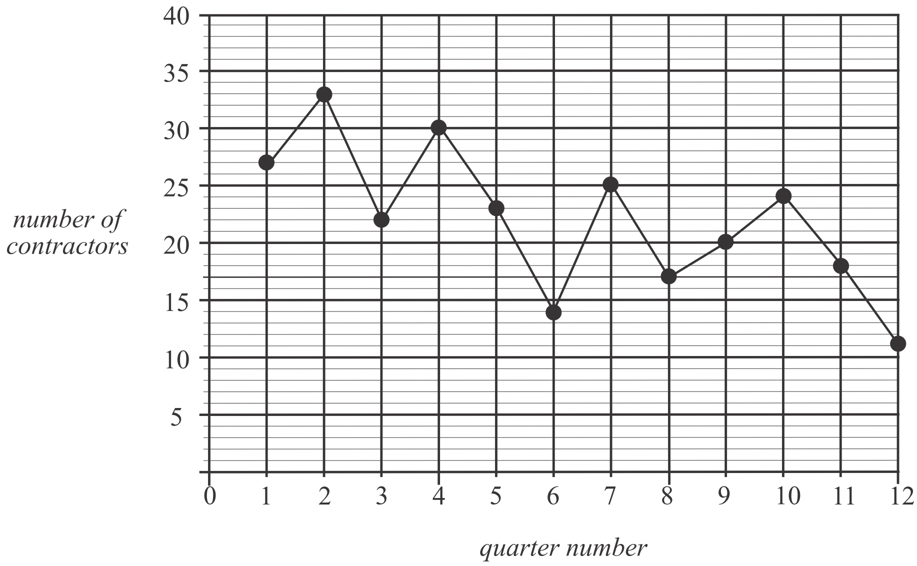
The equation of the least squares line is

$$\textit{pollen count} = 14.3394 + 1.5058 \times \textit{wind speed}$$

Use this information to determine the percentage of variation in *pollen count* that can be explained by the variation in *wind speed*.
Round your answer to the nearest percentage. 2 marks

Question 5 (4 marks)

The time series plot below shows the number of contractors employed by a business each quarter over a period of 12 quarters for the years 2019, 2020 and 2021.



a. Describe the trend in the time series plot. 1 mark

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b. Use nine-median smoothing to find the smoothed number of contractors for quarter number six. 1 mark

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c. Explain why these data do not allow a nine-median smoothed number to be found for the last quarter of 2019. 1 mark

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d. A least squares line is fitted to the plot. The equation of the line is

$$\textit{number of contractors} = 30.05 - 1.24 \times \textit{quarter number}.$$

The predicted *number of contractors* employed by the company in the first quarter of 2024 is 4.
What assumption is being made to predict the *number of contractors* employed by the company in the first quarter of 2024? 1 mark

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Recursion and financial modelling

Question 6 (4 marks)

Jenita deposits \$86 000 into an account that pays compound interest each month.
The balance of Jenita's account in dollars, after n months, V_n, can be modelled by the recurrence relation

$$V_0 = 86\,000, \qquad V_{n+1} = 1.0035 \times V_n .$$

a. Using recursion, the balance of the account after 12 months is calculated to be \$89 682.35.
Write down the recursive calculation that finds the balance of the account after 13 months, V_{13}. 1 mark

b. How many months after the initial deposit is made will the balance of the account first exceed \$91 000? 1 mark

c. The balance of Jenita's account after n months V_n, can also be determined using a rule.

i. Complete the rule below by writing the appropriate numbers in the boxes provided. 1 mark

$$V_n = \boxed{} \times \boxed{}^{\,n}$$

ii. Jenita uses this formula to find the balance of her account after five years.
What value of n does Jenita use? 1 mark

Question 7 (3 marks)

Jenita purchases a car for her business and depreciates it using flat rate depreciation.
The value of the car C_n, in dollars, n years after she purchases it, can be determined using the rule $C_n = 42\,000 - 3570n$.

a. What annual flat rate percentage of depreciation is being used? 1 mark

b. Write down a recurrence relation in terms of C_0, C_{n+1} and C_n that models the value of the car. 1 mark

c. The recurrence relation found in part **b**. could also model the year-to-year value of the car using unit cost depreciation.
The car travels 7000 km each year.
By how much is the value of the car depreciated for every kilometre it travels? 1 mark

Question 8 (5 marks)

Jenita invests an inheritance of \$460 000 in a perpetuity.
She will receive a fortnightly payment of \$736 from this investment.

a. **i.** What is the annual amount of interest, in dollars, that this perpetuity earns? 1 mark

ii. What is the annual percentage rate of interest earned by this perpetuity? 1 mark

Jenita converts the perpetuity to an annuity.
She invests the \$460 000 in an annuity that earns interest of 4.2% per annum compounding monthly. She will receive a regular monthly payment after interest has been added.
The balance of the annuity at the end of the first year is \$438 101.64.

b. What is the monthly payment that Jenita receives? Round your answer to the nearest cent. 1 mark

At the end of the first year of the investment, Jenita changes the monthly payment that she receives to \$3750 a month. The interest rate remains at 4.2% per annum compounding monthly.

c. How much interest will Jenita's annuity earn, in dollars, in the second year of the investment? Round your answer to the nearest cent. 2 marks

Matrices

Question 9 (4 marks)

Matrix C below shows the cost, in dollars, of small (S), medium (M), large (L) and extra large (E) bags of guinea pig food sold at a pet store last week.

$$C = \begin{array}{c} \begin{bmatrix} 8 \\ 13 \\ 25 \\ 42 \end{bmatrix} \end{array} \begin{array}{c} S \\ M \\ L \\ E \end{array}$$

a. What is the order of matrix C? 1 mark

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b. Last week George bought three medium sized and four extra large sized bags of guinea pig food at this pet store.
Write down the row matrix G, for which the matrix product GC gives the total amount George paid for the seven bags of guinea pig food. 1 mark

$$G = [\qquad\qquad\qquad\qquad\qquad\qquad]$$

c. Next week the pet store will

- discount the cost of the medium sized bags by 15%.
- increase the cost of the large bags by 10%.
- leave the cost of the small and extra large sized bags the same.

The matrix product NC produces a 4×1 matrix that gives the cost of the different sized bags of guinea pig food for next week.

Complete matrix N below by entering the four missing elements. 2 marks

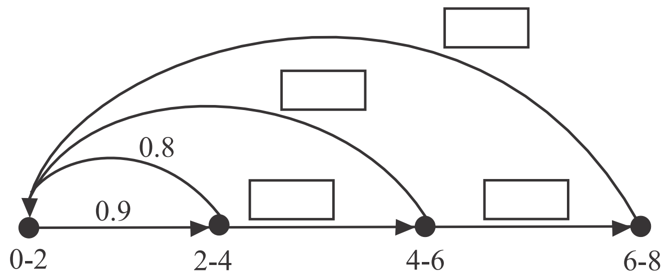
$$N = \begin{bmatrix} \_ & 0 & 0 & 0 \\ 0 & \_ & 0 & 0 \\ 0 & 0 & \_ & 0 \\ 0 & 0 & 0 & \_ \end{bmatrix}$$

Question 10 (4 marks)

A female population of rodents is being studied.
The rodents have a life span of eight years and researchers have classified the population into four age groups of 0–2 years, 2–4 years, 4–6 years and 6–8 years.
The Leslie matrix, L, that models the changes in this female rodent population is given by

$$\begin{array}{cc} & \textit{from} \\ L = \begin{bmatrix} 0 & 0.8 & 0.6 & 0.2 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix} & \textit{to} \end{array}$$

a. An equivalent transition diagram is shown below but is incomplete. Complete the transition diagram by writing the four missing entries in the boxes provided below. 1 mark



The initial state matrix, S_0, for this female rodent population at the beginning of the study, is given by

$$S_0 = \begin{bmatrix} 50 \\ 150 \\ 100 \\ 200 \end{bmatrix}$$

The breeding patterns of this female rodent population can be modelled by the recurrence relation given by

$$S_{n+1} = L\,S_n.$$

where n represents the number of two-year periods since the beginning of the study.

b. Calculate S_1 and use it to determine how many rodents aged 4–6 years there are in the population after one two-year period. 1 mark

c. What percentage of the population of rodents are aged 2–4 years after three two-year periods have lapsed? Round your answer to one decimal place. 1 mark

Twenty-two years after the study begins, researchers notice that each of the four age groups is showing approximately the same percentage increase in numbers from one two-year period to the next.

The state matrices S_{10} and S_{11} are shown below.

$$S_{10} = \begin{bmatrix} 243.16 \\ 206.32 \\ 136.71 \\ 51.47 \end{bmatrix} \qquad S_{11} = \begin{bmatrix} 257.38 \\ 218.84 \\ 144.42 \\ 54.68 \end{bmatrix}$$

d. Using data from these two state matrices, find the percentage increase in the number of rodents in the population from one two–year period to the next.
Round your answer to the nearest whole percentage. 1 mark

Question 11 (2 marks)

A different research institute studies the four stages of the life–and–death cycle of another type of rodent. These stages include young rodents (Y), breeding age rodents (B), elderly rodents (E) and rodents that have died (D).
In this population,

- young rodents may grow into breeding age rodents or they may die.
- breeding age rodents may grow into elderly rodents or they may die.
- elderly rodents may live for a period of time or they may die.

The population of rodents is managed by the institute so that no new young rodents are born. Under this management program, the change in the rodent population can be modelled by the transition matrix

$$
\begin{array}{c}
\text{this month} \\
\begin{array}{cc}
 & \begin{array}{cccc} Y & B & E & D \end{array} \\
T = & \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0.6 & 0.7 & 0 & 0 \\ 0 & 0.2 & 0.4 & 0 \\ 0.1 & 0.1 & 0.6 & 1 \end{bmatrix}
\end{array}
\begin{array}{l} Y \\ B \\ E \\ D \end{array} \text{next month}
\end{array}
$$

The initial state matrix below shows the number of rodents at each stage when the management program begins.

$$S_0 = \begin{bmatrix} 50 \\ 120 \\ 40 \\ 0 \end{bmatrix} \begin{array}{l} Y \\ B \\ E \\ D \end{array}$$

Let S_n represent the state matrix that describes the number of rodents at each stage, n months after the management program begins.
A matrix recurrence relation that generates the values of S_n is

$$S_{n+1} = T \times S_n .$$

a. One month after the program begins, how many of the rodents in the population are at a different stage to the one they were in when the program began? 1 mark

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b. Find the expected number of rodents in the population that will have died over the long term. Round your answer to the nearest whole number. 1 mark

___

___

Question 12 (2 marks)

To extend the period of time when live rodents are in the population described in Question 11, a change to the management program is proposed. A rule that models this proposal is given by

$$R_{n+1} = T \times R_n + E$$

where R_n represents the state matrix that gives the number of rodents in the population n months after the changed management program begins and

$$T = \begin{array}{c} \text{this month} \\ \begin{array}{cccc} Y & B & E & D \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0.6 & 0.7 & 0 & 0 \\ 0 & 0.2 & 0.4 & 0 \\ 0.1 & 0.1 & 0.6 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ Y \\ B \\ E \\ D \end{array} \text{ next month,} \qquad \text{and } E = \begin{bmatrix} 10 \\ 20 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} Y \\ B \\ E \\ D \end{array}$$

According to this model, the state matrix one month after the changed management program begins is given by

$$R_1 = \begin{bmatrix} 40 \\ 150 \\ 40 \\ 50 \end{bmatrix} \begin{array}{c} Y \\ B \\ E \\ D \end{array}.$$

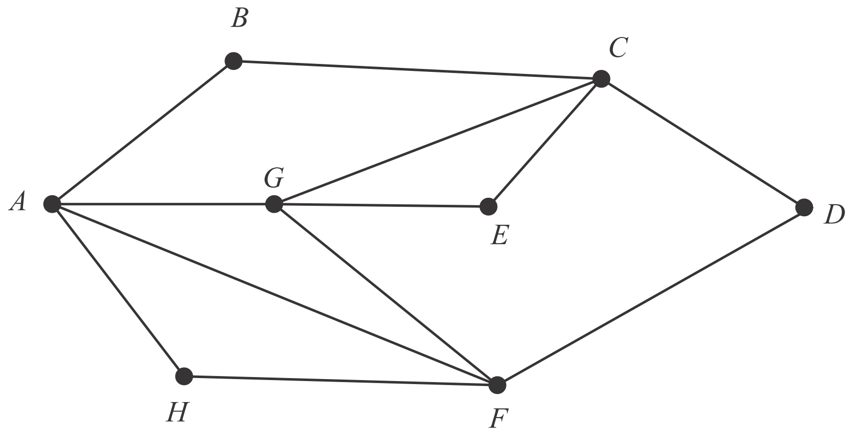
a. How many breeding age rodents are there expected to be in the population two months after the changed management program begins? 1 mark

b. How many elderly rodents would be in the population when the changed management program begins? 1 mark

Network and decision mathematics

Question 13 (4 marks)

The diagram below shows a network of paths joining eight rides at a fun park.
Vertices *A, B, C, D, E, F, G* and *H* represent the rides.



a. What is the degree of vertex *G*? 1 mark

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b. A safety inspector starts at ride *D*, finishes at a different ride, and visits each ride just once.
What is the mathematical term for this route? 1 mark

____

c. Each morning, before the park opens, a mechanical sweeper is driven along all the paths just once, starting at ride *C*.
Which ride will the sweeper finish at? 1 mark

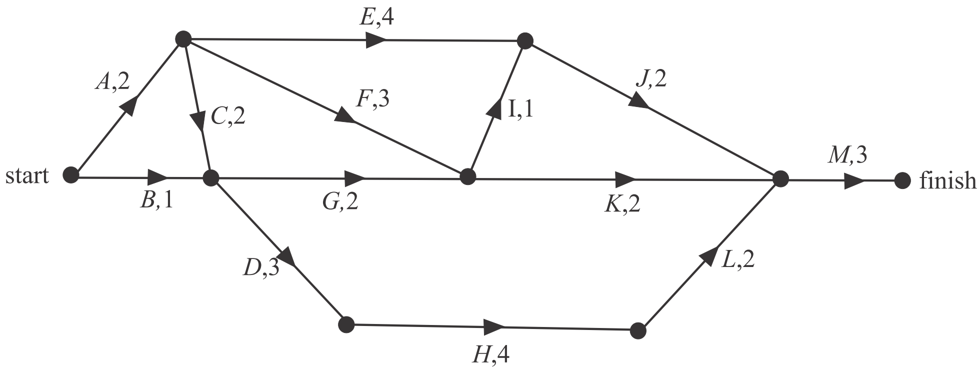
____

d. Euler's formula can be applied to this graph.
Write the appropriate numbers in the boxes provided below to complete this formula. 1 mark

$\square + \square = \square + 2$

Question 14 (5 marks)

A new ride is to be installed at the fun park.
The project will require 13 activities to be completed.
The network below shows these 13 activities and their completion time in days.



a. How many of the activities have three immediate predecessors? 1 mark

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b. What is the earliest start time, in days, for activity *K*? 1 mark

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c. What is the minimum completion time, in days, for this project? 1 mark

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d. How many activities can have their completion time increased by at least three days without affecting the minimum completion time of the project? 1 mark

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e. The project is to be crashed by reducing the completion time of just one activity.
None of the activities can have their completion time reduced to zero days.
What is the minimum completion time possible, in days, for the crashed project? 1 mark

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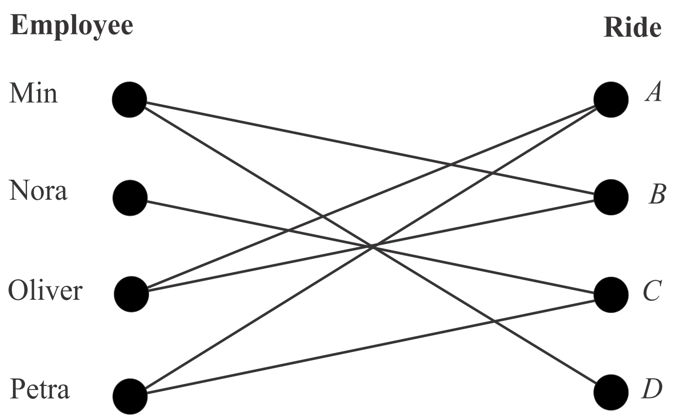
Question 15 (3 marks)

Four of the rides *A, B, C,* and *D*, can be serviced by a fun park employee.
The bipartite graph below shows the rides that each employee is trained to service.



Each employee will be allocated to service just one ride.

a. Complete the table below by allocating an appropriate ride to each employee. 1 mark

Employee	Ride
Min	
Nora	
Oliver	
Petra	

b. The other four rides *E, F, G* and *H* are serviced by external contractors.
The fun park owner obtains quotes from four contractors *W, X, Y* and *Z* to service each of the rides.
The table below shows the time, in hours, it would take for each contractor to service each of the rides.

	Time (hours)			
Ride	*W*	*X*	*Y*	*Z*
E	4	5	3	4
F	8	7	8	7
G	6	5	4	6
H	7	7	5	7

The fun park owner will allocate each contractor to one ride and will minimize the total time it takes to have the four rides serviced.

Find the minimum total time, in hours, for the four rides to be serviced. 2 marks

General Mathematics formulas

Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $\mathrm{Q}_1 - 1.5 \times \mathrm{IQR}$ upper $\mathrm{Q}_3 + 1.5 \times \mathrm{IQR}$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = Ru_n + d$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 =$ initial state, $S_{n+1} = TS_n + B$
Leslie matrix recurrence relation	$S_0 =$ initial state, $S_{n+1} = L\,S_n$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$

END OF FORMULA SHEET