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GENERAL MATHS UNITS 3&4
TRIAL EXAMINATION 1
SOLUTIONS
2023

Answers

Data analysis				Recursion and financial modelling		Matrices		Networks and decision mathematics	
1.	D	**9.**	D	**17.**	A	**25.**	A	**33.**	E
2	E	**10.**	B	**18.**	B	**26.**	C	**34.**	D
3.	A	**11.**	A	**19.**	E	**27.**	E	**35.**	A
4.	D	**12.**	C	**20.**	E	**28.**	D	**36.**	B
5.	B	**13.**	D	**21.**	C	**29.**	E	**37.**	B
6.	B	**14.**	C	**22.**	E	**30.**	A	**38.**	C
7.	B	**15.**	B	**23.**	A	**31.**	E	**39.**	C
8.	E	**16.**	D	**24.**	A	**32.**	C	**40.**	B

Solutions

Data analysis

Question 1

$$\left(\frac{243}{312}\times\frac{100}{1}\right)\% = 77.88...\%$$

The closest answer is 78%.
The answer is D.

Question 2

The variables are both categorical variables, not numerical variables, so reject option A.
The variable *time of entry* is an ordinal variable i.e. the categories can be put in a (time) order.
The variable *type of entry pass* is a nominal variable, i.e. it names the different categories of entry passes, but there is no natural order to these categories.
The answer is E.

Question 3

There are 18 values.
Q_1 occurs in the middle of the lower 9 values i.e. it is the 5th value. So $Q_1 = 28$.
Q_3 occurs in the middle of the upper 9 values i.e. it is the 14th value. So $Q_3 = 43$.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 43 - 28 \\ &= 15 \end{aligned}$$

The answer is A.

Question 4

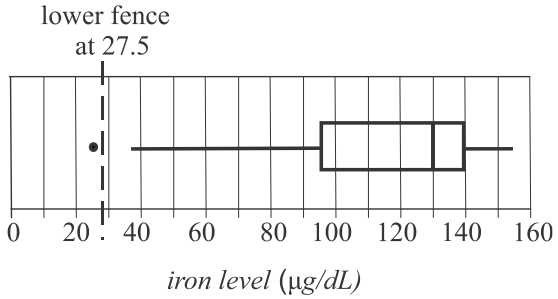
In order to calculate the mean length of the sample, the individual data values must be used.
The only graphical display offered that provides individual data values is the dot plot.
The answer is D.

Question 5

There is a significant difference between the mean of 105 and the median of 130, suggesting that the distribution is skewed and/or there are outliers.
The difference between the median of 130 and Q_3 of 140 is 10 whereas the difference between the median of 130 and Q_1 of 95 is 35, suggesting that the distribution is negatively skewed.

$$\text{lower fence} = 95 - 1.5 \times (140 - 95) \qquad \text{(formula sheet)}$$
$$= 27.5$$

Since $25 < 27.5$, there is at least one outlier.
A **possible** boxplot which displays the five-number summary is shown below. Note that because we don't have the actual data values, we don't know if more than one outlier lies below the lower fence and we don't know where the end of the lower whisker lies.

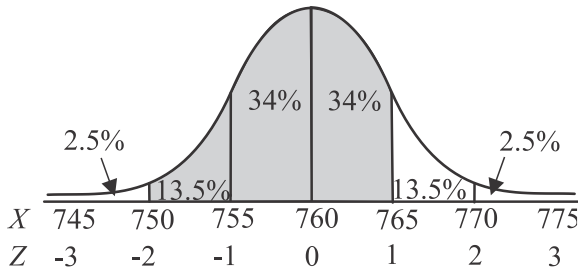


The answer is B.

Question 6

The value 750 is two standard deviations below the mean and the value 765 is one standard deviation above the mean.
Using the bell shaped curve and the $68 - 95 - 99.7\%$ rule,

$$\Pr(750 < X < 765) = 13.5\% + 34\% + 34\%$$
$$= 81.5\%$$



$81.5\% \text{ of } 5600 = 4564$
The answer is B.

Question 7

$$\text{one billion} = 1000000000 \qquad \text{(note the 9 zeros)}$$
$$= 10^9$$

There are $4 + 1 = 5$ countries that emitted more than 10^9 tonnes.

$$\left(\frac{5}{41} \times \frac{100}{1}\right)\% = 12.19...\%$$

The closest answer is 12%.
The answer is B.

Question 8

$$\begin{aligned}\text{upper fence} &= Q_3 + 1.5 \times \text{IQR} \qquad \text{(formula sheet)} \\ &= 15 + 1.5 \times (15 - 11) \\ &= 21\end{aligned}$$

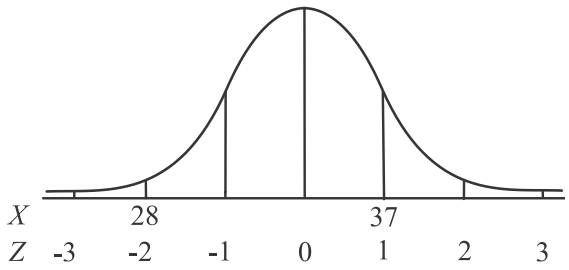
The answer is E.

Question 9

The range of the distribution for 2022 is $22 - 7 = 15$ and NOT $18 - 7 = 11$. Note that the maximum value of the distribution occurs at the outlier with the data value of 22.
Reject option A.
For 50% of weeks in 2021, not 75%, the number of houses sold is less than eight.
Reject option B.
The total number of houses sold each week in 2021 is, on average, **less** than the total number of houses sold each week in 2022.
Reject option C.
Option D is correct. The maximum number of houses sold in a week in 2021 is 11. The median number of houses sold in a week in 2022 is 14. So for more than 50% of the weeks in 2022, the number of houses sold is more than 11.
The number of houses sold each week in 2021 is possibly **less** variable than in 2022 but definitely not more variable.
Reject option E.
The answer is D.

Question 10

Tom's time is one standard deviation above the mean because his standardised time is $z = 1$.
Marco's time is two standard deviations below the mean because his standardised time is $z = -2$.
There are therefore three standard deviations between the boys's times of 28 secs and 37 secs.
$37 \text{ secs} - 28 \text{ secs} = 9 \text{ secs}$ and $9 \text{ secs} \div 3 = 3 \text{ secs}$.
So one standard deviation is 3 secs.
So the mean equals $37 \text{ secs} - 3 \text{ secs} = 34 \text{ secs}$.
Alternatively, mean equals
$28 \text{ secs} + 2 \times 3 \text{ secs} = 34 \text{ secs}$.



The answer is B.

Question 11

$$\begin{aligned}\text{Since } r^2 &= 0.81912 \\ r &= \sqrt{0.81912} \\ &= 0.9050... \\ &= 0.905 \text{ (correct to 3 decimal places)}\end{aligned}$$

When we look at the scatterplot however, we notice that as age increases, the time spent sleeping tends to decrease (i.e. the slope is negative) so the value of r will be negative.
So $r = -0.905$.
The answer is A.

Question 12

The slope of the least squares line is 1.25. This tells us that on average, for every increase of 1 cm in *tibia bone length*, there will be a predicted increase of 1.25 cm in *height*.
The answer is C.

Question 13

Enter the *result* data and the *time* data into your CAS. Now create a column for $\log_{10}(result)$.
Now calculate a least squares equation using *time* as the explanatory variable (*x* variable) and $\log_{10}(result)$ as the response variable (*y* variable).
The equation is given by $\log_{10}(result) = 1.5577... + 0.07029... \times time$.
The closest option is $\log_{10}(result) = 1.558 + 0.0703 \times time$
The answer is D.

Question 14

For February, March, April and May, the mean is $\frac{60+49+32+41}{4} = 45.5$.
For March, April, May and June, the mean is $\frac{49+32+41+36}{4} = 39.5$.
With centering, the mean is $\frac{45.5+39.5}{2} = 42.5$.
The closest answer is 43.
The answer is C.

Question 15

First calculate the seasonal index for November. The seasonal indices for the 12 months should add to 12.

$$\text{November seasonal index} = 12 - 10.87$$
$$= 1.13$$
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}} \qquad \text{(formula sheet)}$$
$$1.13 = \frac{52}{\text{deseasonalised figure}}$$

So deseasonalised figure $= 52 \div 1.13 = 46.0176...$.
The closest answer is 46.
The answer is B.

Question 16

$$\text{seasonal index} = \frac{\text{actual sales}}{\text{deseasonalised sales}} \qquad \text{(formula sheet)}$$
$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$
$$= \frac{\text{actual sales}}{0.625}$$
$$= \frac{1}{0.625} \times \text{actual sales}$$
$$= 1.6 \times \text{actual sales}$$

The deseasonalised sales should be 160% of the actual sales which means the actual sales should be increased by 60%.
The answer is D.

Recursion and financial modelling

Question 17

$T_0 = 1, \quad T_{n+1} = 3T_n - 4$

$T_1 = 3 \times 1 - 4 = -1$
$T_2 = 3 \times -1 - 4 = -7$
$T_3 = 3 \times -7 - 4$
The answer is A.

Question 18

$$\begin{aligned}\text{principal reduction} &= \text{payment} - \text{interest} \\ &= \$9281.10 - \$7974.38 \\ &= \$1306.72\end{aligned}$$
The answer is B.

Question 19

The quarterly interest charged just before the first repayment is made, is \$8000.
$\left(\frac{8000}{400\,000} \times \frac{100}{1}\right)\% = 2\%$.
So the multiplication factor is 1.02.
So $B_0 = 400\,000, \; B_{n+1} = 1.02B_n - 9281.10$
The answer is E.

Question 20

$T_0 = 55\,000, \qquad T_{n+1} = 1.005T_n + 2800$

Karl's initial payment is \$55 000 i.e. $T_0 = 55\,000$.
Karl's monthly payment is \$2800.
The multiplication factor of 1.005 tells us that the monthly interest rate is 0.5% i.e. $(1.005 - 1) \times 100\% = 0.5\%$.
So the annual interest rate is $12 \times 0.5\% = 6\%$ i.e. $(1.005 - 1) \times 12 \times 100\% = 6\%$.
So all four statements are true.
The answer is E.

Question 21

The effective interest rate is given by $r_{effective}\left[\left(1+\frac{r}{100\times n}\right)^{n}-1\right]\times 100\%$ (formula sheet)

where r is the nominal interest rate and n is the number of times the interest compounds in a year.

Option E is already incorrect because when interest is charged yearly, then effective interest rate equals the nominal interest rate so the interest rate would be 6% not 6.01%.

For option A, when interest is charged daily, $n = 365$.

$$r_{effective} = \left[\left(1+\frac{6}{100\times 365}\right)^{365}-1\right]\times 100\%$$
$$= 6.1831...\%$$
$$= 6.18\% \text{ to 2 decimal places}$$

Reject option A.

For option B, when interest is charged weekly, $n = 52$. Copy and paste the calculation from your CAS and replace the 365 used in option A with 52.

$$r_{effective} = 6.1799...\%$$
$$= 6.18\% \text{ to 2 decimal places}$$

Reject option B.

For option C when interest is charged monthly, $n = 12$. Replacing 52 with 12 gives,

$$r_{effective} = 6.1677...\%$$
$$= 6.17\%$$

The answer is C.

Question 22

The equipment is depreciating (or decaying) in value **<u>per quarter</u>** by a factor of $1-\frac{6}{100\times 4}=0.985$.

Since the rate was compounding quarterly, the number of compounding periods in n **years** is $4n$.

So the rule for the value of the equipment after n years is $V_n = 15\,000\times 0.985^{4n}$.

The answer is E.

Question 23

Using TVM

N : 48

I(%): 4.2

PV : −36 000

Pmt: ?

FV : 80 000

PpY: 12

CpY: 12

Pmt : −717.4211...

The closest value is \$717.42.

The answer is A.

Question 24

Using TVM,

N : 60
I(%):7.2
PV : 32 000
Pmt : –636.66
FV:?
PpY:12
CpY:12
FV = –0.16078...

The negative sign on the future value indicates that 16 cents would be outstanding if Katelyn had made 60 repayments of $636.66.

So Katelyn's final payment will need to be $0.16 higher than her previous payments in order to repay the loan to the nearest cent.
The answer is A.

Matrices

Question 25

Method 1 – by hand
The rows of matrix N become the columns of N^T.
So $N^T = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$.
The answer is A.

Method 2 – by CAS
Enter matrix N and navigate the menu to find its transpose.
$N^T = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$.
The answer is A.

Question 26

$\begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is **not** defined because two matrices of a different order cannot be added.

$3 \times \begin{bmatrix} 2 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$ is an example of a scalar product of a matrix and is defined.

Note that $\begin{bmatrix} 3 \end{bmatrix} \times \begin{bmatrix} 2 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$ is an example of a matrix product and gives the same result.

$$\begin{bmatrix} 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \times 1 + 7 \times 4 \end{bmatrix}$$
$$= [33] \text{ i.e. it is defined.}$$

$\begin{bmatrix} 6 \\ 1 \end{bmatrix}^2$ is **not** defined since only square matrices can be raised to a power, in this case squared.

Two of the expressions are defined.
The answer is C.

Question 27

For matrix A, the element:

$a_{11} = 2 \times 1 - 1 = 1$

$a_{12} = 2 \times 1 - 2 = 0$

$a_{13} = 2 \times 1 - 3 = -1$

$a_{21} = 2 \times 2 - 1 = 3$

$a_{22} = 2 \times 2 - 2 = 2$

$a_{31} = 2 \times 3 - 1 = 5$

So all the matrices except the square matrix could be matrix A.
Note that a_{22} in the square matrix equals 1 whereas it needs to equal 2.
The answer is E.

Question 28

The Leslie matrix is given by

$$L = \begin{matrix} \\ \\ \\ \\ next\ year \end{matrix} \begin{matrix} \begin{matrix} this\ year \\ \downarrow \end{matrix} \\ \begin{bmatrix} 0 & 0.7 & 1.2 & 0.3 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{bmatrix} \end{matrix}$$

The birth rate for each of the four age groups appears in the first row of the matrix.
The survival rate for the 0-1 year age group appears as the element in the second row and first column of the matrix.
The survival rate for the 1-2 year age group appears as the element in the third row and second column of the matrix.
The survival rate for the 2-3 year age group appears as the element in the fourth row and third column of the matrix. These three survival rates make a diagonal pattern. (Note however that they do not lie along the leading diagonal of the matrix!)
All the remaining elements in the matrix are zero.
The answer is D.

Question 29

Method 1

Using the diagram, the one-step dominance matrix is given by

$$\begin{matrix} \\ winner \\ \end{matrix} \begin{matrix} \begin{matrix} \\ \rightarrow \end{matrix} \\ \begin{matrix} M \\ N \\ O \\ P \end{matrix} \end{matrix} \begin{matrix} \begin{matrix} loser \\ \downarrow \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \\ M \\ N \\ O \\ P \end{matrix}$$

We square this matrix to obtain the two-step dominance matrix, which is given by

$$\begin{matrix} \\ winner \\ \end{matrix} \begin{matrix} \begin{matrix} \\ \rightarrow \end{matrix} \\ \begin{matrix} M \\ N \\ O \\ P \end{matrix} \end{matrix} \begin{matrix} \begin{matrix} loser \\ \downarrow \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \\ M \\ N \\ O \\ P \end{matrix}$$

The answer is E.

Method 2

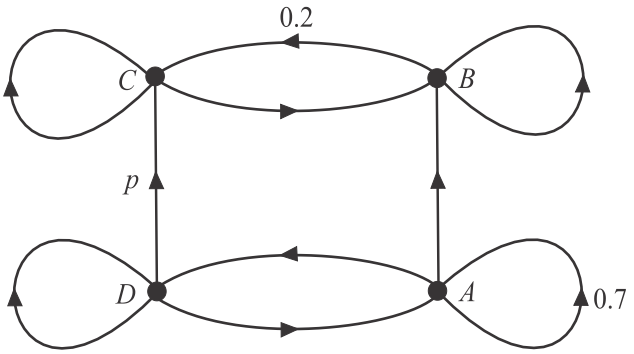
Use the diagram to construct the two-step dominance matrix. For example, *M* has a two-step dominance over *P* (i.e. via *N*).
N has no two-step dominances.
O has a two-step dominance over *N* (via *M*), and two two-step dominances over *P* (one via *M* and one via *N*).
P has no two-step dominances.
The two-step dominance matrix is given by

$$\begin{matrix} \\ winner \\ \end{matrix} \begin{matrix} \begin{matrix} \\ \rightarrow \end{matrix} \\ \begin{matrix} M \\ N \\ O \\ P \end{matrix} \end{matrix} \begin{matrix} \begin{matrix} loser \\ \downarrow \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \\ M \\ N \\ O \\ P \end{matrix}$$

The answer is E.

Question 30

The only transition with a proportion of 0.7 is the transition from *A* to itself.
Note from the matrix that there is no transition from *A* to *C*. So *C* must be the vertex diagonally opposite to *A*.
There is a transition with proportion 0.2 from *B* to *C* but not from *D* to *C* so the vertices are as shown below.



The matrix tells us that the transition from *D* to *C* is 0.1 so $p = 0.1$.
The answer is A.

Question 31

We want either a 1×2 or a 2×1 matrix so that both values can be displayed.
For option A, we have a $(4 \times 1) \times (1 \times 4)$ matrix which results in a 4×4 matrix.
Reject option A.
For option B, we have a $(2 \times 4) \times (4 \times 1)$ matrix which results in a 2×1 matrix.
The first row of this 2×1 matrix will give the sum of the four different payments, not the total value of the payments made so reject option B.
For option C, we have $(1 \times 4) \times (4 \times 1)$ matrix which results in a (1×1) matrix.
Reject option C.
For option D, we have a $(1 \times 4) \times (4 \times 2)$ matrix which results in a (1×2) matrix.
The first column of this matrix gives the sum of the four different payments, not the total value of the payments made so reject option D.
For option E, we have a $(2 \times 4) \times (4 \times 1)$ matrix which results in a (2×1) matrix.
The first row gives the total number of players and the second row gives the total value of the payments.
The answer is E.

Question 32

Since the matrix product *QP* is defined and *P* is a 4×4 matrix and *Q* is a row matrix, then *Q* must be a 1×4 matrix so *R* must be a 1×4 matrix so r_{15} does not exist. Reject option B.

P is a permutation matrix so it can only have elements of 0 or 1 so $p_{24} \neq 3$. Reject option A.

Also since *P* is a permutation matrix, it can only have one "1" in each row and only one "1" in each column. So because p_{12} and p_{32} are in the same column, they cannot both equal 1 so $p_{12} + p_{32} \neq 2$. Reject option E.
Q contains no zero elements so $q_{13} \neq 0$. Reject option D.
The elements in matrix *Q* become the elements in matrix *R* but in a different order so it could be true that $q_{12} = r_{14}$.
The answer is C.

Networks and decision mathematics

Question 33

Dijkstra's algorithm is used to find the shortest distance between two vertices.
The answer is E.

Question 34

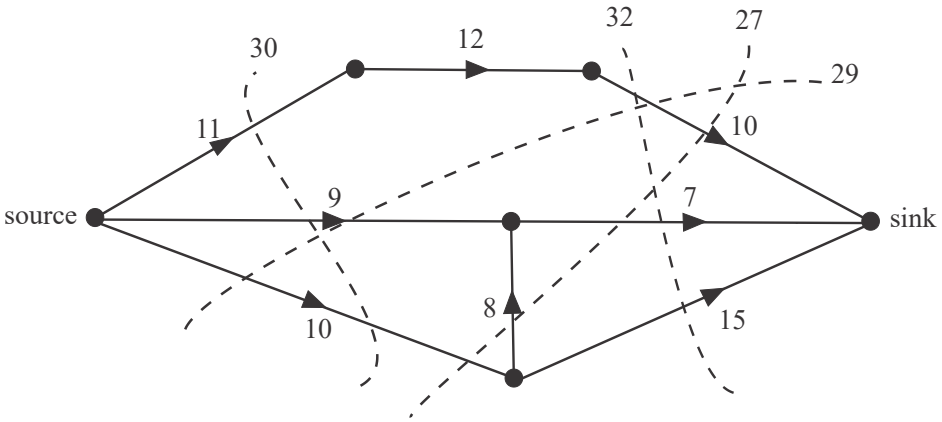
A loop connects a vertex to itself. The graph has no loops. Reject option A.
A bridge is a single edge, which, if it were to be removed would cause the graph to become disconnected. The graph has no bridges. Reject option B.
A complete graph has every vertex connected to every other vertex by an edge. This graph is not complete. Reject option C.
The graph is planar, i.e. it has no edges that cross so option D is correct.
The graph is not a tree because it contains cycles and multiple edges. Reject option E.
The answer is D.

Question 35

Town *X* is the only town where a driver can start at *X* and finish at *X* without having visited another town, i.e. the graph has a loop, so the element in the second row and second column should be 1. Reject options B and C.
There are two ways to go from *Y* to *X* (without visiting another town) so the element in the third row and second column should be 2 not 1. Reject options D and E.
The answer is A.

Question 36

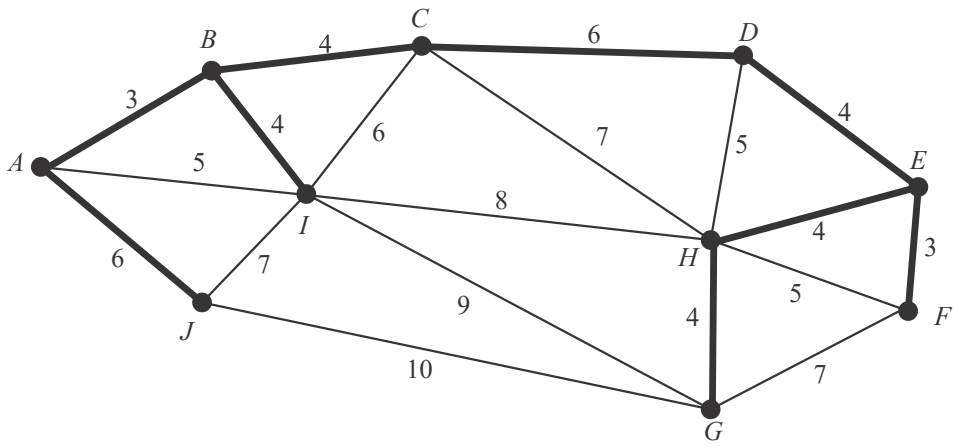
A few cuts, that separate the source from the sink, have been made on the network below.



The minimum cut of 27 gives us the maximum flow possible through the network.
Note that the pipe with a capacity of 8 L/min, flows from the sink side of the cut whereby it should flow from the source side to the sink side and hence we ignore this capacity of 8 L/min.
The answer is B.

Question 37

We use Prim's algorithm to find a minimum spanning tree for this network.



The length of the minimum spanning tree is 38 metres.
The answer is B.

Question 38

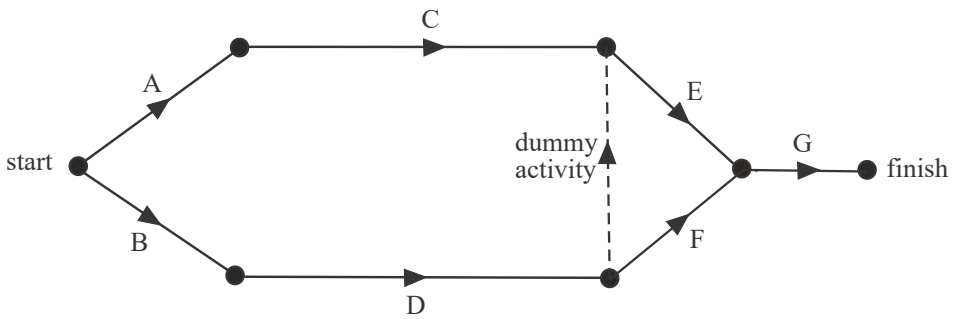
There are 3 possible paths in the network:

A, *C*, *F*, *H* which takes 20 days.
A, *D*, *G*, *H* which takes 21 days.
B, *E*, *G*, *H* which takes 19 days.

The critical path is *A D G H* and the minimum time in which the project can be completed is 21 days. The path that *B* lies on takes 19 days, so the latest start time for activity *B* without delaying the completion of the project is 2 days.
The answer is C.

Question 39

Draw a directed network.



In drawing this network, it becomes apparent that a dummy activity needs to be drawn from the end of activity *D* to the start of activity *E*.
The answer is C.

Question 40

Identify all the paths in the network.

A, *D*, *H*, *K* – 13 weeks
A, *D*, *I*, *J*, *K* – 18 weeks
A, *D*, *I*, *L* – 17 weeks
B, *E*, *H*, *K* – 12 weeks
B, *E*, *I*, *J*, *K* – 17 weeks
B, *E*, *I*, *L* – 16 weeks
B, *F*, *J*, *K* – 16 weeks
B, *F*, *L* – 15 weeks
C, *G*, *J*, *K* – 19 weeks
C, *G*, *L* – 18 weeks

Five of the paths above need to be reduced. They are

A, *D*, *I*, *J*, *K* – 18 weeks
A, *D*, *I*, *L* – 17 weeks
B, *E*, *I*, *J*, *K* – 17 weeks
C, *G*, *J*, *K* – 19 weeks
C, *G*, *L* – 18 weeks

C, *G*, *J*, *K* is the longest path and needs to be reduced by 3 weeks. Since activity *C* cannot be reduced then activities *G, J* and *K* must each be reduced.

Consequently,
A, *D*, *I*, *J*, *K* is reduced to 16 weeks
A, *D*, *I*, *L* remains at 17 weeks
B, *E*, *I*, *J*, *K* is reduced to 15 weeks
C, *G*, *L* is reduced to 17 weeks

Two of these paths still need to be reduced.

By reducing activity *L,* then *A*, *D*, *I*, *L* and *C*, *G*, *L* are both reduced to 16 weeks.

In summary, activities *G, J, K* and *L* have been reduced for a total cost of $4000.

The answer is B.