St Leonard's College

An education for life.

VCE Unit 1 11 General Mathematics

EXAMINATION

Semester 1 2022

Question and Answer Booklet

STUDENT NAME: Solutions

TEACHER(S): Ms Le Mr Rossignolo Mr Toce Ms Yang

TIME ALLOWED:

Reading time 15 minutes

Writing time 90 minutes

D E B C A B
E E D D C D
E D D E A
C
D B

INSTRUCTIONS

A single bound reference and a CAS and scientific calculator permitted.
Answer all questions in the spaces provided.
Round values to 2 decimal places where not specified.
In questions where more than one mark is available, appropriate working must be shown.
Multiple choice questions are worth one mark each.

STRUCTURE OF BOOKLET / MARKING SCHEME

Exam Section	Number of questions to be answered	Total marks
A	20	20
B	6	20
C	2	20
		Total /60

Students are not permitted to bring mobile phones and / or any other unauthorised electronic devices into the examination room.

Section A **Multiple Choice Questions** **20 marks**

Circle the letter corresponding to the correct response.

1. In the geometric sequence $72, 36, 18, 9, \ldots$ the value of the common ratio, $\boldsymbol{r}$ is:

A. 2
B. 4
C. -2
D. 0.5
E. 9

2. The 16^{th} term in the sequence of numbers $121,\ 114,\ 107,\ 100,\ 93, \ldots$ is:

A. -7
B. 7
C. 121
D. 9
E. 16

3. Using the recurrence relation $t_1 = 4, t_{n+1} = 2 \times t_n$, the eighth term would be:

A. 8
B. 32
C. 128
D. 64
E. 512

4. The 5^{th} term, t_5, in the sequence 12, 30, 75, …. is:

A. 150
B. 187.5
C. 468.75
D. 469
E. 1171.875

5. A population of bacteria increases by 10% each hour. If there was an initial population of 1000 bacteria, the recurrence relation for the number of bacteria $\boldsymbol{B_n}$ after $\boldsymbol{n}$ hours is:

A. $B_1 = 1000,\ B_{n+1} = 1.1 \times B_n$
B. $B_1 = 1000,\ B_{n+1} = 0.1 \times B_n$
C. $B_0 = 1000,\ B_{n+1} = 0.1 \times B_n$
D. $B_0 = 1000,\ B_{n+1} = 1.1 \times B_n$
E. $B_1 = 1000,\ B_{n+1} = 1.1 \times B_{n+1}$

6. The histogram below shows the number of people employed per family in a group of families surveyed.

How many families had one person employed?

A. 4
B. 8
C. 12
D. 16
E. 20

Use the following information to answer Questions 7 and 8.

The stem and leaf plot below shows the hourly rate paid to employees who work in the hospitality industry. [Key: 23|6 = \$23.60, n =33]

22							
22	8	9					
23	0	1	2	3	4	4	
23	5	6	6	7	8	9	9
24	0	3	4	4			
24	6	7	8	9			
25	0	1	2				
25	5	7					
26	3	4					
26	6						
27	3	4					
27							

7. For these 33 employees, which one of the following statements is not true:

A. 25% of employees earn \$23.45/hr or less
B. 50% earn \$24.30/hr or more
C. The median hourly rate is \$24.30
D. The range is 5
E. 25% of employees earn above \$25.15/hr

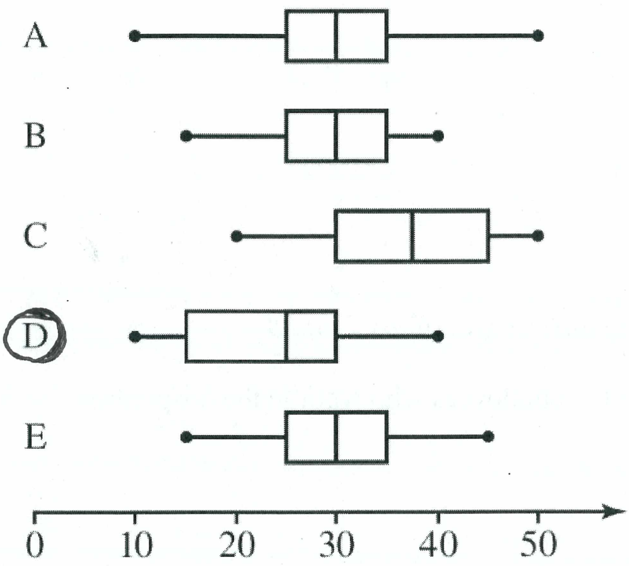
8. The interquartile range for the hourly rate is:

A. \$0.8
B. \$1.70
C. \$4.60
D. \$23.45
E. \$25.15

9. If a data set contains extreme values, which is the most reliable measure of central tendency?

A. Mean
B. Interquartile Range
C. Mode
D. Range
E. Median

10. A distribution has a range of 30, an interquartile range of 15 and a median of 25. Which of the following boxplots could represent this distribution?



11. The value of t in the equation $3t - 11 = 1$ is:

A. 3
B. -11
C. 1
D. 4
E. $-3\frac{1}{3}$

12. If $a = -2$ and $b = 4$, then $5b - 4a$ is equal to:

A. 20
B. 28
C. 8
D. -24
E. 12

13. If we rearrange the expression $A = \frac{h}{2}(a + b)$ to make h the subject of the equation the result is:

A. $h = \frac{A}{2}(a + b)$

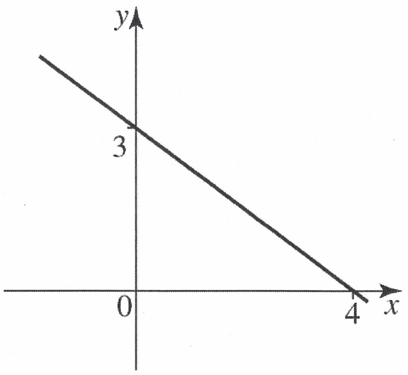
B. $h = 2A - (a + b)$

C. $h = \frac{2A}{(a + b)}$

D. $h = \frac{(a + b)}{2A}$

E. $h = 2A - 2(a + b)$

14. The gradient, m, and y-intercept, c, of the graph below are:



A. $m = \frac{3}{4}, c = 4$

B. $m = -\frac{3}{4}, c = 4$

C. $m = \frac{3}{4}, c = 3$

D. $m = -\frac{3}{4}, c = 3$

E. $m = -\frac{4}{3}, c = 3$

15. The gradient of the line which passes through the points (8, 7) and (2, –5) is:

A. -3

B. -2

C. -1

D. 1

E. 2

16. The solution to the simultaneous equations $y = x + 12$ and $2y + x = 6$ is:

A. $x = -6\,, y = 6$
B. $x = 6\,, y = -6$
C. $x = -12\,, y = 12$
D. $x = 6\,, y = 18$
E. $x = 2\,, y = 14$

17. $\begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -5 \\ 6 & 4 \end{bmatrix}$ is equal to:

A. $\begin{bmatrix} -9 & 1 \\ 3 & 8 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 9 \\ -8 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -9 \\ 8 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 1 \\ -4 & 5 \end{bmatrix}$

E. $\begin{bmatrix} 3 & 8 \\ -9 & 1 \end{bmatrix}$

18. $\begin{bmatrix} a & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & b \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 6 & 3 \end{bmatrix}$

The values of $\boldsymbol{a}$ and $\boldsymbol{b}$ in the above matrix equation are:

A. $a = 1, b = -2$

B. $a = -1, b = -2$

C. $a = -1, b = 2$

D. $a = 1, b = 2$

E. $a = 2, b = 1$

19. If $A = \begin{bmatrix} -2 & 4 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$, then AB is equal to:

A. $\begin{bmatrix} -6 & -6 \\ -8 & -9 \end{bmatrix}$

B. $\begin{bmatrix} 6 & -6 \\ -8 & 9 \end{bmatrix}$

C. $\begin{bmatrix} -8 & 9 \\ 6 & -6 \end{bmatrix}$

D. $\begin{bmatrix} -6 & 6 \\ 9 & -8 \end{bmatrix}$

E. $\begin{bmatrix} 6 & 6 \\ 8 & 9 \end{bmatrix}$

20. Chelsea is a high Court Judge. She charges her clients a fixed charge of \$1000, plus \$550 per hour. The equation that represents the total amount, $\$C$, Chelsea charges for t hours of legal consulting is:

A. $C = 550t$
B. $C = 1000t$
C. $C = 1550t$
D. $C = 1000 + 550t$
E. $C = 450 + 550t$

Section B	**Short Answer Questions**	**20 marks**

Include working throughout.

Recursion and financial modelling (13 marks)

1. State which of the following are Arithmetic or Geometric and state the value of $\boldsymbol{d}$ or $\boldsymbol{r}$

a. 3, 7, 11, 15, ………… $d = 7 - 3 = 4$ (1)

b. 100, −50, 25, −12.5……… $r = \frac{-50}{100} = -\frac{1}{2}$ (1)

2 marks

2. Write down the first five terms of the sequence defined by the recurrence relation

$$t_1 = 3, t_{n+1} = 2t_n - 6$$

Term number, n	1	2	3	4	5
Term value t_n	3	0	−6	−18	−42

(−1) per error

2 marks

3. The following recurrence relation can be used to model the depreciation of a BMW with a purchase price $92 800 and annual depreciation of 15%.

$$V_0 = 92\ 800,\ V_{n+1} = 0.85V_n$$

In the recurrence relation, V_n is the value of the car after n years.

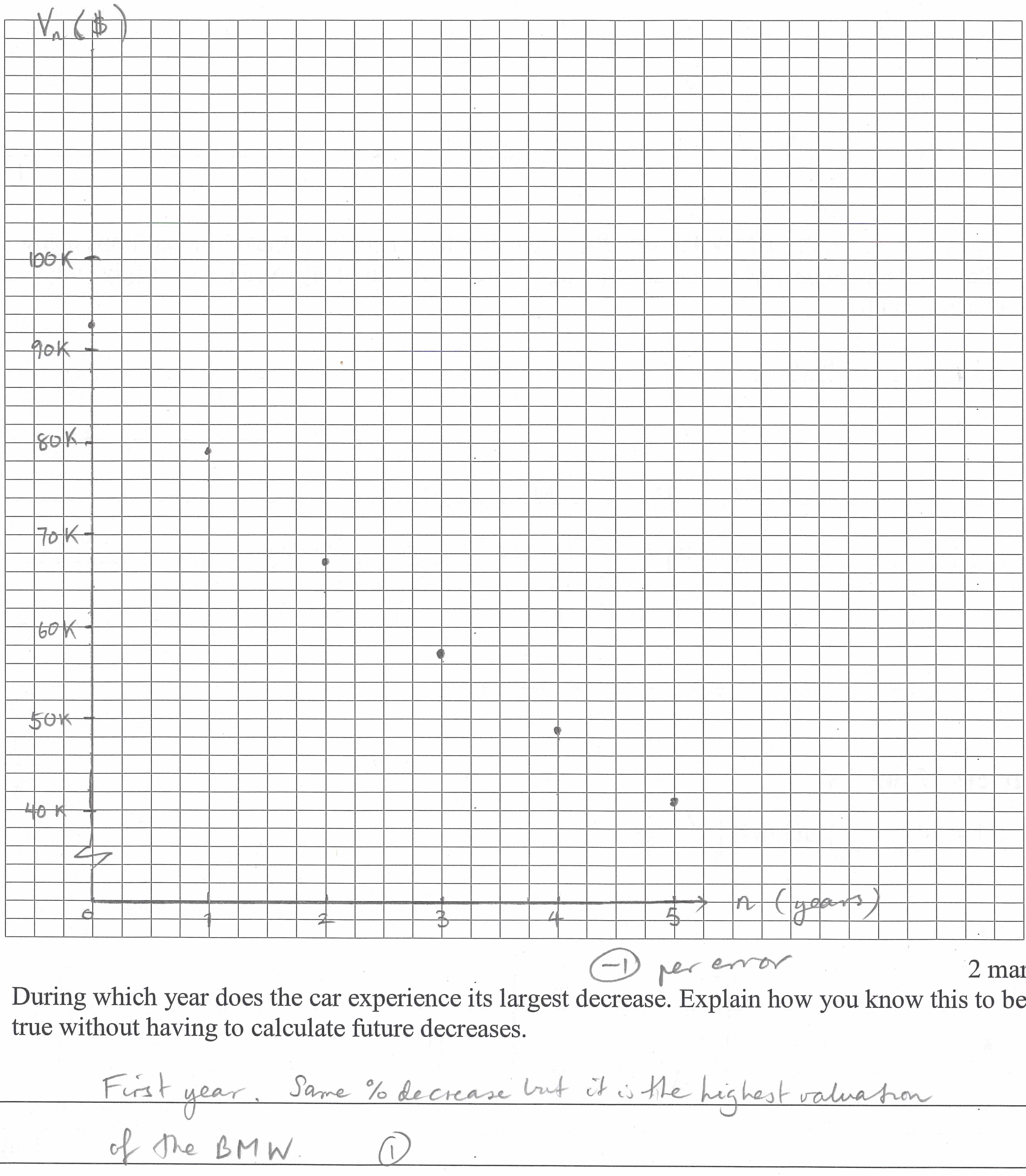
a. **Use the recurrence relation** to find the value of the car over the first 5 years and put the values in the table below.

n (Years)	0	1	2	3	4	5
V_n **(Value)**	92,800	78,8 80	67,048	56990·80	48,442·18	41,175·85

2 marks

(−1) per error

b. Plot the points on the graph below (Place n on the horizontal axis and V_n, the value of the BMW, on the vertical axis. Must use a ruler and an appropriate scale for each axis)).



(−1) per error

2 marks

c. During which year does the car experience its largest decrease. Explain how you know this to be true without having to calculate future decreases.

First year. Same % decrease but it is the highest valuation of the BMW. (1)

1 mark

d. Write a rule for the value, V, of the BMW after n years.

$V_n = V_0 \times 0.85^n$ (1)

1 mark

e. After how many years does the car drop below half its value (give your answer to 2 dp)?

$\frac{92800}{2} = 92800 \times 0.85^n$

$n \simeq 4.26$

after 5 years (accept 4.26 yrs) 1

1 mark

f. Jacqui purchased this BMW January 2018. She decided when the value of the car drops below \$25 000 it would be time to sell and upgrade. When is Jacqui likely to sell her car (give the year and closest month)?

$2500 = 92800 \times 0.85^n$

$n \simeq 8.07$ 1

$0.07 \times 12 = 0.84$ (still Jan)

accept either late Jan 2026

or Feb 2026. 1

2 marks

Matrices (7 marks)

4. Let $A = \begin{bmatrix} 5 & 2 \\ -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 8 \\ 0 & 4 \end{bmatrix}$

Find:

(a) $-5A$

$-5A = \begin{bmatrix} -25 & -10 \\ 15 & 5 \end{bmatrix}$ 1

1 mark

(b) $A^{-1}B$

$A^{-1}B = \begin{bmatrix} -7 & -16 \\ 21 & 44 \end{bmatrix}$ 1

1 mark

5. Given that $-1\begin{bmatrix}3 & 8\\ x & -3\\ 2 & 5\end{bmatrix} + 2\begin{bmatrix}2 & 4\\ -3 & 5\\ 8 & y\end{bmatrix} = \begin{bmatrix}1 & 0\\ 15 & z\\ 14 & 1\end{bmatrix}$, find the values of x, y and z.

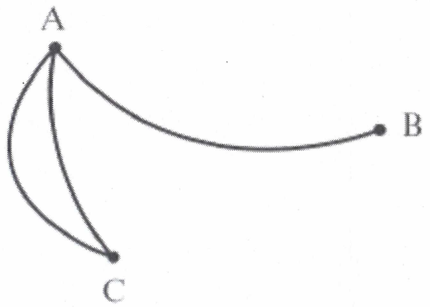
$-x - 6 = 15$ $\qquad$ $-5 + 2y = 1$ $\qquad$ $z = 3 + 10$

$-x = 21$ $\qquad$ $2y = 6$ $\qquad$ $= 13$ 1

$x = -21$ 1 $\qquad$ $y = 3$ 1

3 marks

6. State the matrix that represents the number of ways of travelling directly from one town to another in the diagram below, where **A**, **B** and **C** represent towns.



$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 2 \\ B & 1 & 0 & 0 \\ C & 2 & 0 & 0 \end{array}$$

-1 per error

2 marks

Section C	**Extended Response Questions**	**20 marks**

Include working throughout.

Question 1: Univariate Data (12 marks)

A trout farm was experimenting with two different types of feed for its stock. One group of trout was given feed with a new protein supplement called E43 whilst the other group was given the regular feed. A sample from each batch was collected and their length was measured in centimetres. The results are shown in the following table:

Feed with protein E43. length in cm	Regular feed. length in cm
32	23
26	21
28	21
28	11
28	29
31	29
24	30
24	25
30	19
28	20
29	30
27	27
28	26
30	6
26	22

a. Give the 5 number summaries for each group.

	Feed with protein E43	Regular feed
Min	24	6
Q1	26	20
Median	28	23
Q3	30	29
Max	32	30

(-1) per error

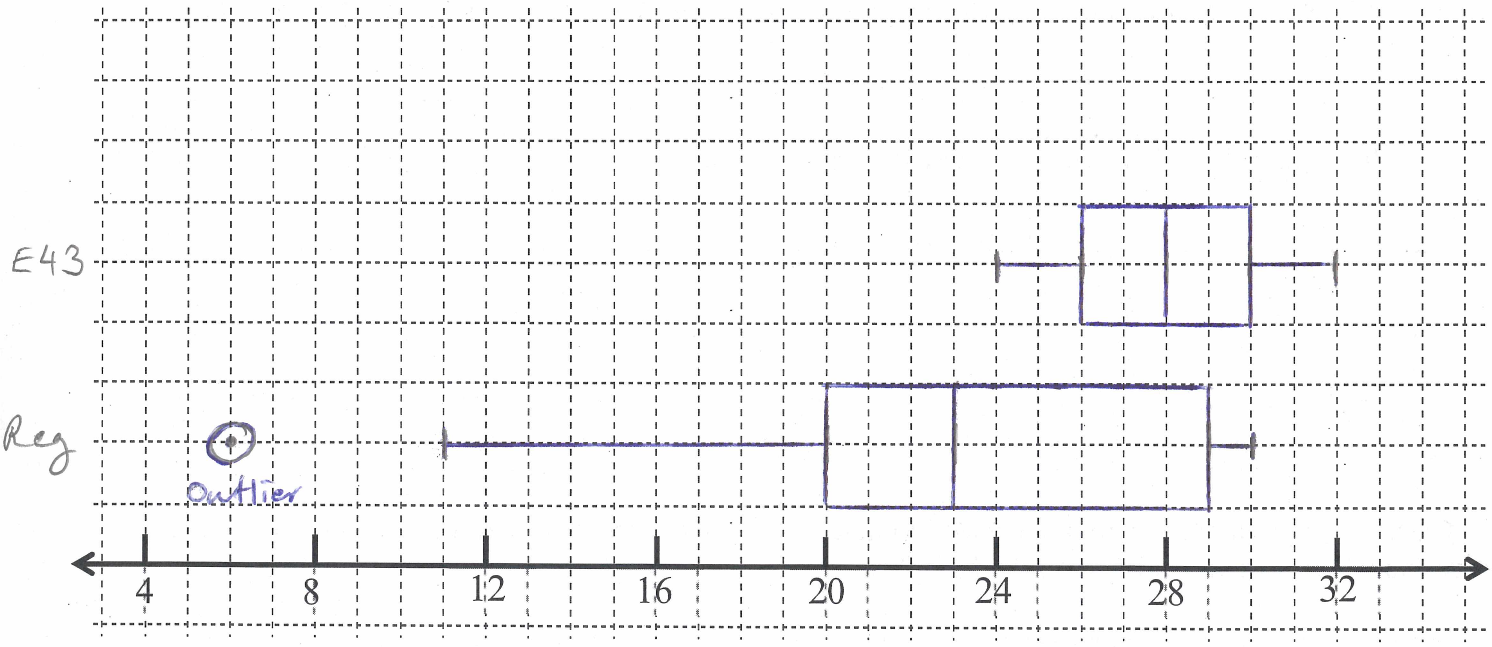
2 marks

b. Perform the relevant calculations to help you identify any outliers. State the value(s) of any outlier and give a possible reason for the occurrence of any outlier in this experiment.

	Lower boundary	Upper boundary
Regular feed	$< 20 - 1.5 \times 9$	$> 29 + 1.5 \times 9$
	< 6.5	> 42.5
	$\Rightarrow$ 6 is an outlier	no outliers
	1	1
E43	$< 26 - 1.5 \times 4$	$> 30 + 1.5 \times 4$
	< 20	> 36
	no outliers	no outliers
	1	1

4 marks

c. Draw boxplots of the scores on **the same scale** below.



4 marks

-1 per error

d. Compare the distributions in terms of ***centre*** and ***spread*** for the lengths of the two different batches of trout.

Median higher for E43

Regular feed has a greater spread (range)

Regular feed has an outlier; E43 doesn't

E43 more consistent, Lower IQR for E43

* others possible

Any 2 will do.

2 marks

Question 2: Linear modelling (8 marks)

Massimo has $12,000 that he wants to invest in a safe way. He decided that the best option would be a term deposit with the Commonwealth Bank. He notices that the value of his investment increases by $800 each year.

a. Set up a linear relation to show the total value, V_1, of Massimo's investment after ***n*** years.

$V_1 = 12000 + 800n$ 1

1 mark

b. What is the value of the investment after 3 years?

$V_1 = 12000 + (800 \times 3)$

$= \$14,400$ 1

1 mark

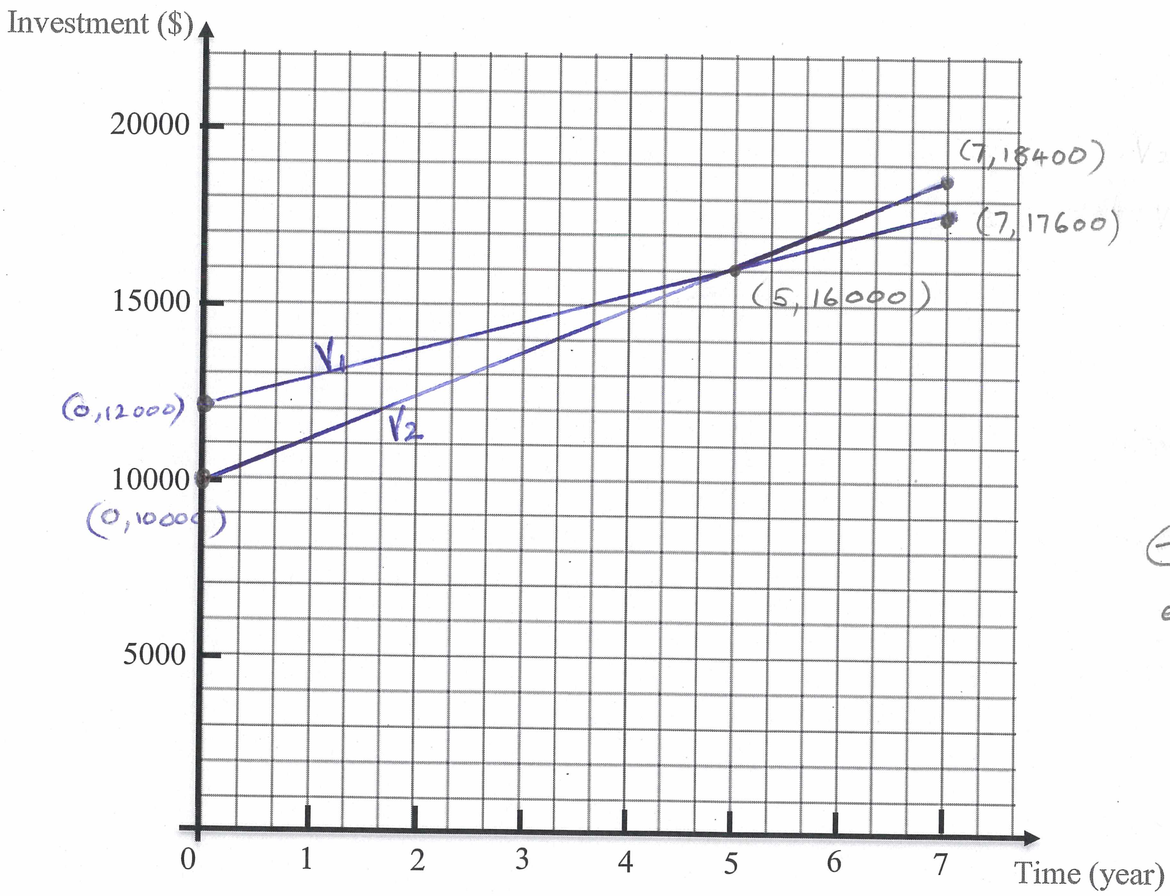
Olivia, who is Massimo's friend, decided to invest her money as well. She has $10,000 to invest and the value of her investment increases by $1200 each year.

c. Set up a linear relation to show the total value, V_2, of Olivia's investment after ***n*** years.

$V_2 = 10,000 + 1200n.$ 1

1 mark

d. Plot the graphs of both investment options over a 7-year period clearly labelling the appropriate graph with V_1 or V_2. Also label the end points for both graphs.



⊖1 per error.

2 marks

e. Using your ***CAS*** (or an alternative method) find the co-ordinates of the point of intersection of the two lines and label the point (include the co-ordinates) on the graph in part ***d***.

(5, 16000) – must show point & co-ordinates on the graph ①

1 mark

f. Interpret the meaning of the point of intersection found in ***e*** and comment on which investment is best on either side of the point of intersection.

< 5 years – Massimo's investment is better ①

same after 5 years

> 5 years – Olivia's investment is better ①

2 marks

END OF EXAMINATION