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Student Name……

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

2022

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 9 short-answer and extended-answer questions from the core.
Section A is compulsory and is worth 36 marks.
Section B begins on page 13 and consists of 4 modules. You should choose 2 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 12 marks.
Section B is worth 24 marks.
There are a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
Formula sheets can be found on pages 31 and 32 of this exam.

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SECTION A - Core

Data analysis

This section is compulsory.

Question 1 (7 marks)

An international car rally is held annually.
The rally involves a number of stages held over a week.
Teams aim to complete each stage as quickly as possible whilst accumulating as many points as possible.
The time taken and points accumulated for stage 1 of the 2019 rally, together with the team number and make of car for 15 of the teams is shown in Table 1 below.

Table 1

Team number	**Make**	**Time** (minutes)	**Points**
46	Toyota	61	135
25	Mercedes	57	119
31	Kia	68	161
6	Alfa Romeo	54	124
17	Honda	59	122
62	Volkswagon	56	134
54	Nissan	64	142
28	Bentley	84	108
71	Chevrolet	75	108
33	Audi	53	153
16	Renault	52	139
85	Buick	92	147
69	BMW	73	170
43	Mazda	66	155
35	Holden	57	136

a. Write down the numerical variables in Table 1. 1 mark

b. How many of the four variables in Table 1 are nominal variables? 1 mark

c. Completing stage 1 of the rally in less than 55 minutes is considered an elite performance.
What percentage of the 15 teams is considered to have produced an elite performance? 1 mark

The ordered stem plot below shows the time taken, in minutes, by each of the 15 teams to complete stage 1.

Key: $5|2 = 52$ $\quad n = 15$
time (minutes)

5	2 3 4 6 7 7 9
6	1 4 6 8
7	3 5
8	4
9	2

d. Determine the median of the distribution. 1 mark

e. Explain why the median is a better measure of the centre of the distribution than the mean. 1 mark

The car rally has been held in its current form over many decades.
The points accumulated by teams for stage 1 of the rally over this time are approximately normally distributed with a mean of 140 and a standard deviation of 12.

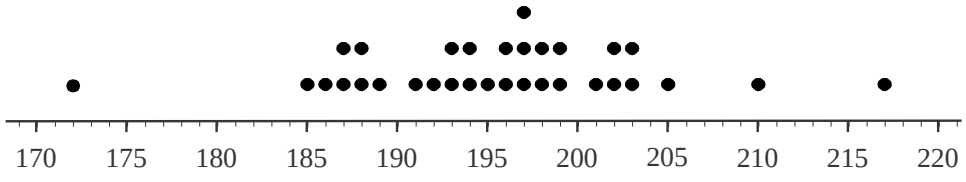
f. In 2021, the Nissan team accumulated 132 points for stage 1 of the rally.
Calculate the standardised score (z) for the Nissan team.
Round your answer to one decimal place. 1 mark

The points accumulated by teams for stage 2 of the rally are also normally distributed but have a different mean and standard deviation to that of stage 1.
For stage 2, 16% of teams accumulated less than 150 points and 0.15% of teams accumulated more than 190 points.

g. Use the $68-95-99.7\%$ rule to find the mean number of points accumulated by teams for stage 2 of the rally. 1 mark

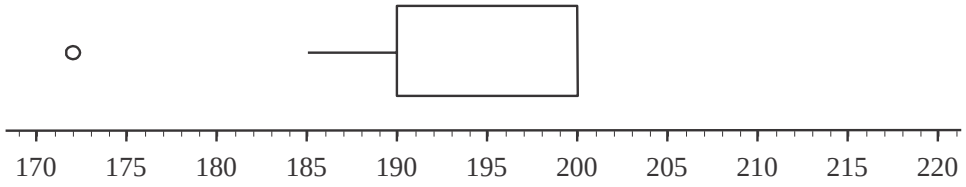
Question 2 (3 marks)

The dot plot below shows the distribution of the time taken, in minutes, to complete stage 5 of the 2018 rally for 32 of the teams.



time (minutes)

An incomplete boxplot displaying this same distribution is shown below.



time (minutes)

a. **i.** Show that the fences for the boxplot are 175 minutes and 215 minutes. 1 mark

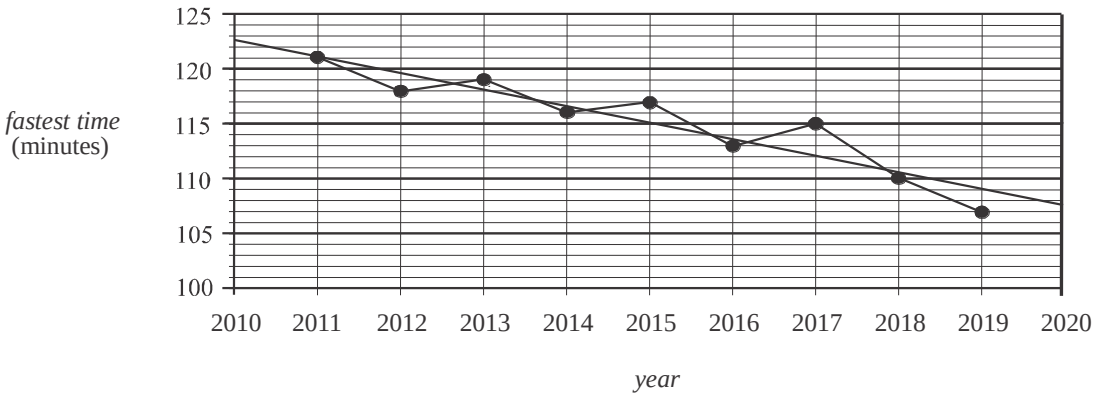
ii. Explain why the data value 217 is an outlier for this distribution. 1 mark

b. Complete the boxplot above, showing any outliers, if appropriate, on the incomplete boxplot above. 1 mark

(Answer on the incomplete boxplot above.)

Question 3 (5 marks)

The time series plot below shows the fastest time, in minutes, recorded for stage 4 of the rally against year for the period 2011-2019.
A least squares line has been fitted to the data.



The equation of the least squares line is $fastest\ time = 3171.19 - 1.5166 \times year$.
The correlation coefficient is -0.9320.

a. Name the response variable in this time series plot. 1 mark

b. Determine the median fastest time, in minutes, for stage 4 of the rally for the period 2011 – 2019. 1 mark

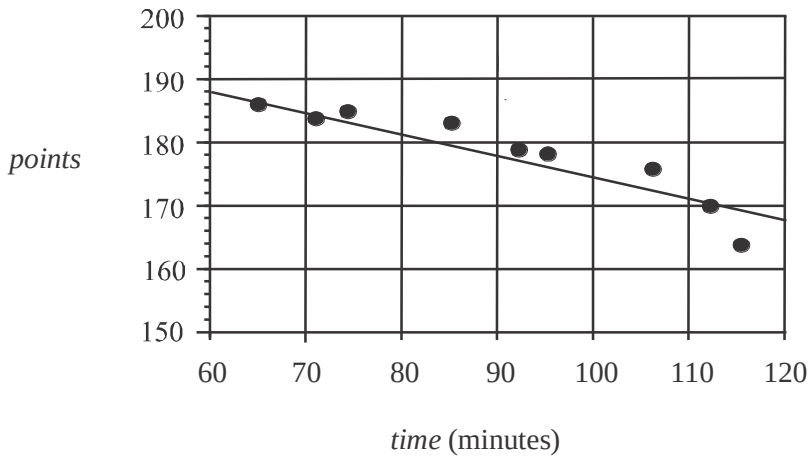
c. Write down the average decrease in fastest time, in minutes per year, for the period 2011 – 2019. 1 mark

d. Write down the percentage of the variation in *fastest time* that is **not** explained by the variation in *year*. 1 mark

e. The fastest time in which a team completed stage four of the rally in 2019 was 107 minutes.
Determine the residual value, in minutes, when the least squares line is used to predict the fastest time in 2019. 1 mark

Question 4 (5 marks)

The scatterplot below shows the *points* accumulated by nine teams during stage seven of the 2015 rally, against the *time*, in minutes, taken to complete it.
As part of an initial analysis, a line has been fitted to the data by eye.

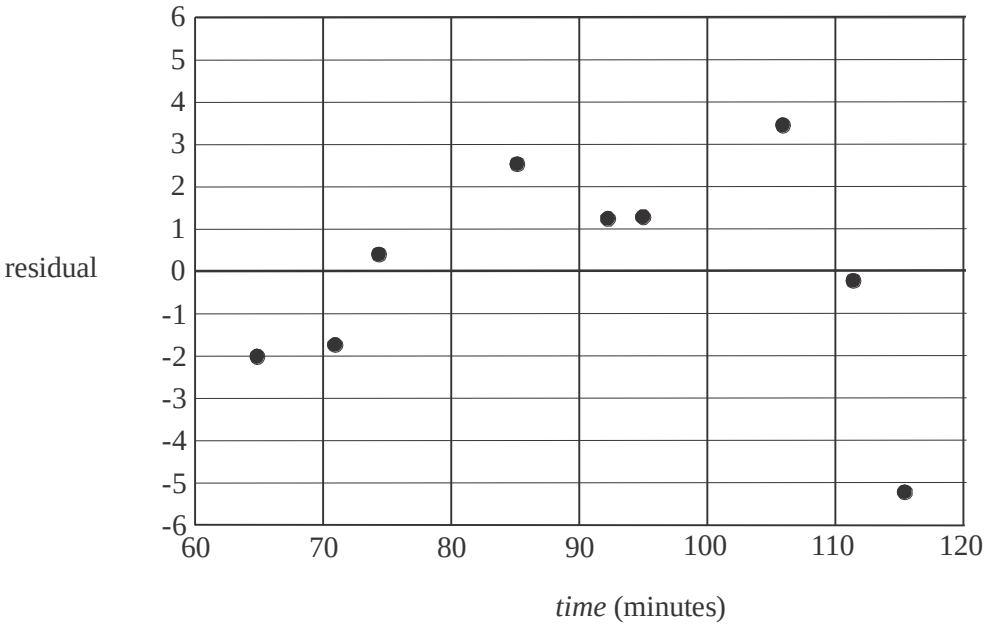


The equation of the line fitted by eye is $points = 208 - 0.33 \times time$.

a. Name the explanatory variable in the scatterplot. 1 mark

b. Describe the association between the variables *points* and *time* in terms of strength and direction. 1 mark

Further analysis of the data involved fitting a least squares line.
The associated residual plot resulting from this is shown below.



c. Explain why the residual plot indicates that the association between *points* and *time* is non-linear. 1 mark

The data used for the scatterplot is shown in Table 2 below.

Table 2

time (minutes)	65	71	74	85	92	95	106	112	115
points	186	184	185	183	179	178	176	170	164

This data can be linearised by applying an appropriate transformation to the variable *points*.

d. Apply an appropriate transformation to the variable *points* and fit a least squares equation to the transformed data.
Write that equation below.
Round the values of the intercept and the slope to one decimal place. 2 marks

Question 5 (4 marks)

At race headquarters there is a museum which showcases the history of the rally.
The number of visitors to this museum in each of the four seasons of 2017 and 2018 is shown in Table 3 below.

Table 3

	Number of Visitors			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2017	416	384	224	256
2018	560	440	276	324

a. Table 4 below shows the seasonal indices for autumn and spring.
Use the values in Table 3 to find the seasonal indices for summer and winter.
Write your answers in Table 4 below. 2 marks

Table 4

	Summer	**Autumn**	**Winter**	**Spring**
Seasonal index		1.15		0.805

b. The number of visitors to the museum in 2019 in each of the four seasons is shown below in Table 5.

Table 5

	Number of Visitors			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2019	648	585	352	415

Use the appropriate seasonal index from Table 4 to deseasonalise the number of visitors to the museum in spring of 2019.
Round your answer to the nearest whole number. 1 mark

c. Use the data in Table 3 to find the four-mean smoothed number of visitors with centring for the winter of 2017. 1 mark

Recursion and financial modelling

Question 6 (4 marks)

Nithin has a reducing balance loan.
The first five lines of the amortisation table for Nithin's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	480 000.00
1	2600.00	1680.00	920.00	479 080.00
2	2600.00	1676.78	923.22	478 156.78
3	2600.00	1673.55	926.45	477 230.33
4	2600.00	-	-	-

Interest is calculated monthly and Nithin makes monthly payments on the loan.

a. How much did Nithin borrow on this loan? 1 mark

b. Show that the interest rate on this loan is 4.2% per annum. 1 mark

c. Calculate the balance of the loan after payment number 4 is made.
Round your answer to the nearest cent. 1 mark

d. Let B_n be the balance of Nithin's loan after *n* months.
Write down a recurrence relation in terms of B_{n+1}, B_n and B_0 that could be used to model the balance of Nithin's loan. 1 mark

Question 7 (3 marks)

Nithin invests a sum of money in a perpetuity from which he will receive a regular monthly payment.
Let V_n be the value of Nithin's perpetuity after *n* months.

The recurrence relation that models the value of this perpetuity over time is

$$V_0 = 120000 \qquad V_{n+1} = 1.004V_n - 480$$

a. What monthly payment does Nithin receive? 1 mark

b. What is the value of the perpetuity after two years? 1 mark

c. Calculate the monthly interest rate percentage for Nithin's perpetuity. 1 mark

Question 8 (3 marks)

Nithin spends $32 000 purchasing equipment for his business.
The value of this equipment will be depreciated by $8 per day of use, using the unit cost method.

The recurrence relation used to model the value of the equipment, E_n, after *n* years is

$$E_0 = 32000 \qquad E_{n+1} = E_n - 2400 .$$

a. Showing recursive calculations, determine the value of the equipment after two years. 1 mark

b. The equipment is used for the same number of days each week for 50 weeks of the year.
For how many days each week is the equipment used? 1 mark

c. The recurrence relation above can also be used to model the value of the equipment from one year to the next if Nithin's equipment was depreciated using flat rate depreciation.
What annual flat percentage rate of depreciation would this represent? 1 mark

Question 9 (2 marks)

Nithin invests $82 000 in an annuity investment which earns interest of 2.8% per annum, compounding quarterly.
For the first five years Nithin makes an additional payment of $3500 each quarter to his investment immediately after the interest is added.
After five years, Nithin increases this regular quarterly payment.
As a result, after a further four years, the balance of Nithin's annuity is $250 000.
What is the value of Nithin's new quarterly payment?
Round your answer to the nearest cent.

SECTION B - Modules

Module 1 - Matrices

If you choose this module all questions must be answered.

Question 1 (4 marks)

Anthony is in charge of merchandise at a sporting club. He places an order with the company that supplies the merchandise for hoodies (*H*), shorts (*S*) and training shirts (*T*).
The number of each item that he orders this year is given in matrix *N* below.

$$N = \begin{bmatrix} 115 \\ 95 \\ 130 \end{bmatrix} \begin{matrix} H \\ S \\ T \end{matrix}$$

The cost of each of the items is given in matrix *C* below.

$$C = \begin{matrix} H & S & T \end{matrix} \begin{bmatrix} 45 & 32 & 24 \end{bmatrix}$$

a. What is the order of matrix *C*? 1 mark

b. Write down a matrix calculation that determines the total cost of Anthony's order. It is not necessary to calculate the total cost. 1 mark

c. The matrix N_{2020} gives the number of hoodies, shorts and training shirts that Anthony ordered in 2020.
The number ordered in 2020 was 20% less than this year and $N_{2020} = k \times N$ where *k* is a scalar. Write down the value of *k.* 1 mark

d. In 2023, Anthony is expecting an increase of 10% in the cost of hoodies, 20% in the cost of shorts and no increase in the cost of training shirts.
The cost matrix for 2023, C_{2023}, is a row matrix and can be determined from the matrix product $C_{2023} = C \times M$ where *M* is a diagonal matrix.
Write down matrix *M.* 1 mark

Question 2 (3 marks)

For the company supplying merchandise to the club, there are five activities in the supply chain.
Those in charge of the five activities are Amanda (*A*), Barry (*B*), Case (*C*), Dimitrov (*D*) and Ely (*E*).
Some of these people have direct communication links with each other and some do not.
The communication matrix *M* below, shows which people can communicate directly with others.

$$M = \begin{array}{c} \\ \text{sender} \\ \\ \end{array} \begin{array}{c} \text{receiver} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ B \\ C \\ D \\ E \end{array}$$

The '0' in row *B*, column *C*, indicates that Barry cannot send information directly to Case.
The '1' in row *A*, column *C*, indicates that Amanda can send information directly to Case.

a. Which two pairs of people can send information directly to each other? 1 mark

b. Case needs to send information to Amanda but it needs to go via Dimitrov.
Write down the sequence of people that this information must be sent to. 1 mark

c. The number of two-step communication links with others is shown in matrix M^2 below. 1 mark

receiver

$$M^2 = \text{sender} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Two pairs of people have two different two-step communication links with each other.
Write down the two-step communication link for one of these pairs that does not involve Dimitrov.

Question 3 (5 marks)

There are three stands, *A*, *B* and *C* at the club's home ground.
Club members can choose to sit in any of these three stands at the club's home games.
The preference for the three stands changes from one home game to the next according to the transition matrix *T* below.

$$T = \begin{array}{c} \text{this home game} \\ \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.8 & 0.05 & 0.05 \\ 0.15 & 0.9 & 0.1 \\ 0.05 & 0.05 & 0.85 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array} \quad \text{next home game}$$

The initial state matrix S_0 shows the number of club members who sat in each of the stands at the end of last season.

$$S_0 = \begin{bmatrix} 2700 \\ 5400 \\ 3900 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

Let S_n represent the state matrix that gives the number of club members expected to sit in each of the stands at the n^{th} home game of this season.

a. How many of the 12 000 club members are expected to sit in the same stand as they had sat in at the end of last season, for the first home game of this season? 1 mark

b. Using the rule $S_{n+1} = T \times S_n$, find how many club members are expected to sit in

i. stand *C* at the second home game of this season? 1 mark

ii. stand *A* in the long term? 1 mark

c. To promote the club, it is proposed that for each home game of this season, a certain number of free tickets will be given away.
People who receive these free tickets can then go to subsequent home games and choose which stand they sit in.
A rule which would model the expected number of club members and free ticket holders, P_n, in each of the stands at the n^{th} home game of this season is given by

$$P_{n+1} = T \times P_n + F,$$

where $T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.8 & 0.05 & 0.05 \\ 0.15 & 0.9 & 0.1 \\ 0.05 & 0.05 & 0.85 \end{bmatrix} \end{array} \begin{array}{l} \text{this home game} \\ \\ \text{next} \\ \text{home,} \\ \text{game} \end{array}$, $P_0 = \begin{bmatrix} 2700 \\ 5400 \\ 3900 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$ and $F = \begin{bmatrix} 75 \\ - \\ 80 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$.

The matrix, F gives the number of free tickets to be given away at each home game for stands A and C. The number of free tickets for stand B has been omitted.

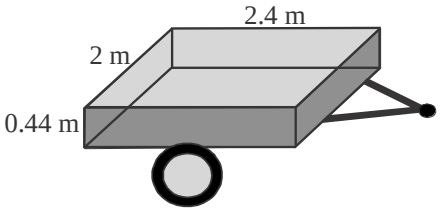
Given that $P_2 = \begin{bmatrix} 2710 \\ 5960 \\ 3730 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$, find the total number of free tickets that the club is proposing to give away at each home game. 2 marks

Module 3 - Geometry and measurement

If you choose this module all questions must be answered.

Question 1 (5 marks)

Tim purchases a box trailer that he can tow behind his car.
The open box part of the trailer is in the shape of a rectangular prism with length 2.4 metres, width 2 metres and height 0.44 metres as shown below.

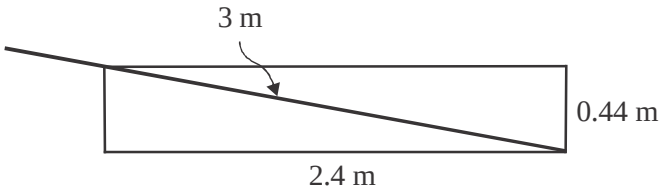


a. Calculate the internal surface area, in square metres, of the box. 1 mark

b. Tim has to transport cylindrical tins of paint of radius 0.1 m and height 0.3 m in his box trailer.
The paint tins must be transported upright and in a single layer.
How many of these tins can fit in Tim's box trailer? 1 mark

c. On another occasion, Tim has to transport 100 of the same sized tins in his box trailer.
A horizontal cover is placed over the box part of the trailer.
Calculate the empty space, in cubic metres, that surrounds the paint tins in the enclosed box. Round your answer to two decimal places. 1 mark

Tim has to transport a piece of straight timber of length 3 metres.
One end is placed at the bottom front edge of the box and the other end extends out the back of the box trailer.
The timber is parallel to the side of the box trailer.
A cross-section of the timber and the box are shown below.

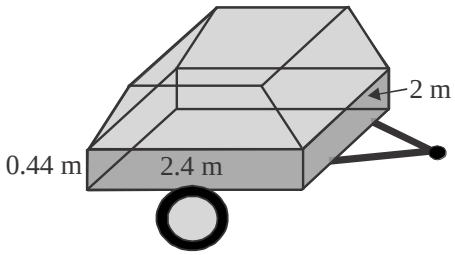


d. What length of timber extends out the back of the box trailer? 1 mark

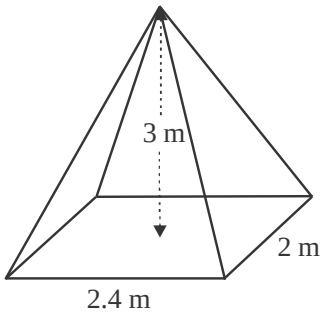
e. How far horizontally does the timber extend beyond the back of the trailer?
Round your answer to two decimal places. 1 mark

Question 2 (2 marks)

Tim wants to make a cover for his trailer in the shape of a truncated (cut-off) rectangular pyramid as shown below.



He starts with a rectangular based pyramid of height 3 m, with a rectangular base with side lengths of 2.4 m and 2 m as shown below.

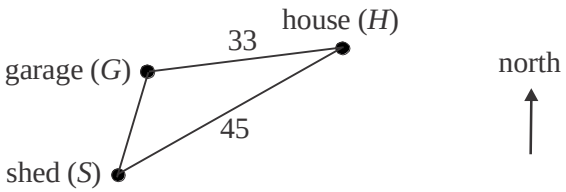


a. Show that the volume of the pyramid is 4.8 cubic metres. 1 mark

b. Tim plans to cut the top off this pyramid. What remains of the pyramid forms his cover.
The part that he will cut off is a pyramid that is similar in shape to the pyramid shown above, and has a height of 2 m.
Find the volume, in cubic metres, of Tim's cover.
Round your answer to two decimal places. 1 mark

Question 3 (2 marks)

Tim stores his box trailer in a shed which is 45 m on a bearing of 242° from Tim's house.
His garage is 33 m from his house on a bearing of 265° as shown in the diagram below.

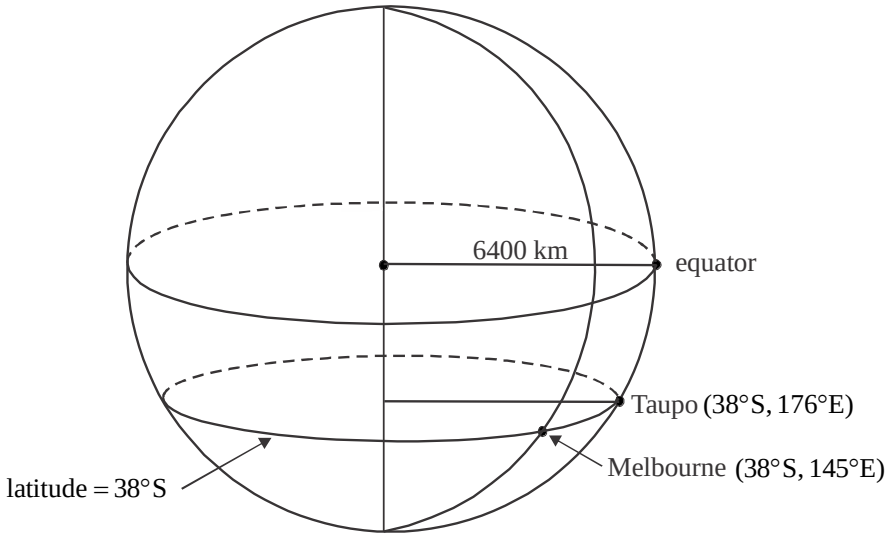


a. What is the distance, in metres, from the garage to the shed?
Round your answer to two decimal places. 1 mark

b. What is the bearing of the garage from the shed?
Round your answer correct to the nearest degree. 1 mark

Question 4 (3 marks)

Tim lives in Melbourne (38°S, 145°E).
He has a cousin who lives in New Zealand in a town called Taupo (38°S, 176°E).
The diagram below shows the locations of Melbourne and Taupo.



a. Find the shortest great circle distance between Taupo and the equator.
Assume the radius of Earth is 6400 km.
Round your answer to the nearest kilometre. 1 mark

b. Calculate the radius of the small circle of Earth at latitude 38°S.
Round your answer to the nearest kilometre. 1 mark

c. Calculate the shortest small circle distance between Melbourne and Taupo.
Round your answer to the nearest kilometre. 1 mark