VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

GREEN STRIPE

FURTHER MATHEMATICS
Written examination 2

Friday 28 May 2021

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 11.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 35 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (8 marks)

Each year, many people participate in a fun run.

The data in Table 1 below was collected for 13 participants who competed in either the 12 km run (women's or men's) or the 6 km run (women's or men's).

The six variables in the table are as follows:

- *number* participant's number
- *name* participant's name
- *event* 1 = 12 km run, 2 = 6 km run
- *gender* F = female, M = male
- *age* age in years
- *time* time taken, in minutes and seconds, to complete the event

Table 1

Number	*Name*	*Event*	*Gender*	*Age*	*Time*
2063	M Jane	1	F	34	41:56
1243	H Roz	2	F	27	26:32
4536	J Nalin	2	M	19	29:05
3429	K Chen	1	M	34	40:58
3657	M French	1	F	56	48:12
987	K Morse	1	M	19	44:48
4897	M Sharif	1	F	29	49:02
356	W Carey	1	M	39	39:51
234	M Chin	1	F	19	55:34
1982	T Khan	1	M	27	46:24
345	R Lu	2	F	46	29:32
2390	N Ghan	2	F	23	28:13
1965	Z Ali	2	M	20	27:12

SECTION A – Question 1 – continued

a. Write down the two numerical variables. 1 mark

b. The variable *number* is a nominal variable.

How many of the other five variables in Table 1 are nominal variables? 1 mark

c. Use the information in Table 1 on page 2 to

i. determine the median *time*, in minutes and seconds, of female participants who completed the 6 km run 1 mark

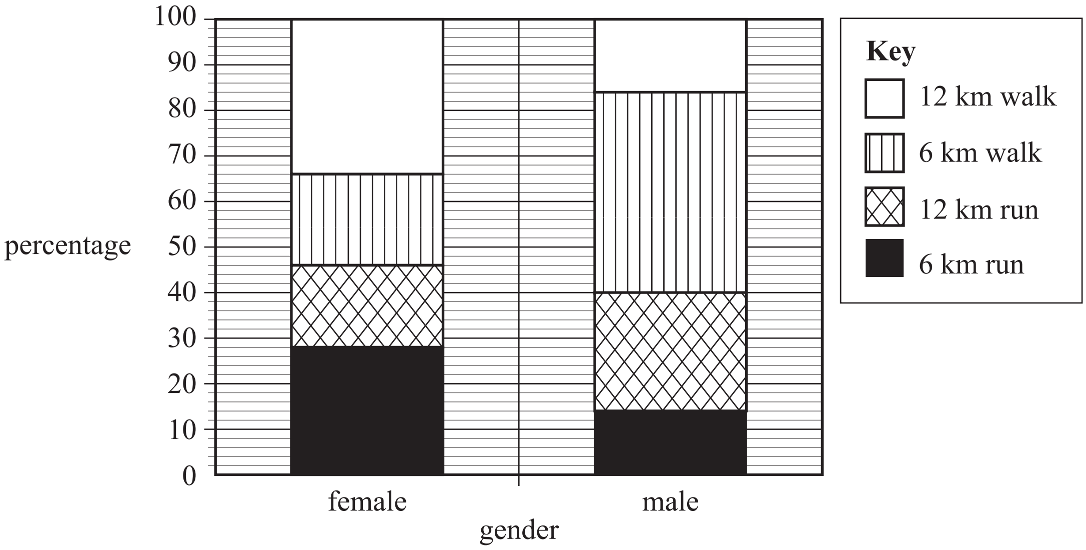
ii. complete the two-way frequency table shown below (Table 2). 2 marks

Table 2

	Gender	
Event	**Female**	**Male**
12 km run		
6 km run		
Total		

d. The fun run included a 12 km walk and a 6 km walk as well.

The percentaged segmented bar chart below shows the percentage of males and females who chose to participate in each of the four events (12 km walk, 6 km walk, 12 km run, 6 km run).

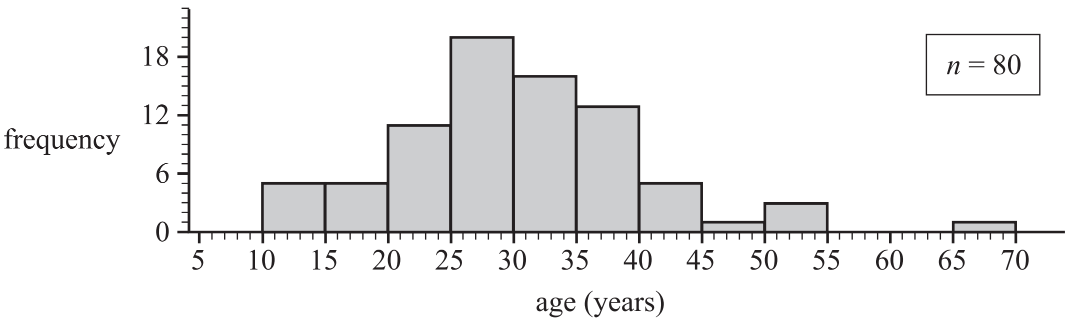


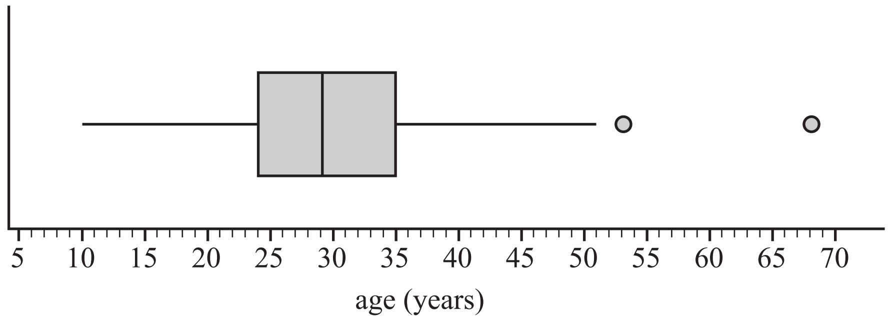
i. What percentage of males participated in a walk? 1 mark

ii. Does the percentaged segmented bar chart support the contention that, for these participants, the event chosen (12 km walk, 6 km walk, 12 km run, 6 km run) is associated with gender? Justify your answer by quoting appropriate percentages. 2 marks

Question 2 (5 marks)

The histogram and the boxplot below display the age distribution of the first 80 participants to finish the women's 12 km run.





Data: City-Bay Fun Run, <https://city-bay.org.au/results/>

a. Describe the shape of the age distribution of these participants, including the number of outliers if appropriate. 1 mark

b. How many of these participants were aged 50 years or older? 1 mark

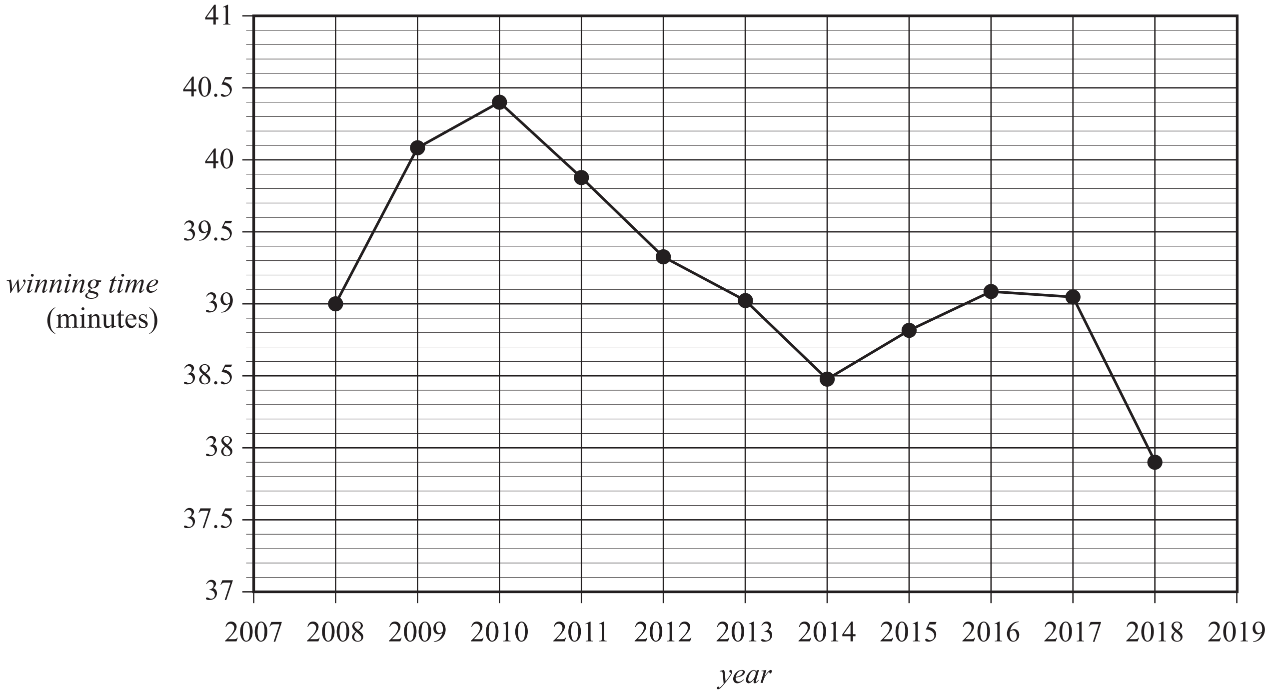
c. Write down the difference in age, in years, between the youngest and oldest of these participants. 1 mark

d. **i.** Show that the fences for the boxplot are 7.5 years and 51.5 years. 1 mark

ii. Use these fence values to explain why the 10-year-olds in this group of participants are **not** shown as an outlier on the boxplot. 1 mark

Question 3 (7 marks)

The time series plot below shows the *winning time*, in minutes, for the women's 12 km run for the period 2008 to 2018.



Data: City-Bay Fun Run, <https://city-bay.org.au/results/>

a. In which of these years was the winning time the largest? 1 mark

b. Use five-median smoothing to smooth the time series plot. Mark each smoothed data point with a cross (✕) on the **time series plot above**. 2 marks

(*Answer on the time series plot above.*)

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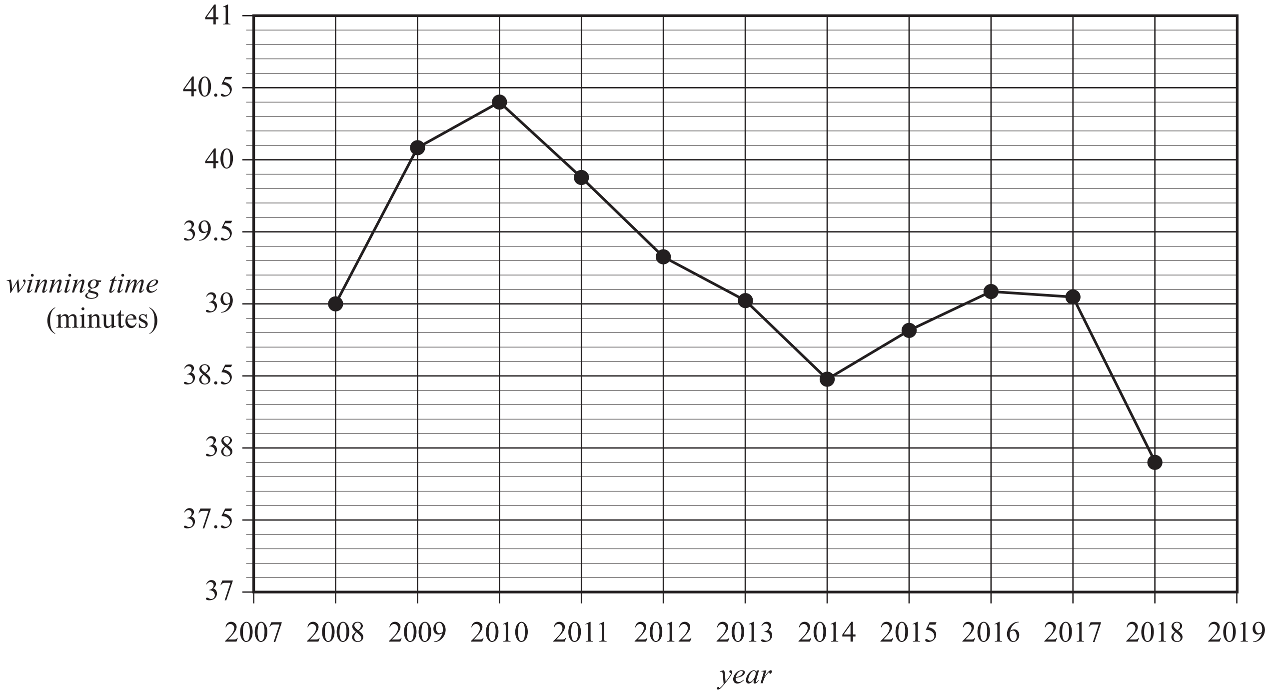
c. With winning times converted to decimal numbers (for example, 39 minutes and 45 seconds = 39.75 minutes) the equation of the least squares line is

$$\textit{winning time} = 340.22 - 0.14955 \times \textit{year}$$

The correlation coefficient is $r = -0.690$

i. Draw this least squares line on the time series plot below. 1 mark



ii. By how many minutes does the least squares line predict that the winning time for the women's 12 km run will decrease each year?

Round your answer to two decimal places. 1 mark

iii. Write down the percentage of the variation in *winning time* that is **not** explained by the variation in *year*. 1 mark

iv. The winning time for the women's 12 km run in 2014 was 38.47 minutes.

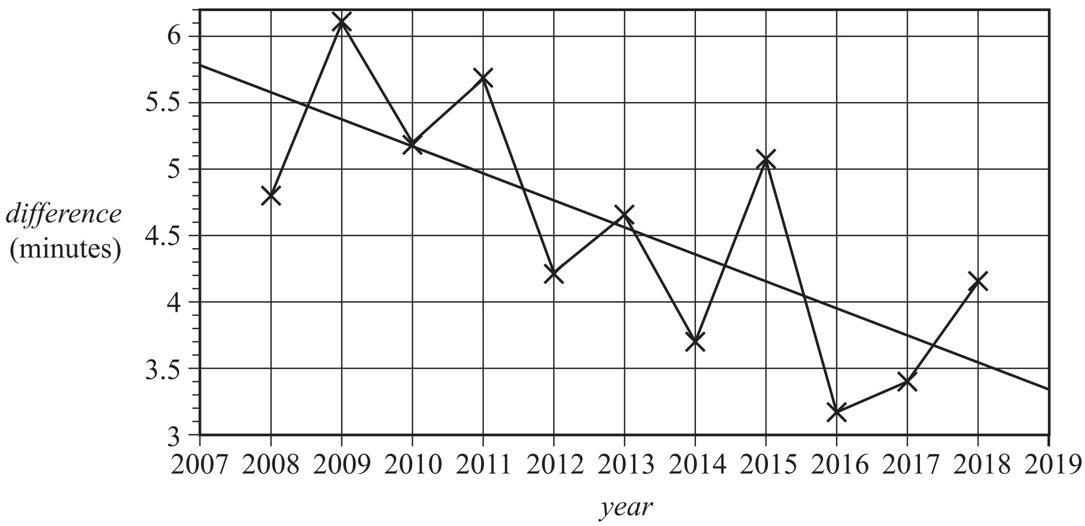
Determine the residual, in minutes, when the least squares line is used to predict the winning time.

Round your answer to two decimal places. 1 mark

Question 4 (4 marks)

During the period 2008 to 2018, the *difference*, in minutes, between the men's winning time and the women's winning time in the 12 km run decreased, as shown in the time series plot below.

Also shown on the time series plot is a least squares line that can be used to model this decreasing trend.



Data: City-Bay Fun Run, <https://city-bay.org.au/results/>

The equation of this least squares line is

$$\textit{difference} = 413.749 - 0.20327 \times \textit{year}$$

a. Determine the predicted *difference*, in minutes, between the men's winning time and the women's winning time in the year 2021.

Round your answer to two decimal places. 1 mark

b. The equation of this least squares line predicts that, sometime in the future, the women's winning time in the 12 km run will be lower than the men's winning time in the 12 km run.

i. In which year is this first predicted to occur? 1 mark

ii. By how many seconds will the predicted women's winning time be lower than the men's predicted winning time in this year.

Round your answer to the nearest second. 1 mark

iii. The equation of this least squares line was calculated using data for the period 2008 to 2018.

What assumption regarding this least squares line is made when the line is used to make predictions for years after 2018? 1 mark

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Recursion and financial modelling

Question 5 (4 marks)

Darrell has a reducing balance loan.

Five lines of the amortisation table for Darrell's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	240 000.00
1	1500.00	800.00	700.00	239 300.00
2	1500.00	797.67	702.33	238 597.67
3	1500.00	795.33	704.67	237 893.00
4	1500.00	P	Q	R

a. What amount did Darrell originally borrow? 1 mark

Interest is calculated monthly and Darrell makes monthly payments.

b. Show that the interest rate for this loan is 4% per annum. 1 mark

c. Write down the values of P, Q and R, rounded to the nearest cent, in the boxes provided below. 2 marks

$P =$ ☐ $Q =$ ☐ $R =$ ☐

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Question 6 (3 marks)

Darrell spent \$120 000 of the money he borrowed on machinery.

The value of the machinery will be depreciated using the unit cost method by \$3.50 per hour of use.

The recurrence relation below can be used to model the value of the machinery, V_n, after n years.

$$V_0 = 120\,000 \qquad V_{n+1} = V_n - 10\,920$$

a. Use recursion to show that the value of the machinery after two years is \$98 160. 1 mark

b. The machinery is used all 52 weeks of the year and for the same number of hours each week.

For how many hours each week is the machinery used? 1 mark

c. The recurrence relation above could also model the year-to-year value of the machinery using flat rate depreciation.

What annual percentage flat rate of depreciation is represented? 1 mark

Question 7 (3 marks)

Darrell takes out a new reducing balance loan of $220 000. The interest rate for the loan is 4.4% per annum, compounding fortnightly.

This loan is to be repaid fortnightly over 10 years.

a. In which of the 10 years will Darrell pay the most interest? 1 mark

The scheduled repayments are $1046.62 per fortnight. However, Darrell finds that he can afford to pay $1200 per fortnight and decides to do so for the duration of the loan.

b. How many of Darrell's repayments will be exactly $1200? 1 mark

c. After five years of repayments, Darrell receives an inheritance of $100 000 and wishes to immediately pay off the remaining balance of the loan.

Will Darrell have enough money to pay off the loan in full?

Justify your answer with a relevant calculation. 1 mark

Question 8 (2 marks)

To fund his retirement Darrell invests $600 000 in a perpetuity.

The perpetuity earns interest at the rate of 3.8% per annum.

Interest is calculated and paid quarterly.

Let V_n be the value of Darrell's investment after n quarters.

Write down a recurrence relation, in terms of V_0, V_{n+1} and V_n, that would model the value of this investment over time.

END OF SECTION A

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SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1 – Matrices

Question 1 (4 marks)

Matrix N lists the number of students enrolled in Year 7, Year 8 and Year 9 at a school.

$$N = \begin{array}{c} \begin{bmatrix} 240 \\ 260 \\ 300 \end{bmatrix} \end{array} \begin{array}{l} \text{Year 7} \\ \text{Year 8} \\ \text{Year 9} \end{array}$$

a. Write down the order of matrix N. 1 mark

Students at these year levels can be awarded a grade of distinction (D), credit (C) or pass (P) at the end of the first semester.

Matrix E lists the proportion of students awarded each grade in each year level for English.

$$E = \begin{array}{c} \begin{array}{ccc} D & C & P \end{array} \\ \begin{bmatrix} 0.25 & 0.55 & 0.20 \end{bmatrix} \end{array}$$

b. Let the matrix $R = N \times E$.

i. Determine matrix R. 1 mark

ii. Explain what the matrix element r_{32} represents. 1 mark

c. The school wants to present a certificate to each student who achieves a distinction (D) in English at the end of the first semester. The printing cost will be $0.25 for each Year 7 certificate, $0.28 for each Year 8 certificate and $0.30 for each Year 9 certificate.

Write down a matrix calculation that determines the total printing cost for distinction (D) certificates. 1 mark

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Question 2 (2 marks)

The year level coordinators for Years 7 to 11 at the school are Amy (A), Brian (B), Cleo (C), David (D) and Ellie (E).

A faulty telephone system means that some of these coordinators **cannot** directly call other coordinators.

The communication matrix, M, below shows which of these coordinators **can** directly call another coordinator.

$$M = \textit{caller} \quad \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \overset{\textit{receiver}}{\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}}$$

The ‘0’ in row D, column A of matrix M indicates that David **cannot** directly call Amy.

The ‘1’ in row D, column C of matrix M indicates that David **can** directly call Cleo.

a. Write the names of the coordinators who can call Brian directly. 1 mark

b. Cleo wants to send a message to Amy using the least number of other coordinators.

Write, in order, the names of the coordinators Cleo must use to send this message to Amy. 1 mark

Question 3 (4 marks)

Students at the school who are studying mathematics in Years 7 to 10 can receive one of three grades at the end of the year: distinction (*D*), credit (*C*) or pass (*P*).

A regular transition matrix *T* that represents how students' grades change from year to year is given below. Four of the values are listed as *j*, *k*, *l* and *m*.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{ccc} D & C & P \end{array} \\ T = & \begin{bmatrix} 0.65 & 0.29 & l \\ j & k & 0.38 \\ 0.03 & 0.09 & m \end{bmatrix} \begin{array}{l} D \\ C \\ P \end{array} \textit{ next year} \end{array}$$

a. Explain the meaning of the value 0.29 in the transition matrix *T*. 1 mark

b. Show that the value of *k* is 0.62 1 mark

c. Let S_0 be the state matrix that shows the number of students achieving D, C or P in Year 7 General Mathematics at the end of 2017.

$$S_0 = \begin{array}{c} \begin{bmatrix} 80 \\ 100 \\ 50 \end{bmatrix} \begin{array}{c} D \\ C \\ P \end{array} \end{array}$$

All of these students completed Year 8 in 2018, with a total of 82 students receiving a distinction.

i. Using this information, determine the value of l in transition matrix T. 1 mark

ii. Students must achieve a distinction in General Mathematics at the end of Year 10 to qualify for Year 11 Advanced Mathematics. All students who completed Year 7 in 2017 also completed Year 10 in 2020.

How many of the Year 7 students from 2017 were predicted to qualify for Year 11 Advanced Mathematics in 2021?

Round your answer to the nearest whole number. 1 mark

Question 4 (2 marks)

A teacher saves examination questions in a file. There are two types of questions: multiple choice (M) and problem solving (P).

Each month she changes some questions from multiple choice to problem solving or from problem solving to multiple choice. She also adds seven new multiple-choice questions and two new problem-solving questions each month.

Let Q_n be the state matrix that shows the number of multiple-choice and problem-solving questions in the file at the end of the nth month.

Let T be the transition matrix that represents how the question types of existing questions are expected to change from month to month.

Let B be the matrix that shows the number of new questions of each type added to the file each month.

The matrix recurrence relation below can be used to predict the expected number of questions in the file at the end of a particular month.

$$Q_{n+1} = TQ_n + B$$

where

$$T = \begin{array}{c} \textit{this month} \\ \begin{array}{cc} M & P \end{array} \\ \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ M \\ P \end{array} \quad \textit{next month} \qquad\qquad B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

The state matrix for the number of questions in the file at the end of the fourth month is $Q_4 = \begin{bmatrix} 38 \\ 14 \end{bmatrix}$.

How many multiple-choice questions were in the file at the end of the second month?

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Module 2 – Networks and decision mathematics

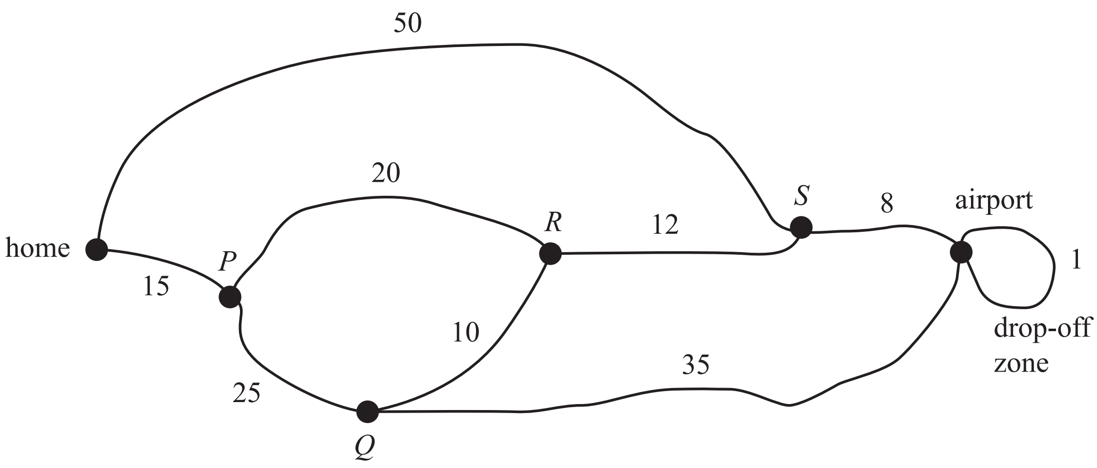
Question 1 (4 marks)

The diagram below shows a network of roads from Leah's home to the airport.

Her home and the airport are labelled as vertices on the network.

The vertices *P*, *Q*, *R* and *S* represent road intersections.

The number on each edge represents the distance, in kilometres, along that section of road.



a. What is the degree of vertex *P*? 1 mark

b. What is the shortest distance, in kilometres, between Leah's home and the airport? 1 mark

c. What is the mathematical name of the edge that forms the drop-off zone? 1 mark

d. An incomplete adjacency matrix for the network on page 20 is shown below.

Write the missing four elements in the spaces provided in the matrix. 1 mark

$$\begin{array}{r c} & \begin{array}{cccccc} \text{home} & P & Q & R & S & \text{airport} \end{array} \\ \begin{array}{r} \text{home} \\ P \\ Q \\ R \\ S \\ \text{airport} \end{array} & \left[\begin{array}{cccccc} \_\_\_ & \_\_\_ & 0 & 0 & 1 & 0 \\ \_\_\_ & 0 & 1 & 1 & 0 & 0 \\ \_\_\_ & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

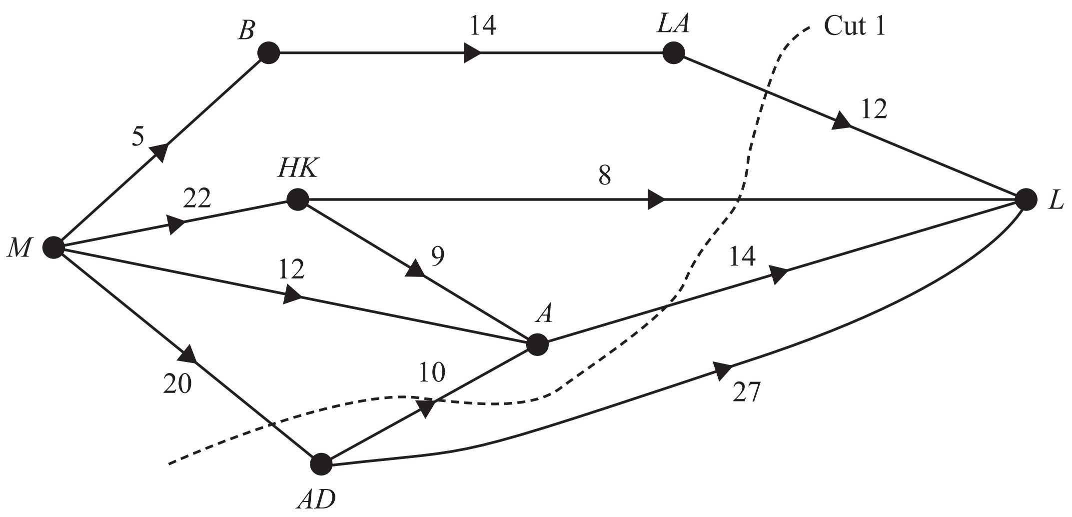
Question 2 (4 marks)

Leah must book a last-minute flight from Melbourne (*M*) to London (*L*).

There are no direct flights. However, Travelsafe Airlines has flights available from Melbourne to London via Brisbane (*B*), Hong Kong (*HK*), Abu Dhabi (*AD*), Amsterdam (*A*) and Los Angeles (*LA*).

The network below shows the maximum number of seats still available on these flights.

A cut, labelled Cut 1, is also shown on the network.



a. How many different flight routes are available from Melbourne (*M*) to London (*L*)? 1 mark

When considering the possible flow through this network, different cuts can be made.

b. What is the capacity of Cut 1? 1 mark

c. What is the maximum number of seats still available to travel from Melbourne (*M*) to London (*L*) on this day? 1 mark

d. Travelsafe Airlines can add eight extra seats to one of its flights in order to allow eight more passengers to travel from Melbourne (*M*) to London (*L*) on this day.

Name **two** cities between which these extra seats could now be available. 1 mark

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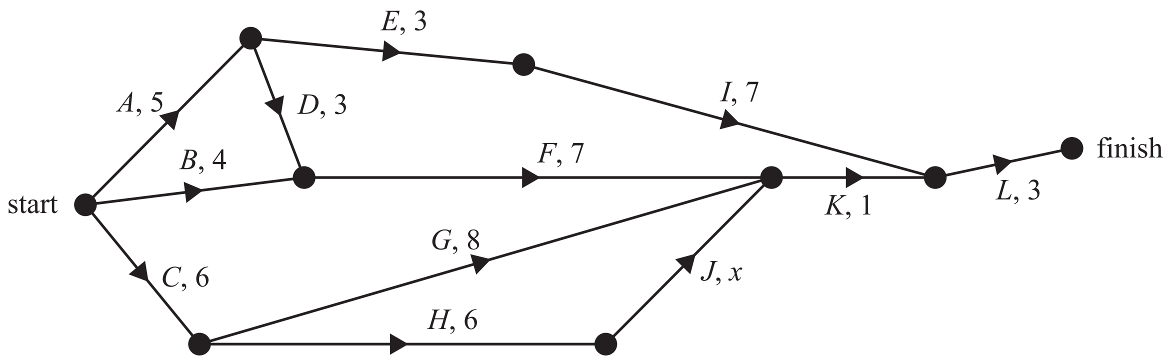
Question 3 (4 marks)

Travelsafe Airlines is planning to renovate a passenger terminal.

This project will involve 12 activities: *A* to *L*.

The directed network below shows these activities and their completion times, in weeks.

The completion time for activity *J* is labelled *x*.



The minimum completion time for this project is 20 weeks.

a. What is the value of *x*? 1 mark

b. Complete the following sentence by filling in the boxes provided. 1 mark

Activity [ ] has the longest float time of [ ] weeks.

c. How many activities could have their completion time increased by two weeks without altering the minimum completion time? 1 mark

d. Travelsafe Airlines has employed more staff to work on the renovation project.

This has ensured that no activity on the network above will have an increase in completion time. However, the completion times of activities *G* and *H* are each reduced by two weeks.

How will this affect the overall completion time of the project? 1 mark

Module 3 – Geometry and measurement

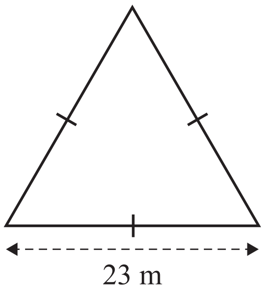
Question 1 (5 marks)

Shona is in charge of decorations for an event.

She wants to hang the decorations from the ceiling.

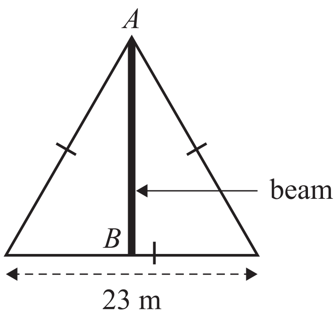
The ceiling is triangular in shape, as shown in the diagram below.

All three sides of the ceiling are 23 m long.



a. What is the perimeter, in metres, of the ceiling? 1 mark

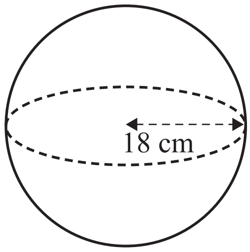
The decorations are to be hung from a beam *AB* that runs across the centre of the ceiling, as shown in the diagram below.



b. Write a calculation that shows that the length of this beam *AB*, rounded to one decimal place, is 19.9 m. 1 mark

c. What is the area, in square metres, of the ceiling?
Round your answer to the nearest whole number. 1 mark

Shona wants to hang spheres from the beam.
Each sphere has a radius of 18 cm, as shown in the diagram below.



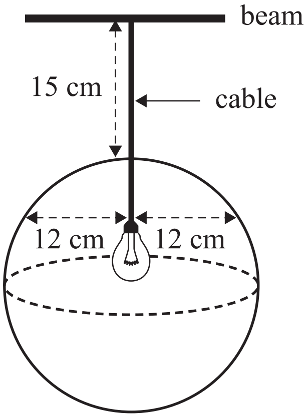
d. What is the volume, in cubic centimetres, of one sphere?
Round your answer to the nearest whole number. 1 mark

e. Each sphere will contain a light globe.

The light globe will be suspended from the beam by a cable.

The lower end of the cable must hang above the centre of the sphere and be 12 cm from each side of the sphere, as shown in the diagram below.

The top of the sphere must be 15 cm from the beam.



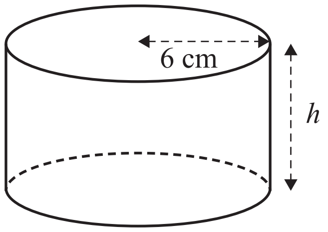
What is the length of the cable, in centimetres, that will be required from the beam to the light globe?

Round your answer to the nearest whole number. 1 mark

Question 2 (4 marks)

Shona will place cylindrical bowls on each table at the event as a centrepiece.

Each cylindrical bowl has a radius of 6 cm, as shown in the diagram below.



Each bowl has a volume of 1244 cm^3.

a. Write a calculation that shows that the height, *h*, of one cylindrical bowl, rounded to the nearest whole number, is 11 cm. 1 mark

b. A candle, also in the shape of a cylinder, is to be placed upright inside each bowl so that it touches the base of the bowl.

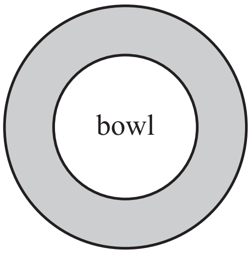
The candle has a radius of 3 cm and a height of 18 cm.

Once the candle has been placed inside the bowl, the remaining volume of the bowl will be filled with sand.

What volume of sand, in cubic centimetres, is required to fill the cylindrical bowl once the candle is placed inside it?

Round your answer to the nearest whole number. 1 mark

c. Each bowl is to be placed on a circular plate.
Below is a diagram of a bowl on top of the plate, as seen from above.
The area of the plate that is not covered by the bowl is shaded.



The ratio of the area of the base of the bowl to the shaded area is 4:5

What is the area, in square centimetres, of one plate?
Round your answer to one decimal place. 2 marks

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Question 3 (3 marks)

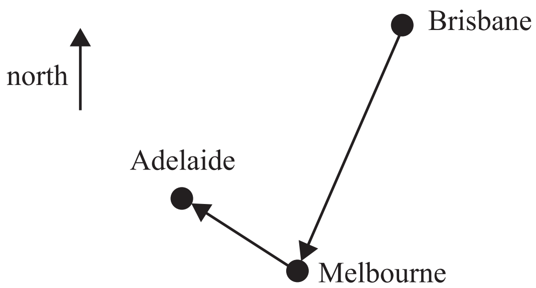
Isha will fly from Brisbane to Adelaide for the event.

She can take a direct flight from Brisbane to Adelaide.

Brisbane is 918 km north and 1310 km east of Adelaide.

a. Show, with calculations, that the bearing of Adelaide from Brisbane, rounded to the nearest degree, is 235°. 1 mark

b. Isha could also take a flight from Brisbane to Adelaide via Melbourne, as shown on the diagram below.



Melbourne is 1370 km from Brisbane on a bearing of 211°.

What is the distance, in kilometres, between Melbourne and Adelaide?

Round your answer to the nearest whole number. 2 marks

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Module 4 – Graphs and relations

Question 1 (4 marks)

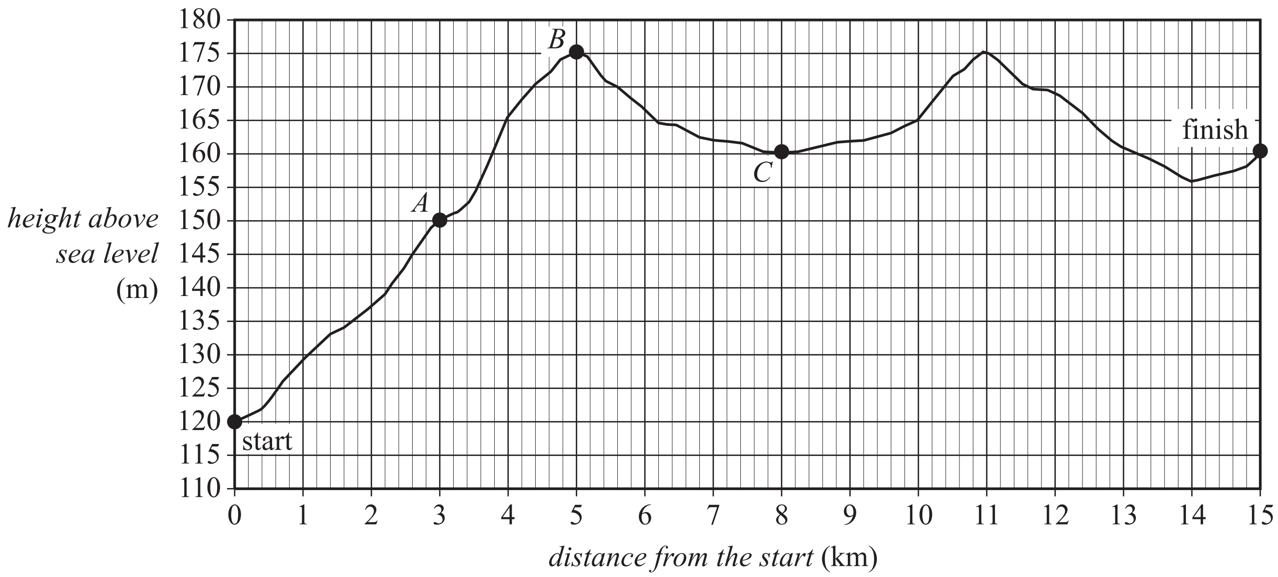
In an annual event, athletes compete against a vintage train in a 15 km race.

A road runs alongside a railway track for the entire length of the race.

There are three rail crossings, at points *A*, *B* and *C*.

Graph 1 below shows the *height above sea level*, in metres, along this road and the railway track.

Graph 1



a. What is the difference in height, in metres, between the starting point of the race and rail crossing *B*? 1 mark

b. Between the start of the race and the finish, what length of road, in kilometres, is downhill? 1 mark

c. Consider the section of road between rail crossings *A* and *B*.

i. Write a calculation that shows that the average slope of this section of road is 12.5 1 mark

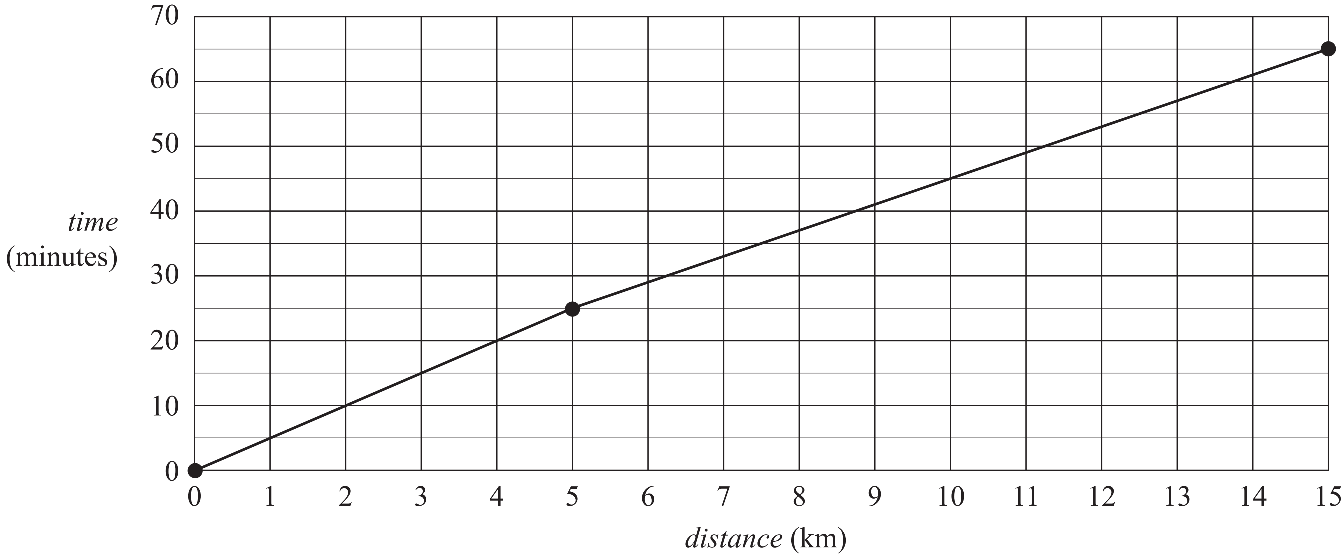
ii. What is the unit in which this average slope of 12.5 is measured? 1 mark

Question 2 (5 marks)

On race day, Jasmine started the race at the same time as the train.

The graph showing the time taken by Jasmine to complete the race is shown on Graph 2 below.

Graph 2



The train runs at a constant speed between the start and point *B* (point *B* is on Graph 1 on page 31) and then also between point *B* and the finish.

The table below shows the times at which the train is at the start, point *B* and the finish of the race.

Location	**Distance travelled (km)**	**Time (minutes)**
start	0	0
point *B*	5	35
finish	15	60

a. On **Graph 2 above**, add the graph that shows the train's time in this race. 1 mark

(*Answer on Graph 2 above.*)

DO NOT WRITE IN THIS AREA

b. The time taken by the train, t minutes, to travel d kilometres is given by

$$t = \begin{cases} 7d & 0 \le d \le 5 \\ ad + b & 5 < d \le 15 \end{cases}$$

i. Using only the numbers given in the table on page 32, write a calculation that shows that $a = 2.5$ 1 mark

ii. Using $a = 2.5$ and the numbers given in the table on page 32, write a calculation that shows that $b = 22.5$ 1 mark

The equation for Jasmine's expected race completion time is

$$t = \begin{cases} 5d & 0 \le d \le 5 \\ 4d + 5 & 5 < d \le 15 \end{cases}$$

c. In kilometres per hour, what is Jasmine's speed for the first 5 km of the race? 1 mark

d. How long after the start of the race does it take for the train to catch up to Jasmine?
Write your answer in minutes and seconds. 1 mark

Question 3 (3 marks)

The organising committee must budget for the race so that it does not make a loss.

Let x be the cost of running the train on race day.

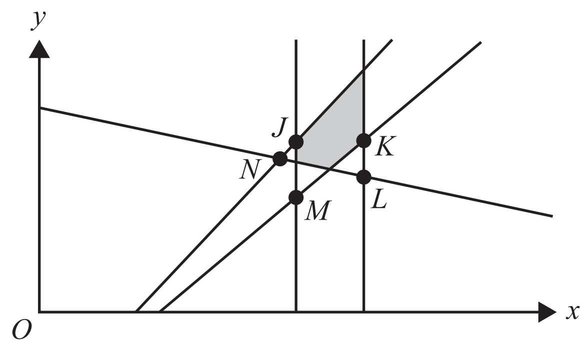
Let y be the cost of other race day operations.

The graph below shows the shaded feasible region for inequalities that constrain the values of x and y.

The feasible region includes the five boundaries shown.

Four of the five inequalities that define the feasible region are:

- Inequality 1 $y \geq 4x$
- Inequality 2 $y \leq 5x$
- Inequality 3 $x \geq 8000$
- Inequality 4 $x + y \geq 45\,000$



The coordinates of the five points shown on the graph are given in the table below.

J	K	L	M	N
(8000, 40 000)	(10 000, 40 000)	(10 000, 35 000)	(8000, 32 000)	(7500, 37 500)

a. Inequality 5 completes the definition of the feasible region.

Write down Inequality 5. 1 mark

An additional 5% of the cost of other race day operations (y) is added as a cost for promotions.

The total cost, $\$C$, of organising and running the race is given by $C = x + 1.05y$.

b. Determine the minimum total cost. 1 mark

DO NOT WRITE IN THIS AREA

c. The race organising committee had a revenue of $75 000 from sponsorships and entry fees for the race.

All costs will be paid from this revenue.

Determine the minimum profit that the organising committee can make from this race. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET