St Leonard's College

An education for life.

VCE Unit 1 11 General Mathematics

EXAMINATION

Semester 1 2021

Question and Answer Booklet

STUDENT NAME: Solutions

TEACHER(S): Ms Daniels Mr Rossignolo Mr Toce Ms Yang

TIME ALLOWED:

Reading time 15 minutes

Writing time 90 minutes

INSTRUCTIONS

A single bound reference and a CAS and scientific calculator permitted.
Answer all questions in the spaces provided.
Round values to 2 decimal places where not specified.
In questions where more than one mark is available, appropriate working must be shown.
Multiple choice questions are worth one mark each.

STRUCTURE OF BOOKLET / MARKING SCHEME

Exam Section	Number of questions to be answered	Total marks
A	20	20
B	6	30
C	1	10

Students are not permitted to bring mobile phones and / or any other unauthorised electronic devices into the examination room.

Section A **Multiple Choice Questions** **20 marks**

Circle the letter corresponding to the correct response.

1. In the arithmetic sequence $27, 20, 13, 6, \ldots$ the value of the common difference, d is:

A. -21
B. 21
C. -7
D. 7
E. -14

2. The 17th term in the sequence of numbers $2.5,\ 5,\ 7.5, 10, \ldots$ is:

A. 40
B. 42.5
C. 45
D. 20
E. 80

3. Using the recurrence relation $t_1 = -1, t_{n+1} = 3 \times t_n$, the sixth term would be:

A. 14
B. 243
C. -81
D. -243
E. -729

4. The 9th term, t_9 in the sequence 1024, 512, 256, … is:

A. 4
B. 8
C. 16
D. 2
E. 1

5. A colony of frogs increases by 15% each year. If there were originally 500 frogs in the colony, the recurrence relation for the number of frogs F_n after n years is:

A. $F_1 = 500,\ F_{n+1} = 1.15 \times F_n$
B. $F_1 = 500,\ F_{n+1} = 0.15 \times F_n$
C. $F_0 = 500,\ F_{n+1} = 0.15 \times F_n$
D. $F_0 = 500,\ F_{n+1} = 1.15 \times F_n$
E. $F_1 = 500,\ F_{n+1} = 1.15 \times F_{n+1}$

Use the following information to answer Questions 6 and 7.

The stem and leaf plot below displays the distribution of *beak length*, in millimetres, of a sample of 33 female birds of the same species.

key: 23 | 9 = 23.9 $n = 33$

Stem	Leaf
22	
22	8 9
23	0 1 2 3 4 4
23	5 6 6 7 8 9 9
24	0 3 4 4
24	6 7 8 9
25	0 1 2
25	5 7
26	3 4
26	6
27	3 4
27	

6. For these 33 female birds, the median *beak length* is:

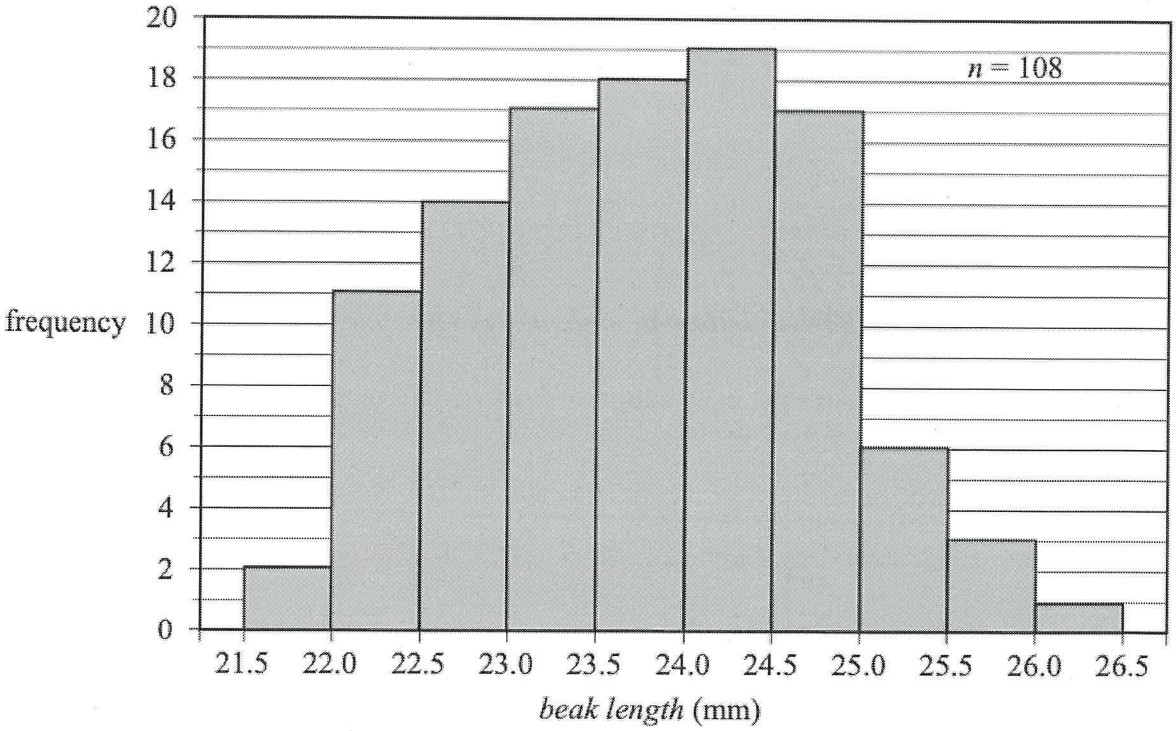
A. 24.0 mm
B. 24.1 mm
C. 24.3 mm
D. 24.6 mm
E. 25.0 mm

7. The percentage of these 33 female birds with a beak length of less than 25.4 mm is closest to:

A. 21.2%
B. 25.4%
C. 27.0%
D. 75.8%
E. 78.8%

Use the following information to answer Questions 8

The histogram below shows the distribution of *beak length*, in millimetres, of a sample of 108 birds of the same species. Both male and female birds are included in this sample.



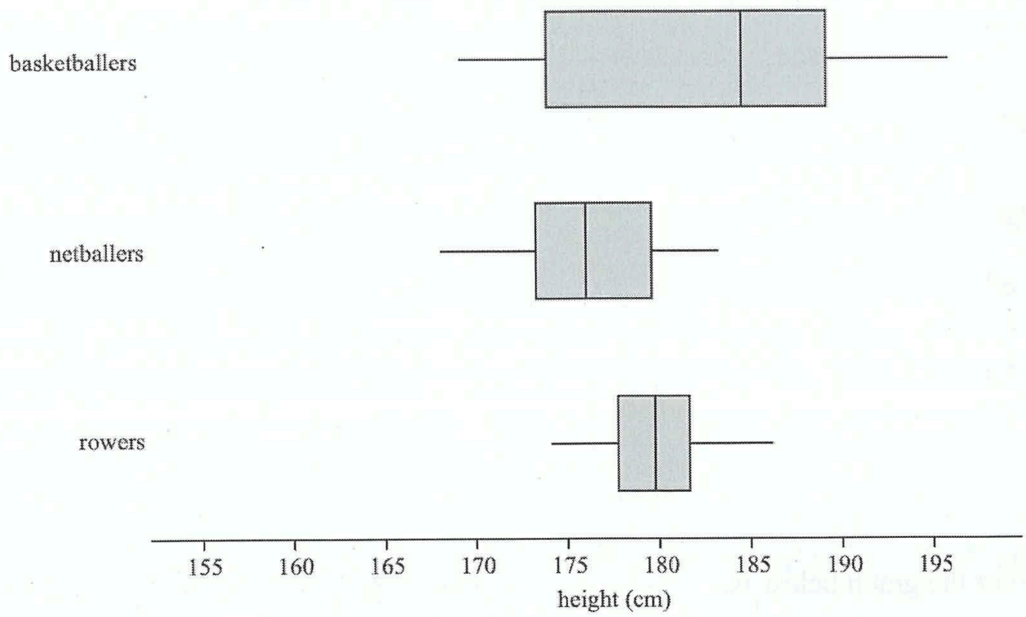
8. The beak length for this sample of 108 birds is most frequently:

A. greater than or equal to 22.5 mm and less than 23.0 mm
B. greater than or equal to 23.0 mm and less than 23.5 mm
C. greater than or equal to 23.5 mm and less than 24.0 mm
D. greater than or equal to 24.0 mm and less than 24.5 mm
E. greater than or equal to 24.5 mm and less than 25.0 mm

9. The *variables height* (less than 1.83 m, 1.83 m and over) and *enthusiasm for playing basketball* (low, medium, high) are:

A. both ordinal variables
B. both nominal variables
C. a nominal and an ordinal variable respectively
D. an ordinal and a nominal variable respectively
E. a numerical and an ordinal variable respectively

10. The parallel boxplots below display the distribution of height for three groups of athletes: rowers, netballers and basketballers:



Which one of the following statements is **NOT** true?

A. The shortest athlete is a netballer

B. The rowers have the least variable height.

C. More than 25% of the netballers are shorter than all rowers.

D. The basketballers are the tallest athletes in terms of median height.

E. More than 50% of the basketballers are taller than any of the rowers or netballers.

11. The value of x in the equation $2x - 5 = 1$ is:

A. -3
B. 3
C. 2
D. -2
E. $\frac{1}{2}$

12. If $x = 3$ and $y = -2$, then $2x - 4y$ is equal to:

A. -2
B. -16
C. -8
D. 14
E. 8

13. The expression $c = \frac{5}{9}(f - 32)$ can be arranged for f as:

A. $f = \dfrac{9c}{5} - 32$

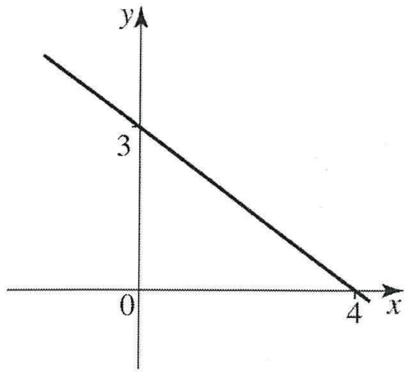
B. $c = \dfrac{9f}{5} + 32$

C. $f = \dfrac{9c}{5} + 32$

D. $c = \dfrac{9f}{5} - \dfrac{160}{9}$

E. $f = \dfrac{9c}{5} - \dfrac{160}{9}$

14. The equation of the graph below is:



A. $y = -\dfrac{3}{4}x + 3$

B. $y = -\dfrac{3}{4}x - 3$

C. $y = -\dfrac{3}{4}x + 4$

D. $y = -\dfrac{3}{4}x - 4$

E. $y = \dfrac{3}{4}x + 3$

15. Which one of the following has the same gradient as $y = 6 - 5x$?

A. $2y - 5x = 0$

B. $y - 5x = 3$

C. $y + 5x = 10$

D. $y = 5x - 1$

E. None of the above

16. The equation of the line which passes through $(-3, 5)$ and $(-1, 15)$:

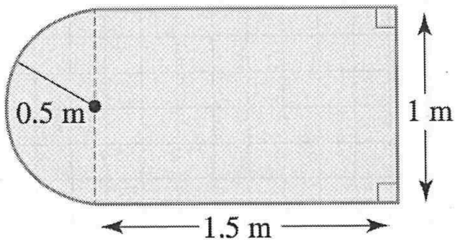
A. $y = -\frac{5}{2}x - \frac{5}{2}$
B. $y = -5x - 10$
C. $y = -5x + 10$
D. $y = 5x + 20$
E. $y = 5x - 20$

17. The circumference of a circle with diameter of 8 mm is closest to:

A. 12.57 mm
B. 25.13 mm
C. 50.27 mm
D. 201.06 mm
E. 6.28 mm

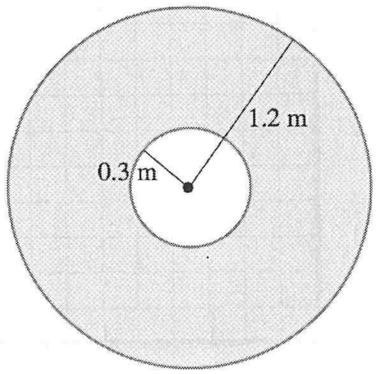
18. The perimeter of the object shown is closest to:

A. 10.28 m
B. 1.89 m
C. 7.14 m
D. 6.57 m
E. 5.57 m



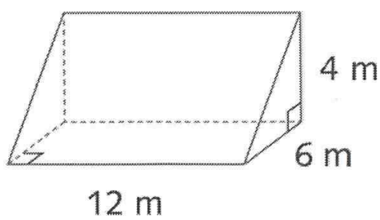
19. The shaded area shown in the diagram is closest to:

A. $0.28\ m^2$
B. $4.52\ m^2$
C. $4.24\ m^2$
D. $5.65\ m^2$
E. $2.83\ m^2$



20. The volume of the given solid is closest to:

A. $144\ m^3$
B. $288\ m^3$
C. $72\ m^3$
D. $36\ m^3$
E. None of the above



Include working throughout.

Recursion and financial modelling (16 marks)

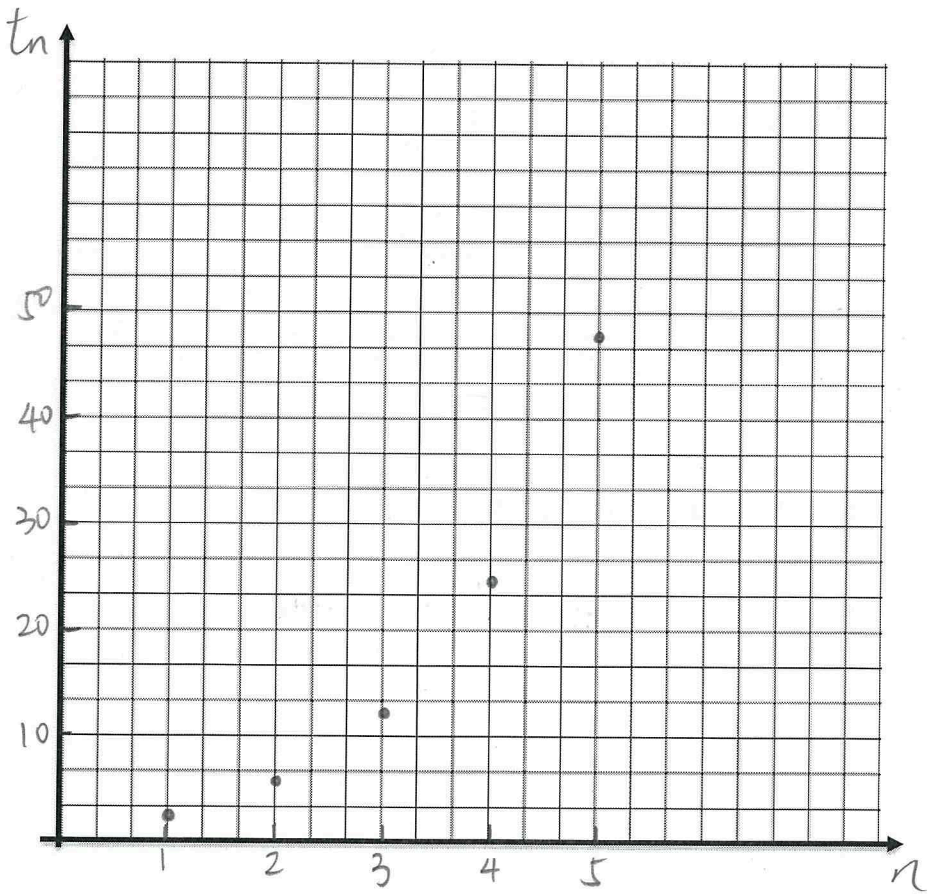
1. (a) Write down the first five terms of the sequence defined by the recurrence relation

$$t_1 = 3, t_{n+1} = 2t_n$$

Term number, n	1	2	3	4	5
Term value t_n	3	6	12	24	48

1 mark

(b) Use the table to plot the graph.



2 marks

2. Alex is a mobile mechanic.

He uses a van to travel to his customers to repair their cars.

The value of Alex's van is depreciated using the flat rate method of depreciation.

The value of the van, in dollars, after n years, V_n can be modelled by the recurrence relation shown below.

$$V_0 = 75\,000, \qquad V_{n+1} = V_n - 3375$$

(a) Recursion can be used to calculate the value of the van after two years.

Complete the calculations below by writing the appropriate numbers in the boxes provided.

$V_0 =$	75 000					
$V_1 =$	75 000	$-$	3375	$=$	71 625	
$V_2 =$	71625	$-$	3375	$=$	68 250	

2 marks

(b) By how many dollars is the value of the van depreciated each year?

$3375

1 mark

(c) Calculate the flat rate of depreciation in the value of the van in the first year. Write your answer as a percentage.

$\frac{3375}{75\,000} \times 100\% = 4.5\%$

1 mark

(d) The value of Alex's van could also be depreciated using the reducing balance method of depreciation. The value of the van, in dollars, after n years, R_n, can be modelled by the recurrence relation shown below.

$$V_0 = 75\,000, \qquad V_{n+1} = 0.943V_n$$

At what annual percentage rate is the value of the van depreciating each year?

$1 - 0.943 = 5.7\%$

1 mark

3. The snooker table at a community centre was purchased for $3000.

After purchase, the value of the snooker table was depreciated using the flat rate method of depreciation.

The value of the snooker table, V_n, after n years, can be determined using the recurrence relation below.

$$V_0 = 3000, \qquad V_{n+1} = V_n - 180$$

(a) Use recursion to show that the value of the snooker table after two years, V_2 is $2640.

$V_1 = 3000 - 180 = 2820$

$V_2 = 2820 - 180 = 2640$

2 marks

(b) After how many years will the value of the snooker table first fall below $2000?

$V_3 = 2460$ $V_4 = 2280$

$V_5 = 2100$ $V_6 = 1920$

$\therefore$ After 6 years

2 marks

(c) The value of the snooker table could also be depreciated using the reducing balance method of depreciation.

After one year, the value of the snooker table is $2760.
After two years, the value of the snooker table is $2539.20

i. Show that the annual rate of deprecation in the value of the snooker table is 8%.

$2760 - 2539.2 = 220.8$

$\frac{220.8}{2760} \times 100\% = 8\%$

2 marks

ii. Let S_n be the value of the snooker table after n years.

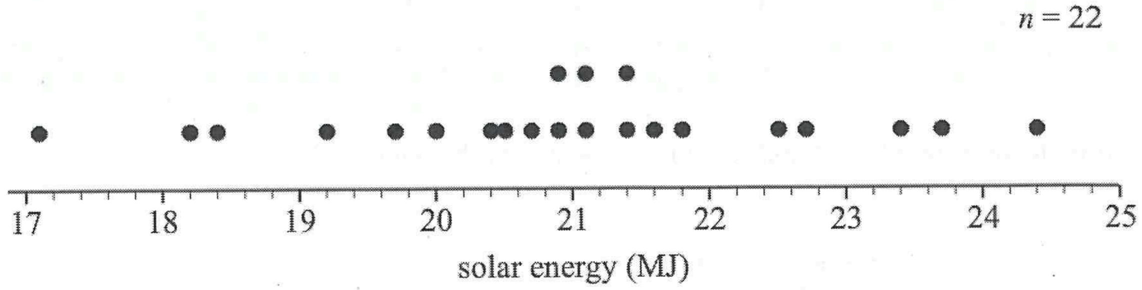
Write down a recurrence relation, in terms of S_{n+1} and S_n, that can be used to determine the value of the snooker table after n years using this reducing balance method.

$S_0 = 3000, \quad S_{n+1} = 0.92\, S_n$

2 marks

Data analysis (10 marks)

4. A 1 m^2 solar panel is located at a weather station.
The total amount of energy generated by the solar panel, in megajoules (MJ), is recorded each month.
The data for the month of February for the last 22 years is displayed in the dot plot below.



(a) Determine the number of years which the energy generated during February was greater than 23 MJ.

3

1 mark

(b) For the data in the dot plot above, the first quartile $Q_1 = 20$ and the third quartile $Q_3 = 21.8$. Show that the data value 17.1 is an outlier.

$IQR = 21.8 - 20 = 1.8$

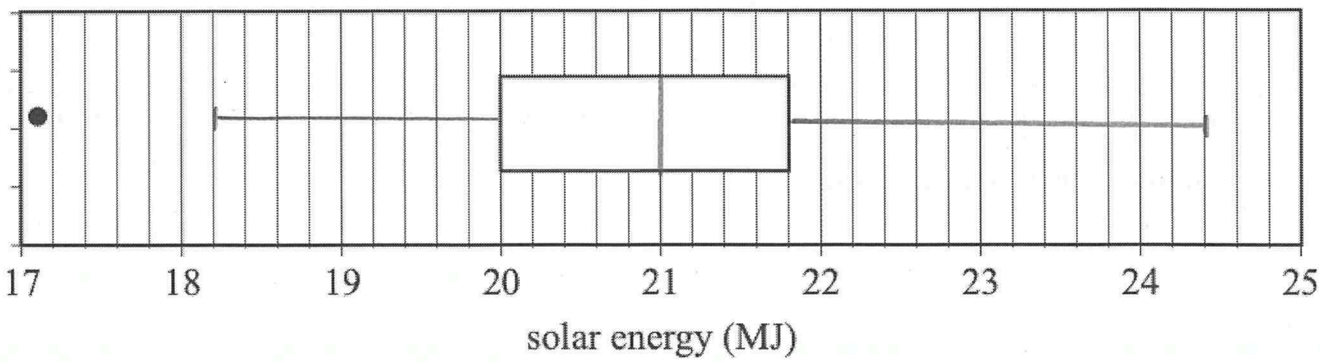
Lower bound: $20 - 1.5 \times 1.8 = 17.3$

$\because 17.1 < 17.3$

$\therefore$ 17.1 is an outlier

2 marks

(c) The box plot below is incomplete. Use the information from the dot plot to complete the following box plot.



1 mark

5. The following data set (of 31 values) shows the maximum daily temperature during the month of January in a particular area.

18	19	22	23	24
25	25	26	26	27
27	27	28	28	28
29	30	31	32	33
34	35	36	37	39
40	41	42	43	44
50	51	60		

(a) Complete an ordered stem and leaf plot to represent the data.

Stem	**Leaf**
1	8 9
2	2 3 4 5 5 6 6 7 7 7 8 8 9
3	0 1 2 3 4 5 6 7 9
4	0 1 2 3 4
5	0 1
6	0

Key: 1|0 = 10

2 marks

(b) Circle the correct distribution of the stem and leaf plot.

Symmetrical distribution. **Positively skewed.** **Negatively skewed.**

1 mark

(c) State the median.

30

1 mark

(d) State the upper and lower quartiles.

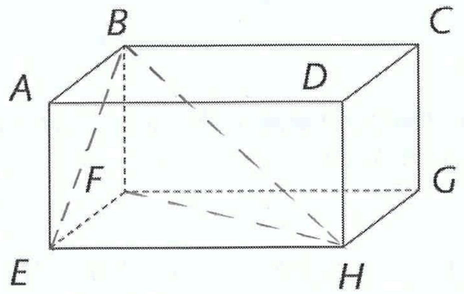
$Q_1 = 26$

$Q_3 = 39.5$

2 marks

Geometry and measurement (4 marks)

6. For the rectangular prism shown below, $EH = 4$ cm, $HG = 2$ cm and $DH = 2$ cm.



(a) Find the length of BE, correct to 2 decimal places.

$BE = \sqrt{2^2 + 2^2}$

$= 2.83 cm$

1 mark

(b) Calculate the length of FH. Answer to 2 decimal places.

$FH = \sqrt{2^2 + 4^2}$

$= 4.47 cm$

2 marks

(c) Hence, calculate the area of triangle BFH. Answer to 2 decimal places.

$A = \frac{1}{2} \times 4.47 \times 2$

$= 4.47 cm^2$

1 mark

Section C **Extended Response Questions** **10 marks**

Include working throughout.

Question 1 (10 marks)

Linear modelling

Jake is a stamp collector. He notices that the value of the rarest stamp in his collection increases by $25 each year. Jake purchased the stamp for $450.

(a) Set up a linear relation to show the total value, V, of the stamp after n years.

$V = 450 + 25n$

1 mark

(b) What is the value of the stamp after 5 years?

$V = 450 + 25 \times 5$

$= \$575$

2 marks

(c) How long does it take for the stamp reach a value of $737.50? Round your answer to the nearest year.

$737.5 = 450 + 25n$

$n = 11.5$

$\therefore$ 12 years

2 marks

Sam, who is Jake's friend, decided to collect stamps as well. He purchased a stamp, worth $290 and the value of his stamp increases by $45 every year.

(d) Write a linear equation that shows the total value, V, of Sam's stamp after n years.

$V = 290 + 45n$

1 mark

(e) How long does it take for both stamps to reach the same value?

$290 + 45n = 450 + 25n$

$n = 8$

$\therefore$ 8 years

2 marks

(f) Lily, Sam's sister, also wants to purchase a stamp at the same time that Jake and Sam purchase theirs. She plans to keep the stamp for 10 years before she sells the stamp. Lily is considering purchasing a stamp identical to either Jake or Sam's stamp. Which stamp should she purchase? Provide your reasons.

$450 + 25 \times 10 = \$700$

$290 + 45 \times 10 = \$740$

$\therefore$ Lily should choose Sam's stamp.

2 marks

END OF EXAMINATION