FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2019 Trial Examination

SOLUTIONS

SECTION A: Core – Data Analysis

Question 1. (9 marks)

a. i. mean $= \dfrac{52.7+45.7}{2} = 49.2\,\text{cm}$

ii. standard deviation $= \dfrac{52.7-49.2}{2} = 1.75\,\text{cm}$

1 + 1 = 2 marks

b. i. $z_1 = \dfrac{49.0-49.2}{1.75} = -0.11428.... \approx -0.11$

$z_2 = \dfrac{50.2-49.2}{1.75} = 0.571428... \approx 0.57$

ii. $x_1 = 165 + 9\times -0.11 = 164\,\text{cm}$

$x_2 = 165 + 9\times 0.57 = 170\,\text{cm}$

2 + 2 = 4 marks

c. i. $residual\,1 = 167 - 164 = 3\,\text{cm}$

$residual\,2 = 175 - 170 = 5\,\text{cm}$

ii. The birth length of these two girls has underestimated adult height by a significant amount. The first was below the mean length at birth and is higher than the mean adult female height. This suggests that estimating adult height based on length at birth is not very reliable.

2 + 1 = 3 marks

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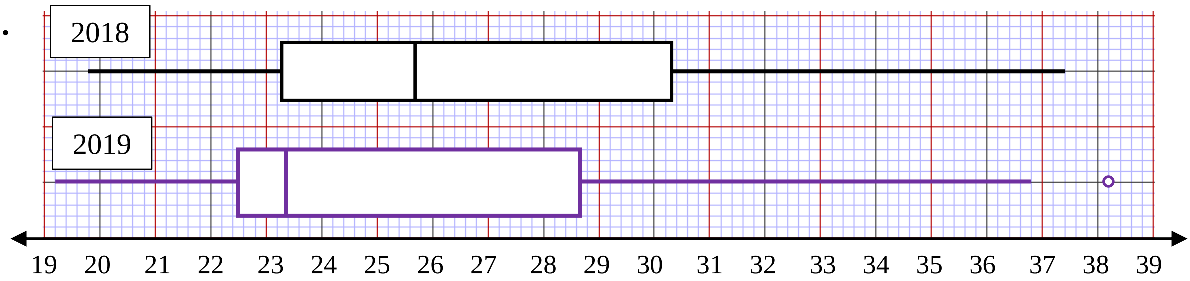
Question 2. (8 marks)

a. i. median = 23.45^{o}

ii. $\frac{7}{28}\times 100 = 25\%$

1 + 1 = 2 marks

b.



2 marks

c. The median February maximum temperature in 2018 was higher (26.1^{o}C) than that in 2019 (23.45^{o}) and 25% of February days were above 30^{o} in 2018 compared to 21% in 2019. The statistics do not support this contention.

1 mark

d. $Q_3 + 1.5 \times IQR = 28.65 + 1.5 \times (28.65 - 22.5) = 28.65 + 1.5 \times 6.15 = 37.875^{o}$

Temperatures above 37.875^{o} would be outliers.

2 marks

e. Date in February is categorical ordinal data.

1 mark

Question 3. (7 marks)

a. *Volume* = $\boxed{3.76}$ + $\boxed{0.105}$ $\times$ *pressure*

2 marks

b. On average, the volume of absorption increases by 0.105 units for each unit increase in pressure.

1 mark

c. Coefficient of determination $r^2 = 0.9974$

1 mark

d. The presence of a clear pattern in the residual plot suggests that the data is not linear.

1 mark

e. Correlation coefficient $r = 0.9853$

1 mark

f. The coefficient of determination for the original data was 0.9974 is higher than the coefficient of determination for the transformed data $(0.9853)^2 = 0.9708$. This means that predictions made using the original regression equation will be more reliable than predictions made with the transformed equation.

1 mark

Core: Recursion and financial modelling

Question 4. (6 marks)

a. $\frac{15}{100} \times \$8420 = \$1263$ deposit

1 mark

b. Recurrence relation

$P_0 = 1263 \qquad P_{n+1} = P_n + 620$

2 marks

c. P_{12} can be calculated using recursion or by the rule $P_n = 1263 + 620 \times n$

$P_{12} = 1263 + 12 \times 620 = \$8703$

1 mark

d. Interest paid $= \$8703 - \$8420 = \$283$

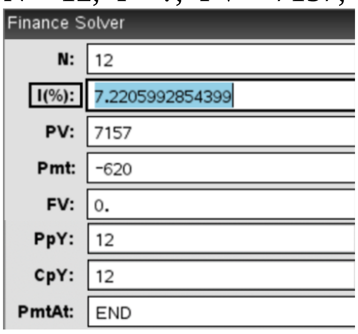
Amount borrowed $= \$8420 - \$1263 = \$7157$

Simple Interest rate $= \frac{283}{7157} \times 100 = 3.95\%$

1 mark

e. Using finance solver

N = 12, I = ?, PV = 7157, Pmt = - 620, FV = 0, Ppy/Cpy = 12



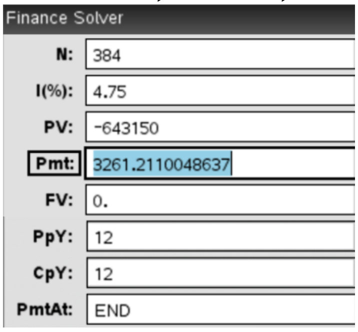
Interest rate = 7.22% p.a.

1 mark

Question 5. (6 marks)

a. Using finance solver

$N = 32 \times 12$, $I = 4.75$, $PV = -643150$, $Pmt = ?$, $FV = 0$, $Ppy/Cpy = 12$



Janet would receive $3261.21 each month.

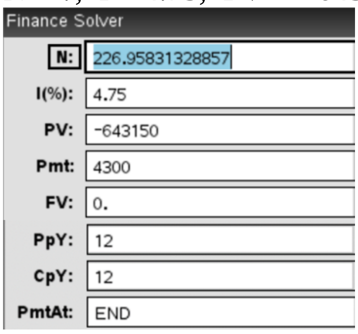
1 mark

b. $4.3 \times \$1000 = \$4300$

1 mark

c. Using finance solver

$N = ?$, $I = 4.75$, $PV = -643150$, $Pmt = 4300$, $FV = 0$, $Ppy/Cpy = 12$



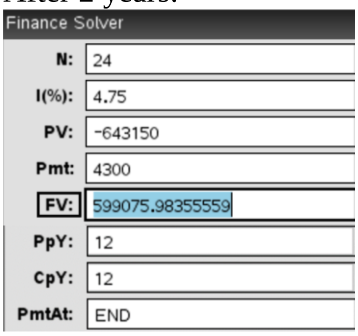
Number of years $= 227 \div 12 = 18.9 \approx 19$ years

Janet will be $68 + 19 = 87$ years of age when her superannuation runs out.

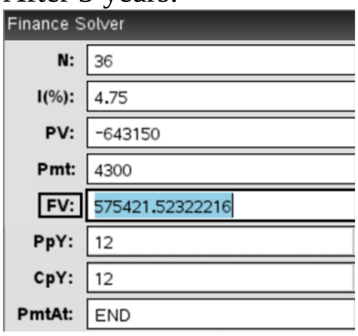
2 marks

d. To determine the amount of interest earned, we need to know the starting value of the investment, the finishing value of the investment and compare the difference to the amount withdrawn over the time.

After 2 years:



After 3 years:



The total amount withdrawn is $12 \times 4300 = \$51600$ during the third year

$(\$575421.52 + \$51600) - \$599075.98 = \$27945.54$

Total interest earned on the investment during the third year is $27,945.54

SECTION B – Module 1: Matrices

Question 1.   (4 marks)

a. On this airline, you can fly directly from Flinders Island to **Essendon** and **Launceston**

1 mark

b. The leading diagonal contains only 0's because there are no flights from a place to itself.

1 mark

c. Matrix multiplication $A \times F$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 & 3 & 3 \end{bmatrix}$$

1 mark

d. The resulting matrix shows the number of flight destinations available from each location.

1 mark

Question 2.   (4 marks)

a. 63% invested in Term deposits stay, so 37% will be transferred $0.37 \times 70000 = \$25900$
74% invested in Shares stays, so 26% will be transferred $0.26 \times 50000 = \$13000$
82% invested in Property stays, so 18% will be transferred $0.18 \times 400000 = \$72000$
The total amount of money transferred into a different investment type is:
$\$25900 + \$13000 + \$72000 = \$110900$

1 mark

b. Missing values shown in bold below:

$$T = \begin{bmatrix} 0.63 & 0.21 & \mathbf{0.06} \\ 0.28 & \mathbf{0.74} & 0.12 \\ \mathbf{0.09} & 0.05 & \mathbf{0.82} \end{bmatrix}$$

1 mark

c. i. Matrix A

$$A = \begin{array}{c} \begin{bmatrix} 0 \\ 5000 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} T \\ S \\ P \end{array}$$

ii. Using recursion

$$S_0 = \begin{bmatrix} 70000 \\ 50000 \\ 400000 \end{bmatrix} \quad S_1 = \begin{bmatrix} 78600 \\ 109600 \\ 336800 \end{bmatrix} \quad S_2 = \begin{bmatrix} 92742 \\ 148528 \\ 288730 \end{bmatrix} \quad S_3 = \begin{bmatrix} 106942.14 \\ 175526.08 \\ 252531.78 \end{bmatrix} \quad S_4 = \begin{bmatrix} 119385.93 \\ 195136.91 \\ 225477.16 \end{bmatrix}$$

The amount invested in shares after four years (at the beginning of the fifth year) is \$195,136.91.

1 + 1 = 2 marks

Question 3. (4 marks)

a. Matrix product

$$H \times C = \begin{bmatrix} 3489 \\ 3335 \\ 2011 \\ 3196 \end{bmatrix}\begin{array}{l} game\ 1 \\ game\ 2 \\ game\ 3 \\ game\ 4 \end{array}$$

This matrix shows the totals made from sales at each of the four games.

2 marks

b. Solving using matrix inverse

$$\begin{bmatrix} s \\ b \\ g \\ p \end{bmatrix} = H^{-1} \times \begin{bmatrix} 1271.5 \\ 1103 \\ 755 \\ 1075 \end{bmatrix} = \begin{bmatrix} 9.5 \\ 6 \\ 15 \\ 15 \end{bmatrix}$$

Seller makes \$9.50 on each scarf, \$6 on each beanie and \$15 on each guernsey and polo sold.

2 marks

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SECTION B – Module 2: Networks and decision mathematics

Question 1. (5 marks)

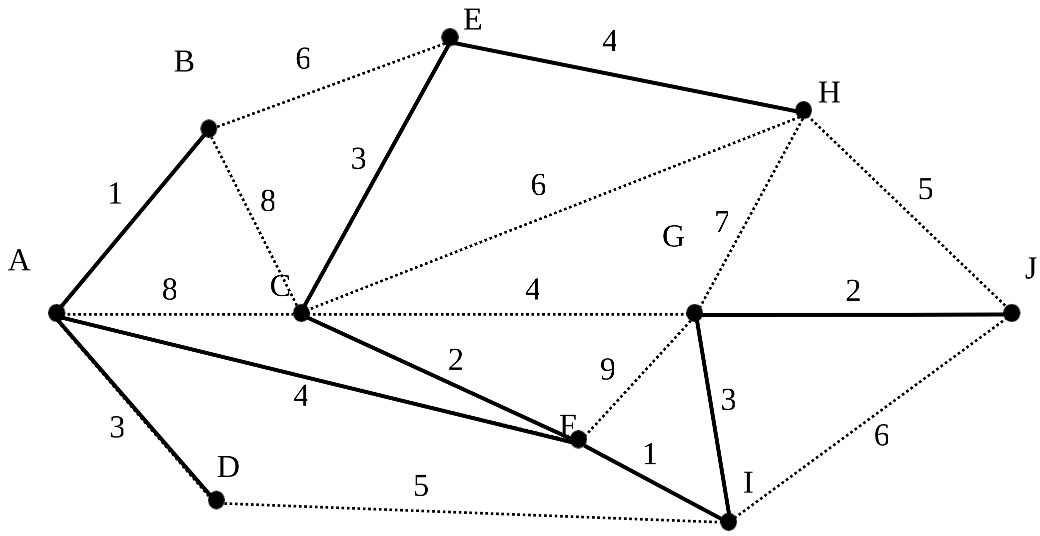
a. i. Shortest distance from A to J is 10 kilometres. (A – F – I – G – J)

ii. 9 couples can be picked up and then dropped off at J where the 10^{th} couple is staying.

iii. Minimum distance to pick up all 9 couples is 38 km following the following route $A-D-I-F-C-B-E-H-G-J$

1 + 1 + 2 = 4 marks

b. Minimum spanning tree shown below:



1 mark

Question 2. (7 marks)

a. The number missing from the table is $28-18=10$

1 mark

b. i. Table with column reduction completed:

	Ryan	Sophie	Dean	Jess
A	3	6	4	0
B	13	5	5	0
C	0	0	0	1
D	5	0	7	0

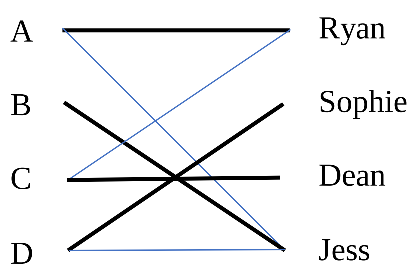
ii. It is not yet possible to determine an optimal allocation because the minimum number of lines required to cover all of the zeros is three. We require a minimum of four lines before an optimal solution can be found.

1 + 1 = 2 marks

c. Completing the Hungarian algorithm:

	Ryan	Sophie	Dean	Jess
A	3	6	4	0
B	13	5	5	0
C	0	0	0	1
D	5	0	7	0

	Ryan	Sophie	Dean	Jess
A	0	6	1	0
B	10	5	2	0
C	0	3	0	4
D	2	0	4	0



Task	Worker
A	Ryan
B	Jess
C	Dean
D	Sophie

2 marks

d. Each worker has 1.5 hours. Total working time is $4 \times 1.5 = 6$ hours

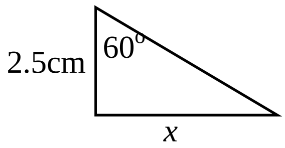
The optimum allocation requires a total of $24+18+23+23=88$ minutes (1 hour 28 mins) for all four jobs to be completed which will leave 4 hour 32 mins for shelves to be restocked.

1 mark for calculating 88 minutes to complete jobs, 1 mark for restocking time
2 marks

SECTION B – Module 3: Geometry and measurement

Question 1. (5 marks)

a. Side length of square is 8.7 cm



$$\tan\left(60^o\right)=\frac{x}{2.5}$$

$$x=2.5\tan\left(60^o\right)=4.330$$

Side length of square $=2x=2\times4.33=8.66\approx8.7$ cm

1 mark

b. $h=2.5+\text{hypotenuse of triangle}=2.5+\dfrac{2.5}{\cos\left(60^o\right)}=7.5\,\text{cm}$

OR

Because the cross-section triangle is equilateral

$h=x\cos60°=4.33\cos60°=7.5$ cm

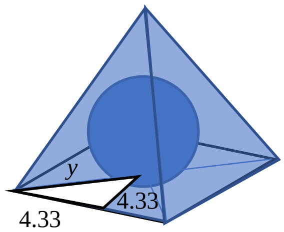
1 mark

c. Find length y first

$$y=\sqrt{(4.33)^2+(4.33)^2}\approx6.124$$

$$\tan\theta=\frac{7.5}{6.124}$$

$$\therefore\theta=\tan^{-1}\left(\frac{7.5}{6.124}\right)=50.767...\approx51^o$$



1 mark

d. $V=\dfrac{4}{3}\pi\left(2.5\right)^3-\dfrac{4}{3}\pi\left(2.5-0.6\right)^3$

$V=36.7189...\approx36.7\ \text{cm}^3$

1 mark

e. $V=V_{pyramid}-V_{sphere}$

$V=\dfrac{1}{3}\times8.66\times8.66\times7.5-\dfrac{4}{3}\pi\times\left(2.5\right)^3$

$=122.039\approx122\,\text{cm}^3$

1 mark

Question 2. (4 marks)

a. i. Longitude of Calgary is West since the sun rise is later than London.

ii. 15^o for every hour and 0.25^o for every minute

longitude $=7\times15^o+36\times0.25^o=114^o\,\text{W}$

1 + 1 = 2 marks

b. **i.** At 6am London (which is earliest time of the 3 locations)
Calgary is 7 hrs 36 minutes later, so time is 1:36 pm in Calgary
Bogota is 74^o which is 4 hrs 56 min later, so time is 10:56 am in Bogota
So yes, Frederick can have a conversation with his friends at that time.

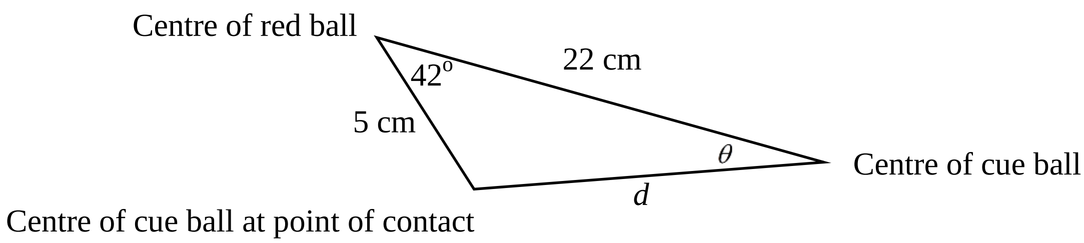
ii. The latest time is in Calgary. The latest time that Trevor can speak is 10pm Calgary time which is 7 hrs 36 min later than Frederick's London time, so at 2:24 pm London time.

Frederick can call his friends any time between 6 am and 2:24 pm London time.

1 + 1 = 2 marks

Question 3. (3 marks)

a. Label the diagram with two lengths and one angle



1 mark

b. Using the cosine rule

$$d^2 = 5^2 + 22^2 - 2 \times 5 \times 22\cos\left(42^o\right) = 345.508..$$

$$d = \sqrt{345.508...} = 18.587... \approx 18.6 \text{ cm}$$

1 mark

c. Using the sine rule

$$\frac{\sin\theta}{5} = \frac{\sin 42^o}{18.587...}$$

$$\theta = \sin^{-1}\left(\frac{5\sin 42^o}{18.587..}\right) = 10.369... \approx 10^o$$

1 mark

SECTION B – Module 4: Graphs and relations

Question 1. (2 marks)

a. Minimum temperature is $13.9\,^\circ\mathrm{C}$ recorded at 6 am.

1 mark

b. The temperature reaches $25\,^\circ\mathrm{C}$ at about 1:15 pm and again at about 8:30 pm. Laurence could work on the bee hives between these times.

1 mark

Question 2. (3 marks)

a. $m = \dfrac{75}{190} = 0.39473... \approx 0.395$

1 mark

b. $\text{inches} = 0.395 \times 56 = 22.12 \approx 22$ inches

1 mark

c. $52 = 0.395 \times x$

$\therefore x = \dfrac{52}{0.395} = 131.645.. \approx 132$ cm

1 mark

Question 3. (7 marks)

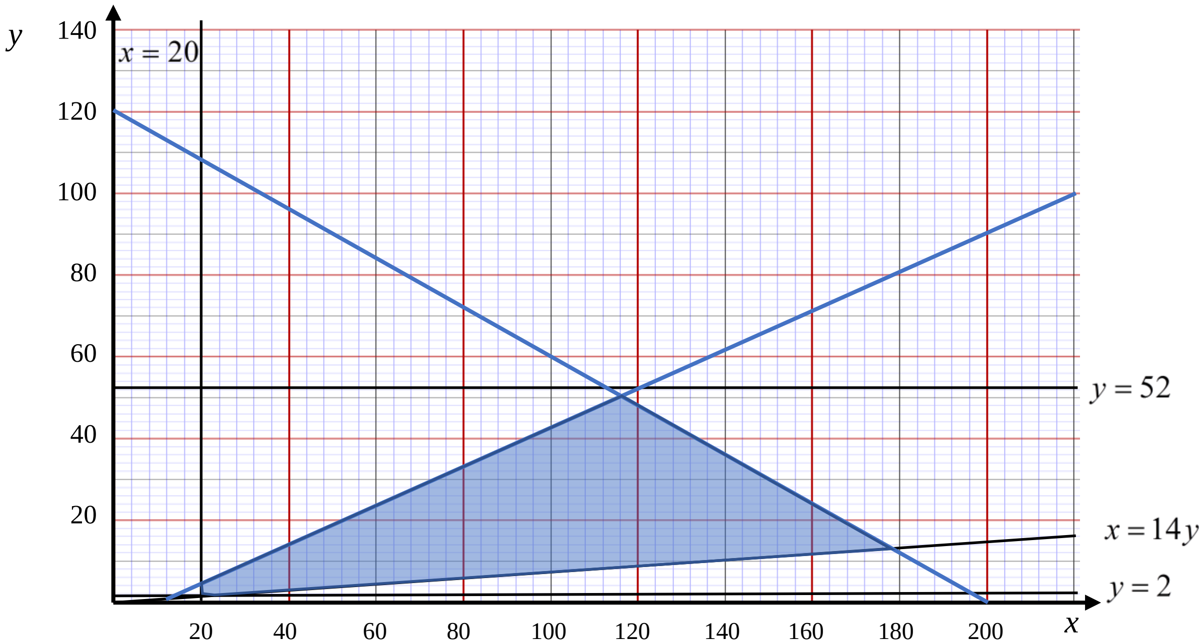
a. The maximum amount of technician time available is 3000 minutes or 50 hours per week.

1 mark

b. If 52 teacher devices have been checked, constraint 4 means there can be a maximum of $14 \times 52 = 728$ student devices that have been checked. However, if not all student devices have been checked there must be more than 728 students. So the minimum number of students is 729.

1 mark

c. Lines added to graph and feasible region shaded



2 marks

d. The objective function will be optimized at the intersection of the lines $x = 14y$ and $15x + 25y = 3000$. These intersect at $x = 178.72$ and $y = 12.766$ but only whole numbers of devices can be serviced. All constraints are satisfied when $x = 178$ and $y = 13$
The number of student devices checked is 178 and the number of teacher devices updated is 13.

1 mark

e. It will take $52 \div 13 = 4$ weeks for all teacher devices to be updated if the maximum number of devices is checked each week.

1 mark

f. Constraint 3 will now become $15x + 25y \le 1500$
The point of intersection between this line and $x = 14y$ now changes to $x = 89.36$ and $y = 6.38$. The maximum number of devices serviced happens when 88 student devices are serviced and 7 s devices are updated.

1 mark

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