The Mathematical Association of Victoria

Trial Examination 2019

FURTHER MATHEMATICS

Trial Written Examination 2 - SOLUTIONS

SECTION A

CORE – Data Analysis

Question 1 (4 marks)

    a. Four.   A1

        The numerical variables are age, distance travelled, amount spent, wait time.

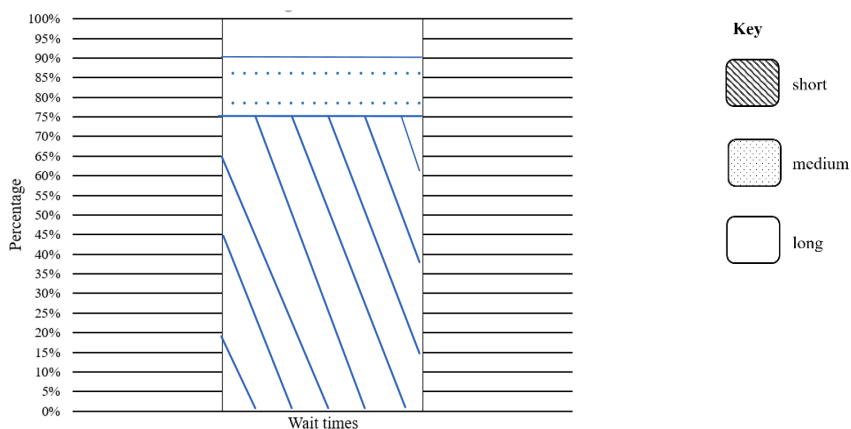   **b.** $\frac{2}{12} \times 100 = 16.7\%$ A1

    c.

wait time	Percentage
short	**75%**
medium	**15%**
long	**10%**

All answers must be correct   A1

    d.

                                                                     A1

Question 2 (4 marks)

a.   12 is the median so 50%   A1

b.   15 is the upper quartile so 25% of the data.

          25% of 168 = 42

c.   IQR = 15 − 5 = 10

          Q_3 +1.5(10) = 15 + 15 = 30   A1

          As 21< 30 (upper fence) it is not an outlier   H1

Question 3 (3 marks)

   a. 93.6+2(21.20) = 136
          136 is 2 standard deviations above the mean.

   So **2.5%** of the sales are expected to be above $136   A1

   b. 93.6 - 3(21.20) = 30 so 0.15%   which is **18** customers   A1
   c.

               $-1.6 = \frac{x-93.6}{21.2}$ $x = 59.68$

               59.68/4 = $14.92   A1

Question 4 (3 marks)

   a. *Items purchased*   A1

   b. The *items purchased* are associated with *age* because there is a difference in the percentages
          of *items purchased* for each *age group*   A1

          21% of the under 25 group bought drinks which is more than 10% for the 25 to less than 45
          group and 15% for the 45 or more group.
          Answers may vary   A1

Question 5 (3 marks)

   a. $b = 0.61 \times \frac{21.2}{6.7} = 1.93$ M1
             $a = 93.6 - 1.93(37.2) = 21.80$ H1

   b.   On average the amount spent at the supermarket is predicted to increase by $1.93 for
             every one year increase in age of customers.
                                                                                                    A1

Question 6 (4 marks)

   a. $\log_{10}(sales) = 211.1 - 0.1045 \times year$

b.   Actual = 19.4

     $\log_{10}(predicted) = 211.1 - 0.1045(2008) = 1.264$ predicted = 18.365...   M1

     Residual = actual – predicted = 19.4-18.4 = **1.0**   H1

Question 7 (3 marks)

a. $4 - (1.85 + 0.79 + 1.12) \ = \ 0.24$ A1

b.   *time = 12 for Spring 2019*

     Deseasonalised sales = 3.4 + 0.3 × 12 = 7.0   M1

     $1.12 = \frac{x}{7}$ $x = 7.84$. This is \$7800 correct to the nearest \$100.   H1

CORE - Recursion and financial modelling

Question 8 (5 marks).

a. $V_1 = 0.852 \times 6640 = \$5657.28$
$V_2 = 0.852 \times 5657.28 = \$4820.00$ A1

b. $1 - 0.852 = 0.148$, therefore 14.8% A1

c. $6640 \times 0.852^n = 1300$ $n = 10.181 \ldots$ so in 11 years. A1

Alternatively if students have a ClassPad:

d. i. $\dfrac{6640-1300}{6} = \$890$ A1

ii. $\dfrac{890}{6640} \times 100 = 13.4\%$ H1

Question 9 (7 marks)

a.

Payment Number	Payment	Interest	Principal Reduction	Balance of Annuity
0	0.00	0.00	0.00	120 000.00
1	6000.00	1860.00	4140.00	115860.00
2	6000.00	1795. 83	**4204.17**	111655.83
3	6000.00	**1730.67**	4269.33	**107386.50**

two answers correct A1, all answers correct A1

Quarterly rate is $\dfrac{6.2}{4} = 1.55\%$.

$A = 6000 - 1795.83 = 4204.17$

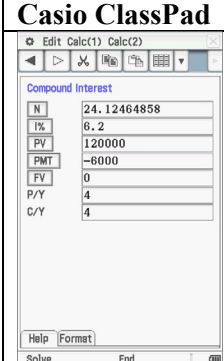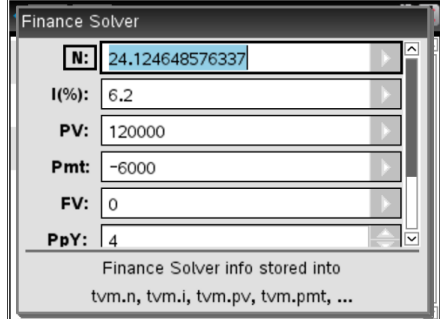$B = \dfrac{1.55}{100} \times 111655.83 = 1730.67$ or $B = 6000 - 4269.33 = 1730.67$

$C = 111655.83 - 4269.33 = 107386.50$

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b. $\frac{6.2}{4} = 1.55 \quad 1 + \frac{1.55}{100} = 1.0155$ A1
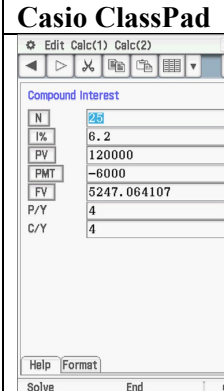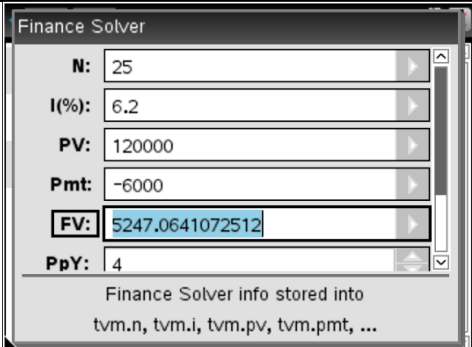
   $A_0 = 120\,000, \quad A_{n+1} = 1.0155 \times A_n - 6000$

c.   25 payments.   A1

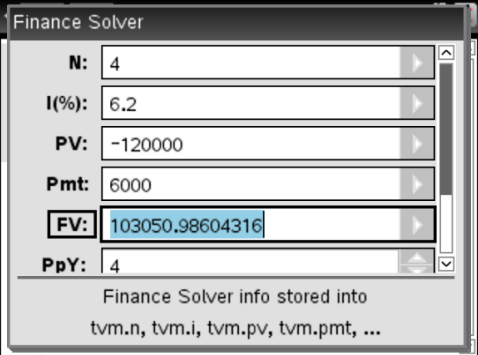There are 24 full payments and 1 part payment.

Casio ClassPad	TI-Nspire

ii. $6000 - 5247.06 = \$752.94$ A1

Casio ClassPad	TI-Nspire

d.   The balance of the annuity after one year (4 quarters) is \$103050.99.   M1

For a perpetuity the balance of the annuity stays the same; the interest equals the payment.

$\frac{\left(\frac{7.8}{4}\right)}{100} \times 103050.99 = \$2009.49$ A1

	Casio ClassPad	TI-Nspire
The perpetuity starts after one year, so the balance after one year must be determined.		

| Alternatively to using a percentage calculation, the finance solver can be used with PV and FV both at $103050.99 | **Compound Interest** N 1 I% 7.8 PV −103050.99 PMT 2009.494305 FV 103050.99 P/Y 4 C/Y 4 | **Finance Solver** N: 1 I(%): 7.8 PV: −103051. Pmt: 2009.494305 FV: 103051. PpY: 4 Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, … |

MODULES - Module 1: Matrices

Question 1

a. $80 + 50 = 130$ A1

b. $[1\ 1]$ is a summing matrix and sums the columns. A1

$Q = [1 \quad 1] \times \begin{bmatrix} 80 & 50 \\ 10 & 20 \end{bmatrix} = [90 \quad 70]$

The total number of standard phones is 90 and the total number of Android phones is 70.

A1

c. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 & 14 \\ 43 & 30 \end{bmatrix}^{-1} \begin{bmatrix} 33800 \\ 72600 \end{bmatrix}$

$R = \begin{bmatrix} 20 & 14 \\ 43 & 30 \end{bmatrix}^{-1} = \begin{bmatrix} -15 & 7 \\ \frac{43}{2} & -10 \end{bmatrix}$ A1

Question 2 (2 marks)

$$E = \begin{bmatrix} & S & B & J & W & C & M & \\ & 0 & 0 & 0 & 0 & 1 & 0 & \end{bmatrix} \begin{matrix} S \end{matrix}$$

$$E = \begin{array}{c} \\ \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} S \\ B \\ J \\ W \\ C \\ M \end{matrix}$$

a. Read down the columns to get the check in the list.

$S \to J \to W \to B \to M \to C$ A1

b. $V = \begin{bmatrix} 1.15 & 0 & 0 & 0 \\ 0 & 1.10 & 0 & 0 \\ 0 & 0 & 0.90 & 0 \\ 0 & 0 & 0 & 0.95 \end{bmatrix}$ A1

Question 3 (7 marks)

a. 113 A1
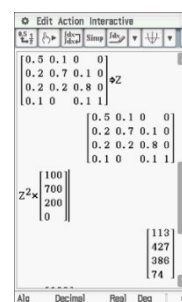
$G_4 = Z^2 \times G_2$

G_4 gives the values for the end of April. There are two transitions.

$G_4 = \begin{bmatrix} 0.5 & 0.1 & 0 & 0 \\ 0.2 & 0.7 & 0.1 & 0 \\ 0.2 & 0.2 & 0.8 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{bmatrix}^2 \begin{bmatrix} 100 \\ 700 \\ 200 \\ 0 \end{bmatrix} = \begin{bmatrix} 113 \\ 427 \\ 386 \\ 74 \end{bmatrix}$

There will be 113 people paying the top plan at the end of April

b. 1000 customers A1

To work backwards to January we must use the inverse matrix

$$G_1 = \begin{bmatrix} 0.5 & 0.1 & 0 & 0 \\ 0.2 & 0.7 & 0.1 & 0 \\ 0.2 & 0.2 & 0.8 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 700 \\ 200 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix}$$

All the customers, 1000, were on the standard plan at the end of January.

c. First find the matrix for March. $G_3 = \begin{bmatrix} 120 \\ 530 \\ 320 \\ 30 \end{bmatrix}$ M1

The number of customers in total on the low plan in April is given by

$(0.2 \times 120) + (0.2 \times 530) + (0.8 \times 320) + (0 \times 0) = 386$

The number of customers changing from standard to low plan in April is $0.2 \times 530 = 106$

$\frac{106}{386} \times 100 = 27.5\%$ A1

d. 40% of 1000 = 400

$$G_{10} = \begin{bmatrix} 51.888 \\ 211.605 \\ 368.663 \\ 367.844 \end{bmatrix} \quad \text{and} \quad G_{11} = \begin{bmatrix} 47.104 \\ 195.367 \\ 347.629 \\ 409.899 \end{bmatrix}$$

Hendricke will have first lost more than 40% of his customers at the end of November A1

e. $G_3 = Z_1 \times G_2$

$$\begin{bmatrix} 0.5 & 0.1 & 0 & 0 \\ 0.2 & 0.7 & x & 0 \\ 0.2 & 0.2 & y & 0 \\ 0.1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 700 \\ 200 \\ 0 \end{bmatrix} = \begin{bmatrix} 120 \\ 590 \\ 280 \\ 10 \end{bmatrix}$$

$(0.2 \times 100) + (0.7 \times 700) + (x \times 200) = 590 \qquad x = 0.4$ M1

$y = 1 - 0.4 = 0.6$ A1

MODULES - Module 2: Network and Decision Mathematics

Queston 1 (4 marks)

a. Activity H cannot be started until A, B and D are completed

A takes 6 weeks, B and D take a total of 5 weeks.

The earliest starting time for activity H is therefore 6 weeks. A1

b. There are only four paths for this activity network.

Path	Time	
AEG	12	Critical path
AH	11	
BDH	10	
CF	10	

The critical path is AEG A1

c. The dummy goes from the end of activity D to the start of H.
Activity H has both activities A and D as immediate predecessors whereas Activity E only has
activity D as a predecessor. A1

d. Two activities have float times of 2. A1

Activity	EST	LST	Float
A	0	0	0
B	0	1	1
C	0	2	2
D	2	3	1
E	6	6	0
F	7	9	2
G	9	9	0
H	6	7	1

Question 2 (3 marks)

a. The degree of the vertex is the number of cities that can be reached by a direct flight from that
vertex. For example the degree of the Nairobi vertex is 5 and there are 5 cities (Cape Town,
Harare, Dar Es Salaam, Maputo and JHB) that can be reached by direct flight. Examples may
vary. A1

b.

$$\text{to}$$

		C	D	G	H	J	M	N	W
	C	0	0	1	0	1	0	1	1
	D	0	0	0	1	1	0	1	0
	G	1	0	0	0	0	0	0	1
from	H	0	1	0	0	1	0	1	1
	J	1	1	0	1	0	1	1	**1**
	M	0	0	0	0	1	0	1	0
	N	1	1	0	1	1	1	0	0
	W	1	0	1	1	1	0	**0**	0

A1

c.

Path	
JHB – CT - G	115+120 = 235
JHB – CT – W- G	115 + 125 + 90 = 330
JHB – W - G	115 + 90 = 205
JHB – H –W- G	100 + 140 + 90 = 330

The shortest path is JHB – W – G taking 205 minutes   A1

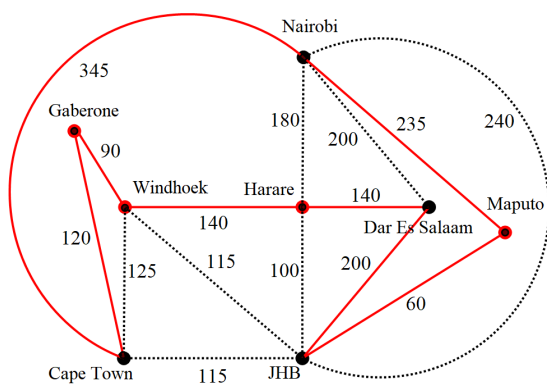**Question 3** (3 marks)

    a. i. Hamiltonian Cycle   A1

       ii. JHB **-** Dar Es Salam - Harare – Windhoek – Gaborone - Cape Town – Nairobi – Maputo – JHB (Could also go in the opposite direction).   A1
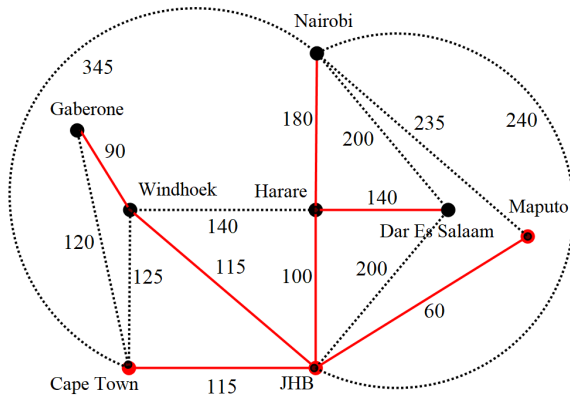

    b. Nairobi   A1
    This is an Eulerian trail; all the vertices are of even degree except at the start (Dar Es Salaam) and the finish (Nairobi).

Question 4 (2 marks)

a.

A1

b.   Weight is $115 + 115 + 90 + 100 + 180 + 140 + 60 = 800$ H1

H mark is for the correct weight of an incorrect spanning tree with working shown.

MODULES - Module 3: Geometry & Measurement

Question 1 (4 marks)

    a.   63.6 m^2 A1

$A = \pi r^2$

$A = \pi(4.5)^2 = 63.6 \, m^2$

    b.   12.6 cm   H1

$V = area\ of\ base\ \times height$

$8 = h \times 63.6$

$h = 0.12575m \approx 12.6cm$

    c.   4.2m = 420cm   2.2m = 220cm

         Volume = $\pi \times (420)^2 \times 220 = 121918928cm^3$

         Volume = $121918928 \div 1000 = 121919L$ M1

    d. $1.2^3 = 1.728$ A1

Question 2 (3 marks)

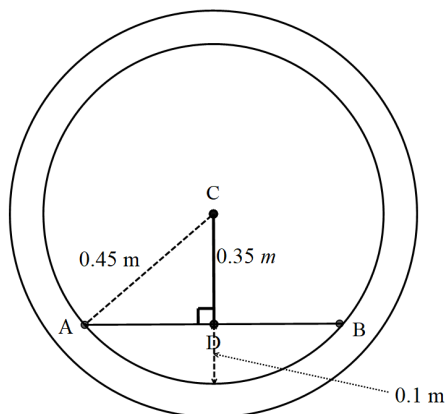   **a.** $3^2 + 3.2^2 = 19.24$ $\sqrt{19.24} \approx 4.4m$ A1

    b. $\tan(\theta) = \frac{3.2}{3}$ $\theta = 46.8°$ This is greater than $12°$.   A1

    c. $x \leq \frac{3.2}{\tan(12)}$ $x = 15.1 \, m$ A1

Question 3 (3 marks)

a.

$\sqrt{0.45^2 - 0.35^2} = 0.2828$

Chord length = $2 \times 0.2828 = 0.57$ metres (correct to two decimal places)

Chord length = $0.57 \times 100 = 57$ cm   A1

b.

Area of the cross section of pipe is $\pi \times (0.45)^2 = 0.63617$

$\cos(\theta) = \frac{0.35}{0.45}$

$\theta = 38.94244 \ldots$ so the angle of the sector at the centre is $2 \times 38.94244 = 77.88488°$

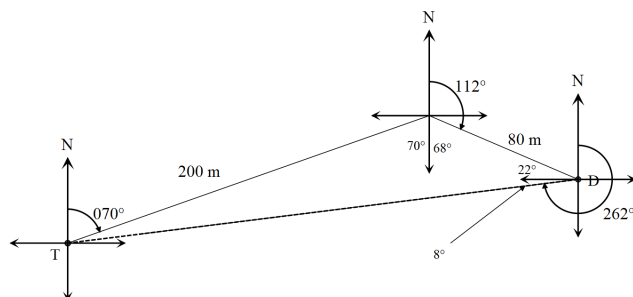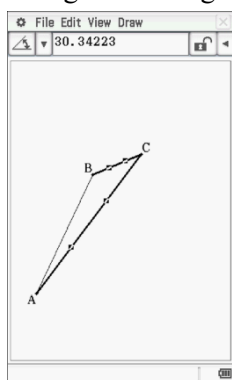Area of the sector is $\pi \times (0.45)^2 \times \frac{77.88488}{360} = 0.1376$ M1

Area of the triangle $0.5 \times 0.35 \times 0.57 = 0.09975$

Percentage $= \frac{0.1376 - 0.09975}{0.63617} \times 100 = 6\%$ (correct to the nearest whole number)   H1

Question 4 (2 marks)

Using ClassPad geometry:

$Bearing = 270 - 8 = 262°$

Alternatively use the cosine rule to find the length DT

$DT^2 = 200^2 + 80^2 - 2(200)(80)\cos(138) \quad DT = 265$

Then use the side rule to get angle $\angle D$

$\frac{\sin(D)}{200} = \frac{\sin(138)}{265}$ $\angle D = 30°$ M1

$\angle D = 30°$

Using alternate angles the 30º angle is split into 22º and 8º.

$Bearing = 270 - 8 = 262°$ A1

MODULES - Module 4: Graphs and Relations

Question 1 (2 marks)

a. 0.2 m/s   A1
Reading off the graph for 400 m the average speed is 0.2 m/s.

b. $k = 80$ H1
Using the point found in part (**a**) $0.2 = \dfrac{k}{4000}$ so $k = 80$

Question 2 (5 marks)

a.   0.25 hours or 15 minutes   A1

b.   **i.** 10 km   A1

$40 \times 0.25 = 10$

ii. On the graph below   A1

Maggie's father leaves home 15 minutes (0.25 hours) after Maggie, so the line passes through (15, 0). As found in the previous question 15 minutes later he is 10 km from home, so the line also passes through the point (30,10).

c.   For Maggie gradient of the line $=\dfrac{7.2}{0.75} = 9.6$

Equation is $d = 9.6t$
For Maggie's dad $d = 40t - 10$ (both equations)   M1

$9.6t = 40t - 10$ $t = 0.3289\ldots$

$d = 9.6 \times 0.3289\ldots = 3.16km = 3160m$ H1

Question 3 (5 marks)

a. $x = 10$ and $y = 10$

Substituting we get $10 \geq 20$ is untrue so this does not satisfy the constraint.   A1
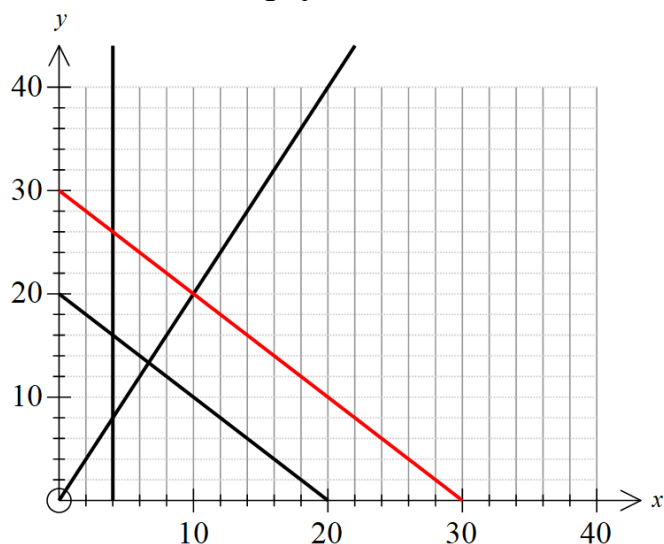
b. Inequality 3: $x + y \geq 20$
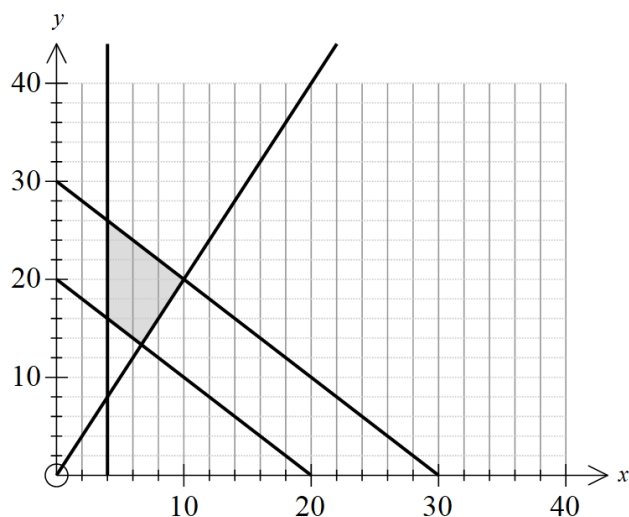Inequality 4: $x + y \leq 30$ (both correct)   A1

c. Answer is on the graph below.   A1

d. Answer is on the graph below.   A1

e. 13 hours and 20 minutes   A1

The question requires the lowest y value.

This occurs when at the intersection of $y = 2x$ and $x + y = 20$

The intersection occurs at $x = 6\frac{2}{3}$ $y = 13\frac{1}{3}$

Answer is when $y = 13\frac{1}{3}$ or 13 hours and 20 minutes.

END OF SOLUTIONS