2019 PRACTICE EXAM

FURTHER MATHS UNITS 3&4

Exam 1 Question Booklet
Exam 2 Question Booklet
Worked Solution Booklet

ATARNotes

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ATARNotes

STUDENT NUMBER | | | | | | | | | | Letter | |

FURTHER MATHEMATICS

Written examination 2

2019

Reading time: 9:00 a.m. to 9:15 a.m. (15 minutes)
Writing time: 9:15 a.m. to 10:45 a.m. (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	6	6	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 31 pages
- Data book
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds, or fractions.
In 'Recursion and financial modelling,' all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (11 marks)

The table below displays the number of lollies collected by 9 children during Halloween

Number of lollies	5	15	25	34	29	22	18	32	3

a. From the information given, determine

i. The range 1 mark

_____

_____

ii. The median 1 mark

_____

_____

b. Write down the five number summary for this set of data 2 marks

_____

_____

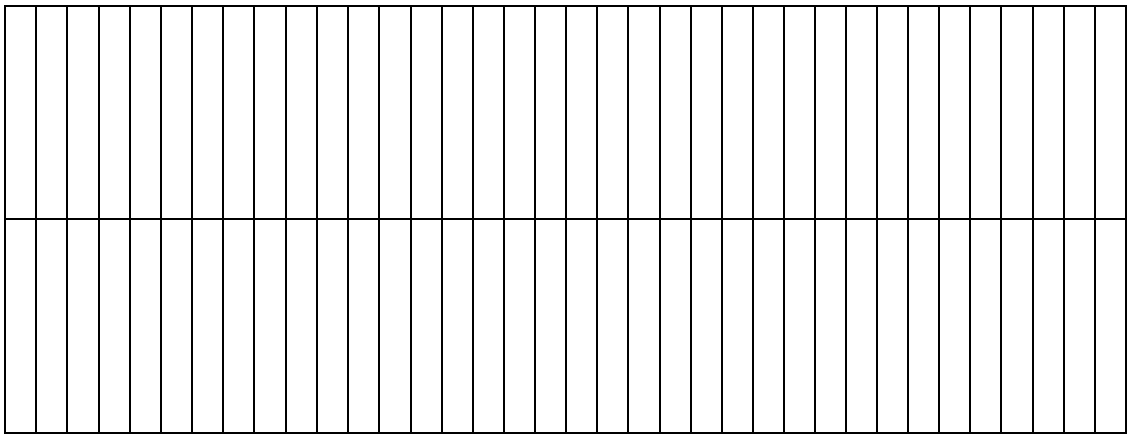
c. Are there any outliers? If so what are they? Show all working out. 2 marks

_____

_____

d. Using the information you have found, construct a boxplot for this data on the grid below. Label the information you used to construct the boxplot.

2 marks



In a larger population of children, the number of lollies collected is approximately normally distributed with a mean of 35 and a standard deviation of 7.

e. Using the 68-95-99.7% rule, determine

i. The percentage of children who collect less than 28 lollies

1 mark

_____

_____

ii. The number of children who collect more than 42 lollies

1 mark

_____

_____

f. The standardised number of lollies collected by a girl is z = -3. Determine the number of lollies she collected

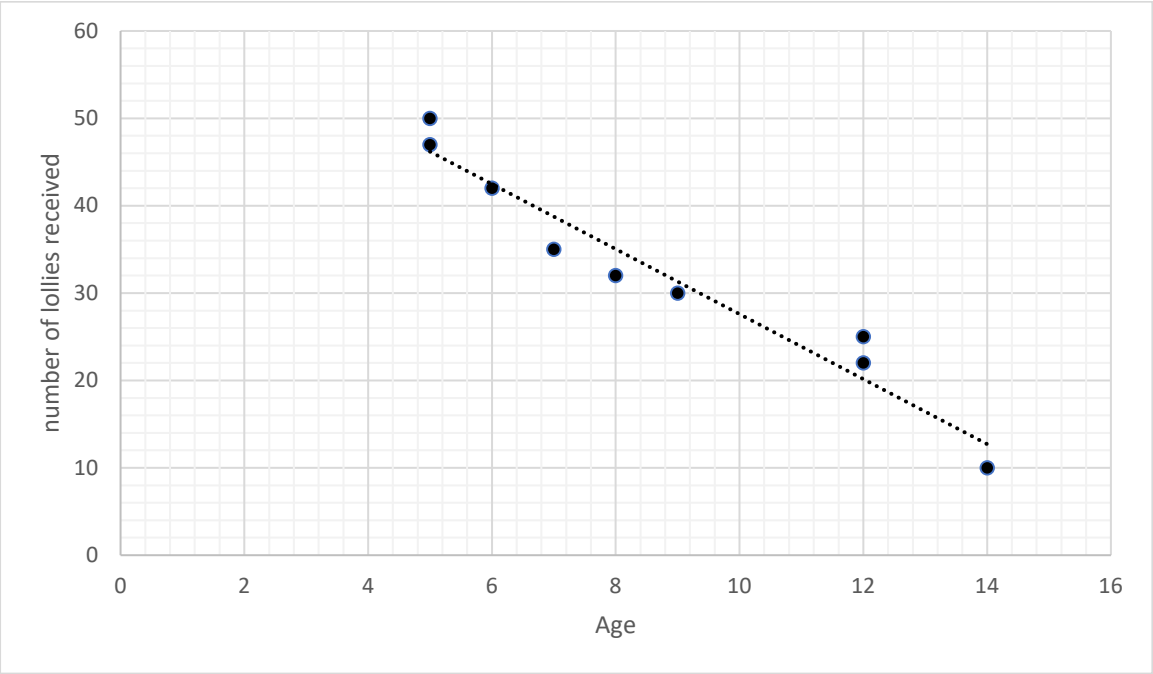
1 mark

_____

_____

Question 2 (9 marks)

An inquisitive 14 year old boy wants to find out whether there is an association between the *age* of children and *the number of lollies received*. He conducts a study by asking children of different ages how many lollies they collected during Halloween. The following scatterplot displays the data he collected.



a.

i. Determine the median age 1 mark

_____

_____

ii. Determine the median number of lollies received 1 mark

_____

_____

The included linear regression line has the equation

$$number\ of\ lollies\ received = -3.72 \times age + 64.79$$

b. Name the explanatory variable in this equation 1 mark

_____

_____

c. Use the equation of the least squares regression line to predict the number of lollies a child aged 10 receives.

1 mark

_____

_____

d. Describe the association between the age and the number of the lollies that this group of children receive in terms of strength, direction and form.

1 mark

_____

_____

e. The value of the correlation coefficient is -0.97067.
Determine and interpret the coefficient of determination.

2 marks

_____

_____

_____

_____

f. Determine the residual value when the equation of the least squared line is used to predict the number of lollies received by the child aged 9 years old.

2 marks

_____

_____

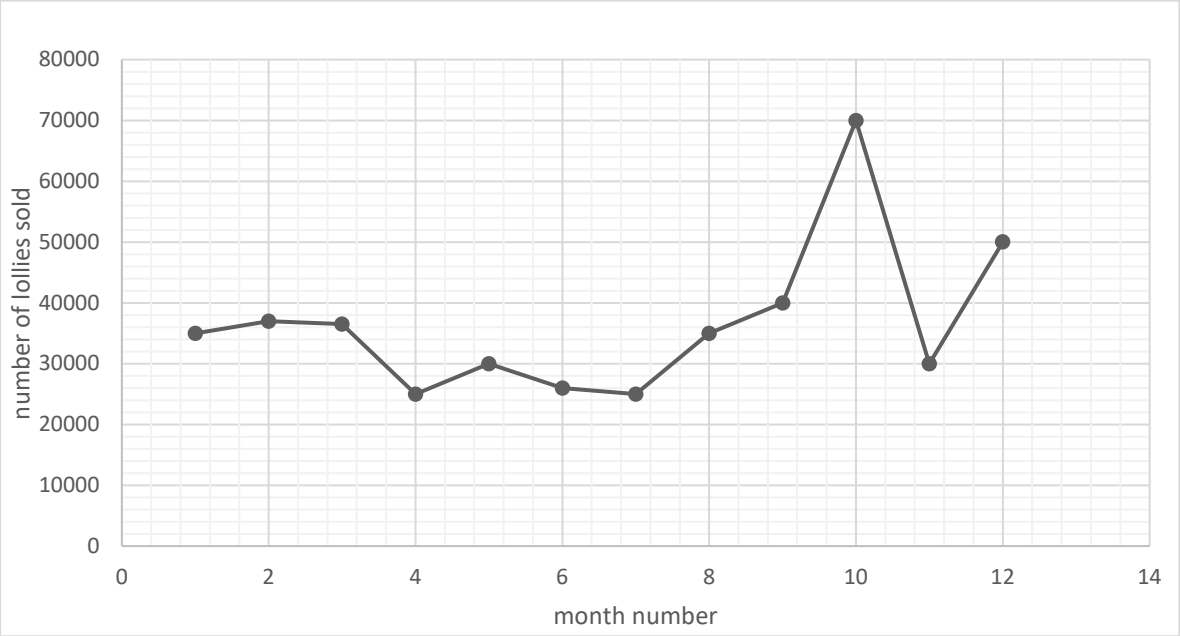
_____

_____

Question 3 (4 marks)

The time series plot below shows the number of lollies sold by a certain lolly shop each month plotted against the month number (1 = January, 2 = February, and so on).

The data was collected over a period of a year.



a. Use a cross (X) to mark the five median smoothed number of lollies sold for month 9 (Answer on the time series plot above)

1 mark

The table below displays the data plotted in the time series plot.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Number of lollies sold	35000	37000	36500	25000	30000	26000	25000	35000	40000	70000	30000	50000

b. What is the seasonal index for month number 3? Show all working out 2 marks

_____

_____

c. Interpret the seasonal index for month number 3 1 mark

_____

_____

Recursion and financial modelling

Question 5 (5 marks)

Phil wishes to purchase an Audi R8 which will cost $367,000 (pray for him). He deposits some money into a savings account that pays compound interest annually. The value of the investment after n years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 15000,\ V_{n+1} = 1.03 \text{ x } V_n$$

a. How much money does Phil initially deposit into the savings account? 1 mark

_____

b. Use the recursion to write down calculations that show the amount of money in Phil's savings account after 2 years. 1 mark

_____

_____

c. What is the annual percentage compound interest rate for his savings account? 1 mark

_____

_____

d. The amount of money in the account after n years, V_n, can also be determined using a rule.

i. Complete the rule below by writing the appropriate numbers in the box provided.

V_n ☐^{n} × ☐ =

ii. How much money will be in Phil's savings account after 10 years? 1 mark

_____

_____

iii. How long will Phil have to leave his money in the savings account before he can afford his dream car? 1 mark

_____

_____

Question 6 (3 marks)

Phil's opts for a cheaper first car, which has a purchase price of $26,000.

After 3 years, the value of the car is $20,0000.

a. Calculate the average depreciation in the value of the car per year.

1 mark

_____

_____

b. Calculate the annual percentage depreciation rate of his car during these 3 years.

1 mark

_____

_____

c. Write down a recurrence relation to calculate the value of Phil's car after n years.

1 mark

_____

_____

Question 7 (4 marks)

After his experience with purchasing his first car, Phil decides on a new investment strategy in order to be more likely to be able to afford his dream car before he dies.

He discovers a new bank with a higher interest rate and adds monthly deposits to his investments. The bank offers an interest rate of 4.5% a year compounding monthly. He deposits $500 into the account each month. He intends to initially deposit $15,000.

a. What is the value of the investment after 10 years?

1 mark

_____

_____

b. If Phil does not add the additional monthly payment of $500, what would be the value of the investment after 10 years?

1 mark

_____

_____

c. After 10 years, to his dismay, interest rates are reduced by the RBA and his bank now offers a 3% interest a year compounding monthly. He decides to increase his monthly deposits to reach his goal of $367,000 before he turns 50. After the first 10 years, he is 29 years old.
What is the value of his new monthly payment? Round your answer to the nearest cent.

2 marks

_____

_____

_____

_____

SECTION B

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds, or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents

Module 1 – Matrices

Question 1 (4 marks)

Matrix A has 3 columns and 5 rows. Matrix B has 5 columns and 3 rows.

a. What is the order of matrix AB? 1 mark

_____

b. What is the order of matrix BA? 1 mark

_____

c. Can matrix A and matrix B be added? Why/why not? 1 mark

_____

_____

Question 2 (6 marks)

A local hardware store specialises in three types of items, nails (N), rivets (R) and bolts (B).

The number of packets containing each item sold over a 3 week period is shown in matrix *S* below.

$$S = \begin{array}{c} \begin{matrix} \text{N} & \text{R} & \text{B} \end{matrix} \\ \begin{bmatrix} 37 & 22 & 59 \\ 27 & 31 & 44 \\ 30 & 31 & 57 \end{bmatrix} \end{array} \begin{matrix} \textit{week 1} \\ \textit{week 2} \\ \textit{week 3} \end{matrix}$$

a. In total, how many bolt packets were sold over the three-week period? 1 mark

_____

b. What does the number in row 2 column 1 indicate? 2 marks

_____

_____

c. Consider the matrix equation provided below. 2 marks

$$\begin{bmatrix} 37 & 22 & 59 \\ 27 & 31 & 44 \\ 30 & 31 & 57 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 500.25 \\ 428 \\ 488.20 \end{bmatrix}$$

Where a = the price of a packet of nails, b = the price of a packet of rivets, c = the price of a packet of bolts.

Find the value of a.

_____

_____

_____

d. What is the order of matrix S? 1 mark

_____

Question 3 (3 marks)

Anthony may need to withdraw from the next election at the end of June.

Matrix E, shown below, shows the percentage of voters who change their preferred candidate, from month to month, before Anthony would withdraw from the election.

$$E = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.70 & 0.20 & 0.20 \\ 0.15 & 0.45 & 0.40 \\ 0.15 & 0.35 & 0.20 \end{bmatrix} \end{matrix}$$

Matrix E_1, shown below, shows the percentage of voters who change their preferred candidate from June to August, after Anthony would withdraw from the election.

$$E_1 = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0.75 & 0.39 & 0.45 \\ 0.25 & 0.61 & 0.55 \end{bmatrix} \end{matrix}$$

Consider the voters who preferred Boris in June and who were expected to prefer Anthony in August.

a. What percentage of these voters are now expected to prefer Boris in August?

1 mark

_____

The state matrix that indicates the number of voters who are expected to have a preference for each candidate in January, P_1, is given below.

$$P_1 = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{bmatrix} 5000 \\ 3459 \\ 2160 \end{bmatrix}$$

b. If Anthony withdraws, how many votes is Boris expected to receive in the election in August?

Write your answer, correct to the nearest vote.

2 marks

_____

_____

_____

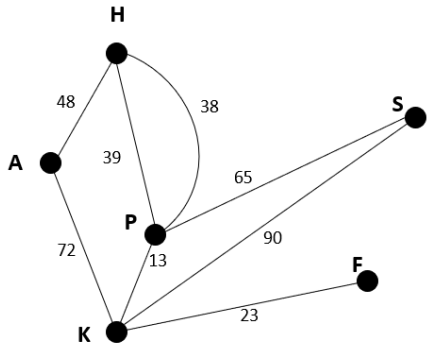
_____

Module 2 – Networks and decision mathematics

Question 1

Kira searches up flights between 6 cities—A, H, K, P, S, and F.

The flight routes connecting these 6 cities are shown in the diagram below. The cost of the plane tickets, in dollars, are also shown.



Kira travels F-K-A-H-P-S-K-P-H.

a. Give a mathematical term that describes this travel route.

1 mark

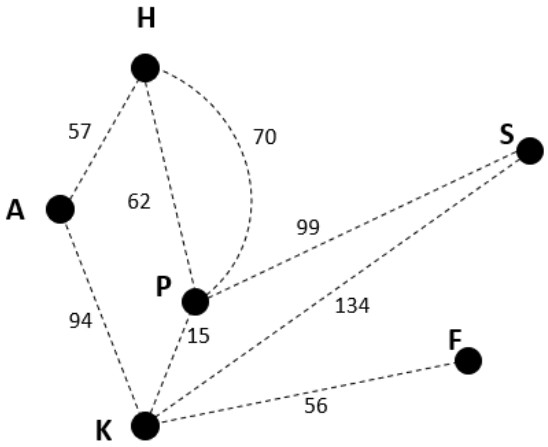
_____

b. If Kira wants to travel to S from her hometown, A. What is the cheapest path she can take?

1 mark

_____

The travel distances, in km, between the 6 cities is shown in the diagram below.

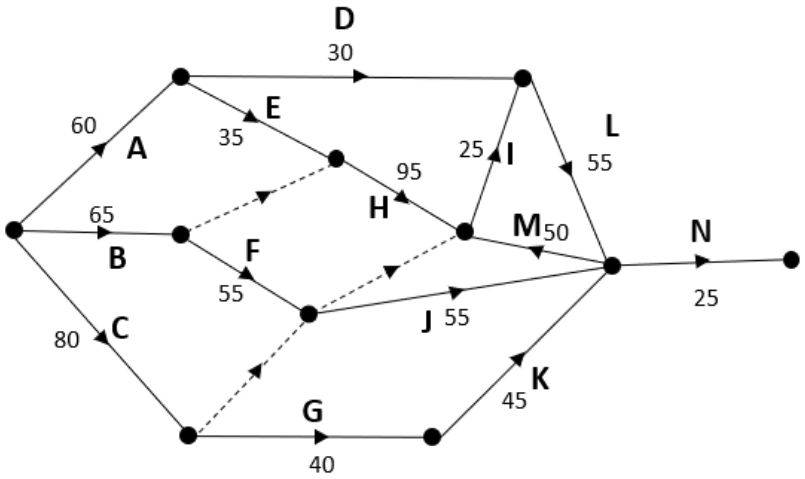


c. Draw the minimum spanning tree on the diagram above.

1 mark

Question 2 (3 marks)

The network for flight set-up is shown below. A-N represents the activities leading up to departure.



a. Which activities are immediate predecessors of activity J?

1 mark

_____

_____

b. The network has a minimum completion time of 295 minutes.

i. What is the critical path?

1 mark

_____

_____

ii. Which activities has a float time of 105 minutes?

1 mark

_____

_____

iii. Determine the latest starting time for activity J.

1 mark

_____

_____

d. The completion times of activities E, F, G, and H can all be shortened by 10 minutes once for $35 per activity.

i. What is the new minimum completion time?

1 mark

_____

_____

ii. Determine the cost to reduce the activities to the new minimum completion time.

1 mark

_____

_____

Question 3 (4 marks)

Mohmed, a staff manager at an airline, hires new staff and is in the middle of organising tasks. The new workers and the time they require for each activity, in minutes, is shown in Table 1.

Table 1

	A	B	C	D	E
Nathan	25	13	18	21	28
Callum	36	12	32	11	9
Rom	12	24	27	16	8
Rimon	23	28	19	20	21
Kenny	13	17	12	13	25

The airline aims to minimise the time it takes for all staff to complete their tasks.

Mohmed applies the Hungarian algorithm to table 1 to produce table 2.

Table 2:

	A	B	C	D	E
Nathan	10	0	4	6	5
Callum	25	3	22	0	0
Rom	2	1	x	6	0
Rimon	3	10	0	0	3
Kenny	0	6	0	0	4

a. What is the value of x?

1 mark

_____

_____

b. Determine the minimum total completion time, in minutes, for all tasks.

1 mark

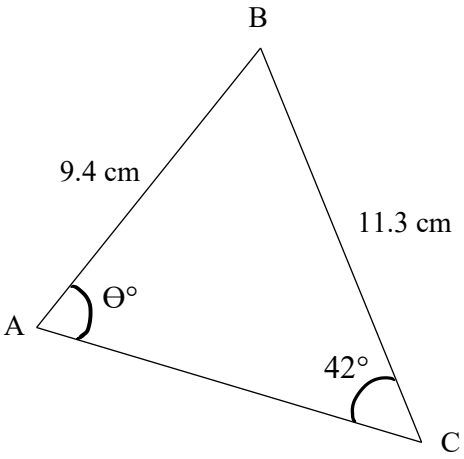
_____

_____

Module 3 – Geometry and Measurement

Question 1 (4 marks)

A diagram of one side of a samosa is shown below. Assume the sides shown are flat.



a. Calculate the possible angle(s) for Θ, rounded to one decimal place.

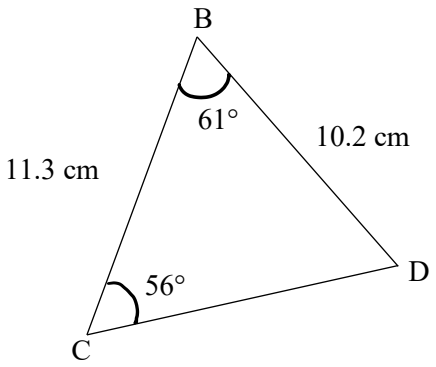
2 marks

_____

_____

_____

b. A diagram of another triangular side of the samosa is shown below.



i. Calculate the length of CD, rounded to one decimal place.

1 mark

_____

_____

_____

ii. Using your answer calculated in **part i.**, calculate the surface area encompassed by the side of the samosa in cm^2. Round your answer to two decimal places.

1 mark

_____

_____

_____

_____

Question 2 (4 marks)

Maria takes a flight from Singapore (1° N, 104° E) to Luxembourg City, Luxembourg (50° N, 6° E). Singapore is 6 hours ahead of Luxembourg City.

a. If Maria departs Singapore at 2:32 am (Singapore time) and arrives in Luxembourg at 11:17 am (Luxembourg time), how long was Maria's flight?

1 mark

_____

_____

_____

Maria then takes a 12 hour and 45-minute flight to Chengdu, China (31° N, 104° E).

Chengdu is 6 hours ahead of Luxembourg.

b.

i. If she departs at 9:53pm, what time does Maria arrive in Chengdu (Chengdu local time)?

1 mark

_____

_____

_____

Chengdu and Singapore are on the same longitude (104° E).

ii. What is the angle between Chengdu and Singapore from the centre of the Earth?

1 mark

_____

_____

_____

iii. What is the shortest distance between Chengdu and Singapore?

1 mark

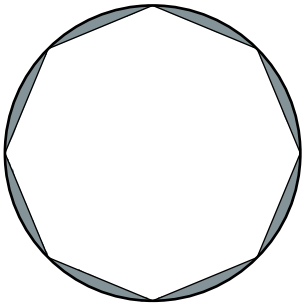
_____

_____

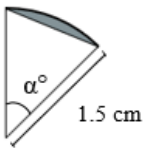
_____

Question 3 (3 marks)

A 2D model of a watch face is shown below. The face has gold border around the edges, the shaded area shown below. The diameter of the watch face is 3 cm.



The watch face can be split up into even sections. A section of the watch face is shown below.



a. Calculate the angle $\alpha°$.

1 mark

_____

_____

b. Calculate the shaded area of the whole watch face, in cm^2, rounded to two decimal place.

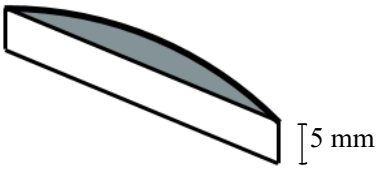
1 mark

_____

_____

_____

c. The gold border rises 5 mm above the face. A diagram of this 3D structure is shown below.



Calculate the amount of gold, in cm^3, required for one section of the watch face.

1 mark

_____

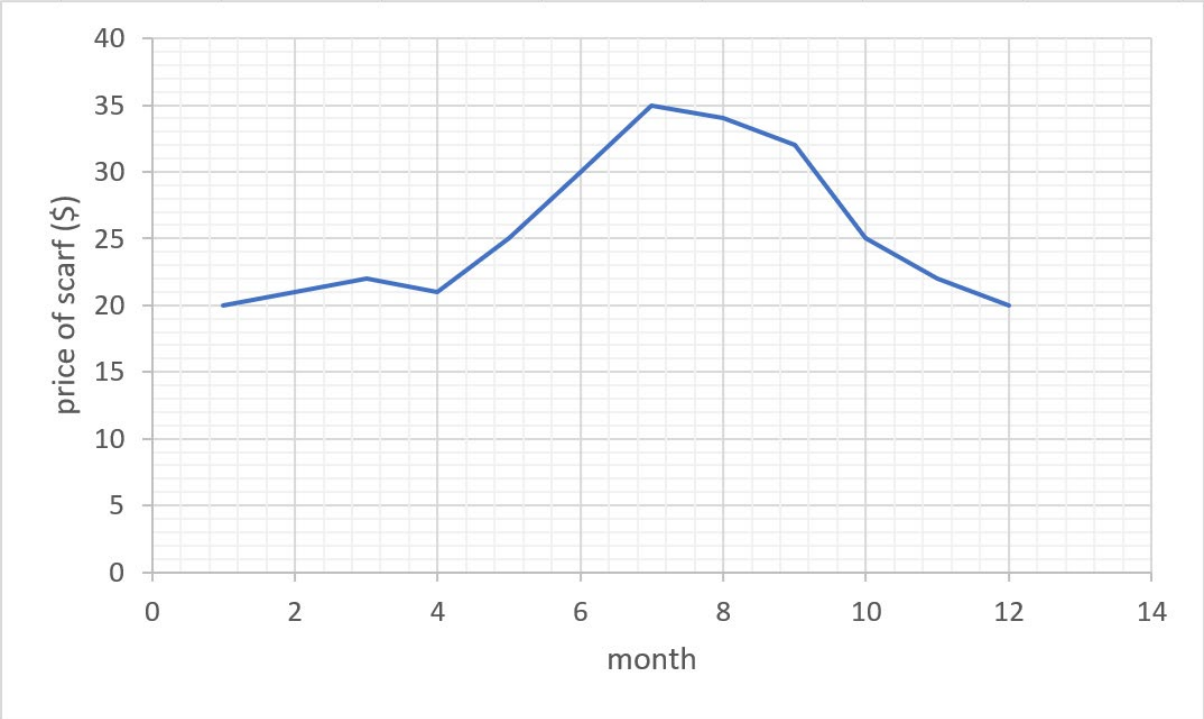
_____

_____

Module 4 – Graphs and relations

Question 1 (2 marks)

Jasmine runs a scarf selling business in her free time. She prices the scarves differently throughout the year according to varying demand. The graph below displays the price of each scarf over a year.



a. What is the price of a scarf in month 4?

1 mark

_____

_____

b. For how many months does Jasmine price her scarves above $25?

1 mark

_____

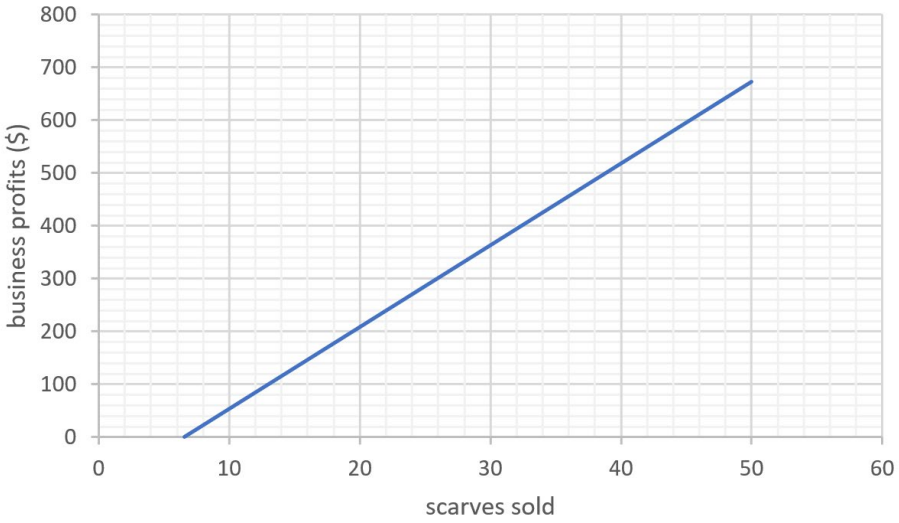
_____

Question 2 (4 marks)

One year, Jasmine has more free time and chooses to buy the equipment necessary to make her scarves from scratch. She also buys the yarn needed to produce the scarves. The following graph shows her total business profits against the number of scarves she sells in month 4.

The equation of the graph is

$$\textit{Business profits} = 15.5 \times \textit{scarves sold} - 102$$



a. How much profit does Jasmine make from selling 15 scarves? Correct your answer to 2 decimal places.

1 mark

_____

_____

b. What is the fixed cost of Jasmine's business for this month?

1 mark

_____

_____

c.

i. Find the x-intercept of the graph. Correct your answer to 2 decimal places.

1 mark

_____

ii. Interpret the x-intercept of the graph.

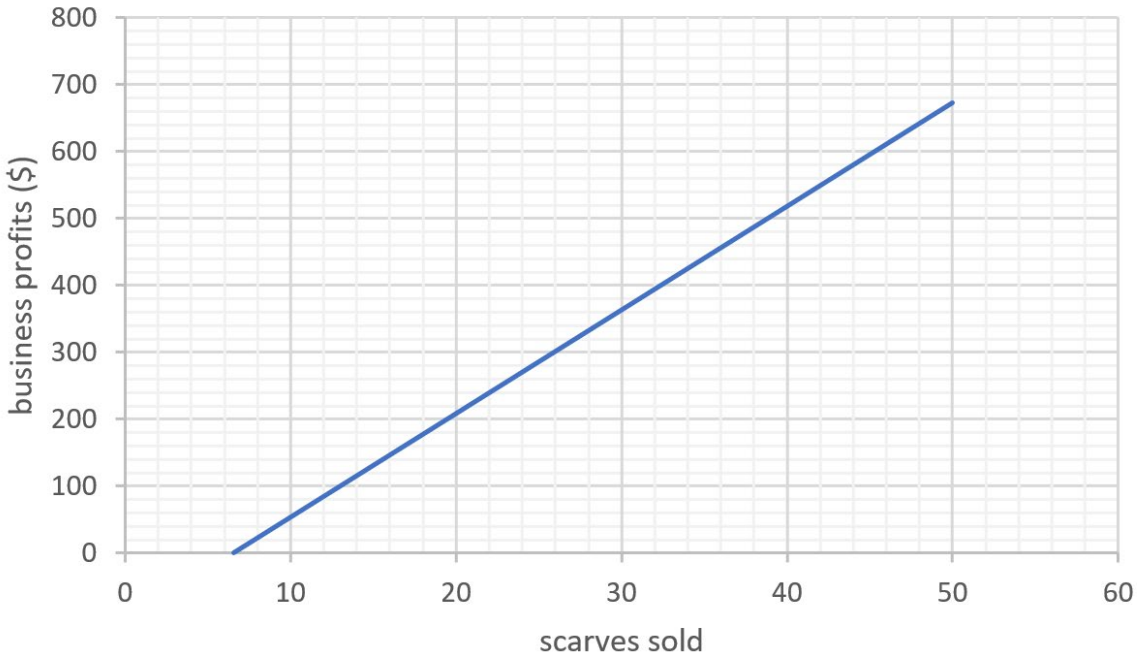
1 mark

_____

Question 3 (3 marks)

In month 4, Jasmine charges $21 for each scarf. In month 5, she charges $23 for each scarf. The price of yarn happens to differ across the two months such that her costs are altered. Her fixed cost of production remains the same.

The following graph displays her profits for month 4.



In the other month, the cost of yarn for producing each scarf is $9.

a. On the graph above, sketch the graph displaying her profits for month 5. Label two coordinates you used to plot the graph. Correct the coordinates to 2 decimal places if necessary.

2 marks

b. Interpret the point of intersection of the two graphs. No calculations are required.

1 mark

_____

_____

Question 4 (5 marks)

Jasmine also starts to sell beanies.

- Let x be the number of scarves she sells in a month.
- Let y be the number of beanies she sells in the same month.
- She makes a maximum of 40 scarves per month
- She makes a maximum of 20 beanies per month
- She produces a maximum of 50 scarves and beanies in total

This information can be represented by inequalities 1.2 and 3

Inequality 1 $x \leq 40$

Inequality 2 $y \leq 20$

Inequality 3 $x + y \leq 50$

There is another constraint given by

Inequality 4 $y \leq \frac{x}{2}$

a. Describe inequality 4 in terms of the number of scarves and beanies Jasmine produces in a month.

1 mark

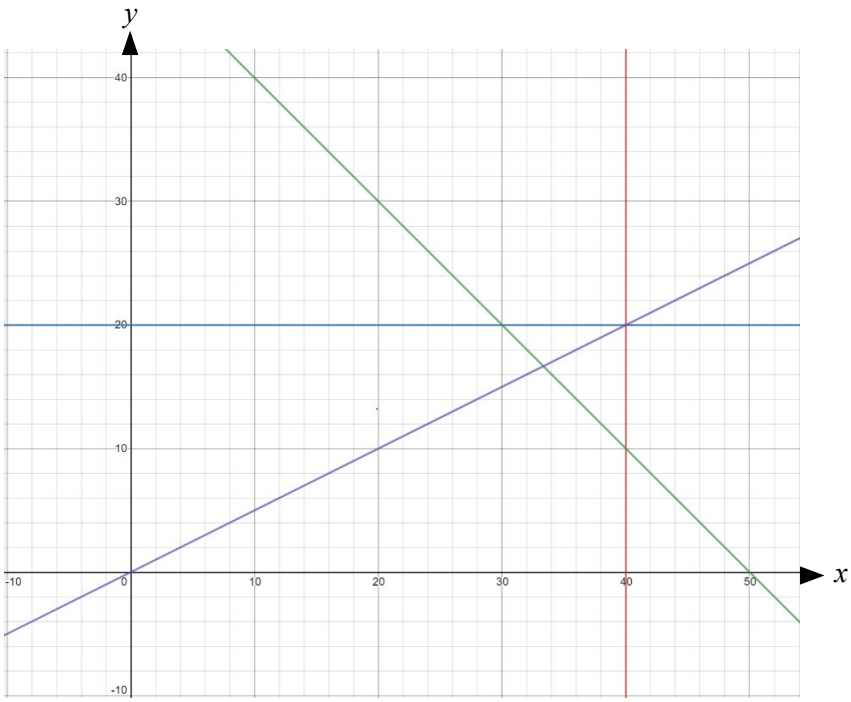
_____

_____

_____

b. The graph below shows the lines that represent the boundaries of inequalities 1 to 4. On the graph below, shade the region that contains the points that satisfy these inequalities

1 mark



The profit, P, that Jasmine makes from the sale of her handmade winter apparel is given by

$$P = 15x + 12y$$

c. Find the maximum profit that jasmine can make from the sale of her scarves and beanie in a month.

1 mark

d. Jasmine wants to change the selling price of her scarves and beanies in order to increase her maximum profit to $850.

All of the constraints on the numbers of scarves and beanies that can be produced each month remain the same.

The profit, Q, that is made from the sale of scarves and beanies is now given by

$$Q = mx + ny$$

The profit made on the scarves is m dollars per scarf

The profit made on the beanies is n dollars per beanie.

The maximum profit is made by selling 36 scarves and 14 beanies.

What are the values of m and n?

2 marks

END OF QUESTION AND ANSWER BOOK