Neap:

Trial Examination 2018

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 (9 marks)

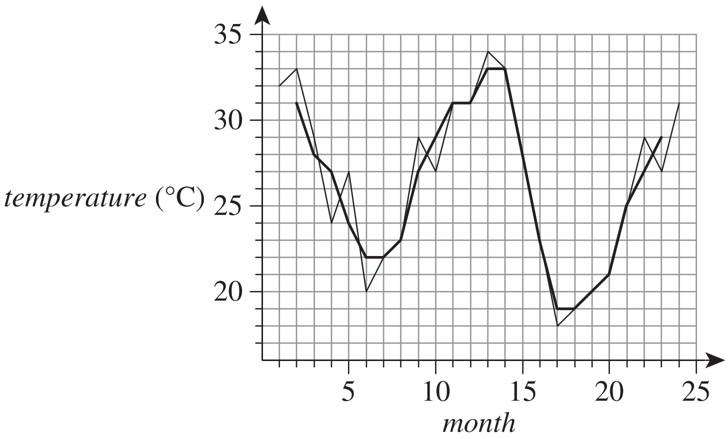
a. **i.** 1 + 3 + 7 + 2 + 4 = 17 months A1

ii. 10 000 = 10^4, so on the log scale we are looking for the number of scores of 4 or more. This score has a frequency of 4 which is a percentage of $\frac{4}{17} \times 100 = 23.5$. Since 23.5 is less than 30 the island is suitable. A1

Note: Demonstration of knowledge of the log scale is required for mark.

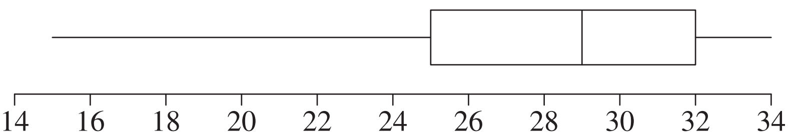
b. **i.** There is a seasonal pattern. A1

ii.



correct method used A1
accurate graph A1

c. **i.**



five-figure summary accuracy A1
accurate boxplot and scale A1

ii. IQR = 32 – 25 = 7, the lower fence will be at 25 – 1.5 × 7 = 14.5. M1

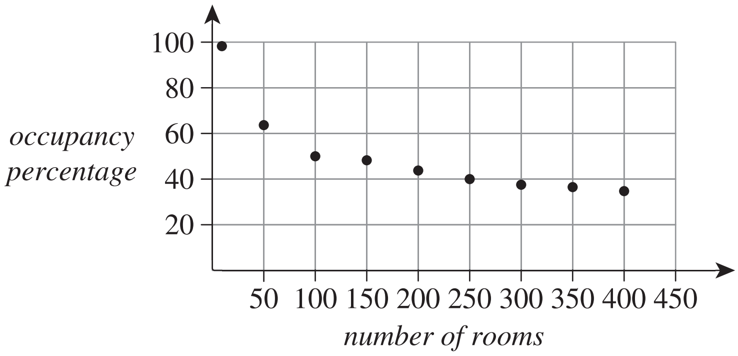
The value of 15 is inside the fence so not an outlier. A1

Question 2 (7 marks)

a. The **number of rooms,** is the explanatory variable because **the percentage occupancy varies in response. The occupancy rate does not cause extra rooms to be built or not built.** A1

Note: An explanation similar to the above is required for mark.

b.



correct labelling A1
correct scatterplot information A1

c. There is a strong non-linear trend. A1

d. correlation coefficient = –0.84 A1

e. **i.** correlation coefficient = –0.99 A1

ii. The data more closely fits a log x relationship than a linear one. A1

Question 3 (3 marks)

a. 95% of the data lies within two standard deviations, which leaves 5% outside of this range; 2.5% at each of the higher and lower ends. $125 + 2 \times 40 = 205$.

The occupancy will exceed 205 rooms 2.5% of the time. A1

b. 85 is one standard deviation below the mean of 125. 68% lies within one standard deviation of the mean (similar to **part a.**), so there will be $\frac{32}{2} = 16\%$ of the data below 85.

$16\% \times 365 = 58.4$, so 58 days. A1

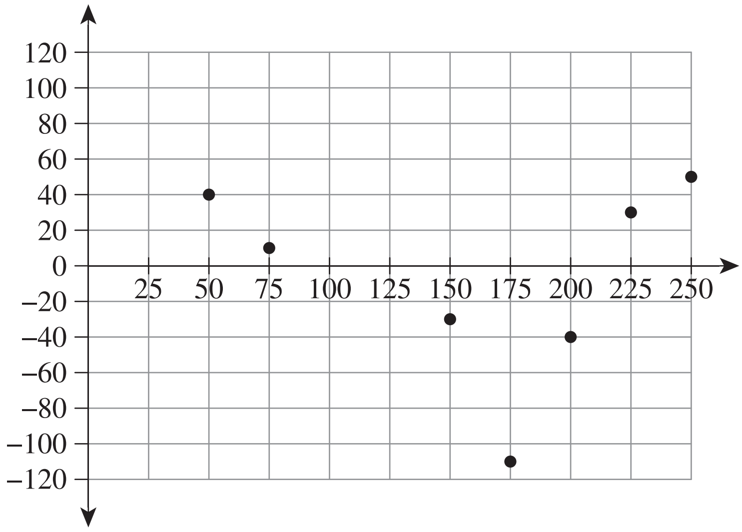
c.
$$z = \frac{x - \mu}{\sigma}$$
$$1.8 = \frac{x - 125}{40}$$
rearranging gives $x = 197$ A1

Question 4 (5 marks)

a. Reading from the graph the y-intercept is +100. A1

Using the points (0, 100) and (250, 400) the gradient is $\frac{300}{250} = 1.2$. The line of best fit equation is $y = 1.2x + 100$. A1

b.



residual plot A1
accuracy A1

c. There is a pattern in the residuals, so the assumption of a linear trend is not supported. A1

Recursion and financial modelling

Question 5 (3 marks)

a. Use a calculator with $N = 1$, $I = 5.2\%$ and with the balance remaining at \$7 000 000.

Alternatively solve $A = 7\,000\,000\left(1 + \frac{\frac{5.2}{12}}{100}\right)$.

The monthly interest only payment is \$30 333.33. A1

b. **i.** $t_{n+1} = t_n \times 1.15$, $t_0 = 300\,000$ A1

ii. Applying the regression formula shows that in year four the profit passes \$400 000. A1

Year 1	t_0	\$300 000
Year 2	t_1	\$345 000
Year 3	t_2	\$396 750
Year 4	t_3	\$456 262.50

Question 6 (4 marks)

a. The 1.06 factor is $1 + \frac{6}{100}$, so 6%. A1

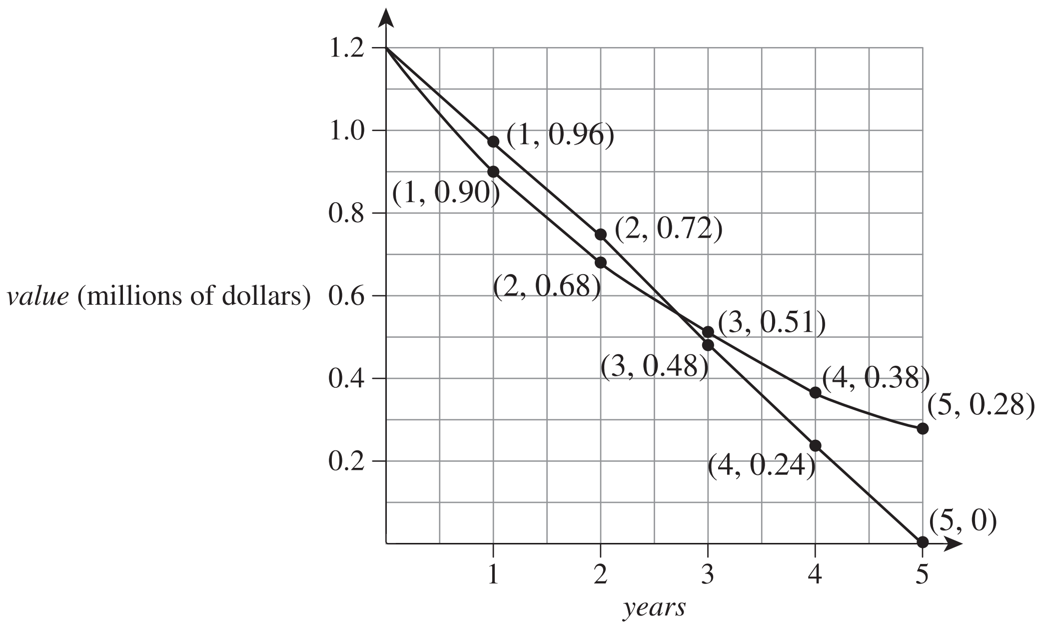
b. Using financial functions on your calculator with $N = 12$, amount borrowed is \$650 000, interest is 4.1% with zero payments. The current value is \$677 156.55, so the interest earned is \$27 156.55. A1

c. Each year the rental figure has increased and the principal upon which the interest is earned has decreased, so the interest will be reduced. A2

1 mark for each reason given.

Question 7 (5 marks)

a.



method A1
accuracy A1

b. From the graph or otherwise, the flat depreciation model becomes the lower value between years 2 and 3. A1

c. After five years the scrap value is \$100 000. Since the purchase price was \$1 200 000, the air-conditioning system has lost \$1 100 000 in value over five years or 5 000 000 hours use (based on 1 million hours per year.) A1

$$\text{depreciation per hour} = \frac{1\,100\,000}{5\,000\,000} = 22 \text{ cents per hour}$$

d. After four years of use, the value will be 1 200 000 – 4 000 000 × 0.22 = \$320 000. A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (5 marks)

a. This diagonal represents a player playing against themselves. A1

b. Both W and Z have a '1' recorded for their game. Since they cannot both win, one of the numbers should be a '0'. A1

c. 20 + 30 + 20 +30 = 100 dresses A1

d.
$$\begin{bmatrix} 8 & 20 & 9 & 100 \\ 10 & 30 & 20 & 140 \\ 20 & 20 & 15 & 150 \\ 10 & 30 & 20 & 180 \end{bmatrix}\begin{bmatrix} 12 \\ 10 \\ 8 \\ 1.20 \end{bmatrix} = \begin{bmatrix} 8 \times 12 + 20 \times 10 + 9 \times 8 + 100 \times 1.20 \\ 10 \times 12 + 30 \times 10 + 20 \times 8 + 140 \times 1.20 \\ 20 \times 12 + 20 \times 10 + 15 \times 8 + 150 \times 1.20 \\ 10 \times 12 + 30 \times 10 + 20 \times 8 + 180 \times 1.20 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 488 \\ 748 \\ 740 \\ 796 \end{bmatrix}$$

$= \$2272$ A1

Note: If a calculator is used, then the matrices being multiplied must be shown to get full marks.

Question 2 (3 marks)

a. *For example:*

Commenting upon both reasons is not necessary. The second set of information is a multiple of the first. The inverse of the matrix produced cannot be found. A1

b.
$$\begin{bmatrix} 3 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 27\,500 \\ 22\,000 \end{bmatrix}$$ A1

c. $P_1 = 2500$, $P_2 = 4000$ A1

Note: Both correct answers required for full marks.

 FMU34EX2_SS_2018.FM

Question 3 (4 marks)

a. This can be done more efficiently on the calculator. Enter the transition matrix as matrix A and S_1 as matrix B and calculate A^2B. A1

OR

$$S_2 = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 850 \\ 1510 \\ 340 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 \times 850 + 0.3 \times 1510 + 0.4 \times 340 \\ 0.3 \times 850 + 0.4 \times 1510 + 0.4 \times 340 \\ 0.5 \times 850 + 0.3 \times 1510 + 0.2 \times 340 \end{bmatrix}$$

$$= \begin{bmatrix} 759 \\ 995 \\ 946 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 759 \\ 995 \\ 946 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 \times 759 + 0.3 \times 995 + 0.4 \times 946 \\ 0.3 \times 759 + 0.4 \times 995 + 0.4 \times 946 \\ 0.5 \times 759 + 0.3 \times 995 + 0.2 \times 946 \end{bmatrix}$$

$$= \begin{bmatrix} 828.7 \\ 1004.1 \\ 867.2 \end{bmatrix}$$ A1

b. Find $S_{60} = A^{60}B$ and $S_{61} = A^{61}B$ are both equal to $\begin{bmatrix} 816.8 \\ 998.3 \\ 884.9 \end{bmatrix}$. A1

Since S_{60} and S_{61} are equal a steady state situation has been reached. A1

c. $S_{n+1} = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} S_n + \begin{bmatrix} 30 \\ 30 \\ 50 \end{bmatrix}$ A1

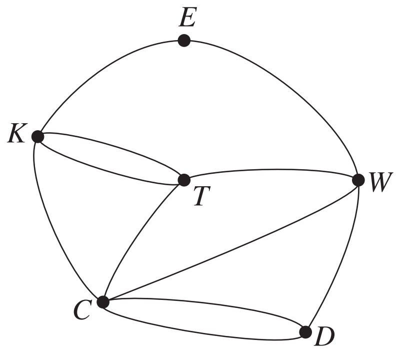
Module 2 – Networks and decision mathematics

Question 1 (2 marks)

a. echidnas and cassowaries A1

Note: Students can also select Tasmanian devils and not have marks deducted.

b. Consider this graph representing the map:

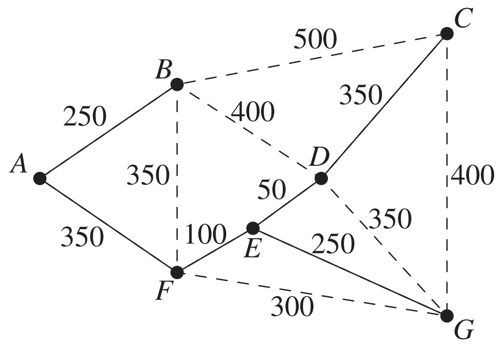


The two pairs are *K*-*T* and *C*-*D*. A1

Question 2 (2 marks)

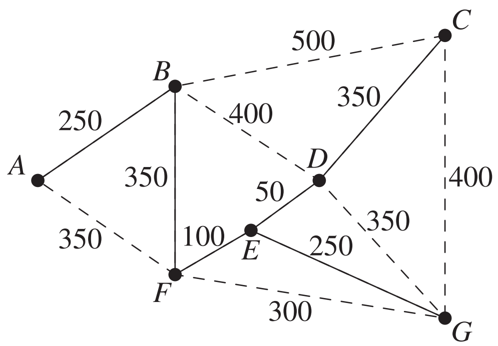
a. minimum length = 250 + 350 + 100 + 50 + 250 + 350 = 1350 A1

b.



A1

OR



A1

Question 3 (3 marks)

a. 30 A1

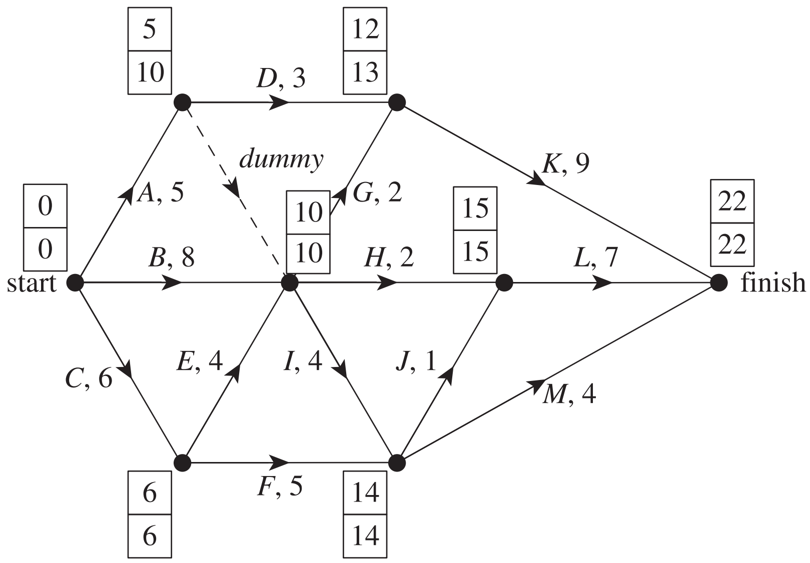
b. The number of lines required to cover the '0's in the table is 4. A1

c.

Worker	Task
A	1
B	4
C	3
D	2

A1

Question 4 (5 marks)



(The earliest start time and latest start time at each vertex is shown.)

a. *A*, *B* and *E* A1

b. *C–E–I–J–L* A1

c. Activity *D* could have its time extended since it has earliest start time = 5 days and latest start time = 10 days (a float time of 5 days), and similarly activity *A* with earliest start time = 0 and latest start time = 5. A1

d. **i.** Reducing *C* by 2 days would result in *C–E–I–J–L* remaining a critical path and *B–I–J–L* becoming a critical path, both with a completion time of 20 days. Reducing *L* by 2 days would result in *C–E–G–K* becoming the critical path with a completion time of 21 days. Therefore, the critical paths resulting in the minimum completion times are *C–E–I–J–L* and *B–I–J–L*. A1

ii. The completion time would be 20 days. A1

Module 3 – Geometry and measurement

Question 1 (2 marks)

a. $\frac{3}{10} \times \text{circumference} = \frac{3}{10} \times \pi \times 9$

$= 8.4823$

$= 8.48 \text{ cm}$ A1

OR

$\frac{4.5 \times \pi}{180} \times 108 = 8.4823$

$= 8.48 \text{ cm}$ A1

b. $\frac{1}{3} \times \pi \times (4.5)^2 \times 21 = 445.321$

$= 445.32 \text{ cm}^3$ A1

Question 2 (2 marks)

Use Pythagoras to find the base length of the right-angled triangle with *AB* as the hypotenuse:

$\sqrt{10^2 - 8^2} = 6$ M1

Create a right-angled triangle using *CD* as the hypotenuse.

The base length is 27 – (18 + 6) = 3

Using trigonometry (adjacent = 3, opposite = 8):

$\tan(x) = \frac{8}{3}$

$\tan^{-1}\left(\frac{8}{3}\right) = 69.444$

$= 69.4°$ A1

Question 3 (3 marks)

a. area scale factor = 2

length scale factor = $\sqrt{2}$

volume scale factor = $(\sqrt{2})^3$ M1

$(\sqrt{2})^3 = 2.82843$

$30 \times 2.8284271247462 = 84.8528$

$= 85.0 \text{ L}$ A1

b. $h = 60$

$r = 15$

$2 \times \pi \times 15^2 + 2 \times \pi \times 15 \times 60 = 7068.58$

$= 7068.6 \text{ cm}^2$ A1

Question 4 (3 marks)

a. Use cosine rule:

$\sqrt{7^2 + 6^2 - 2 \times 7 \times 6 \times \cos(70)} = 7.50135$

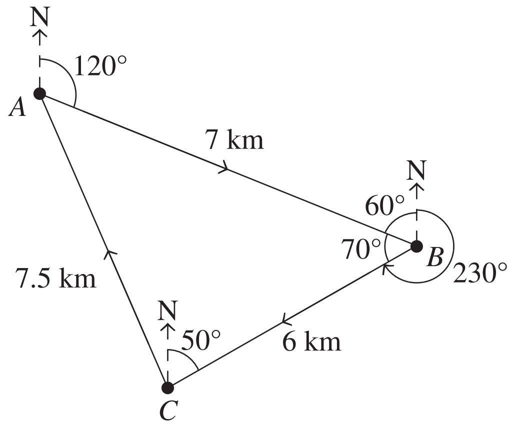
$= 7.5$ km A1

b. Find the angle at point *C* using sine rule:

$\frac{7 \times \sin(70)}{7.5} = 0.877046$

sin(0.87704644606683) = 61.2881 M1

Find the bearing:



Using the north direction as a parallel lines and then using co-interior angles in parallel lines summing to 180.

60 + 70 = 130

180 – 130 = 50

Subtract 50 from the angle at *C* to get the anti-clockwise angle.

61.288104878574 – 50 = 11.2881

Subtract this value from 360 to get clockwise bearing.

360 – 11.288104878574 = 348.712

= 349°

A1

Question 5 (2 marks)

Find the radius at 38° S using trigonometry:

6400 × cos(38) = 5043.27 M1

Find the length of the arc:

174 – 144 = 30

$\frac{5043.268823083 \times 30 \times \pi}{180} = 2640.65$

= 2640 km A1

Module 4 – Graphs and relations

Question 1 (3 marks)

a. $\frac{50}{1.6} = 31.25$ km/h A1

b. John = 10 + 17t

Priti = 26t

Solve 10 + 17t = 26t for t:

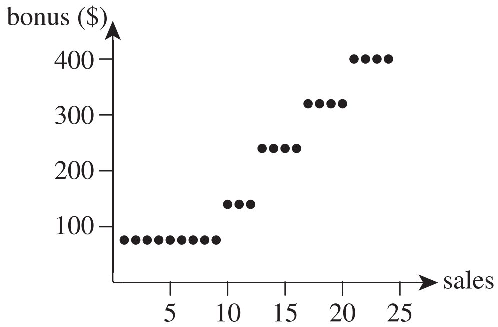
$t = 1.11111$ A1

$60 \times 1.11111 = 66.66667$

Therefore 67 minutes. A1

Question 2 (4 marks)

a.



A1

b. Divide bonus amount by 15 to find an integer solution as sales need to be a whole number.

$\frac{80}{15} = 5.33$

$\frac{140}{15} = 9.33$

$\frac{240}{15} = 16$

$\frac{320}{15} = 21.33$

$\frac{400}{15} = 26.67$

The answer will therefore be 16 sales. A1

c. $800 + 300(11) = \$4100$ A1

d. Solve $800 + 300n = 500n$ for n.

$n = 4$ weeks A1

FMU34EX2_SS_2018.FM

Question 3 (5 marks)

a. *For example:*

The number of freezer units produced must be no more than one-third of the number of fridge units produced. A1

b. $y \geq 10$ A1

c. Maximum profit occurs at (90, 30).

$130 \times 90 + 150 \times 30 = \$16\ 200$ A1

d. Use the sliding rule concept:

Maximum profit now occurs at (100, 20).

The profit line equation has the same gradient (–1).

Therefore, m and n are equal.

$Q = 100m + 20n$

$Q = 120m$

$120m = 18\ 000$ M1

$m = \frac{18\ 000}{120}$

$m = n = 150$ A1