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Year 12 *Trial Exam Paper*

2017

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students must write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 35 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (5 marks)

A sample of travellers on an evening train were asked how many hours (to the nearest hour) they work in a week. The data were sorted into two categories, based on their age.

The results of this survey are shown in the back-to-back stem plot below.

Key: 1|0 = 10 hours

18 – 24 years	**Stem**	*25 – 30 years*
4 4 3 3	**0**	
9 9 8 8 8 6 6 5	**0**	
4 4 2 2 2 0 0	**1**	0 1 2 2
9 9 9 8 8 7 6 5	**1**	8 9
3 3 2 1 1	**2**	2 3 4
7 7 6 5 5 5 5 5	**2**	5 5 6 8 9
2 2 1	**3**	1 3 3 4
	3	5 5 6 7 8 8
	4	0 0 2 2 4 4
5	**4**	5 5 6 7 8
	5	2 2 3 3 4
	5	8

a. What type of data is the variable *hours worked*?

1 mark

_____

b. What percentage of 18–24 year olds work more than 20 hours a week?

Round your answer to one decimal place.

1 mark

_____

c. Find the median for the data gathered on *hours worked* for those in the 18–24 years category.

1 mark

_____

d. The statistician conducting the survey suspects that the data for the 18–24 years age group contain an outlier at 45 hours.

Is the statistician correct?

Explain your answer, showing a calculation to support your argument.

2 marks

_____

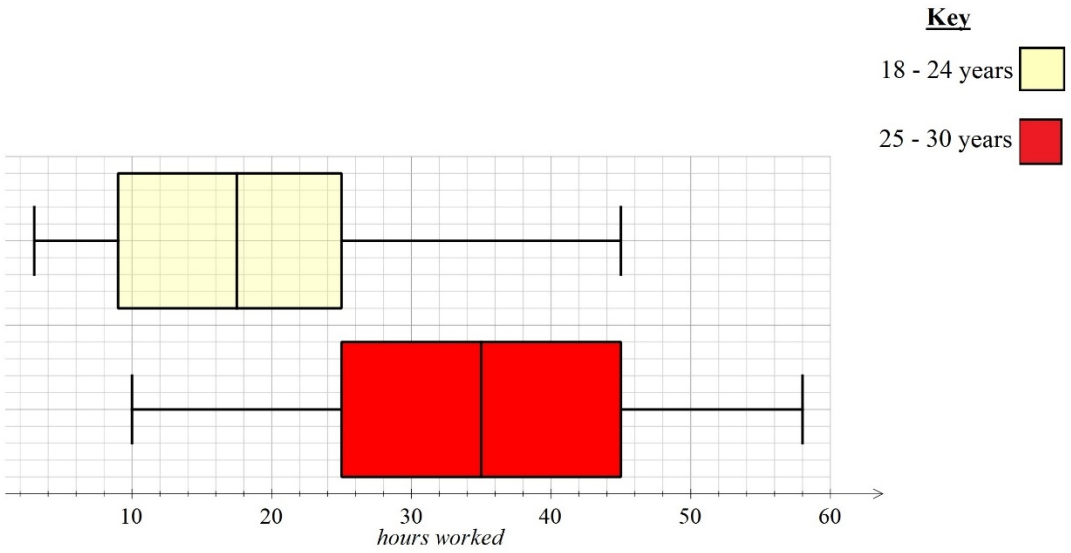
_____

_____

_____

Question 2 (5 marks)

A boxplot has been drawn for the *hours worked* data gathered. These boxplots are shown below.



a. Describe the boxplot for the 25–30 years age group, in terms of shape, centre and spread.

1 mark

_____

_____

b. Which measure of centre, **mean** or **median**, would be most appropriate for a boxplot showing a positive skew?

Explain your answer.

2 marks

_____

_____

c. Is there a correlation between the variables *age group* and *hours worked*?

Quote appropriate statistics in your explanation.

2 marks

_____

_____

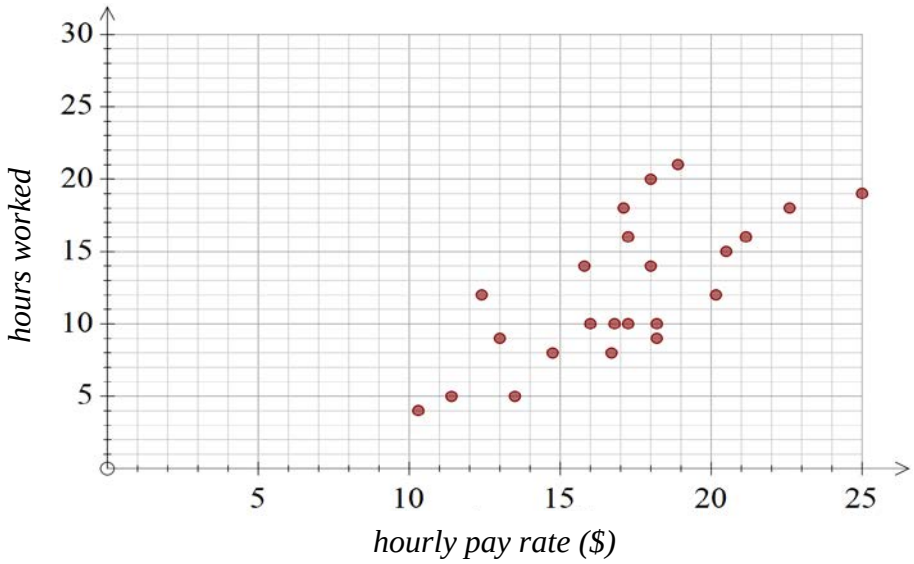
_____

Question 3 (9 marks)

A smaller group of train travellers were surveyed on the morning train. Those who specified that they were in the 18–24 years age group were also asked what their *hourly pay rate* was, with the intention of identifying an association between *hours worked* and *hourly pay rate*.

The data are shown in the table below and are also displayed as a scatterplot.

Hourly pay rate ($)	***Hours worked***
11.40	5
18.20	10
10.30	4
12.40	12
13.50	5
16.00	10
13.00	9
14.75	8
16.80	10
16.70	8
18.20	9
25.00	19
17.25	10
15.80	14
20.16	12
21.15	16
22.60	18
18.00	14
20.50	15
17.25	16
18.90	21
17.10	18
18.00	20



a. Use the scatterplot to describe the relationship between *hourly pay rate* and *hours worked* in terms of strength, direction and form.

1 mark

_____

_____

b. Find the value of Pearson's correlation coefficient, rounding your answer to two decimal places.

1 mark

_____

_____

c. Determine the equation of the least squares regression line that can be used to predict the number of *hours worked* from the *hourly pay rate*.

Round the values for the slope and intercept to two significant figures.

2 marks

_____

_____

d. Interpret the **slope** of the least squares regression line in terms of the variables *hourly pay rate* and *hours worked*.

1 mark

_____

_____

e. The value of the coefficient of determination for the association between the variables *hourly pay rate* and *hours worked* is 0.5159.

Interpret the coefficient of determination in terms of these variables.

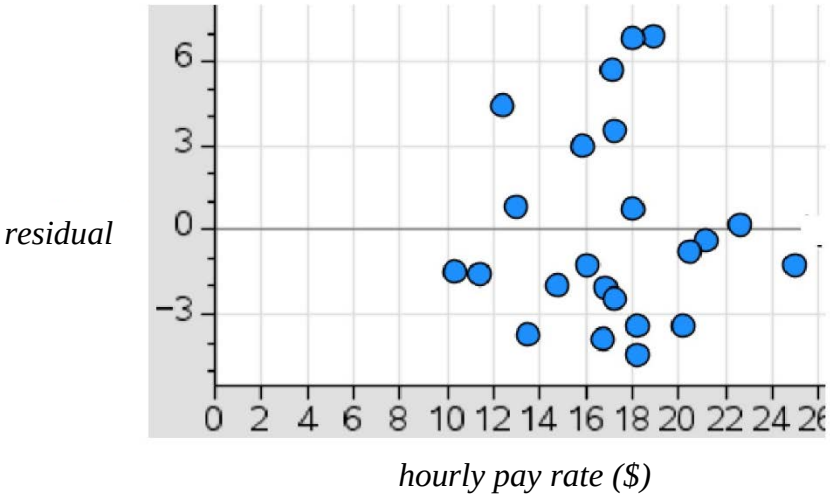
1 mark

_____

_____

_____

f. A residual plot was obtained when the least squares regression line was fitted to the data. This is shown below.



i. Does the residual plot support the assumption that there is a linear association between *hourly pay rate* and *hours worked*?

Explain your answer.

2 marks

_____

_____

_____

ii. Find the residual value for a train passenger who is paid $25.00 per hour, and works 17 hours in a week.

Round your answer to one decimal place.

1 mark

_____

_____

_____

Question 4 (5 marks)

A Year 12 student recorded the amount of sleep that she had in the months leading up to her November exams.

The data she recorded, shown to the nearest hour, are given in the table below.

A time series graph has also been produced.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct
Month number	1	2	3	4	5	6	7	8	9	10
Sleep per month (hours)	282	252	265	257	262	224	218	198	172	165

a. Describe the time series plot above.

1 mark

_____

b. Find the equation for the least squares trend line for this time series data, using the variables *sleep per month* and *month number*.

Round your coefficients correct to one decimal place.

1 mark

_____

c. Explain why it would **not** be appropriate to use this trend line to predict the hours for *sleep* for this student in January of the following year.

1 mark

_____

_____

d. Using the data provided, show that the seasonal index for July (Month 7) is 0.95.

2 marks

_____

_____

_____

_____

Recursion and financial modelling

Question 5 (6 marks)

Casey has opened a simple interest savings account in order to save money to buy a new motorbike. Interest is paid monthly into this account.

The amount of money in the savings account after *n* years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 6000, \qquad V_{n+1} = V_n + 216$$

a. How much money did Casey initially deposit in his savings account?

1 mark

_____

b. What is the annual percentage interest rate for this savings account?

2 marks

_____

_____

_____

c. The amount of money in the account after *n* years, V_n, can also be determined using a rule.

i. Complete the rule by writing the appropriate numbers in the boxes provided below.

1 mark

$$V_n = \square + \square \times n$$

ii. How much money will be in Casey's savings account after 5 years?

1 mark

_____

_____

_____

Casey decides to compare a compound interest savings account to his savings in the simple interest account.

He invests \$6000 at an interest rate of 5.4% compounding annually.

The amount of money in the savings account after n years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 6000, \qquad V_{n+1} = 1.054 \times V_n$$

d. Use recursion to write down calculations that show that the amount of money in Casey's compound interest savings account after 2 years, V_2, will be \$6665.50.

1 mark

Question 6 (3 marks)

Casey's first motorbike had a purchase price of $37 000.

After five years, the value of the motorbike was $29 600.

a. Let C_n be the value of the motorbike *n* years after it was purchased.

Assume that the value of the motorbike has depreciated using the **flat rate** method of depreciation.

Write a recurrence relation, in terms of C_n and C_{n+1}, that models the value of the motorbike.

2 marks

_____

_____

_____

_____

b. The motorbike has travelled an average of 4000 km in each of the five years since it was purchased.

Assume that the value of the motorbike has been depreciated using the **unit cost** method of depreciation.

By how much is the value of the motorbike reduced per kilometre travelled?

1 mark

_____

_____

_____

Question 7 (3 marks)

Katerina has borrowed $45 000 to buy a new car.

She will be charged interest at a rate of 7.2% per annum, compounding monthly.

a. For the first year (12 months), Katerina will make monthly repayments of $550.
Find the amount that Katerina will owe on her loan after she has made 12 repayments.

1 mark

_____

_____

b. After four years, Katerina will increase her repayments in order to fully repay her loan in a further two years.

Find the value of Katerina's new repayments, in order to repay the loan during these two years.

2 marks

_____

_____

_____

_____

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (4 marks)

A postal delivery company charges fees for postage based on the size of the parcel: small (*S*), medium (*M*), large (*L*) and overweight (*O*).

Matrix *C* contains the cost of each size parcel.

$$C = \begin{matrix} S & M & L & O \end{matrix} \begin{bmatrix} 4 & 5.60 & 7.20 & 9 \end{bmatrix}$$

The number of each type of parcel that was delivered in January 2016 is shown in the matrix *P* below.

$$P = \begin{bmatrix} 250 \\ 135 \\ 180 \\ 65 \end{bmatrix} \begin{matrix} S \\ M \\ L \\ O \end{matrix}$$

a. Write down the order of matrix *P*.

1 mark

_____

b. **i.** Calculate the matrix product $M = C \times P$.

1 mark

_____

_____

_____

ii. What does matrix *M* represent?

1 mark

_____

_____

_____

iii. Explain why the matrix product $P \times C$ also exists.

1 mark

_____

_____

_____

Question 2 (8 marks)

The postal company has four distribution points within a small town: Abbey (*A*), Buron (*B*), Cally (*C*) and Dovan (*D*).

The company is studying the choice made by its customers between the four distribution points within the town.

Matrix *T*, shown below, contains the percentages of customers who are expected to change where they post their parcel from month to month.

$$T = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \textit{this month} & & & \\ A & B & C & D \end{array} \\ \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \end{array} \quad \textit{next month}$$

Let S_n be the matrix that shows the number of parcels delivered from each distribution centre *n* months after January 2016.

Matrix S_0 below shows the number of parcels delivered from each distribution centre in January 2016.

$$S_0 = \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array}$$

Matrix S_1 below shows the number of parcels delivered from each distribution centre in February 2016.

$$S_1 = TS_0 = \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array}$$

a. Find the value of *f*, which is missing from the matrix S_1.

1 mark

b. Write a calculation that shows that 134 parcels were expected to be delivered to Cally in February 2016.

1 mark

_____

_____

c. Consider the customers who chose the Dovan distribution centre in January 2016.

i. What percentage of these customers were expected to choose Dovan in February 2016?

1 mark

_____

ii. How many of these customers are expected to change their parcel distribution centre in February 2016?

1 mark

_____

d. In the long term, how many customers are expected to choose Abbey as their preferred distribution centre? Show matrix calculations to support your answer.

2 marks

_____

_____

_____

_____

In January 2017, the postage company increases its business by extending the zone in which it will collect and deliver parcels.

The matrix J_{2017} shows the number of customers who chose each distribution centre in January 2017.

$$J_{2017} = \begin{bmatrix} 350 \\ 225 \\ 270 \\ 240 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

The company intends to continue to grow its customer base through increased advertising initiatives.

The matrix that contains the number of customers who are expected to choose each distribution centre in February 2017 (F_{2017}) and March 2017 (M_{2017}) can be determined using the following matrix equations

$$F_{2017} = T \times J_{2017} + B \qquad M_{2017} = T \times F_{2017} + B$$

where

$$T = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \text{this month} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \\ \text{next month} \end{array}$$

and $B = \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$

e. Determine the number of customers who are expected to choose the Burton distribution centre in March 2017.

Round your answer to the nearest whole number.

2 marks

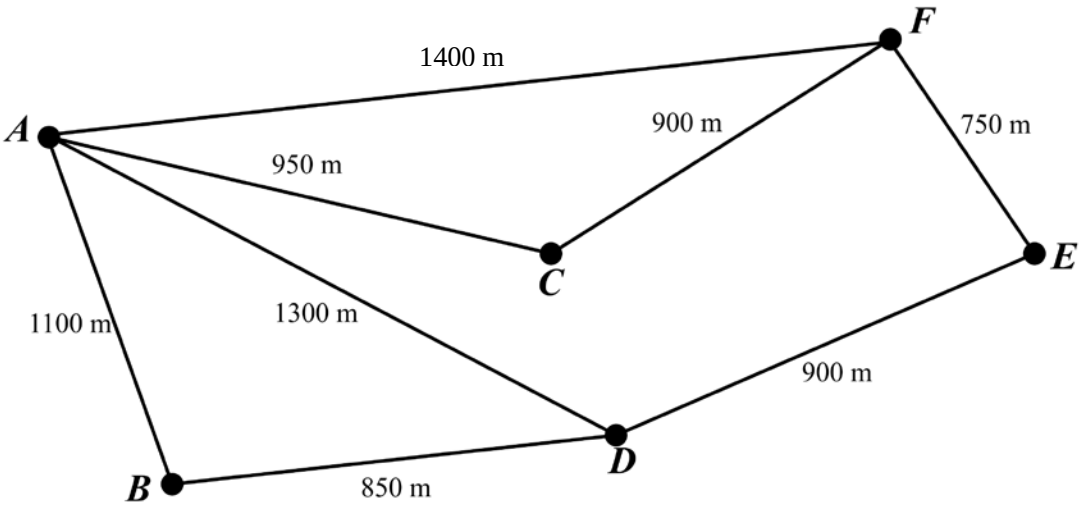
Module 2 – Networks and decision mathematics

Question 1 (4 marks)

A local postal company has six distribution points within a small town: Abbey (*A*), Buron (B), Cally (*C*), Dovan (*D*), Ellery (*E*) and Fova (*F*).

On the graph below, the distribution centres are shown as vertices *A*, *B*, *C*, *D*, *E* and *F*.

The edges are roads connecting the distribution centres. Distances between distribution centres are also marked on the graph.



a. What is the shortest path from the Abbey distribution centre to Ellery?

1 mark

_____

b. A postal worker wishes to form a Hamiltonian circuit, starting at Burton, in order to efficiently visit each of the distribution centres.

Write down a possible circuit.

1 mark

_____

c. Explain why it is not possible to find an Eulerian circuit for this network.

1 mark

_____

_____

_____

d. If the postal worker wished to form an Eulerian trail through this network, at which distribution centres should he start and finish?

1 mark

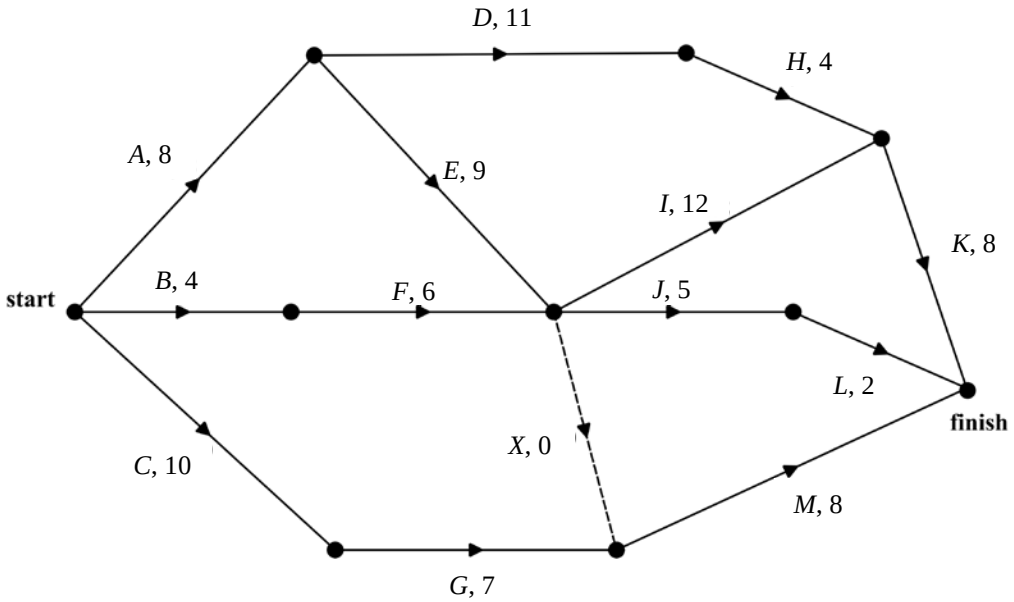
_____

Question 2 (5 marks)

A new distribution centre will be built by the postage company.

This project involves 12 activities, *A* to *L*.

The directed network below shows these activities and their completion times, in days.



a. Determine the earliest start time for activity *F*.

1 mark

_____

b. Find the critical path and minimum completion time for the project.

1 mark

_____

_____

c. Which activities have a float time of 7 days?

1 mark

_____

_____

d. Explain the purpose of the dummy activity, X_0, in the context of the project.

1 mark

_____

_____

_____

e. The completion times for activities *C, E, F, I* and *M* can each by reduced by one day.

The cost of reducing the completion time by one day for these activities is shown in the table below.

Activity	**Cost ($)**
C	2000
E	4000
F	1000
I	4000
M	3000

What is the minimum cost to complete the project in the shortest time possible?

1 mark

_____

_____

_____

_____

Question 3 (3 marks)

Four postage workers are required to complete one job each at the Buron distribution centre.

The postal workers each can complete the required jobs in a different time, recorded to the nearest minute.

This information is shown in the table below.

	Addressing	**Weighing**	**Packaging**	**Sorting**
Alistair	22	32	35	14
Brianna	28	31	32	18
Christine	25	36	37	16
Danielle	26	30	32	17

a. Complete the Hungarian algorithm, using the boxes below as required, in order to find the minimum allocation for the four workers to complete one job each.

1 mark

b. What is the allocation that will result in the minimum completion time, and what is the minimum completion time?

2 marks

_____

_____

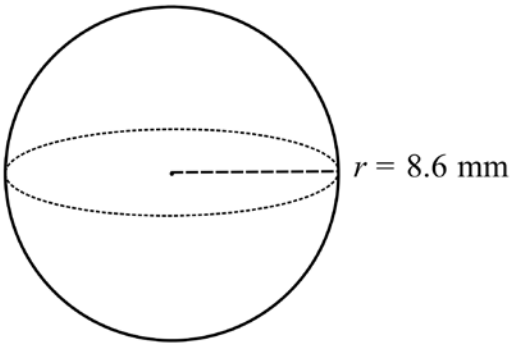
_____

_____

Module 3 – Geometry and measurement

Question 1 (2 marks)

A shot-put is spherical in shape and has a radius of 8.6 mm, as shown in the diagram below.



Assume that the surface of the shot-put is smooth.

a. What is the **surface area** of the shot-put shown?

Round your answer to the nearest square millimetre.

1 mark

_____

_____

_____

_____

b. What is the **volume** of the shot-put shown?

Round your answer to the nearest millimetre cubed.

1 mark

_____

_____

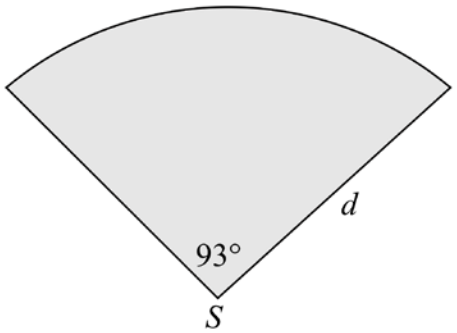
_____

_____

Question 2 (3 marks)

The athletics club plans to update the shot-put pitch.

Plans for the new pitch have been drawn as shown below.



a. The shaded area on the plan will be filled with sand.

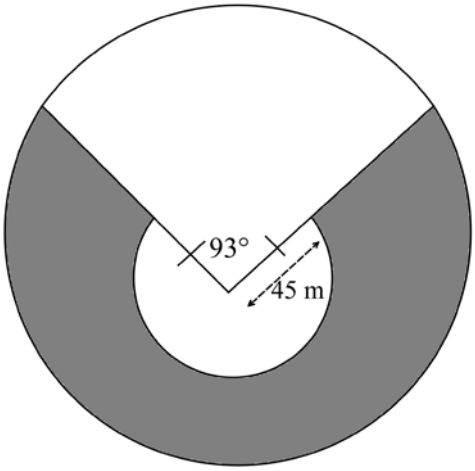
If the athletics club has access to sufficient sand to cover an area of 9642.4 m^2, what is the maximum distance, d metres, from the point S to the end of the pitch?

Round your answer to the nearest whole number.

1 mark

b. The new pitch will be surrounded by a spectator's area to allow for family and friends to watch the shot-put.

A diagram of this area is shown below



The spectator's area will be paved. This area is shaded on the diagram above.

What is the area that requires paving?

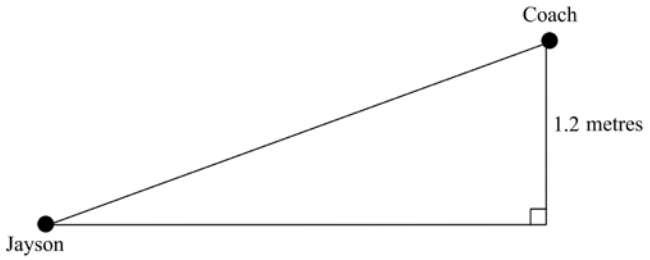
Round your answer to the nearest square metre.

2 marks

Question 3 (2 marks)

Jayson has asked his coach to observe his shot-put from the spectator area. The coach brings a high stool, which is 1.2 metres off the ground, in order to better critique Jayson's technique.

The position of the coach's stool is shown in the diagram below.



a. If the coach's direct line of sight to Jayson is a length of 2.2 metres, how far away from Jayson has the coach placed his stool?

Round your answer to one decimal place.

1 mark

_____

_____

_____

b. What is the angle of depression from the coach to Jayson's position?

Round your answer to the nearest degree.

1 mark

_____

_____

_____

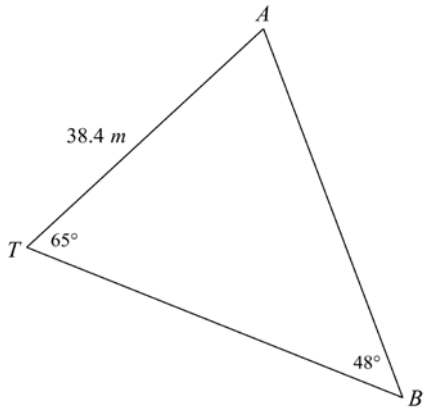
Question 4 (5 marks)

Michaela practises her technique on the pitch by throwing her shot-put from point *T*.

The first shot-put that Michaela throws lands at point *A*.

The second shot-put that she throws lands at point *B*.

The diagram below shows the positions of the two shot-puts after they have landed.



a. How far apart are the two shot-puts, to the nearest metre?

1 mark

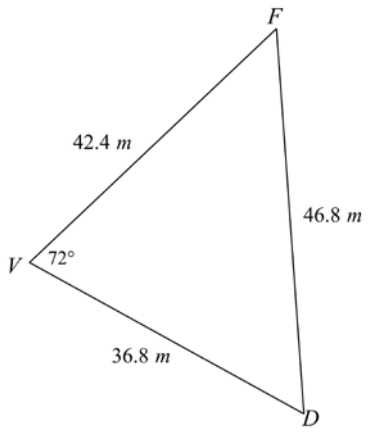
b. Michaela walks to point *A* first, before turning to retrieve the shot-put from point *B*.

If point *T* is at a bearing of $227°$ from point *A*, what is the bearing of point *B* from *A*?

2 marks

A second shot-putter, Karen, is also practising her shot-put.

She throws two shot-puts that land as shown on the diagram below.



c. Use the cosine rule to show that the angle $\angle VFD$ is equal to $48°$.

2 marks

Module 4 – Graphs and relations

Question 1 (5 marks)

An athletics club must pay its workers a minimum of $25 a day, plus an additional $7.50 per hour.

The *cost*, in dollars, of paying an athletics club worker for the number of *hours worked* can be found using the equation:

$$cost = 25 + 7.50 \times hours\ worked$$

a. **i.** What is the hourly rate for an athletics club worker?

1 mark

_____

ii. How many hours has Monique worked at the athletics club if she is paid $130 in one day?

1 mark

_____

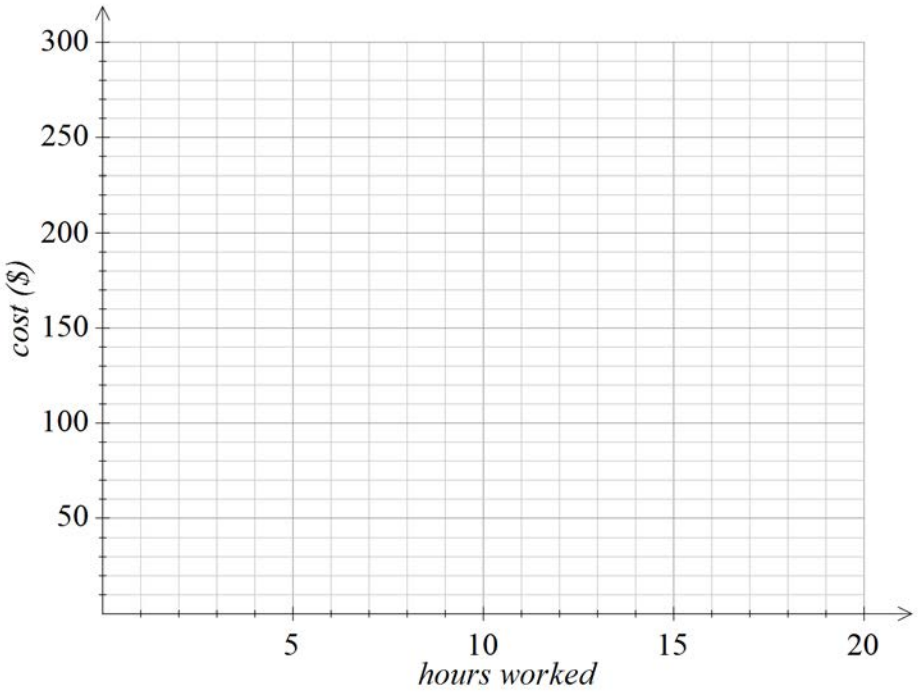
_____

_____

b. On the grid below, sketch the graph of the relationship between payroll *cost* and the number of *hours worked*.

1 mark

(*Answer on the grid below.*)



The athletics club will break even on their employee costs, with employees being paid for 150 hours in total on this day, when they sell 300 tickets to their weekly athletics tournament.

c. Find the price of one ticket.

2 marks

Question 2 (3 marks)

The athletics manager implements a pay scale, based on the age of employee.

The graph below shows the age and hourly pay rate for employees.

a. Mary is 18 years old. What is her hourly pay rate?

1 mark

_____

b. What is the minimum age for the athletics company to hire an employee, based on the information provided on the graph?

1 mark

_____

The following is the rule for the step graph above, outlining the hourly pay rate based on age.

$$\text{pay rate} = \begin{cases} 9.00, & 15 < age \leq x \\ 12.00, & 18 \leq age \leq 21 \\ 15.00, & 22 \leq age \leq 24 \\ 18.00, & 25 \leq age \leq 27 \\ 22.00, & age \geq 27 \end{cases}$$

c. What is the value of x in the rule above?

1 mark

_____

Question 3 (4 marks)

The athletics club is considering upgrading its shot-puts and must purchase two sizes: 1.5 kg and 4 kg. They have found a company that produces both weights.

Let x be the number of 1.5 kg shot-puts produced by the company, each month.

Let y be the number of 4 kg shot-puts produced by the company, each month.

Each month, up to 600 shot-puts in total can be produced.

The inequalities below represent constraints on the number of each type of shot-put that can be produced each month.

Constraint 1: $x \geq 50$

Constraint 2: $y \leq 250$

Constraint 3: $x + y \leq 600$

a. Interpret Constraint 3 in terms of the number of 1.5 kg and 4 kg shot-puts produced each month.

1 mark

_____

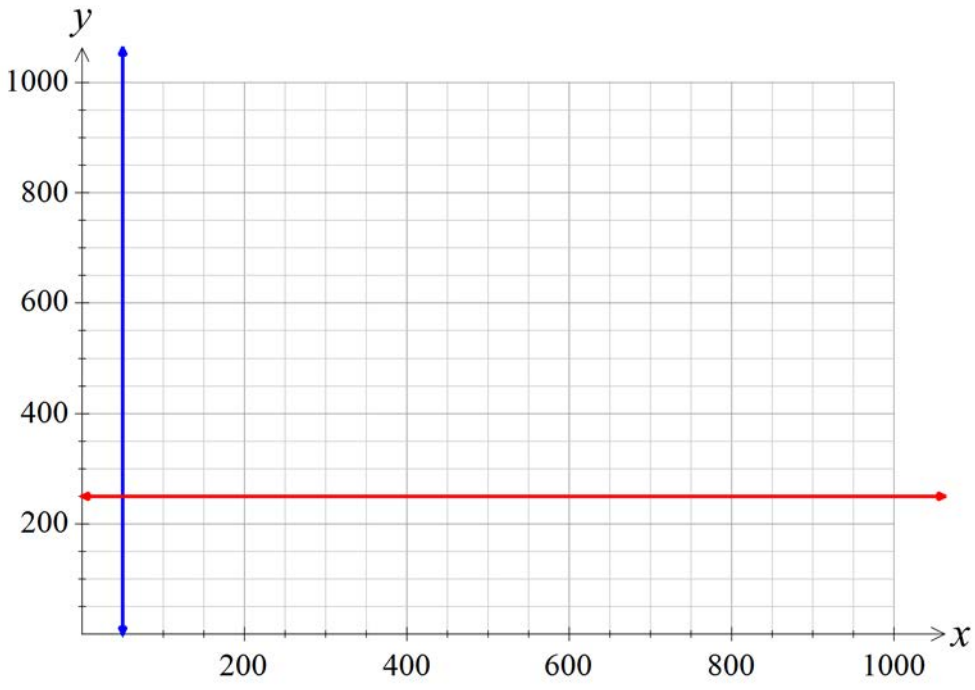
_____

_____

b. Complete the graph below by sketching Constraint 3 and shading the region specified by Constraints 1, 2 and 3.

2 marks

(*Answer on the graph below.*)



The revenue, R, that the company makes from the sale of the shot-puts is given by:

$$R = 45x + 68y$$

c. Find the maximum revenue that the company can make from the sale of shot-puts.

1 mark

END OF QUESTION AND ANSWER BOOK