2016 VCAA Further Mathematics Sample (v3 July) Exam 2 Solutions

SECTION A - Core

Data analysis

Q1a 19%

Q1b 23% of 128 000 000 = 29 440 000

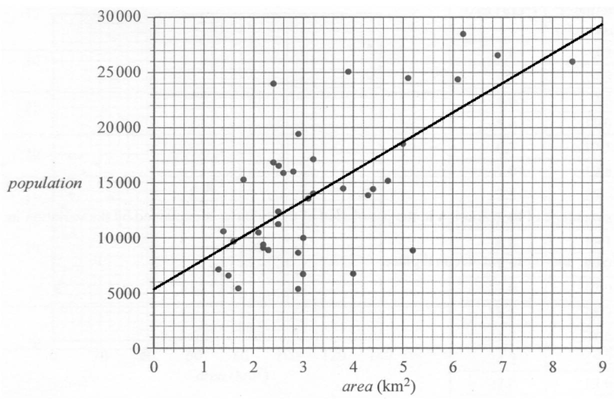
Q1c Little difference in the percentages of people in the 15-64 group among the 3 countries.
Australia: 33%; India: 36%; Japan: 36%

Q2a The mode: 78 The range: 9

Q2b Lower fence $= Q_1 - 1.5\times IQR = 70.5$
$70 < 70.5$, .: 70 is an outlier

Q3a Response variable: Population

Q3b



Q3c For each km^2 greater in area of an inner suburb the population of the inner suburb is higher by 2680.

Q3di $\textit{predicted population} = 5330 + 2680\times 4 = 16050$
$\textit{residual} = \textit{observed value} - \textit{predicted value}$
$= 6690 - 16050 = -9360$

Q3dii $r^2 = 0.668^2 \approx 0.446 = 44.6\%$

Q4a $\textit{population} = 7700 + 7700\log(\textit{area})$

Q4b $\textit{population} = 7700 + 7700\log(90) \approx 23\,000$

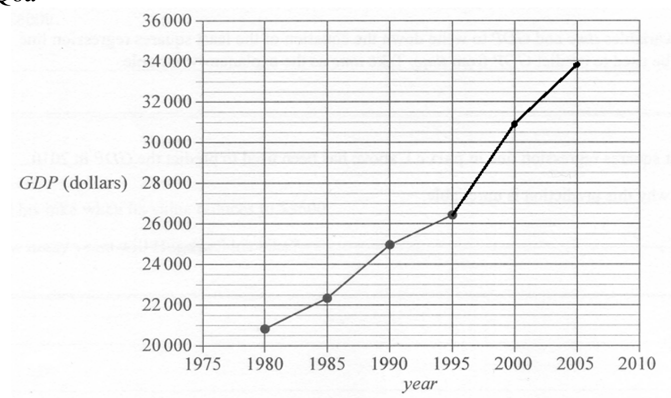
Q5a $\textit{population density} = \dfrac{\textit{population}}{\textit{area}}$, inverse relationship between population density and area, .: $r = -\sqrt{0.141} \approx -0.375$

Q5bi $z = \dfrac{3082-4370}{1560} \approx -0.8$

Q5bii $z = -0.8$ indicates that the suburb's population density is 0.8 of the standard deviation lower than the mean population density.

Q5biii 2.5% of 38 ≈ 1

Q6a



Q6b General trend: Positive, approximately linear

Q6ci $GDP = 20000 + 524\times \textit{time}$

Q6cii GDP depends on the world economy and government policies. Predictions based on past figures are unreliable.

Recursion and financial modelling

Q7a $V_1 = 8400 - 1200 = 7200$, $V_2 = 7200 - 1200 = 6000$ dollars

Q7b Number of years $= \dfrac{8400-3600}{1200} = 4$

Q7c \$ 0.25 per kilometre

Q7d $8400 - 0.25n = 6000$, $n = 9\,600$

Q8a $0.004\times 12 = 0.048 = 4.8\%$ per annum

Q8b $S_{12} = 1.004^{12}\times 5\,000 \approx 5\,245.35$ dollars

Q8ci $T_0 = 3\,000$, $T_{n+1} = 200 + 1.0035\,T_n$

Q8cii $T_1 = 200 + 1.0035\times 3\,000 = 3\,210.50$
$T_2 = 200 + 1.0035\times 3\,210.50 = 3\,421.73675$
$T_3 = 200 + 1.0035\times 3\,421.73675 = 3\,633.712829$
$T_4 = 3\,846.430824$, $T_5 = 4\,059.893331$, $T_6 \approx 4\,274.10$
Total interest $= 4\,274.10 - 3\,000 - 6\times 200 = 74.10$ dollars

Q9a Amount owing after n months
$= 7500\left(1 + \dfrac{5.76}{12\times 100}\right)^n = 7500\times 1.0048^n$
Total repaid after n months $= 430n$
Let $7500\times 1.0048^n = 430n$, $n \approx 19.114$, .: 19 repayments

Q9bi $7500\times 1.0048^{12} = 430\times 5 + Q\times 7$, $Q = 827.65$ dollars

Q9bii $7500\times 1.0048^{12} = 430\times 5 + 827.65\times 6 + R$
$R = 827.70$ dollars

SECTION B

Module 1: Matrices

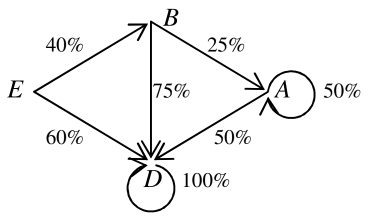
Q1a The sum of the elements in row 3 of matrix W is 2.
It represents the direct connections of 2 ponds by pipes to pond R.

Q1b The number 2 indicates that pond R is connected to pond Q through another pond by pipes in 2 different ways.

Q2ai $0.6 \times 10\,000 = 6\,000$

Q2aii

$E \xrightarrow{40\%} B$, $E \xrightarrow{60\%} D$, $B \xrightarrow{25\%} A$, $B \xrightarrow{75\%} D$,
$A \xrightarrow{50\%} A$, $A \xrightarrow{50\%} D$, $D \xrightarrow{100\%} D$



Q2bi

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

Q2bii

$$S_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}^4 \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 331 \\ 11469 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$

.: 331 adult trouts

Q2ci 13 years

Q2cii The largest number is predicted to be 1325 after 2 years.

Q2d From S_0 to S_1, $E = -10000$, $B = +3000$ and $A = -150$.
To maintain a constant population, add 10 000 eggs, remove 3000 baby trouts and add 150 adult trouts.

Q2e $S_2 = (T + 500M)^2 S_0$

$$= \begin{bmatrix} 0 & 0 & 250 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}^2 \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 162500 \\ 80000 \\ 1325 \\ 130475 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$

Module 2: Networks and decision mathematics

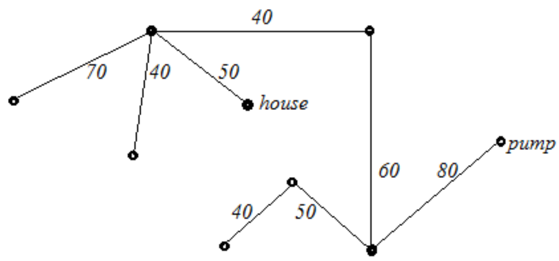
Q1ai $70 + 90 = 160\,\text{m}$

Q1aii 2 vertices, the house and the top right location

Q1aiii An Eulerian path exists, starting from the house (odd degree) and finishing at the other odd-degree vertex with a distance of 1180 m. There is a another 70 m distance from this odd-degree vertex back to the house.
Total distance $= 1180 + 70 = 1250\,\text{m}$

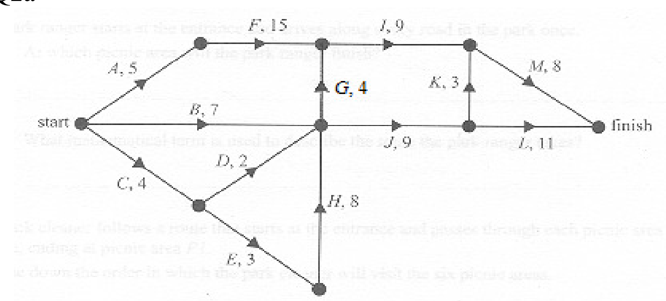
Q1aiv 8 edges

Q1bi



Q1bii Minimum spanning tree

Q2a



Q2b *EST of H* $= 4 + 3 = 7$ hours after starting

Q2ci *AFIM*

Q2cii *LST for D* $= 20 - (2 + 4) = 14$ hours after starting

Q2d The crashed activity is one of the four activities on the critical path.

Q2e There is a new critical path (*CEHGIM*) when *F* is crashed by 2 hours, .: minimum completion time for the project is 36 hours.

Module 3: Geometry and measurement

Q1a $\ell = \frac{34.92}{360} \times 2\pi \times 65 \approx 39.62$ m

Q1b $A = \frac{34.92}{360} \times \pi \times 65^2 \approx 1288\,\text{m}^2$

Q2a $d = \frac{90-34}{360} \times 2\pi \times 6400 \approx 6255\,\text{km}$

Q2bi $r = 6400\sin(90-34)^\circ \approx 5305.84$ km

Q2bii $d = \frac{151-142}{360} \times 2\pi \times 5305.84 \approx 833\,\text{km}$

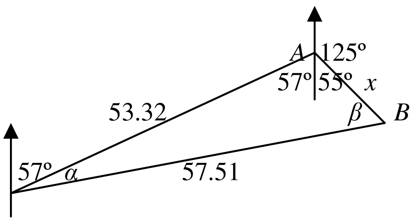
Q2c $\frac{151-12}{360} \times 24 = 9.2666...\,\text{hours} = 9\,\text{hours}\,16\,\text{minutes}$

Q2d Leaving Sydney airport on Sunday, 6 March at 10.20 (i.e. Sunday, 6 March at 00.20), arriving Rome on Monday, 7 March at 02.30. Flight time is 26 hours 10 minutes.

Q3 Let k be the value such that $V_{sen} = kV_{\text{int}}$.

$\therefore\ k = \frac{V_{sen}}{V_{\text{int}}} = \left(\sqrt{\frac{720}{500}}\right)^3 = 1.728$

Q4



$57^\circ + 55^\circ = 112^\circ$, $\frac{\sin\beta}{53.32} = \frac{\sin 112^\circ}{57.51}$, $\beta \approx 59.2753^\circ$

$\therefore\ \alpha = 180^\circ - 112^\circ - 59.2753^\circ \approx 8.7247^\circ$

$\frac{x}{\sin 8.7247^\circ} = \frac{57.51}{\sin 112^\circ}$, $x \approx 9.4$ m

Module 4: Graphs and relations

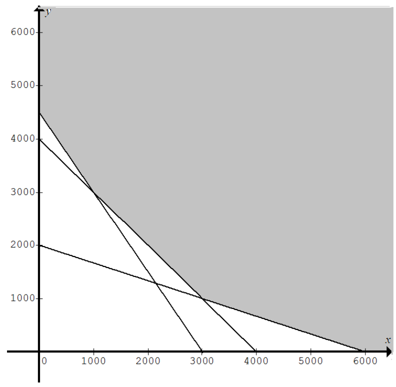
Q1a $0.02 \times 2 = 0.04\,\text{kg}$

Q1b $0.05 \times 100 + 0.05 \times 400 = 25\,\text{kg}$

Q1c $0.06x + 0.04y \geq 180$ kg

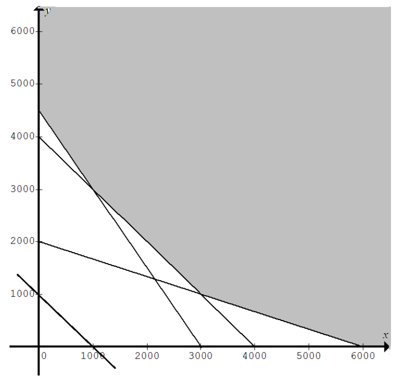
Q1di $y = 2\,000 - \frac{1}{3}x$, it can be expressed as $0.02x + 0.06y = 120$

Q1dii

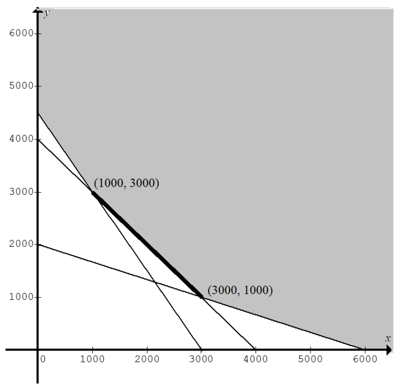


Q1ei $gradient = -1$

Q1eii



Q1eiii The points are shown as a thick solid line in the following diagram. Vertices $(1\,000,\ 3\,000)$ and $(3\,000,\ 1\,000)$ are included.



Q3a Revenue $= 10.8 + 4(8-2) = 34.8$ dollars

Q3b Assuming the revenue is a continuous function of n, $10.8 + 4(n-2) = a + 2(n-10)$ when $n = 10$

$\therefore\ a = 10.8 + 4(10-2) = 42.8$

Q3c When $n \leq 10$, $R - C > 0$. $R - C$ decreases to zero as n increases above 10.

Let $R - C = 0$, i.e. $42.8 + 2(n-10) - 3.5n = 0$, $n = 15.2$.

$\therefore$ the maximum number of kg is 15.2 to break even.

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