

2016 Trial Examination

STUDENT NUMBER

Figures										Letter
Words										

FURTHER MATHEMATICS

Written Examination 2

Reading Time: 15 minutes
Writing Time: 1 Hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	36
Module		
<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
4	2	24
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 32 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This paper consists of a core and six modules. Students are to answer all questions in the core and then select **three** modules and answer all questions within those modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve e.g. π , surds, fractions.

Diagrams are not drawn to scale unless specified otherwise.

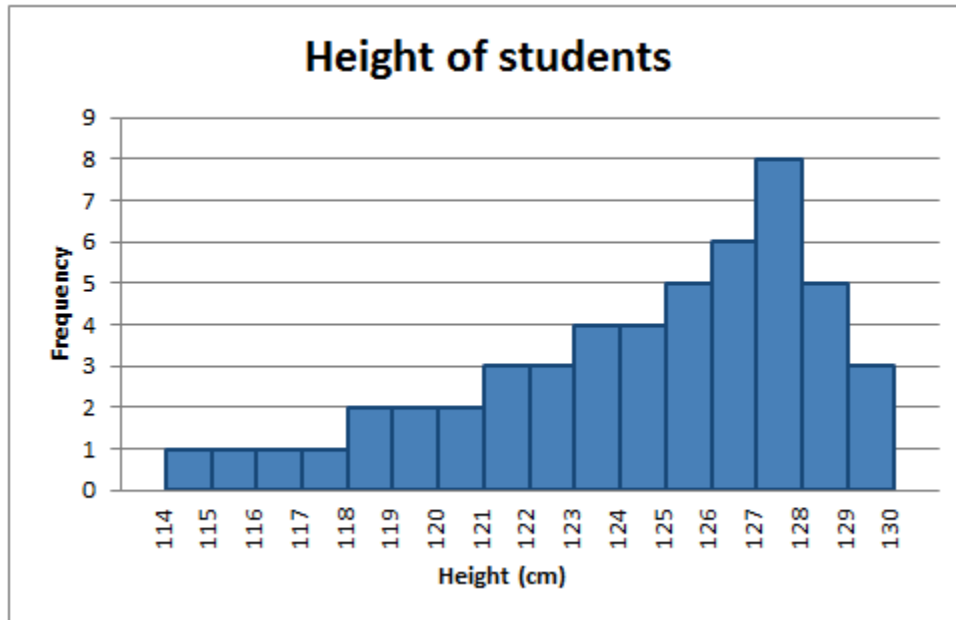
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TURN OVER

Core – Data Analysis

Question 1 (4 marks)

The following histogram shows the distribution of heights of 51 students.



- a. For this distribution, find the modal class.

1 mark

- b. Find the median range of the heights of students in this sample.

1 mark

- c. What percentage of students have a height of at least 123cm? Give your answer to the nearest whole number.

1 mark

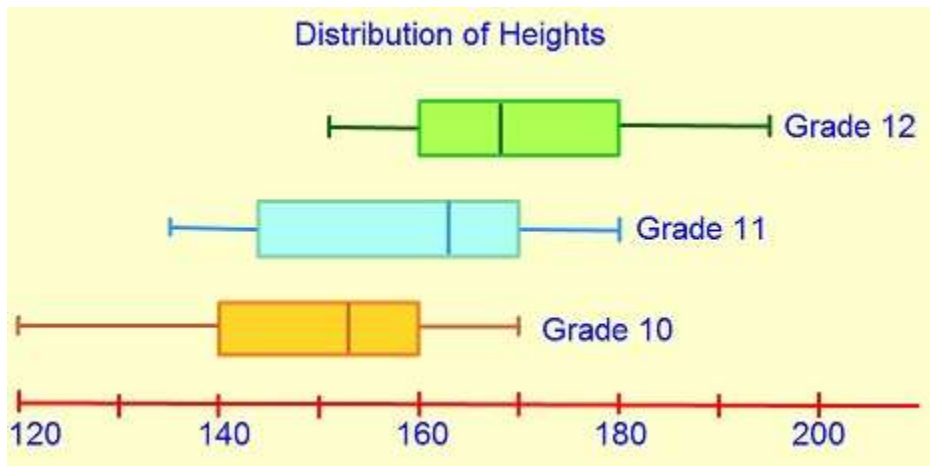
CORE - continued

d. What does the histogram suggest about the distribution of the heights?

1 mark

Question 2 (5 marks)

The following parallel box plots show the distribution of heights of grade 10, 11 and 12 students.



a. What is the minimum height of a Grade 11 student who is in the top 25% of his/her group?

1 mark

b. Explain why a new Year 12 student with height 120 cm will be an outlier to the group.

1 mark

**CORE - continued
TURN OVER**

- c. Compare the variability in the heights of Grade 10 and 11 students.

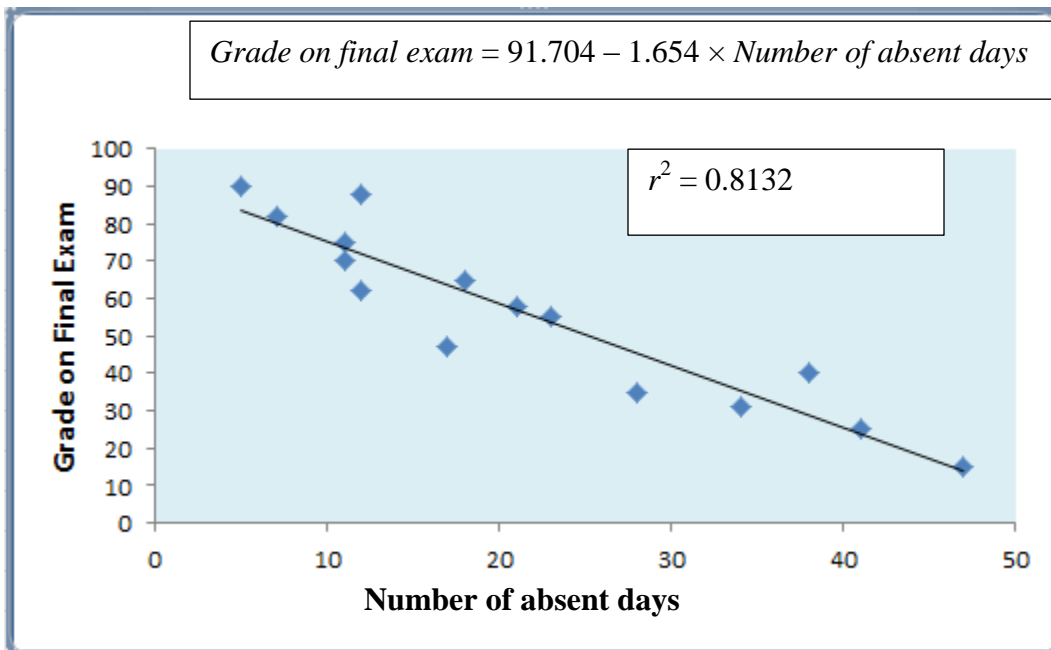
2 marks

- d. What percent of students in Grade 12 have height above 180cm?

1 mark

Question 3 (4 marks)

Mr. Holmes wants to predict grade on final exam from the number of absent days. A least squares regression line is fitted to the scatterplot as shown below. The coefficient of determination is also given.



CORE - continued

2016 FURTHERMATH EXAM 2

- a. Interpret the slope in terms of the variables *grade on the final exam* and *number of absent days*.

2 marks

- b. Interpret the coefficient of determination in terms of the variables *grade on the final exam* and *number of absent days*.

1 mark

- c. Mr. Holmes records 32 days of absence for a particular student. Predict the final grade of this student to the nearest whole number.

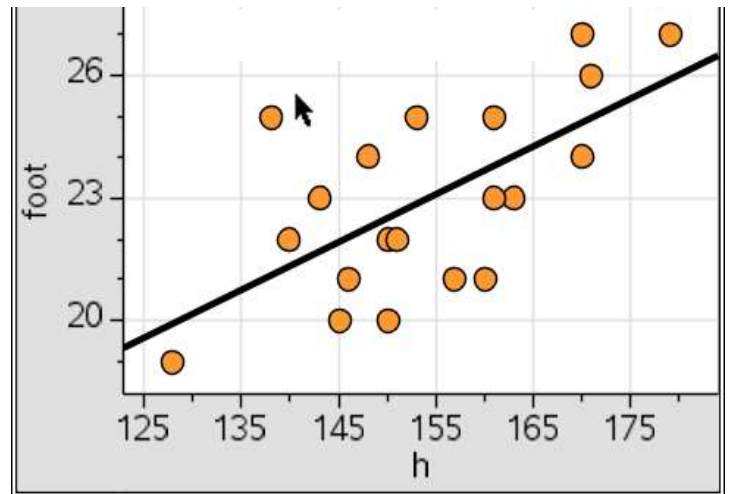
1 mark

CORE - continued
TURN OVER

Question 4 (5 marks)

The table below shows the height and right foot length of 20 students.
The scatterplot is also shown below on the right.

Height (in cm)	Right foot length (in cm)
148	24
161	25
171	26
163	23
138	25
146	21
150	22
170	27
151	22
128	19
153	25
157	21
179	27
160	21
150	20
143	23
140	22
170	24
161	23
145	20



- a.** Use the scatterplot to describe the association between height and foot length in terms of form, direction and strength.

1 mark

- b.** Find the least squares regression line to predict the foot length from the height. Write the slope and intercept correct to two decimal places.

2 marks

CORE - continued

- c. Predict the foot length of a student whose height is 190cm. Explain why this prediction may be of limited reliability.

2 marks

Question 5 (6 marks)

Consider the data from a memory retention experiment in which 13 students were asked to memorize a list of words. The students were then asked to recall the words at various times up to a week later. The proportion of items correctly recalled at various times is given below in the table-

Time (in minutes)	Proportion
1	0.84
5	0.71
15	0.61
30	0.56
60	0.54
120	0.47
240	0.45
480	0.38
720	0.36
1440	0.26
2880	0.20
5760	0.16
10080	0.08

- a. If there were 80 words in the list, how many words could a student recall after four hours?

1 mark

**CORE - continued
TURN OVER**

The scatterplot suggests that the proportion of recalled items is not linearly related to time. For linearity, a logarithmic transformation is performed on the time variable.

- b.** Find the least squares regression line of the transformed data. Write the equation in terms of the variables and leave the coefficients correct to three decimal places.

2 marks

- c.** Predict the proportion of words recalled after 2880 minutes using the transformed regression line. Give your answer correct to two decimal places.

1 mark

- d.** Using the linear model, calculate the residual proportion after 2880 minutes.

1 mark

- e.** What does the residual plot tell us about the transformed data?

1 mark

CORE - continued

Core – Recursion and financial modelling

Question 1 (6 marks)

Mr. Holmes takes out a loan of \$11000 to buy his new car. He pays 9% per annum compounded monthly on his loan amount. He will repay his loan in 12 monthly payments of \$950.

- a. Find the loan amount after one payment is made.

_____ 1 mark

Let V_n represent the amount of his loan after n months.

A recurrence relation $V_0 = 11000$, $V_{n+1} = aV_n - b$ is used to model this loan.

- b. Write down the values of a and b

_____ 2 marks

- c. Use the recurrence relation to determine how much Mr. Holmes still owes after 5 months.

_____ 1 mark

- d. What is the value of the last payment?

_____ 1 mark

**CORE - continued
TURN OVER**

- e. How much interest did Mr. Holmes pay in total?

1 mark

Question 2 (6 marks)

Mr. Holmes bought his new car for \$14000. The car depreciates each year by \$1050.

- a. Find the value of his car after two years.

1 mark

- b. In how many years will the value of the car be \$5600?

1 mark

Mr. Holmes buys a machine for \$8500. After five years, the value of the machine depreciated to \$7644 using the reducing balance method.

- c. Find the annual percentage rate at which the machine depreciated. Give your answer correct to one decimal place.

1 mark

CORE - continued

2016 FURTHERMATH EXAM 2

Mr. Holmes takes a home loan of \$480 000. He is charged interest at the rate of 8.1% per annum, compounded monthly.
The loan is repaid in 30 years.

- d. Find the minimum monthly repayment to the nearest dollar.

1 mark

- e. How much interest is paid by Mr. Holmes on this loan?

1 mark

- f. The loan could be modelled by the recurrence relation
 $V_0 = 480000$, $V_{n+1} = aV_n - b$. Show that $a = 1.00675$.

1 mark

Total 36 marks

**END OF CORE
TURN OVER**

Module 1: Matrices**Question 1 (2 marks)**

The matrix N below shows the number of students who attend Music club, Art club or Dance club. Each student is allowed to attend only one club each month.

$$N = \begin{array}{cc} & \begin{array}{cc} \text{Yr7} & \text{Yr8} \end{array} \\ \begin{array}{c} \left[\begin{array}{cc} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{array} \right] & \begin{array}{l} \text{Music} \\ \text{Art} \\ \text{Dance} \end{array} \end{array}$$

Let n_{ij} represent an element of matrix N .

- a. What does the element n_{32} represent in the matrix?

1 mark

- b. How many students from year 7 attend a club?

1 mark

Question 2 (5 marks)

Let S_0 represent the matrix showing the number of students attending a club.

- a. The matrix $S_0 = \begin{bmatrix} a & \text{Music} \\ 83 & \text{Art} \\ 32 & \text{Dance} \end{bmatrix}$, find the value of a .

1 mark

MODULE 1 - continued

The number of students attending a club changes each month according to the following transition matrix.

$$T = \begin{array}{ccc|c} & \textit{this month} & & \\ & M & A & D \\ \begin{array}{l} 0.45 \\ 0.15 \\ 0.40 \end{array} & \begin{array}{l} 0.60 \\ 0.30 \\ 0.10 \end{array} & \begin{array}{l} 0 \\ 0 \\ 1 \end{array} & \begin{array}{l} M \\ A \textit{ next month} \\ D \end{array} \end{array}$$

- b. What does the number 1 in the above matrix represent?

1 mark

- c. Calculate $S_1 = T \times S_0$. Write the elements of this matrix to the nearest whole number.

1 mark

- d. How many Art students have chosen to attend Music club after one month?

1 mark

- e. How many students are expected to attend Art club in the seventh month?

1 mark

MODULE 1 - continued
TURN OVER

Question 3 (5 marks)

The fee, in dollars, associated with attendance in each club is given by the following matrix.

$$A = \begin{matrix} & \begin{matrix} Music & Art & Dance \end{matrix} \\ \begin{bmatrix} 28 & 19 & 13 \end{bmatrix} & \end{matrix}$$

For example, every student who attends Music club pays a fee of \$28 each month. The matrix N below shows the number of students who attend Music club, Art club or Dance club.

$$N = \begin{matrix} & \begin{matrix} Yr7 & Yr8 \end{matrix} \\ \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} & \begin{matrix} Music \\ Art \\ Dance \end{matrix} \end{matrix}$$

- a. Explain why the product matrix NA is not defined.

1 mark

- b. Find the product matrix AN . Explain what this matrix represents.

1 mark

2016 FURTHERMATH EXAM 2

After the first four months, the school subsidises the fee to attend each club and reduces it by 30%.

- c. Show a matrix calculation that will find the new fees.

1 mark

- d. How much will the school collect as fees in the first six months from year 8 students?

2 marks

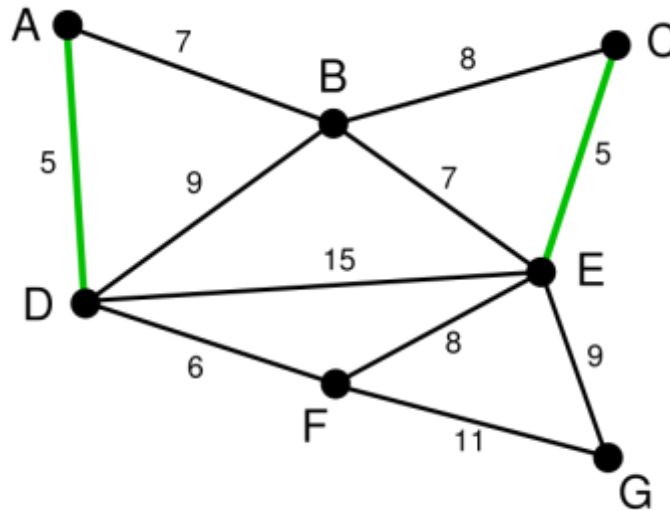
Total 12 marks

**END OF MODULE 1
TURN OVER**

Module 2: Graphs and Networks

Question 1 (5 marks)

A courier man has to travel to seven suburbs to deliver parcels. The suburbs A, B, C, D, E, F and G are shown as vertices on the graph below.



The distances, in kilometres, between each suburb is written along the edges above.

- a. How much distance does the courier man have to travel from D to B?

1 mark

- b. Find the shortest distance that the courier man has to travel between A and G.

1 mark

- c. Show that the network is a connected planar graph.

1 mark

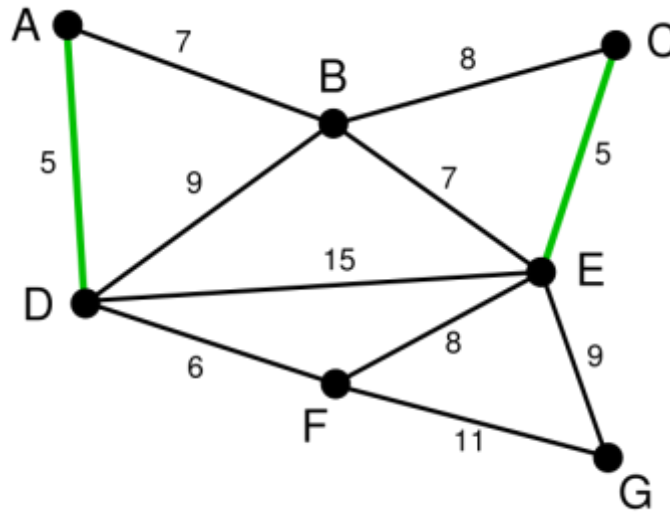
MODULE 2 - continued

- d. Write down one path that the courier man could take to visit each suburb. What is the name given to this trail?

2 marks

Question 2 (3 marks)

The courier man decides to follow the path which gives a minimal spanning tree.



- a. Draw the minimum spanning tree below.

2 marks

**MODULE 2 - continued
TURN OVER**

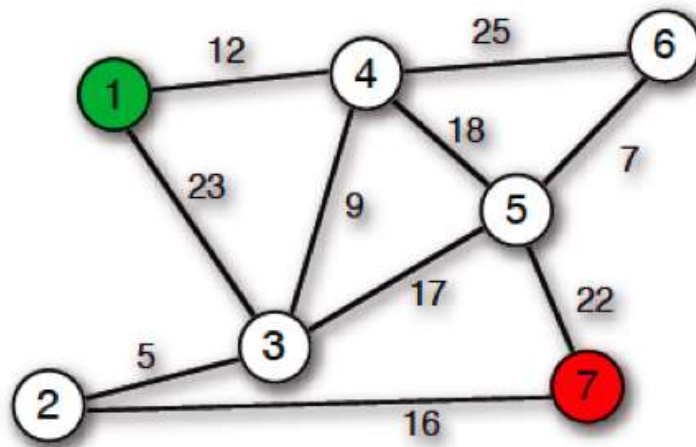
The courier man gets a call from the courier company not to go to suburb G.

- b. How many kilometres less will the courier man travel to deliver all parcels from A to F?

1 mark

Question 3 (2 marks)

The following network shows the road links between vertices 1 to 7. The courier man starts from the post-office at 1 and has to drop off all parcels at post-office 7. The numbers on the edges represent the distance between the vertices.



- a. Find the trail that the courier man should take to cover minimum distance from 1 to 7.

1 mark

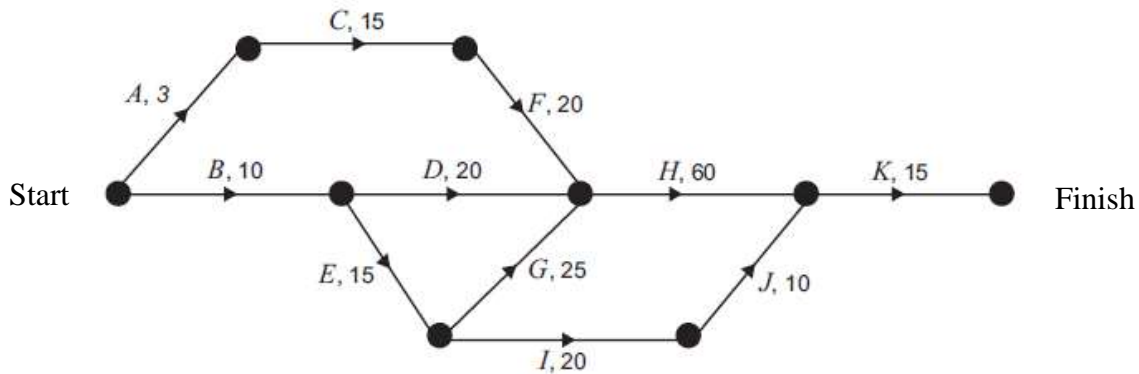
- b. What is the shortest distance travelled from 1 to 7?

1 mark

MODULE 2 - continued

Question 4 (2 marks)

The courier man's van goes for servicing. This usually involves every activity shown in the network diagram below. The duration of each activity, in minutes, is also shown.



The incomplete table below shows this same information and includes predecessor activities and the earliest starting times (EST).

Activity	Predecessor(s)	Duration of activity (minutes)	Earliest starting time EST (minutes)
<i>A</i>	–	3	0
<i>B</i>	–	10	0
<i>C</i>	<i>A</i>	15	3
<i>D</i>	<i>B</i>	20	10
<i>E</i>	<i>B</i>	15	10
<i>F</i>	<i>C</i>	20	18
<i>G</i>	<i>E</i>	25	25
<i>H</i>		60	
<i>I</i>	<i>E</i>	20	25
<i>J</i>	<i>I</i>	10	45
<i>K</i>	<i>H, J</i>	15	110

- a. Fill the two shaded cells for activity *H* in the table above

1 mark

**MODULE 2 - continued
TURN OVER**

b. What is the minimum time needed to complete this service?

1 mark

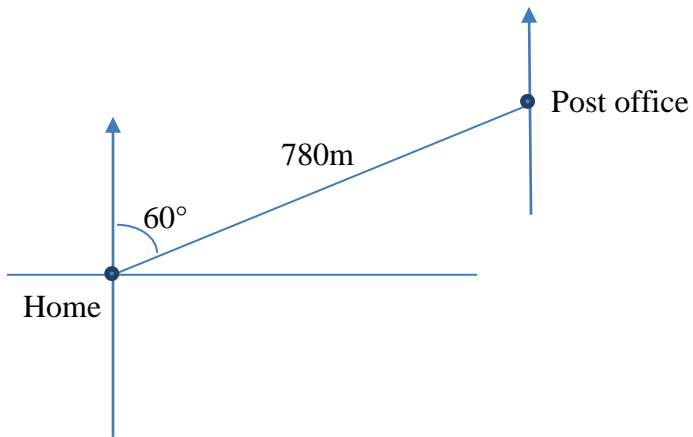
Total 12 marks

END OF MODULE 2

Module 3: Geometry and measurement

Question 1 (6 marks)

Alex starts from home and decides to go to the post office first. She walks on a bearing of 060° for 780m and reaches the post office as shown below.



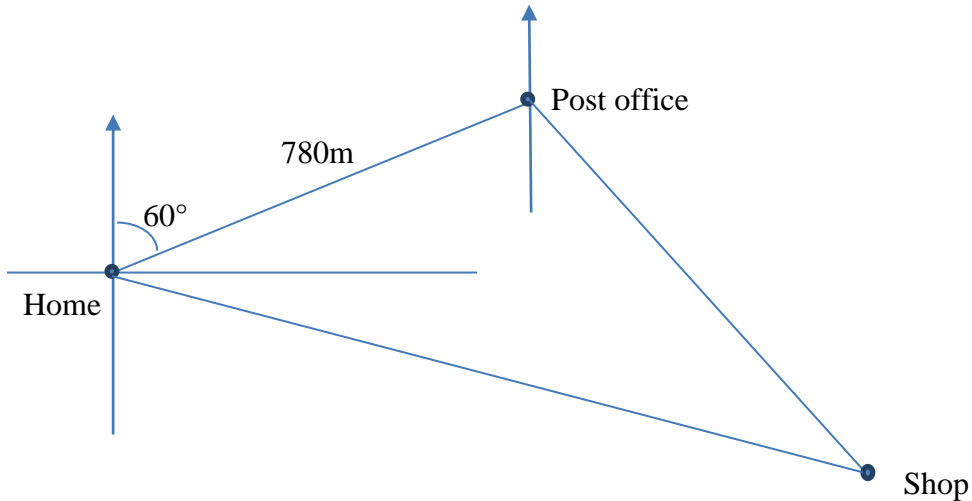
- a. What is the bearing of Alex's home from the post office?

1 mark

**MODULE 3 - continued
TURN OVER**

From the post-office Alex walks to the shop to buy some stationery. She walks on a bearing of 120° to reach the stationery shop.

From the shop she goes back home on a bearing of 285° as shown in the diagram below.



b. Label all the interior angles of the above triangle.

1 mark

The area of the triangular course that Alex walked is 0.71975 km^2 .

c. Find the area of the triangular course in square metres.

1 mark

- d. What distance, to the nearest metre, did Alex travel from the post office to the stationery shop?

1 mark

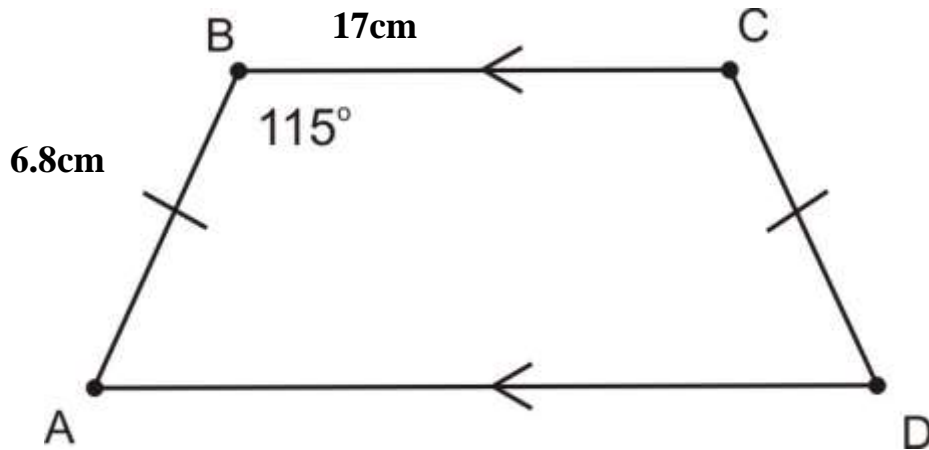
- e. What distance, to the nearest metre, did Alex travel altogether? Give your answer correct to one decimal place.

2 marks

MODULE 3 - continued
TURN OVER

Question 2 (3 marks)

Alex buys some cardboard and paint colours from the stationery shop. She cuts the cardboard in the shape of a trapezium with dimensions as shown below to draw a scale drawing of her table top.



- a. How much paint does she require to paint the shape above?

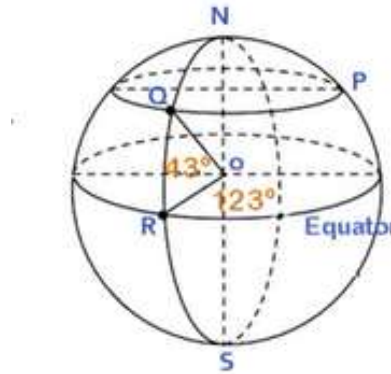
2 marks

- b. If the scale used to construct the table top is 1:400, find the area of the real table top to the nearest square metres.

1 mark

Question 3 (3 marks)

Alex sees the location of three points P, Q, R on the Earth as shown below in the diagram.



a. What is the latitude of Q?

1 mark

b. State the location of R if the longitude of P is 80° E.

1 mark

c. Calculate the distance, in nautical miles, from P due west to Q measured along the common parallel of latitude. Give your answer to the nearest whole number.

1 mark

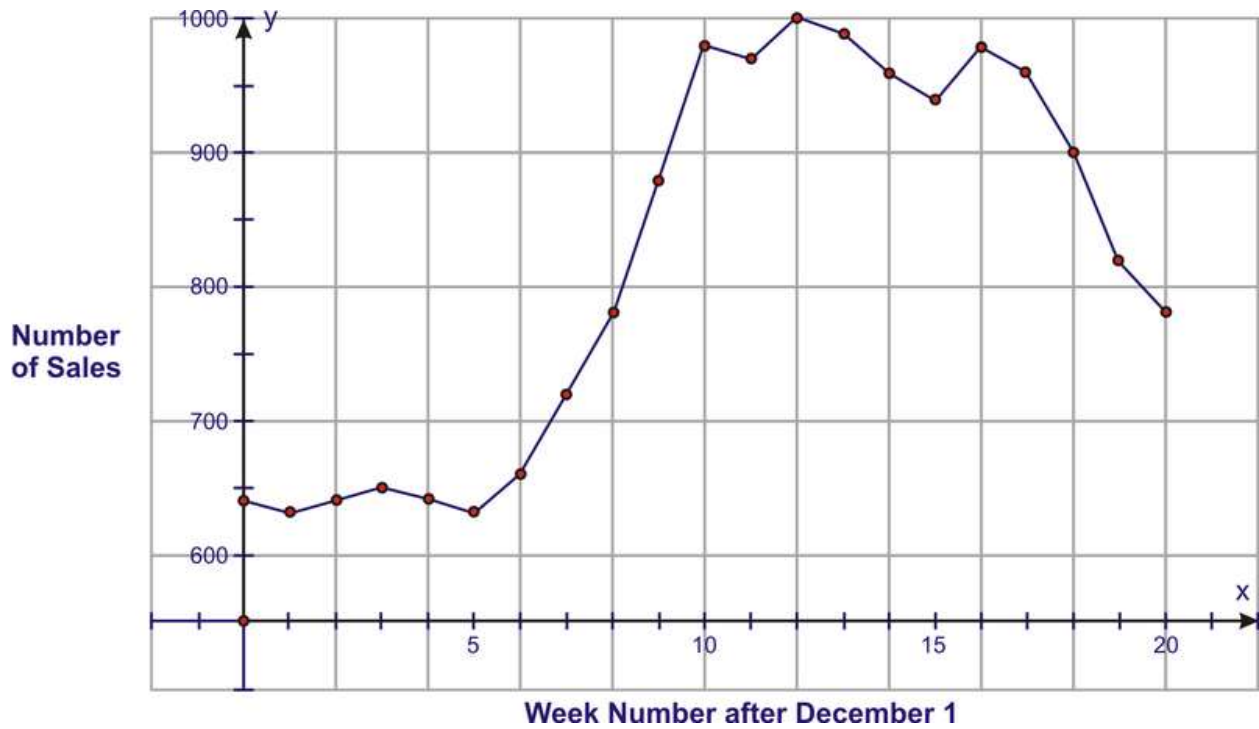
Total 12 marks

**END OF MODULE 3
TURN OVER**

Module 4: Graphs and Relations

Question 1 (2 marks)

The graph below shows the number of sales of pairs of shoes each week after December 1.



a. How many pairs of shoes are sold in week 18 after December 1?

1 mark

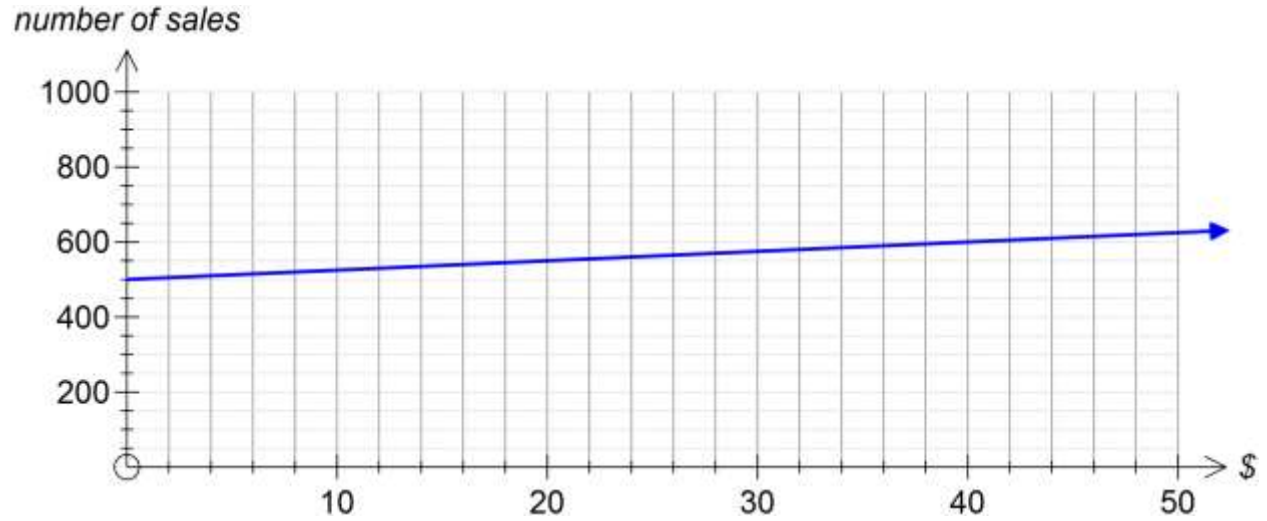
b. For how many weeks were the sales more than 700?

1 mark

MODULE 4 - continued

Question 2 (4 marks)

The following graph shows the relationship between the number of sales and amount of money spent on advertising the shoes.



- a. What is the number of sales without any advertising costs?

1 mark

The relationship between the number of sales and amount spent on advertising can be modelled as-

$$Sales = a + b \times advertising\ cost$$

- b. Write down the values of a and b .

1 mark

- c. What does the value of b represent here?

1 mark

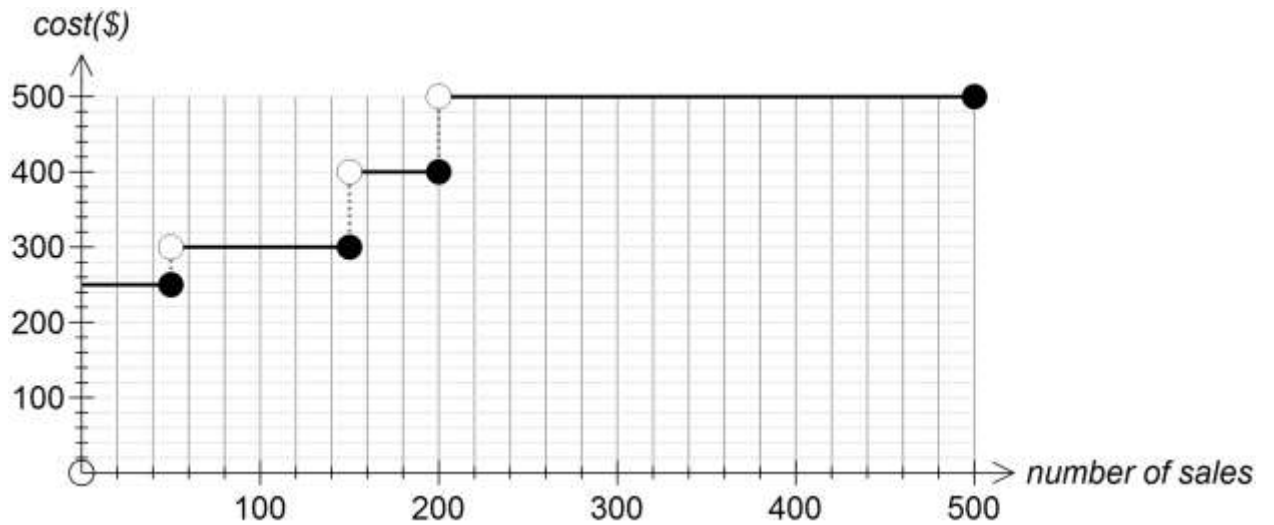
MODULE 4 - continued
TURN OVER

- d. Use the relationship to find the number of sales when the advertising cost is \$30.

1 mark

Question 3 (2 marks)

The shoes are transported from a warehouse to the retailer’s shop. The graph below shows the transportation cost depending upon the sales order.



- a. How much does the retailer pay as transportation cost for an order of 160 pairs of shoes?

1 mark

MODULE 4 - continued

The function that describes the transportation cost based on the sales order can be represented as-

$$f(n) = \begin{cases} 250, & 0 < n \leq 50 \\ 300, & 50 < n \leq a \\ 400, & a < n \leq 200 \\ 500, & 200 < n \leq b \end{cases}$$

- b. Write down the values of a and b .

1 mark

Question 4 (4 marks)

The retailer decides to find the most effective way to make a profit under some restrictions. The table below shows the constraints.

	Space (in square units)	Cost (in dollars)	Profit per pair (in \$)
Boots (x)	4	60	80
Shoes (y)	2	20	30
Available	1600	17000	

The constraints follow the inequalities

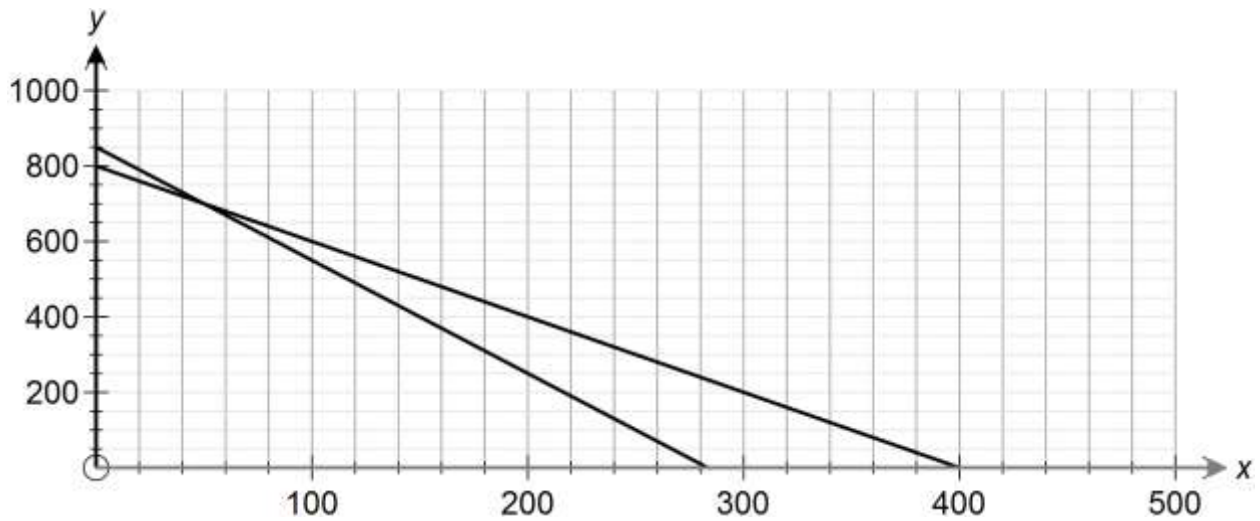
$$4x + 2y \leq 1600, \quad 0 < x \leq 200, \quad y > 0$$

- a. Write down one more inequality that represents a constraint.

1 mark

MODULE 4 - continued
TURN OVER

The graph below shows the region bounded by these inequalities. One of them is missing in the graph below.



b. Sketch the last inequality and shade the feasible region above.

2 marks

c. Find the number of shoes and boots that the retailer should purchase to make the maximum profit.

1 mark

Total 12 marks

END OF QUESTION AND ANSWER BOOK