Neap:

Trial Examination 2016

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 (4 marks)

a. Order the values.

The median is between 27 000 and 29 800.

$\frac{27\,000 + 29\,800}{2} = 28\,400$ A1

b. mean profit = $\frac{\text{profit}}{\text{nights rented}}$

mean profit for 2013 = $\frac{20\,850}{200}$

$= 104.3$

All other years are lower than 104.3, so 2013 had the highest profit per night rented. A1

c. **i.** The missing figure is below the 280.

$\frac{280 + 360 + 200}{3} = 280$ A1

ii. A 4-point moving mean would result in only 3 points remaining. There are 6 points to be considered. 2-point and 3-point moving means could be considered because they are factors of 6. A1

Question 2 (13 marks)

a. The profit is the **response** variable because **it reacts to the number of guests who have stayed**.

identifying variable A1
correct explanation A1

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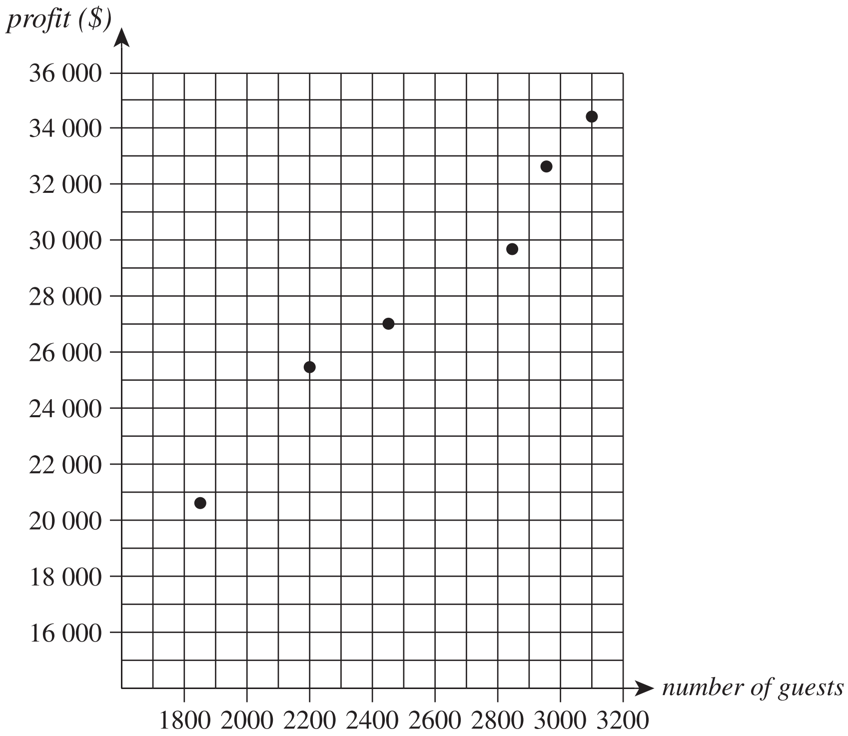
b.



correct labels A1
correct points A1
correct x-axis and y-axis A1

c. Enter data into the statistics section of the calculator and use the linear regression component.

i. 0.98 A1

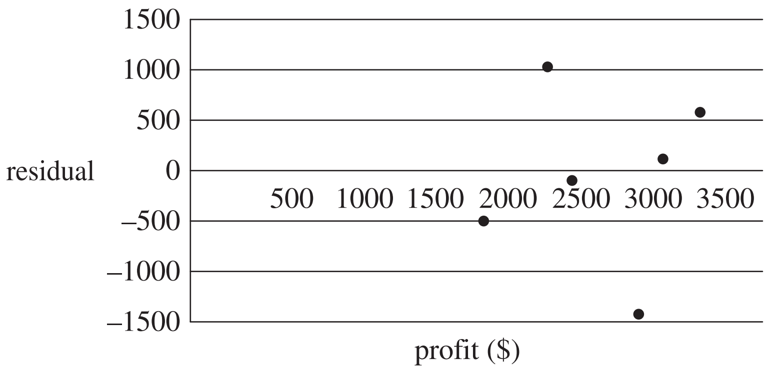
ii. $y = 2238.97 + 10.15x$ A2

d.

	2010	2011	2012	2013	2014	2015
Number of guests	2450	2200	3100	1880	2850	2960
Actual profit ($)	27 000	25 600	34 340	20 850	29 800	32 500
Predicted profit ($)	27 106	24 569	33 704	21 321	31 166	32 383
Residual	**–106**	**1031**	**636**	**–471**	**–1366**	**117**

uses correct formula: residual = actual – predicted M1
A1

e.



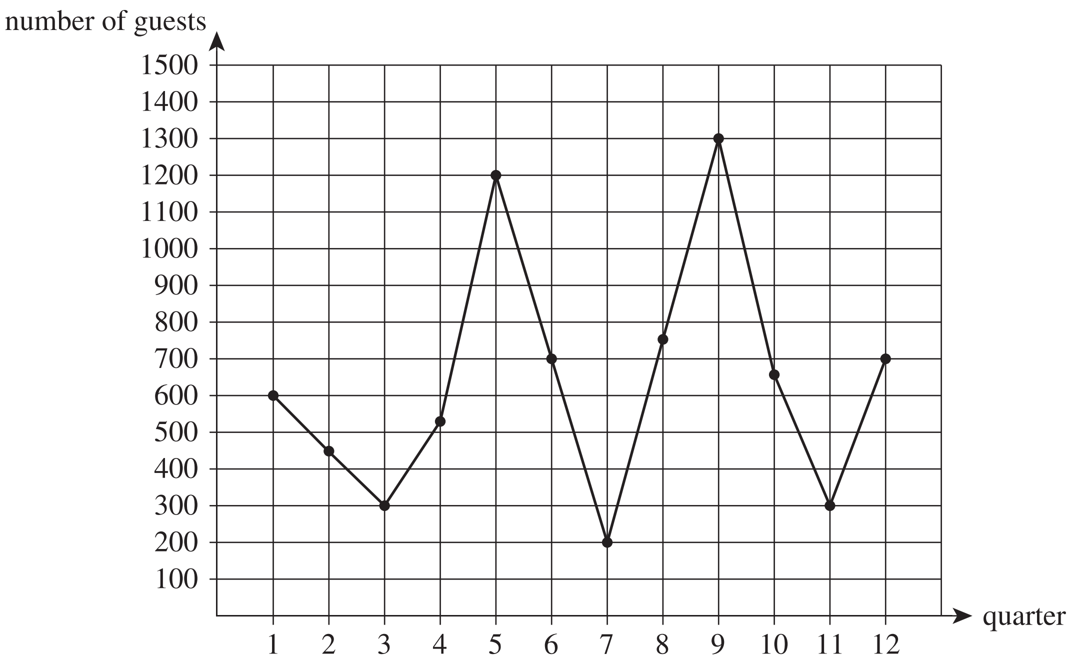
correct labels A1
correct points A1

f. There is a pattern, so linearity is not confirmed. A1

Question 3 (7 marks)

a. Q1; range Q1 = 1300 – 600 = 700 A1

b.



correct labels A1
correct points A1

c. There is a seasonal pattern. A1

d. Q3 has the lowest number of guests staying. A holiday at that time would have the least effect. A1

e. Find the mean of each year and divide each figure in Q1 by that number. Find the mean of these three figures. M1

seasonal index = 1.57 A1

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Recursion and financial modelling

Question 4 (3 marks)

a. Use the financial package of your calculator or $800\ 000 \times \dfrac{\left(\frac{4.8}{12}\right)}{100}$.

monthly repayment = \$3200

annual repayment = 12 × 3200

= \$38 400 A1

b. After three years the income exceeds the repayment.

P_0	35 000
P_1	36 400
P_2	37 856
P_3	39 370
P_4	40 946

M1 A1

Question 5 (4 marks)

a.

Year	Deposit (\$)	Interest earned (\$)	Balance (\$)
0	50 000	0.00	50 000
1	35 000	**2600**	**87 600**
2	35 000	**4555.20**	**127 155.20**
3	35 000	**6612.07**	**168 767.27**

M1 A1

b.

Year	Deposit (\$)	Interest earned (\$)	Balance (\$)
0	50 000	0.00	50 000
1	35 000	2600	87 600
2	35 000	4555.20	**127 155.20 – 20 000 = 107 155.20**
3	35 000	**5572.07**	**147 727.27**

168 767.27 – 147 727.27 = \$21 040 less M1 A1

Question 6 (5 marks)

a. $N_{n+1} = 1.05N_n + 100,\ N_0 = 3000$

correct form M1
correct answer A1

b.

N_0	3000
N_1	$1.05 \times 3000 + 100 = 3250$
N_2	$1.05 \times 3250 + 100 = 3512.50$
N_3	$1.05 \times 3512.50 + 100 = 3788.125$

During the fourth year, the number of guests budgeted for is 3788. M1 A1

c.

N_0	3000	3000
N_1	$1.05 \times 3000 + 100$	3250
N_2	$1.05 \times 3250 + 100$	3512.50
N_3	$1.05 \times 3512.50 + 100$	3788.125
N_4	$1.05 \times 3788.125 + 100$	4077.53
N_5	$1.05 \times 4077.53 + 100$	4381.41
N_6	$1.05 \times 4381.41 + 100$	4700.48
N_7	$1.05 \times 4700.48 + 100$	5035.50

After 7 years the guest numbers will be expected to exceed 5000. A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (9 marks)

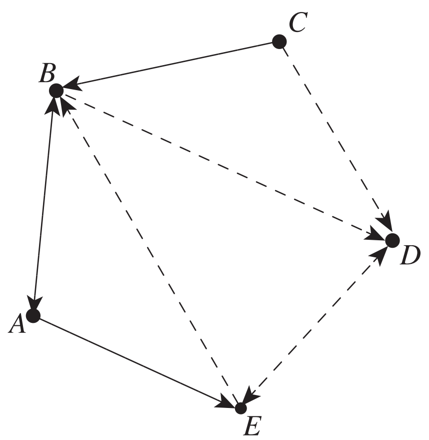
a. Beechworth to Echuca is a known road and thus $b = e = 1$. Chiltern and Deniliquin are also joined by tourist routes and so $a = 1$. Clearly there is no need for any road to the same town and thus the leading diagonal is all zero, so $d = 0$. Deniliquin and Echuca are linked and thus $c = 1$.

values a and b A1
values c and d A1
Note: 1 mark is to be deducted for each incorrect value.

b. 2 A1

c. This was the sum of the Chiltern row. It shows the number of tourist routes through Chiltern. The diagram confirms this. A1

d.



A1

e. $C = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 3 \\ 2 \end{bmatrix}$ A1

f. $F^2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}^2$

$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ 0 & 2 & 1 & 0 & 1 \end{bmatrix}$ A1

g. The matrix that we require will add all of the elements in the corresponding row.

$$G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$ A1

h. The result is $\begin{bmatrix} 4 \\ 6 \\ 0 \\ 6 \\ 4 \end{bmatrix}$.

Both Beechworth and Deniliquin have the most second-level tourists and thus the most traffic. A1

Question 2 (3 marks)

a. We require the row from Beechworth (the second column) to Echuca (the fifth row). The figure is 0.05, so that is 5%. A1

b.
$$S_1 = \begin{bmatrix} 0.90 & 0.10 & 0.10 & 0.05 & 0.05 \\ 0.05 & 0.80 & 0 & 0 & 0.05 \\ 0 & 0 & 0.75 & 0 & 0 \\ 0.05 & 0.05 & 0.10 & 0.95 & 0.05 \\ 0 & 0.05 & 0.05 & 0 & 0.85 \end{bmatrix} \begin{bmatrix} 900 \\ 500 \\ 700 \\ 600 \\ 1200 \end{bmatrix}$$

$$= \begin{bmatrix} 1020 \\ 505 \\ 525 \\ 770 \\ 1080 \end{bmatrix}$$

Town	Albury	Beechworth	Chiltern	Deniliquin	Echuca
No. favouring in 2016	**1020**	**505**	**525**	**770**	**1080**

A1

c. $$S_{50} = \begin{bmatrix} 0.90 & 0.10 & 0.10 & 0.05 & 0.05 \\ 0.05 & 0.80 & 0 & 0 & 0.05 \\ 0 & 0 & 0.75 & 0 & 0 \\ 0.05 & 0.05 & 0.10 & 0.95 & 0.05 \\ 0 & 0.05 & 0.05 & 0 & 0.85 \end{bmatrix}^{50} \begin{bmatrix} 900 \\ 500 \\ 700 \\ 600 \\ 1200 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3667 & 0.3667 & 0.3667 & 0.3667 & 0.3667 \\ 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \\ 0 & 0 & 0 & 0 & 0 \\ 0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.5000 \\ 0.0333 & 0.0333 & 0.0333 & 0.0333 & 0.0333 \end{bmatrix} \begin{bmatrix} 900 \\ 500 \\ 700 \\ 600 \\ 1200 \end{bmatrix}$$

$$= \begin{bmatrix} 1430 \\ 390 \\ 0 \\ 1950 \\ 130 \end{bmatrix}$$

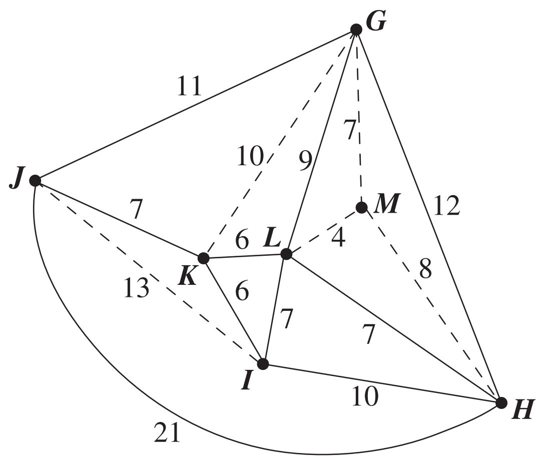
Town	Albury	Beechworth	Chiltern	Deniliquin	Echuca
No. favouring long-term	**1430**	**390**	**0**	**1950**	**130**

A1

Module 2 – Networks and decision mathematics

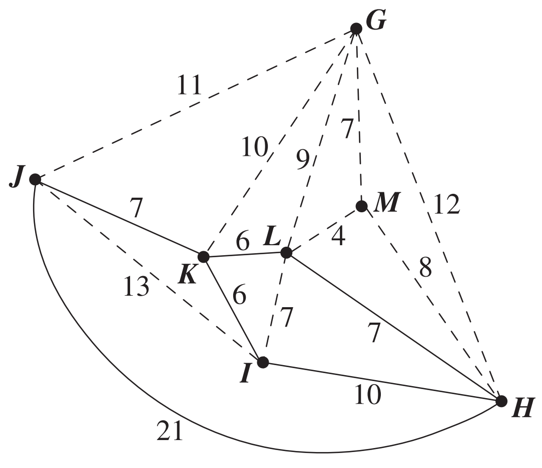
Question 1 (5 marks)

a. Students may do this just by looking at the diagram. A formal method is shown below.



Edges will be removed when a better route is available. The ones being removed appear dotted. The *IJ* serves no purpose. *GK* also serves no purpose for us as we are travelling between *J* and *H* and *JG* is better than *JKG*. If we pass through *G*, we can get to *H* directly in 12 km instead of through *M* so *GM* can go. If we pass through *L* we can go to *H* in 7 km so passing through *M* is not needed.

We can now see that passing via *G* to *H* is not optimal as it has length 23 km whereas the direct route is 21 km. *GL* now has no purpose as vertex *G* is dead. *LI* is also useless as *L* to *I* takes us backward and *I* to *L* is not useful since *KIL* is longer than *KL* directly.

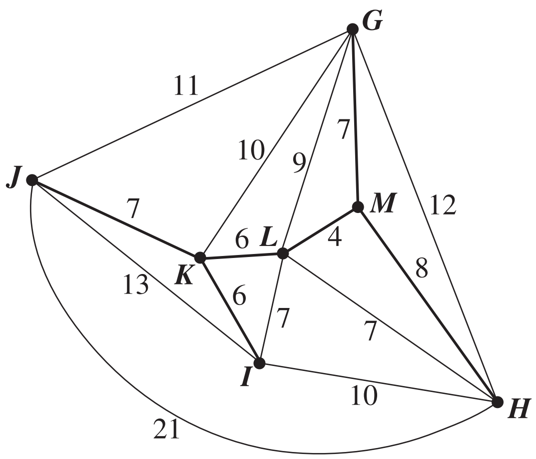


There are now two routes possible. *JKLH* is the shorter one with length 20 km. A1

b. We are looking for an Euler path here since we need to pass along each edge. Thus we need to check which vertices have odd degree. M1

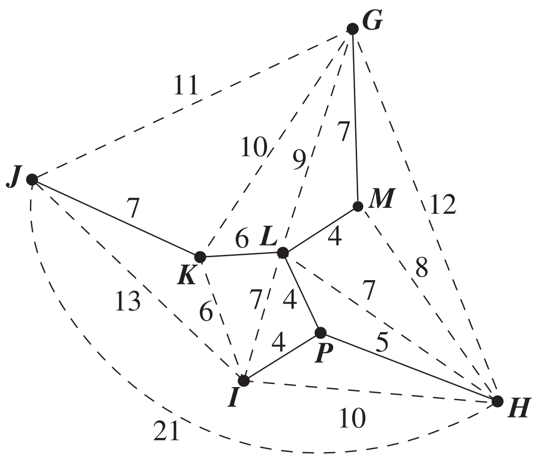
Vertex *G* has a degree of 5. Once *LM* is removed, *H* is the only remaining odd vertex. Thus we must start at *G* and end at *H* (or vice-versa). A1

c. **i.** We will select edges one at a time, choosing the smallest first while remaining connected. Thus we choose the 4 km edge. The smallest edge connected to *L* or *M* is now chosen. It is the 6 km edge of *KL*. The *K*, *L* and *M* are all now active and so the 6 km edge of *KI* is now the best option. The process continues. We now have three choices all of length 7 km. Any of *GM*, *LH* and *JK* can be chosen. We will choose *JK* here but all three options are valid. Note that *IL* is not valid as it forms a loop.



Only *G* and *H* remain to be connected now. *MG* is the shortest of these. *MH* is next best. Any other tree of length 38 km is also appropriate. A1

ii. We need to add one more vertex.

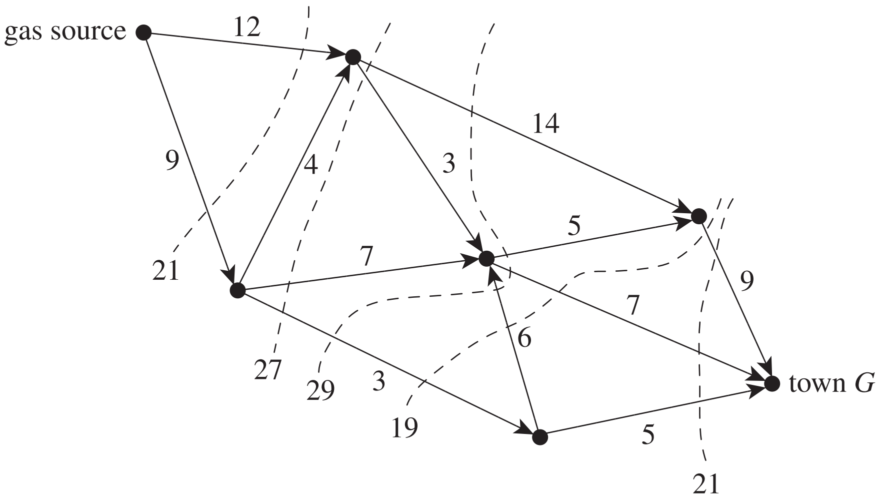


The first three connections are now *LM* (as before), followed by *LP*, then *IH*. Adding *P* has thus altered the existing order, not just added an extra connection. *PH* follows, which is also a change, and then either *KL* or *KI*. Finally, *JK* and *GM* finish the tree.

The length of this tree is 37 km, compared to 38 km for the original network. We actually ended up with a shorter total length of pipes. A1

Question 2 (3 marks)

a.



Various cuts are shown. The one labelled 19 is the least. Thus the maximum flow is 19 GL per hour. A1

b. The relevant pipes are those involved in the minimum cut. These are the 3, the 7 going to town *G* and the 9 going to town *G*. Any of these can be increased by 4. Increasing the 6 is not acceptable, as this pipe is away from the destination, town *G*. A1

The corresponding maximum flow is 21, as the cut labelled 21 near the source is unchanged. A1

Question 3 (4 marks)

a.

Activity	Predecessors	Duration	EST
A	none	4	0
B	none	3	0
C	*A*	3	4
D	*B*	5	3
E	*B*	6	3
F		2	**8**
G	*F*, *E*	4	10
H	*C*, *D*	5	8
I	*G*, *H*	3	8
J	*C*, *D*	3	**13**

both correct A1

b. First we check the completion time of the entire process. It is 16.

Activity	Predecessors	Duration	EST	LST
A	none	4	0	**1**
B	none	3	0	**0**
C	*A*	3	4	**5**
D	*B*	5	3	**3**
E	*B*	6	3	**5**
F		2	8	**9**
G	*F*, *E*	4	10	**11**
H	*C*, *D*	5	8	**8**
I	*G*, *H*	3	8	**13**
J	*C*, *D*	3	13	**13**

Shows LST calculations or values given in table M1

LST of activity *E* is 5 hours. A1

c. The critical activities are those for which LST and EST match.

Thus the correct answer is *BDHJ*. A1

Module 3 – Geometry and measurement

Question 1 (4 marks)

a. The great-circle radius is 6400 km.

Thus $d = 2 \times \pi \times 6400 \times \frac{2}{360}$

$= 223$ km — A1

b. $\frac{4}{360} \times 24 = \frac{4}{15}$ hours — A1

$= 16$ minutes — A1

Note: Students who provide time in minutes only should be awarded both marks.

c. Use Pythagoras.

$d_3 = \sqrt{223^2 + 427^2}$

$= 482$ km — A1

Question 2 (8 marks)

a. Use cosine rule. — M1

$\cos(\theta) = \frac{60^2 + 65^2 - 40^2}{2 \times 60 \times 65}$

$= 0.7981$

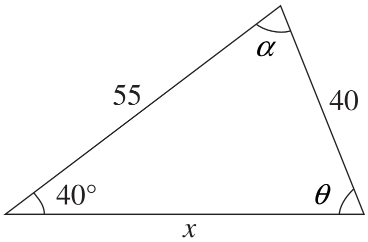
$\theta = 37°$ — A1

b. We require sector area.

$A = \pi \times 25^2 \times \frac{64}{360}$ — M1

$= 13.96$ cm^2 — A1

c. The manager's plan is shown below.



Find angle θ.

$$\frac{\sin(\theta)}{55} = \frac{\sin(40°)}{40}$$

$$\sin(\theta) = 0.8838$$

$$\theta = 62.1$$

The third angle must thus be:

$$\alpha = 180 - 62.1 - 40$$
$$= 77.9°$$

Thus, all angles are acute.

Can the information allow for a different interpretation? Yes.

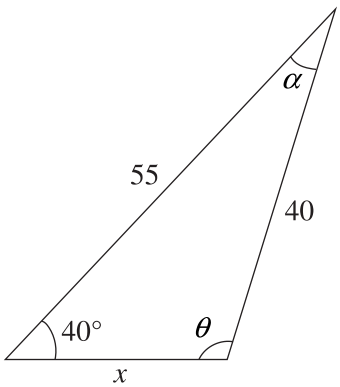
If $\sin(\theta) = 0.8838$

$$\theta = 180 - 62.1$$
$$= 117.9°$$

A1

Both designs are required for mark.

This is an obtuse angle, and so the designer could draw a triangle with an obtuse angle that complied with the specifications. It is shown below.



A1

d. We are being asked to find x. This can be done using the sine rule again.

Manager's design:

$$\frac{x}{\sin(77.9°)} = \frac{40}{\sin(40°)}$$

$$x = 60.85 \text{ cm}$$ A1

Designer's design:

$$\alpha = 180 - 40 - 117.9$$
$$= 22.1$$

$$\frac{x}{\sin(22.1°)} = \frac{40}{\sin(40°)}$$

$$x = 23.41 \text{ cm}$$ A1

Alternatively:

Manager's design:

$$x^2 = 55^2 + 40^2 - 2 \times 40 \times 55 \cos(77.9°)$$
$$x = 60.85$$ (A1)

Designer's design:

$$x^2 = 55^2 + 40^2 - 2 \times 40 \times 55 \cos(22.1°)$$
$$x = 23.41$$ (A1)

Module 4 – Graphs and relations

Question 1 (5 marks)

a. From the graph, the amount is \$4400. A1

Note: An error of 100 in either direction would also obtain the mark.

b. Firstly, we need to obtain estimates of the New Zealand currency obtained for each of \$4000 and \$5000 Australian. Students must use their result from part **a.** for the first of these. We will use \$4400 for this purpose. Our result for the conversion of \$5000 Australian is \$5600 New Zealand. M1

Thus the difference = 5600 – 4400 = 1200. A1

Note: A variation of 100 more or less would also obtain the mark.

c. If the graph is straight, then the gradient at all points will be the same. We will pick one section near the left and one section near the right of the graph.

Consider points (200, 0) and (1000, 800).

$$m = \frac{800 - 0}{1000 - 200}$$

$= 1.00$ (the gradient on the left side)

Consider points (7000, 8000) and (8500, 10 000).

$$m = \frac{10\,000 - 8000}{8500 - 7000}$$

$= 1.33$ (the gradient on the right side) M1

Note: Both calculations are required for the mark.

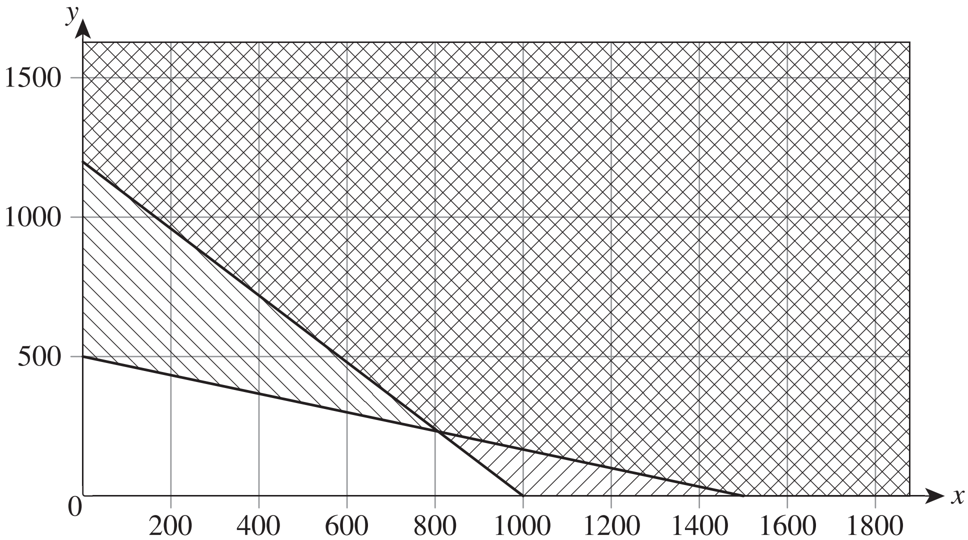
These results are quite different and thus it is clear that the line is not straight. A1

Question 2 (7 marks)

a. The other constraint relates to technology shares.

$5x + 4y \leq 5000$ A1

b.



A1

c. $P = 100x + 120y$ A1

d.

Point	Coordinates	Value of *P*
A (*y*-intercept)	(0, 500)	$60 000
B (intersect)	**(818, 227)**	**$109 040**
C (*x*-intercept)	(1000, 0)	**$100 000**

correctly identifies point B coordinates A1
correctly determines both missing P values A1

e. We can see from the table that the best outcome is when 818 basic and 227 advanced packages are sold. A1

The associated profit is $109 040. A1