**The Mathematical Association of Victoria**

# Trial Examination 2016

# FURTHER MATHEMATICS

## Written Examination 2

**STUDENT NAME:** _____

**Reading time: 15 minutes**
**Writing time: 1 hour 30 minutes**

## QUESTION AND ANSWER BOOK

**Structure of Book**

| Section A - Core | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| | 10 | 10 | 36 |
| Section B - Modules | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
| | 4 | 2 | 24<br>Total 60 |

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 34 pages
- Formula sheet.
- Working space is provided throughout the book.

**Instructions**

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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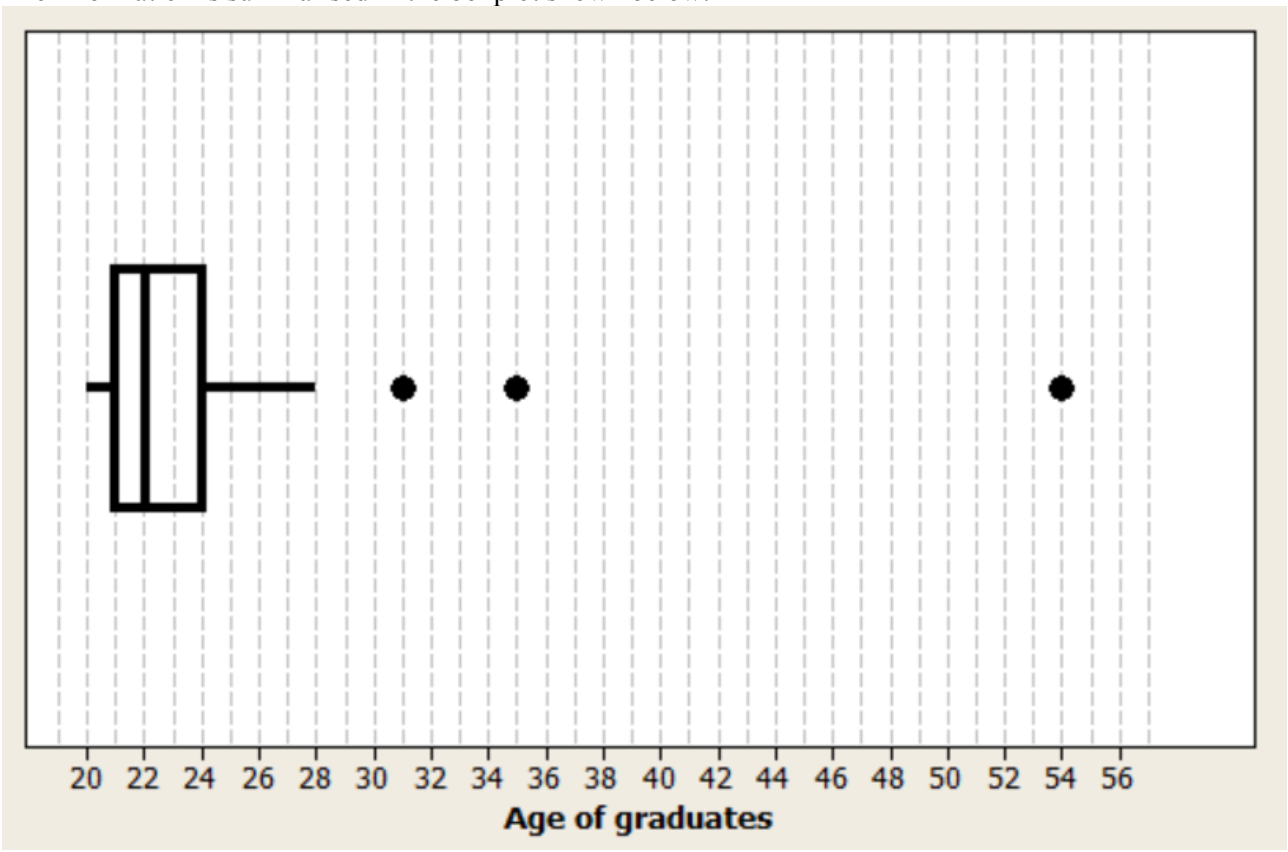
## SECTION A - Core

**Instructions for Section A**

Answer **all** questions in the space provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include for example, $\pi$, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.

## Core Data Analysis

**Question 1 (5 marks)**
The ages of the 54 students graduating from an Australian university at a recent ceremony were recorded. The information is summarised in the boxplot shown below:




a) What type of data is the age of graduates? **1 mark**

b) State the median age of graduates at this ceremony. **1 mark**

c) Describe the shape of the "Age of graduates" distribution shown in the boxplot.

**1 mark**

d) There are a number of outliers in the "Age of graduates" data. Show an appropriate calculation and use it to explain why these values are outliers. **2 marks**

**Question 2 (3 marks)**

Students who attend university in Australia must pay fees known as HECS. Students can pay their fees as they go or they can defer their fees until the end of their course, when they will have a HECS debt. The distribution of HECS debt is bell shaped and the average HECS debt for students at the end of their degree is $24 000 with a standard deviation of $5 500.

a) What percentage of students will have a HECS debt of less than $13 000 at the end of their degree?

**1 mark**

b) What percentage of students will have a HECS debt between $29 500 and $35 000 at the end of their degree?

**1 mark**

c) James has a HECS debt that has a standardised score for HECS debt of $-1.73$. Determine the amount of James' HECS debt correctly rounded to 3 significant figures.

**1 mark**

**Question 3 (2 marks)**
Table 1 below shows the percentages of males and females who completed undergraduate degrees in the United States during three different years (Source: www.statista. com).

**Table 1**

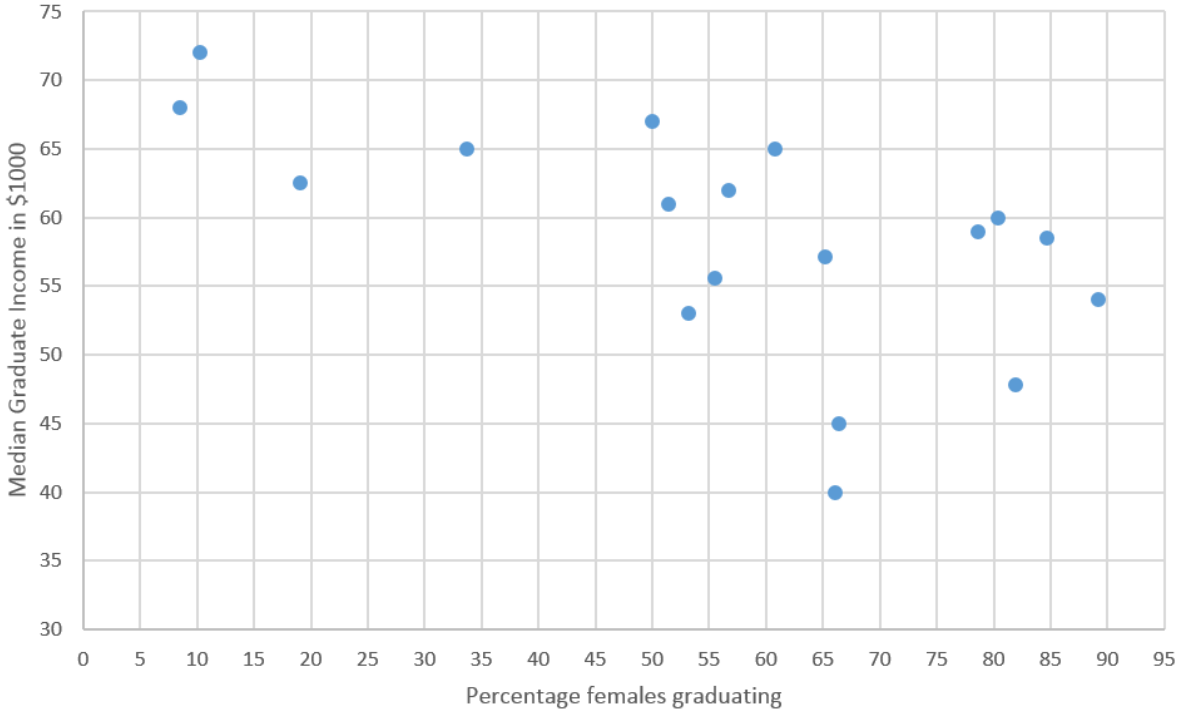
| Gender of Graduates | Year of Graduating | | |
|---|---|---|---|
| | **1975** | **1995** | **2015** |
| **Male** | 54.7% | 45.4% | 42.6% |
| **Female** | 45.3% | 54.6% | 57.4% |

Does the information in Table 1 support the contention that the gender balance is associated with the year of graduating? Explain your answer, quoting appropriate statistics to support your answer.

**2 marks**

**Question 4 (7 marks)**

A scatterplot is shown below showing the median graduate income in thousands of dollars against the percentage of female graduates for a number of different career paths.




The equation for the least squares relationship between Median graduate income in thousands of dollars (*MGIT*) and percentage of females graduating (*PFG*) is $MGIT = 69.893 - 0.203PFG$ correct to 3 decimal places.

The coefficient of determination for this relationship as a percentage is 36.47% correct to 2 decimal places.

a) Predict the median graduate income if 50% of the graduates are female. **1 mark**

b) Add the least squares regression line to the graph above. **1 mark**

**ANSWER ON GRAPH**

c) Interpret the meaning of the vertical intercept for this relationship. **1 mark**

d) Interpret the meaning of the slope for this relationship. **1 mark**

e) State the value of the correlation coefficient for this relationship.
Give your answer correctly rounded to 2 decimal places. **1 mark**

f) One course that appears in this data is Pharmacy where 66.1% of the graduates were female. Calculate the residual of *MGIT* if the median graduate earned $40 000. Give your answer correct to the nearest dollar. **1 mark**

g) The standard deviation of *PFG* is 24.6 correct to three significant figures. Determine the value of the standard deviation for *MGIT* correct to three significant figures.
**1 mark**

**Question 5 (3 marks)**

Table 2 gives the number of female graduates in thousands in the years 2001 to 2015 in the United States.

**Table 2**

| Year | Number of females graduates in 1000's |
|---|---|
| 2001 | 712 |
| 2002 | 742 |
| 2003 | 776 |
| 2004 | 804 |
| 2005 | 826 |
| 2006 | 855 |
| 2007 | 875 |
| 2008 | 895 |
| 2009 | 916 |
| 2010 | 943 |
| 2011 | 982 |
| 2012 | 1026 |
| 2013 | 1053 |
| 2014 | 1065 |
| 2015 | 1063 |

a) Determine the equation of the least squares regression line for the relationship between the number of female graduates in thousands and the year of graduation. Write the equation below with the coefficients correctly rounded to two decimal places. **1 mark**

b) Use your equation to predict the number of female graduates in the year 2030, correct to the nearest whole thousand. **1 mark**

c) Explain why this prediction may not be reliable. **1 mark**

**Question 6 (4 marks)**
A university shop records its sales in dollars for each quarter of the year between 2011 and 2016. The sales are seasonal and the seasonal indices for each quarter (except Quarter 3) are shown below.

| Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 |
|---|---|---|---|
| 1.65 | 0.8 | | 0.45 |

a) Determine the seasonal index for Quarter 3. **1 mark**

b) Interpret the meaning of the seasonal index for Quarter 1. **1 mark**

The university shop determines the following relationship for the deseasonalised sales in dollars, given that Quarter 1, 2011 is $t = 1$:

$$deseasonalised\ sales = 32\,500 + 215t$$

c) Determine the actual sales at the university shop in Quarter 2, 2015. Give your answer correctly rounded to 3 significant figures. **2 marks**

## Core Recursion & Financial Modelling

**Question 1 (3 marks)**
Kylie is in Year Ten, and is saving money for her "Schoolies" activity at the end of Year Twelve. She has started an account with $850, and this account will be paid 3.6 % p.a. interest, compounded quarterly.

(a) What is the interest rate per quarter, expressed as a decimal number ?

**1 mark**

(b) Using $B_n$ to represent the balance of the account after $n$ quarters, write down a recurrence relation to model this investment situation.

**1 mark**

(c) How much money will be in Kylie's account after two years, correct to the nearest cent ?

**1 mark**

**Question 2 (4 marks)**
Kylie's elder sister, Sophia, needs a new computer costing $1100 to assist her university studies. To pay for it she will have to take out a personal loan.

Her Student Credit Union offers her a personal loan which charges interest at 9 % per annum, compounding monthly. The loan is intended to be repaid with 6 equal monthly payments of $190.00.

(a) Using $B_n$ to represent the balance of the loan after *n* months, write down a recurrence relation to model this loan situation.

**1 mark**

A partially-completed amortisation table for Sophia's loan is shown below.

| Payment number | Payment made | Interest paid | Principal reduction | Balance of loan |
|---|---|---|---|---|
| 0 | | | | 1100.00 |
| 1 | 190 | 8.25 | 181.75 | 918.25 |
| 2 | 190 | 6.89 | 183.11 | 735.14 |
| 3 | 190 | 5.51 | 184.49 | 550.65 |
| 4 | 190 | 4.13 | 185.87 | 364.78 |
| 5 | 190 | 2.74 | 187.26 | 177.52 |
| 6 | | | | |
| Total | | | | |

Looking at this partially completed amortisation table, we see that the balance owing after the fifth payment is quite a lot less than $190, meaning that the final payment to amortise this loan will be less than $190.

(b) Determine the amount of the final payment, correct to the nearest cent.

**2 marks**

(c) What is the effective rate of interest that Sophie is paying on her loan, correct to two decimal places ?

**1 mark**

**Question 3 (3 marks)**
Kylie's father, George, has a superannuation account which currently has a balance of $350 000. He pays $650 per month into the account which is earning 4.8 % p.a. compounded monthly. If George continues this level of payments to the superannuation account he estimates that he will have approximately $665 000 when he retires in ten years.

(a) George wants to have at least $750 000 in the account when he retires.
How much **extra** will he have to pay each month for the next ten years to meet this new financial goal, correct to the nearest cent ?

| N | I(%) | PV | Pmt | FV | PpY, CpY |
|---|---|---|---|---|---|
| | | | | | |

**2 marks**

(b) If George retires with $750 000 and invests it in a perpetuity earning 5.4 % p.a. compounded and paid monthly, how much will remain in the account after five years of retirement ?

**1 mark**

**Question 4 (2 marks)**
Kylie's big brother Sam has borrowed $350 000 to buy a house and will repay $2600 per month for 20 years, with interest being charged at 6.45% p.a., compounded monthly. His accountant has suggested that Sam repay his loan at half the monthly amount ($1300) each fortnight to repay the loan sooner. When Sam checks the numbers, he realises that it will take just over 17 years to repay the loan this way.

Explain, using the total amount paid each year and showing all calculations, why this method works.

**2 marks**

**END OF SECTION A**
**TURN OVER**

## SECTION B - Modules

**Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include for example, $\pi$, surds or fractions.
Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.

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## Module 1 MATRICES

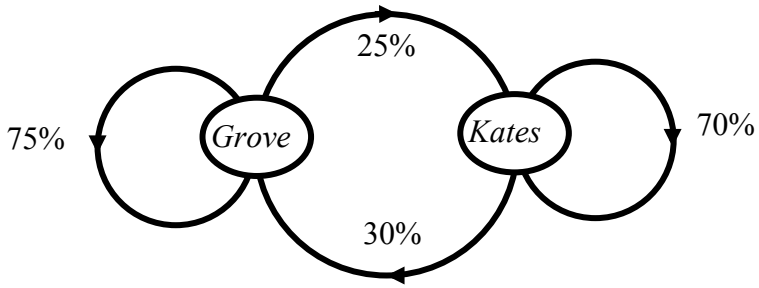
**Question 1 (5 marks)**
A supermarket owner in a country town is keeping an eye on ice cream sales where two major brands – *Grove* and *Kates* – are battling for market supremacy. There are 225 regular shoppers each week, and at the start of summer ice cream sales period, 125 customers are buying *Grove*.

(a) Write the initial state matrix as a column matrix, putting *Grove* in the first row.

$$S_0 = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$$

**1 mark**

Market research conducted by both companies has shown how buyers' preferences change from week to week, and this is shown in the transition diagram below.




(b) Write down the transition matrix $T$ that represents this situation

**1 mark**

(c) Show that the number of *Grove*'s customers after four weeks, writing your answer correct to the nearest whole number, is 123.

**1 mark**

At this point in time, tourists start coming to town to shop and the situation can be described in the following recurrence relation :

$$S_{n+1} = T_1 S_n + B \qquad \text{where } T_1 = \begin{bmatrix} 0.77 & 0.27 \\ 0.23 & 0.73 \end{bmatrix},\ S_0 = \begin{bmatrix} 123 \\ 102 \end{bmatrix} \text{ and } B = \begin{bmatrix} 19 \\ 13 \end{bmatrix}$$

(d) Write the state matrix after two weeks of tourist shopping, giving values correct to the nearest whole number.

**2 marks**

**Question 2 (5 marks)**
The supermarket owner is a big employer in the country town as she employs 50 people on her staff. In 2015 there were 20 junior casuals ($J$), 20 senior casuals ($C$) and 10 permanent staff ($P$).

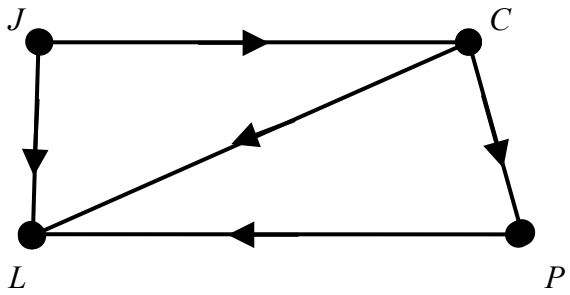
Each year, staff will remain in the same job, move to a different level because of age (junior staff become senior staff when they turn 18) or promotion to a permanent position or leave the job. ($L$). The expected number of staff in each category each year can be determined using the transition matrix, $T$, shown below.

$$T = \begin{array}{c} \\ J \\ C \\ P \\ L \end{array} \begin{array}{c} \begin{array}{cccc} J & C & P & L \end{array} \\ \begin{bmatrix} 0.65 & 0 & 0 & 0 \\ 0.10 & 0.75 & 0 & 0 \\ 0 & 0.05 & 0.90 & 0 \\ 0.25 & 0.20 & 0.10 & 1.00 \end{bmatrix} \end{array} \begin{array}{l} \text{this year} \\ \\ \\ \\ \\ \text{next year} \end{array}$$

**(a)** Explain the meaning of the entry in the fourth row and fourth column of transition matrix $T$.

**1 mark**

(b) An incomplete transition diagram for matrix $T$ is shown below.
Complete the transition diagram by adding the missing information.




**2 marks**

The supermarket owner promotes senior casual staff to fill any vacancies caused by permanent staff leaving, so she needs to find junior casual and senior casual staff to fill the job vacancies.

(c) In 2016, how many junior casual staff and senior casual staff will need to be found, to replace staff who have left or been promoted, to maintain the total staff numbers as in 2015 ?

**2 marks**

**Question 3 (2 marks)**
Four of the permanent staff played a round-robin darts tournament during their lunch breaks, in which every staff member played against every other staff member once. In each game there was a winner and a loser.

A table of one-step and two-step dominances was prepared to summarise the results.

| Employee | one-step dominances | two-step dominances |
|---|---|---|
| Ken (*K*) | 2 | 2 |
| Leanne (*L*) | 1 | 1 |
| Maggie (*M*) | 2 | 3 |
| Neil (*N*) | 1 | 2 |

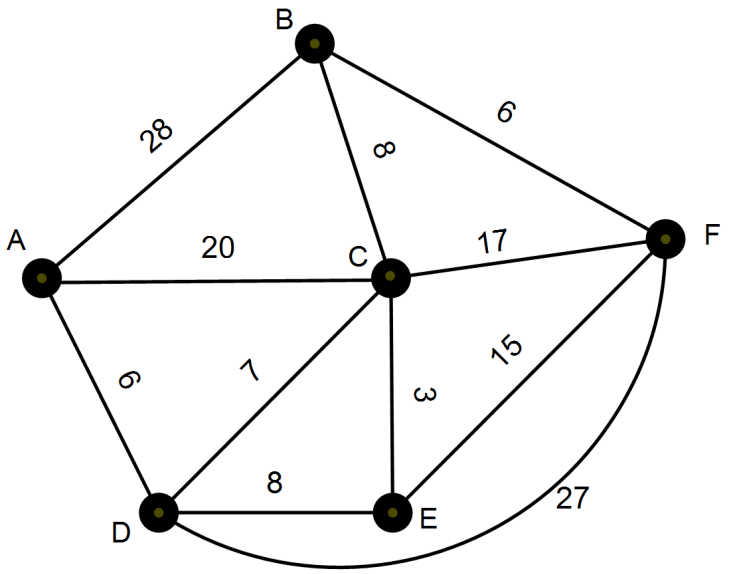
Use the one-step and two-step dominance values to construct a one-step dominance matrix. Label the rows as Winners and the columns as Losers.

**2 marks**

**End of Module 1 - SECTION B** – continued
**TURN OVER**

## Module 2 Networks and decision mathematics

**Question 1 (6 marks)**
A network showing the length in kilometres of a number of roads between towns Aden (A), Billings (B), Charleton (C), Dalton (D), Eventown (E) and Fairburn (F) is shown below.




a) The shortest distance from Aden to Fairburn using these roads is_____. **1 mark**

b) A travelling salesman is based at Aden. He leaves Aden and drives to Charleton, Billings, Fairburn, Eventown, Dalton and then back to Aden in that order. Give the name of the route taken by the travelling salesman. **1 mark**

_____

The towns in the region are to have the NBN connected for better internet services.
The company installing the cabling needs to determine where they place the cables, so they drive along all the roads in the region. The driver wants to start at Aden and travel along every road exactly once, returning to Aden.

c) Explain why this is not possible in this network. **1 mark**

_____

_____

_____

_____

d) What is the minimum number of roads that he would need to travel along twice if he is to start and finish at Aden and travel the minimum distance? **1 mark**

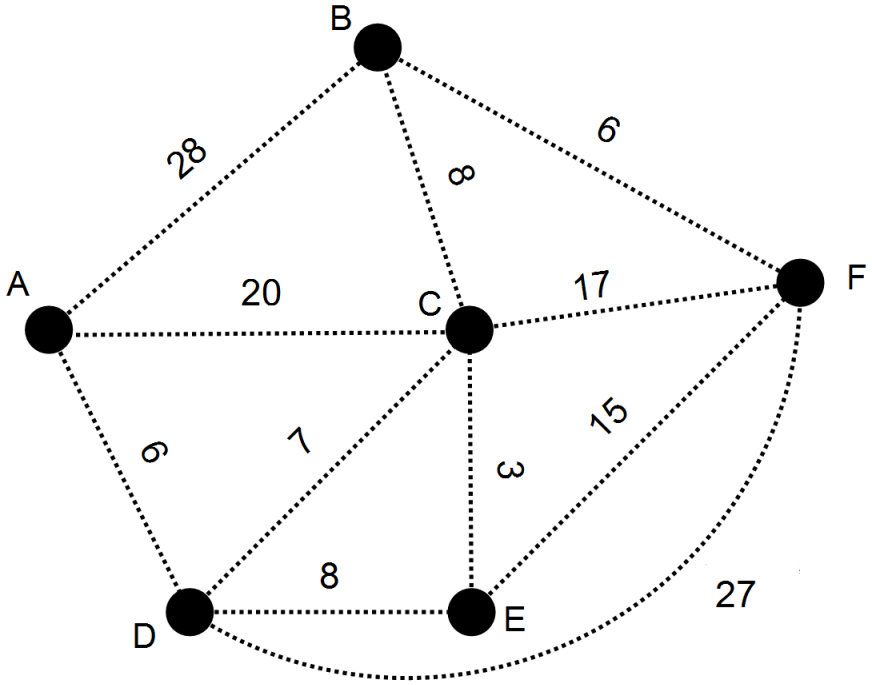
_____

_____

_____

After the initial inspection it is decided that any road in the network would be suitable to use. All towns in the region are to be connected. The NBN cabling is to be laid alongside the roads and the minimum length of cabling would be most cost efficient.

e) Show where the cabling should be laid to ensure that the minimum length of cabling is used. Another network has been given below for you to show the roads used. **1 mark**




f) What is the minimum length of cabling required to connect the towns? **1 mark**

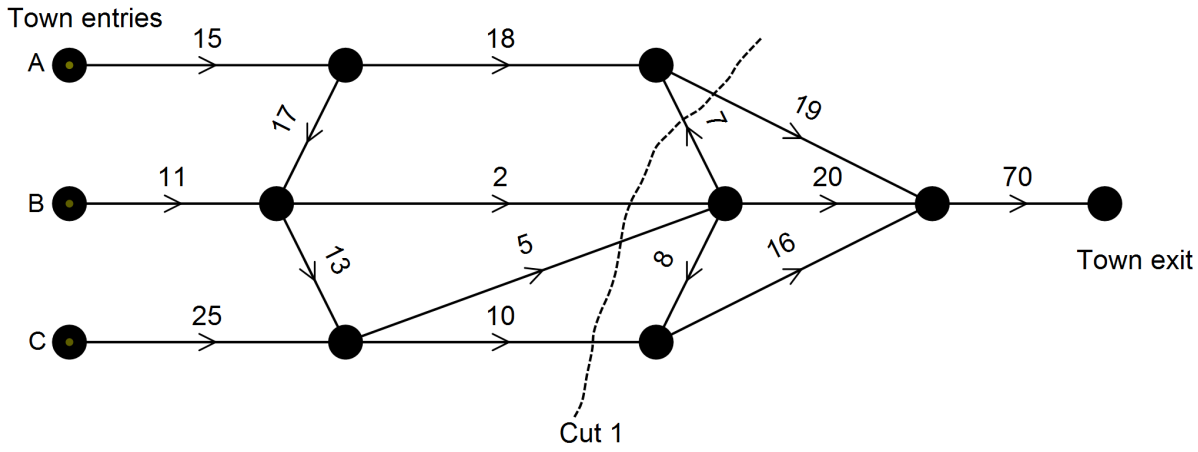
_____

_____

**SECTION B - Module 2**- continued
**TURN OVER**

**Question 2 (5 marks)**

Charleton is a major town and there are a number of traffic flow problems. One section of the road network that passes through the town centre of Charleton from the town entries A, B and C in the East to the town exit in the West is shown below. The numbers on the edges are the maximum number of vehicles that can pass along each road per minute.

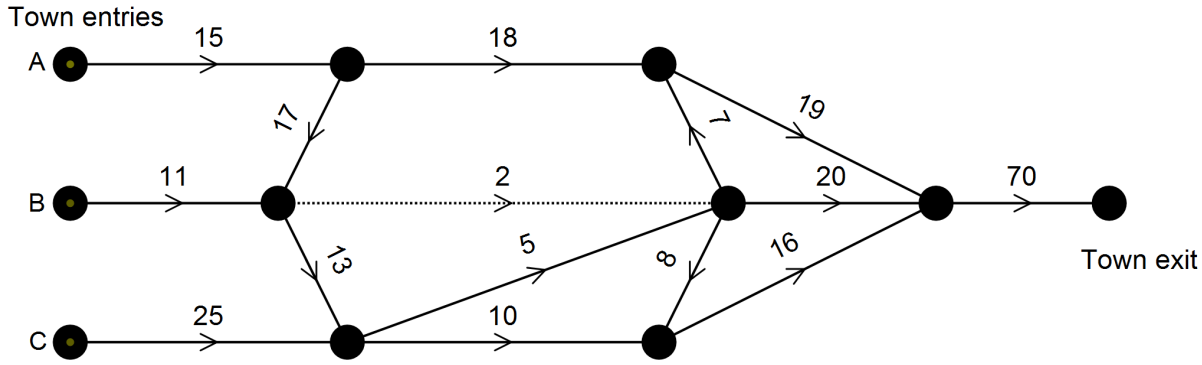



a) Cut 1 is shown on the network. What is the capacity of Cut 1? **1 mark**

b) The town planners want to know the maximum flow of vehicles per minute through this network. What is the maximum flow from the East to the West of the town?

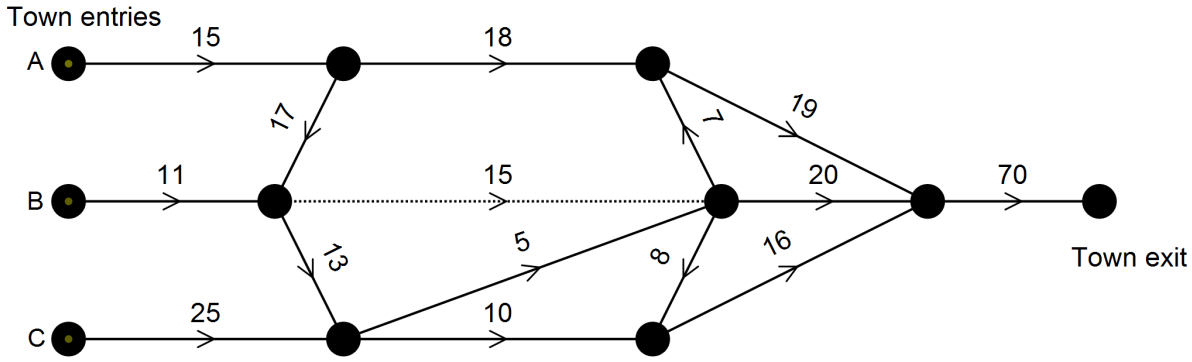
**1 mark**

The dotted road in the diagram below can be widened, allowing the flow to increase to 15 vehicles per minute.




c) How does this affect the flow through this network? Explain your answer. **2 marks**

During the annual Harvest Festival the people of Charleton make floats to parade through the town centre. The parade must pass along the widened road shown as a dotted edge below.




d) On the network above, show the route that the floats should take, from one of the town entries in the East through to town exit in the West, passing along the dotted road, that would ensure the greatest number of floats per minute. Note that the floats must all pass along the same route.

**1 mark**

**ANSWER ON GRAPH**

**Question 3 (1 mark)**
One float carries the Harvest Queen. Her dress is made by four people, Thomas (T), Ursula (U), Victoria (V) and Xavier (X) who work in a factory. They each are allocated a task for making the dress based on their efficiency. The tasks are cutting (C), fitting (F), sewing (S) and ironing (I). Part of the Hungarian Algorithm has been completed and is shown below. Use this information to complete the table of allocations.

**1 mark**

$$\begin{array}{c} \\ T \\ U \\ V \\ X \end{array} \begin{array}{c} \begin{array}{cccc} C & F & S & I \end{array} \\ \begin{bmatrix} 0 & 3 & 4 & 0 \\ 0 & 4 & 5 & 2 \\ 0 & 7 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

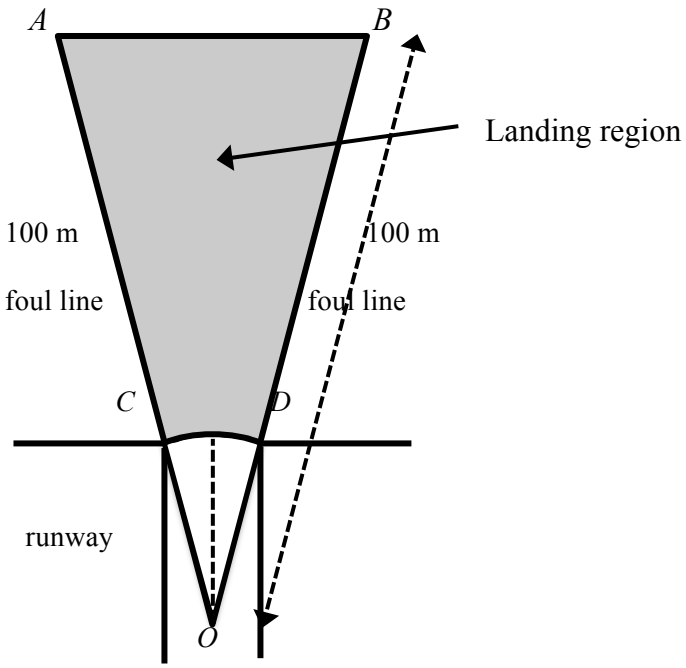
| Worker | Task |
|---|---|
| Thomas | |
| Ursula | |
| Victoria | |
| Xavier | |

**End of Module 2 - SECTION B** – continued

## Module 3 GEOMETRY and MEASUREMENT

**Question 1 (6 marks)**
Marcus works for the Mildura City Council, and is preparing the Mildura Athletics Stadium surface for a javelin competition.

The field markings for the javelin event consist of a 4m wide runway, ending in a curved arc from points *C* to *D* (part of a circle 8.0 m in radius), foul lines and the edge of the landing region, as shown in the diagram below. The shaded area on the diagram is the landing region for a javelin throw.
The foul lines meet the edge of the landing region at points *A* and *B*, 100 m from the apex of the triangle (and centre of the arc) at *O*.
The angle *AOB* is 28.96°.




(a) What is the distance between points *A* and *B,* correct to the nearest metre ?

**1 mark**

(b) What is the length of the arc from point *C* to point *D* ?
Write your answer in metres, rounded to two decimal places.

**1 mark**

**SECTION B - Module 3- Question 1** - continued
**TURN OVER**

The current men's world champion is booked to run a javelin-throwing clinic at the stadium, and the promoters have asked for an extra 20 m to be added to the foul lines on each side.

(c) How much extra grassed area will Marcus need to prepare, correct to the nearest square metre ?

**2 marks**

**Question 2 (6 marks)**
In this question, assume that the radius of Earth is 6400 km.

People are travelling from many interesting places to Mildura for this competition.
The American champion has been taking a little holiday break in Normanton, Queensland (17º S, 142º E), before flying direct to Mildura (34ºS, 142º E) in a charter flight.

(a) If his plane flies the great circle route along the 142º E meridian, how far will it travel, correct to the nearest ten (10) kilometres ?

**1 mark**

Kai, the Australian champion, has been training at Margaret River in Western Australia prior flying to Mildura for the competition.

(b) The flight from Margaret River (34º S, 115º E) to Mildura (34° S, 142° E) travels along a small circle.

(i) Find the radius of this small circle.
Round your answer to two decimal places.

**1 mark**

(ii) Find the distance the plane travels between Margaret River (34º S, 115ª E) and Mildura (34° S, 142° E) along the small circle.
Round your answer to the nearest kilometre.

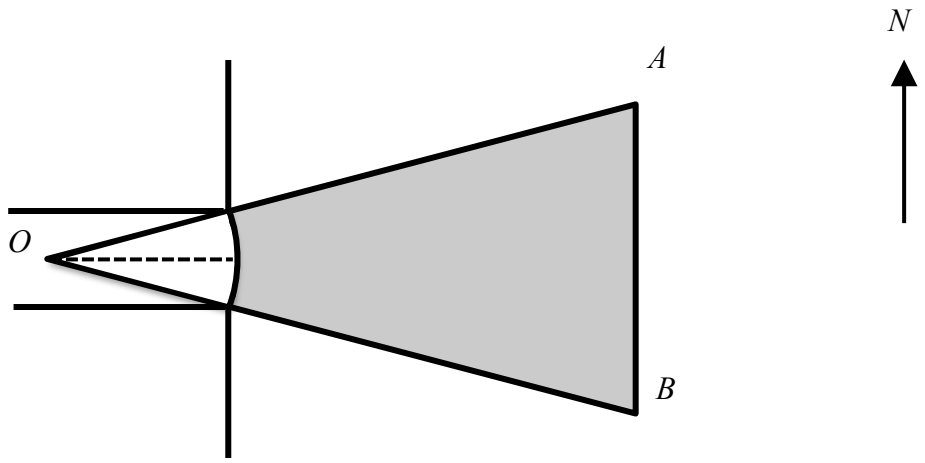
**1 mark**

The world champion is a Kenyan, and Kenyan TV has sent a reporter and crew to cover his appearance at this competition. The reporter will make live reports from Mildura (34º S, 142º E) to Nairobi (2º S, 37º E) every evening to air at 6.30 p.m. local time in Nairobi.

(c) At what time will the reporter be making their report in Mildura, assuming that there will be one (1) hour time difference for each 15º difference in longitude ?

**2 marks**

**Question 3 (3 marks)**




The actual orientation of the javelin field at the Mildura Athletics Stadium is West-East.

Kai's first throw travelled 85.41 m on a bearing of 078ºT.
Kai's second throw landed at a spot which was 38.00 m on a bearing of 187ºT from the landing spot of his first throw.

(a) How far, in metres correct to the nearest centimetre, did Kai throw the javelin on his second throw ?

**2 marks**

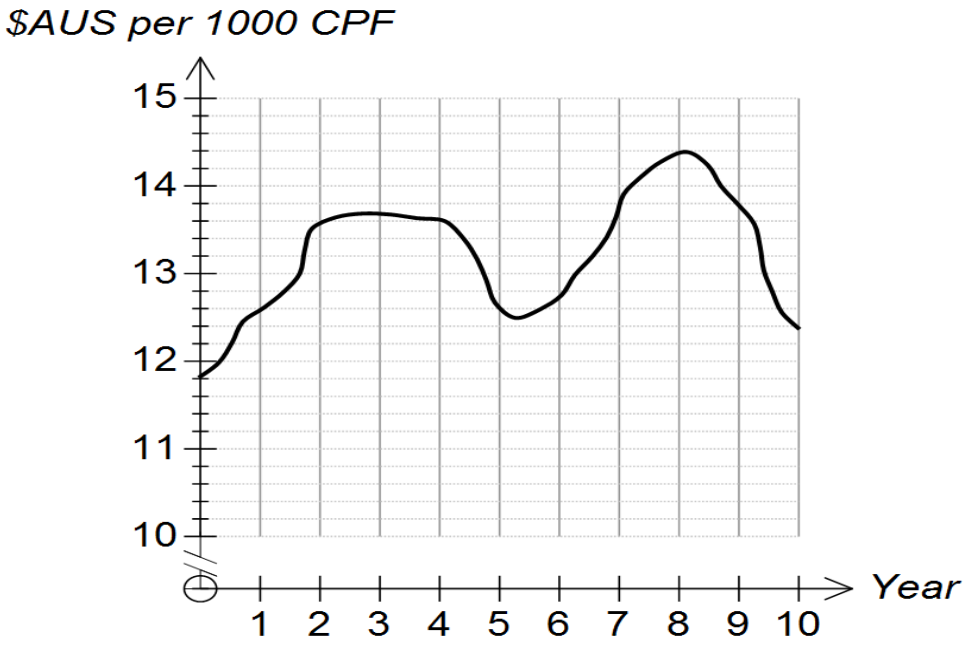
(b) On what bearing, correct to the nearest whole degree, did Kai's second throw travel ?

**1 mark**

**End of Module 3 - SECTION B** - continued

## Module 4 Graphs and Relations

**Question 1 (3 marks)**
The graph below shows the number of Australian dollars that would be equivalent to 1000 Central Pacific Francs (CPF) over a 10 year period, where the value on the x- axis is the time in years after recording started.




Amy had 10 500 CPF at the end of the first year.

a) How many Australian dollars was 10 500 CPF worth at the end of the first year?

**(1 mark)**

Amy plans to travel to Noumea (where CPF is the currency) at the end of the 8th year.

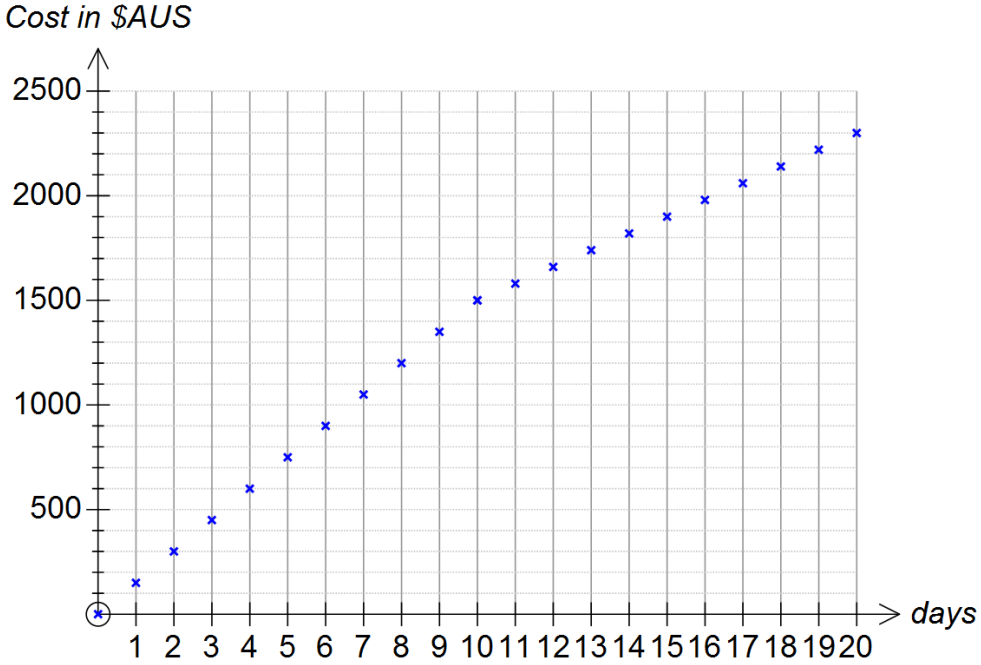
(b) What was the average yearly increase in dollars per 1000 CPF between the end of the first year and the end of the eighth year? Give your answer correct to the nearest cent.

**(1 mark)**

(c) How much extra money did Amy have if she left her money as CPF until the end of the 8th year? Give your answer correct to the nearest cent. **(1 mark)**

**Question 2 (4 marks)**

Amy plans to stay in a motel in Noumea that charges Australian dollars for her accommodation. The motel charges a cheaper rate per day when customers stay after a certain number of days. A graph showing the cost to stay at this motel over a period of days is given below. It is plotted as a series of points because only whole days are charged:




The points can be joined using two straight lines that would have the relationship between cost ($C$) and number of days ($n$):

$$C = \begin{cases} 150n & 0 < n \leq a \\ b + dn & a < n \leq 20 \end{cases}$$

a) Determine the values of $a$, $b$ and $d$ and write them below. **(2 marks)**

Another motel that Amy considers for her holiday charges an upfront cost of $1500, but then only charges $30 per day for stays up to 20 days long.
The equation that models this relationship is $C = 30n + 1500$, $0 \leq n \leq 20$.

b) Add the line for the relationship in part b) to the graph above. **(1 mark)**

**ANSWER ON GRAPH**

c) How long would Amy need to stay for the price to be cheaper at the second motel?

**(1 mark)**

**Question 3 (3 marks)**
Amy takes her mobile phone to Noumea with a charger and notices that her phone is quicker to charge than usual. She records the percentage charge ($p$) at different times in minutes ($m$) from a completely flat battery. The results are shown in the table below:

| $m$ (minutes) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| $p$ (percentage charge) | 2 | 8 | 18 | 32 |

a) The relationship between $p$ and $m$ is of the form, $p = km^2$.
Determine the value of $k$. **(1 mark)**

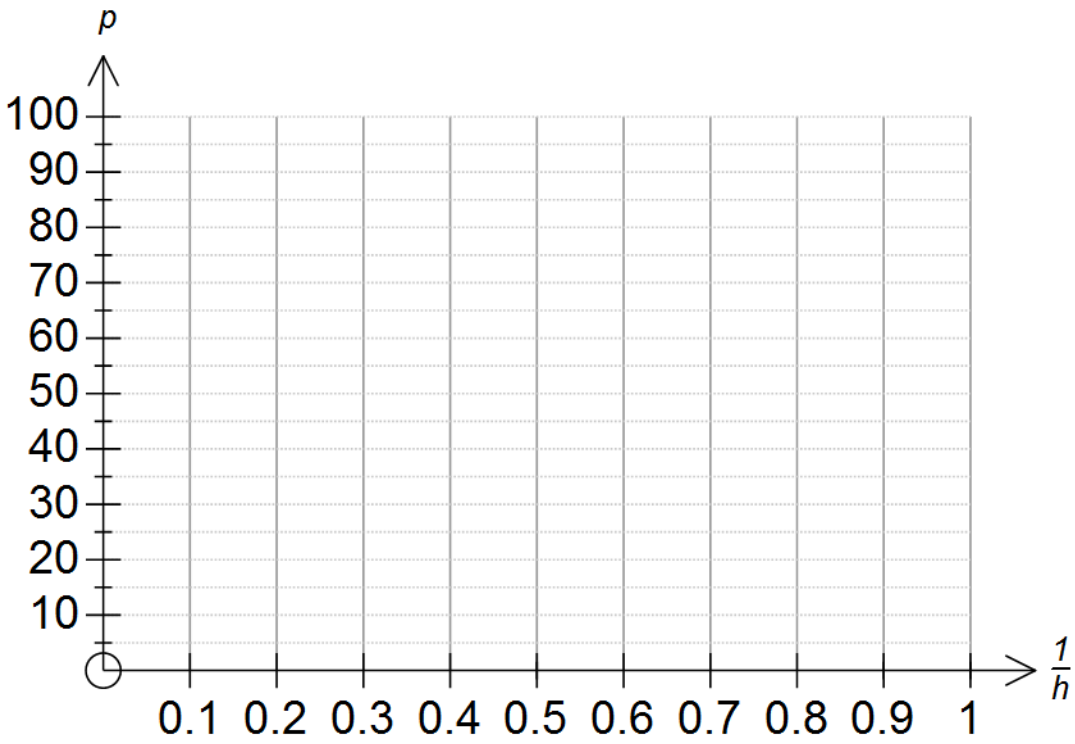
b) How long will it take for Amy's phone to reach full charge (100%)?
Give your answer in minutes correct to one decimal place. **(1 mark)**

**SECTION B - Module 4 - Question 3** - continued

Amy also notices that, due to international roaming, her phone discharges faster than usual. She determines that the relationship between the percentage charge (p) and hours (h) without using the phone is:

$$p = \frac{90}{h} \qquad 0 < h \leq 10$$

c) Draw the relationship between p and $\frac{1}{h}$ on the axes below. **(1 mark)**



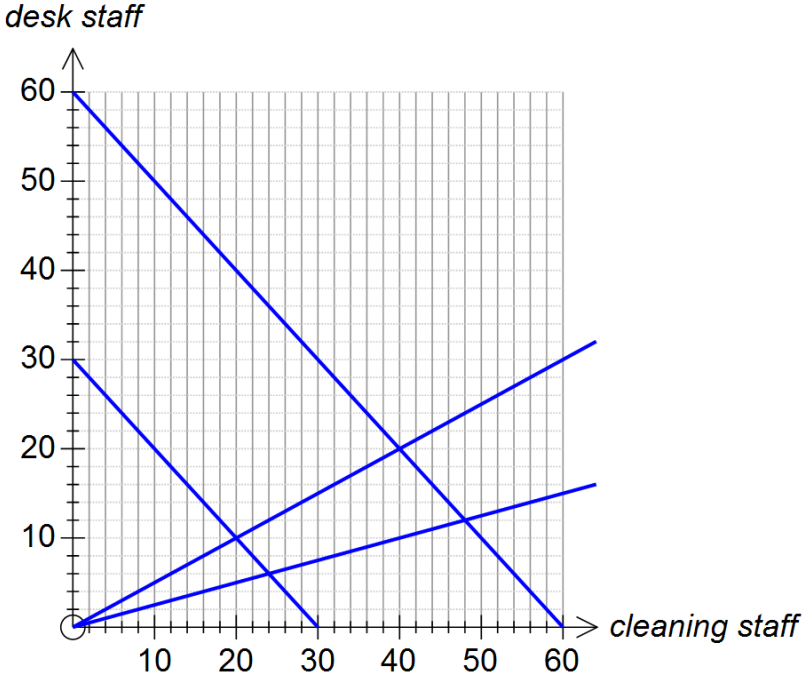
**Question 4 (2 marks)**

The motel that Amy stays at have worked out the ideal conditions for the motel to operate based on the numbers of cleaning staff ($x$) and desk staff ($y$) working at the motel. These can be described by the constraint inequations shown below:

$$30 \leq x + y \leq 60$$

$$\frac{x}{4} \leq y \leq \frac{x}{2}$$

The lines that form this feasible region have been graphed below:




a) Clearly identify the feasible region by shading the appropriate region on the graph above.

**(1 mark)**

**ANSWER ON GRAPH**

b) Explain the meaning of the inequation $\frac{x}{4} \leq y \leq \frac{x}{2}$ in the context of this problem.

**(1 mark)**

**END OF QUESTION AND ANSWER BOOKLET**