Engage Education Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Core

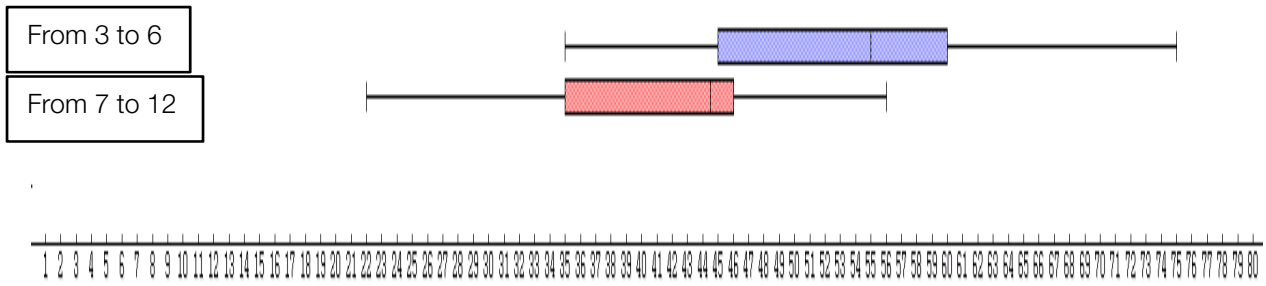
Data Analysis

Question 1a

Parallel boxplots and/or back-to-back stemplots are appropriate responses [1].

Question 1b i

Both labels need to be correct for [1].



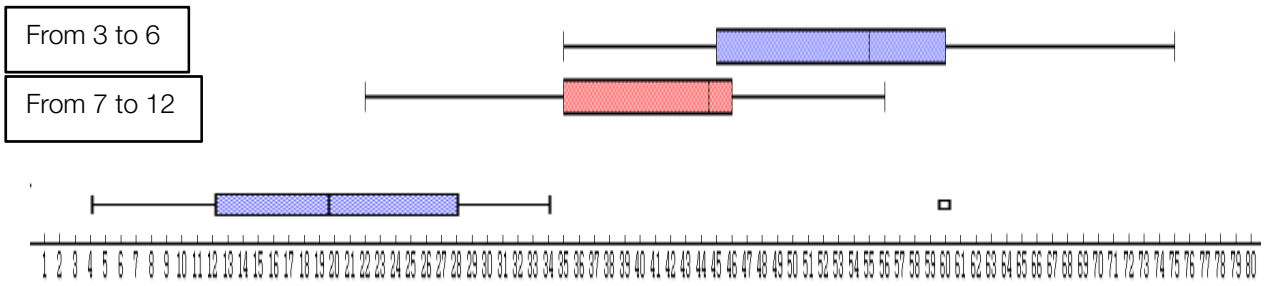
Question 1b ii

All the values of min, Q1, median, Q3, max and outlier have to be correctly shown for [1] mark.

Question 1c

There is an association between the time spent outdoors and age groups, with the time spent outdoors **generally** decreases as the children get older. The most suitable statistics to use for support is the median. The median decreases from 55 hours in 3-6 group, to 44.5 hours in 7-12, to 19.5 hours in 13-15 group.

Both the statement of association AND the supporting statistics are needed for 2 marks. The statement alone is not sufficient for 1 mark (50-50 chance of guess).You can also support the statement using the



position of the IQR.

Question 2a

Comments can be made about either the **centre** of distribution **or** the **spread** of distribution.
School A has a lower centre that school B **[1]**. Median of 154 cm (A) compared to 175 cm (B) [1].

School B has larger spread than school A [1]. Range of 96 cm (B) compared to 45 cm (A) [1].

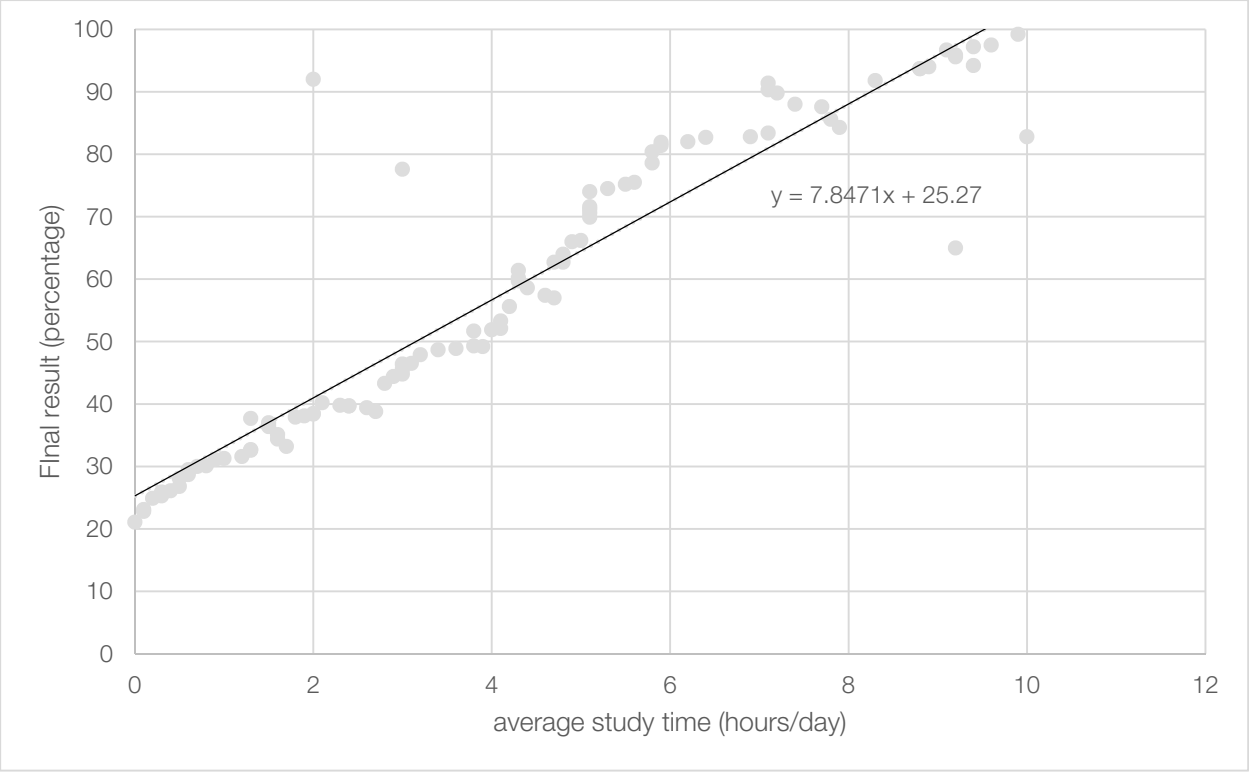
Question 2b

Calculation is required.

IQR = 26.
Lower fence = 157.5 – 1.5 x IQR = 118.5 [1]
112 is smaller than lower fence [1] and hence, 112 is the outlier.
Both the calculation of the lower fence and the comparison against the value 112 are needed for 2 marks to be awarded.

Question 3a

[1] for the correct line.



Question 3b

For each increase of an hour in study time, there **tends to be** an increase of 7.8471 marks/percent in final result [1].
It is important that student does not make a definitive statement about this e.g. …causes…, …leads to…, etc.

Question 3c

Final results (predicted) = 7.8471 x 5 + 25.27 = 64.5 ~ 65.
Residual = 85 -65 = 20 [1]

Question 3d

R^2 = 0.88 = 88% [1].

Question 3e

The gradient will be more positive/increase [1].
One or both of the following explanations will be acceptable for [1].

The gradient of a least squares regression line is

$$\boldsymbol{m = \frac{rs_x}{s_y}}$$

If (9.2 , 65) is changed into (9.2 , 95), the points adhere more closely to a line. This increases the correlation coefficient r. Hence, m will increase.
2.

Since the changed value is of y, or the final result, there is overall less variation in the final results, or the y-values, and hence, $\boldsymbol{s_y}$ decreases. This will also contribute to make the gradient more positive.

Question 4a

This plot can be linearised by either expanding the y-axis or compressing the x-axis. Hence, y-squared and 1/x can also work [1]. No explanation is needed.

Question 4b

Student can opt for y-squared or 1/x transformation.

y-squared: $R^2 = 0.6815 < 0.8845$ [1]. Hence, y-squared transformation is not as effective [1].

1/x: $R^2 = 0.4887 < 0.8845$ [1]. Hence, 1/x transformation is not as effective [1].

Question 5a

Science marks show more variations. The standard deviation of science marks (9) is larger than that of English (3) [1].

Question 5b

$z\ (English) = \frac{76-70}{3} = 2$ [0.5]

$z(Science) = \frac{67-50}{9} = 1.89$ [0.5]

The standard score for science (1.89) is smaller than that for English (2). Hence, Patrick performed better in English than Science [1].

Question 6a

$deseasonalised\ stock\ price = \ 1.21 \times time + 14.35.$

The time code starts at 0 for Summer '10. Winter '12 will have the code of 10.
$deseasonalised\ stock\ price = \ 1.21 \times time + 14.35 = 26.45$ [1]
$actual = 26.45 \times 0.84 = 22.22$ [1].

Question 6b

The time code for Spring '20 is 43. This is too far off in the future compared to existing data set (from 2010 to 2012) and hence this prediction will not be reliable [1].

Recursion and financial modelling

Question 7a i

$V_n = 36000 - n \times 6000$ [1]

Question 7a ii

Half the value would be 18000.
Solve for n in the equation $18000 = 36000 - n \times 6000$. $n = 3$ years.

Question 7b i

$\boldsymbol{V_n = 36000 - s \times 0.75}$

$18000 = 36000 - s \times 0.75$

$s = 24000\ km$ [1]

Question 7b ii

It takes 3 years to depreciate to half the value by flat rate depreciation.
It takes 24000/7500 = 3.2 years to depreciate to half the value by unit cost depreciation. Hence, it is better to go with unit cost depreciation [1].
Reasoning in some way is needed for awarding of mark.

Question 8a i

$$r_{\text{effective}} = \left(\left(1 + \frac{r/n}{100}\right)^n - 1\right) \times 100\%$$

Hence $r_{effective} = \left(\left(1 + \frac{\frac{4}{12}}{100}\right)^{12} - 1\right) \times 100 = 4.07\%$ [1]

Question 8a ii

The amount of money in her investment after 3 years is

$$1000 \times \left(1 + \frac{\frac{4}{12}}{100}\right)^{36} = \$1127.27$$

Interest is:

$1127.27 - 1000 = \$127.27$ [1]

Correct result with some appropriate working is awarded 2 marks.

Question 8b i

Each correct answer is awarded [1].

Withdrawal number	Withdrawn amount	Interest earned	Principal reduction	Balance of annuity
0	0	0	0	1000
1	80	20	60	940
2	80	18.8	61.2	878.8
3	80	17.58	62.42	878.8 – 62.42 = 816.38
4	80	16.33	63.67	752.71
5	80	$\frac{752.71 \times 4/2 \times 1}{100} = 15.05$	64.95	687.76
6	80	13.76	66.24	621.52
7	80	12.43	67.57	553.95
8	80	11.08	68.92	485.03
9	80	9.7	80 – 9.7 = 70.3	414.73
10	80	8.29	71.71	343.02
11	80	6.86	73.14	269.88
12	80	5.4	74.6	195.28

Question 8b ii

After 3 years, Anna has only earned $101.52 of interest [1], and therefore the savings account is a better strategy.

Module 1 – Matrices

Question 1a

Matrix L is as follows

$$L = \begin{array}{c} \begin{matrix} J & I & P & A & \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} J \\ I \\ P \\ A \\ B \end{matrix}$$

Correct matrix with appropriate labelling is awarded [1].

Question 1b

$$L^2 = \begin{array}{c} \begin{matrix} J & I & P & A & B \end{matrix} \\ \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 0 & 1 \\ 1 & 1 & 3 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} J \\ I \\ P \\ A \\ B \end{matrix}$$

$$L + L^2 = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 \\ 2 & 3 & 2 & 1 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Correct matrix is awarded [1].

L^2 + L represents a two-step communication, i.e. with one intermediate.

The zero values represent people who would need to go beyond this point to communicate. Those are Aleksei and Benkei [1].

Question 2a i

According to the transition matrix, 0.4 or 40% of customers will stay. 60% will leave. Hence 600 customers [1].

Question 2a ii

In total, 0.1 + 0.05 + 0.1 = 0.25 or 25% of customers will leave permanently. Hence 25% x (1000+600+500) = 525 customers [1].

0.1x1000 + 0.05x600 + 0.1x500 = 180

Question 2b i

$$\begin{bmatrix} 473.5 \\ 708 \\ 583.25 \\ 335.25 \end{bmatrix}$$ [1]

Question 2b ii

The state matrix on the 4th day is

$$\begin{bmatrix} 418.4875 \\ 654.3375 \\ 550.85 \\ 476.325 \end{bmatrix}$$

There are 654 customers going to shop B on the 4th day [1].

Question 2c i

82nd day [1].

$$\boldsymbol{T^{81} \times S_0 = day\ 82}$$

Question 2c ii

735 customers [1]. This occurs on the 2nd day.

Question 2d

The goal of adding/removal of customers is to restore the state matrix to the initial state.
The state matrix on the second day is

$$\begin{bmatrix} 595 \\ 735 \\ 590 \\ 180 \end{bmatrix}$$

Hence, 405 customers have to be added to shop A, 135 removed from shop B, 90 removed from shop C [2].
If only one mistake is made in the answer compared to above, [1] is awarded.

Question 2e

Matrix M is as follows

$$\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ [1]

In order to answer this correctly, student needs to be aware that numbers in the transition matrices represent percentage or probability.

Module 2 – Networks and decision mathematics

Question 1a

Activity	Earliest starting time	Latest starting time
A	0	11
B	0	0
C	0	12
D	2	13
E	3	3
F	5	17
G	10	10
H	10	23
I	8	20
J	8	23
K	18	18
L	26	26
M	18	33

Table is completed as above. [0.5] marks for each completed box.

Question 1b

38 hours [1].

Question 1c

B -> E -> G -> K -> L [1].

Question 1d

Reduction of time for activity B is less expensive. So the extra earning from shortening B by 2 hours is ($70 - $50) x 2 = $40 [1].

Question 2a

Yes. All vertices are connected and C and H have odd degrees [1].

Question 2b i

For a Eulerian trail with 2 odd-degree vertices, choose one of them as the starting point and the other vertex will be the endpoint.

Question 2b ii

C -> D -> E -> A -> B -> C -> A -> H -> F -> C -> E -> F -> G -> H [1].
A -> B -> C -> D -> E -> F -> G -> H. [1] Any other order that visits each vertex once will also work.

Question 2c

41 km. (A -> C -> F -> G) [1]

Question 3

Company	Community
M	D
N	B
O	A
P	C

Correct allocations of two companies are awarded [1].

Module 3 – Geometry and measurement

Question 1a

30° [1].

Question 1b

The area of the sector:

$$A(sector) = \pi 100^2 \times \frac{30}{360} = 2617.99\, m^2$$

$$A(triangle) = \frac{1}{2} \times 100 \times 100 \times \sin(30) = 2500\, m^2$$

$$A(segment) = A(sector) - A(triangle) = 117.99 \sim 118\, m^2$$

Question 1c i

Given that the sum of the angles inside a triangle is 180°, Angle = 180 – 90 – 30/2 = 75° [1].

Question 1c ii

Height = 100 x sin(75) = 96.59 m^2 [0.5].

Radius = 100 x cos(75) = 25.88 m^2 [0.5].

Volume = $\frac{1}{3} \times \pi \times height \times radius^2 = 67758.67\, m^3$ [1]

Question 1c iii

The top part of the cone is a similar cone. Hence the similarity ratio *k* can be used here.

The volume ratio is 30% or 3:10.

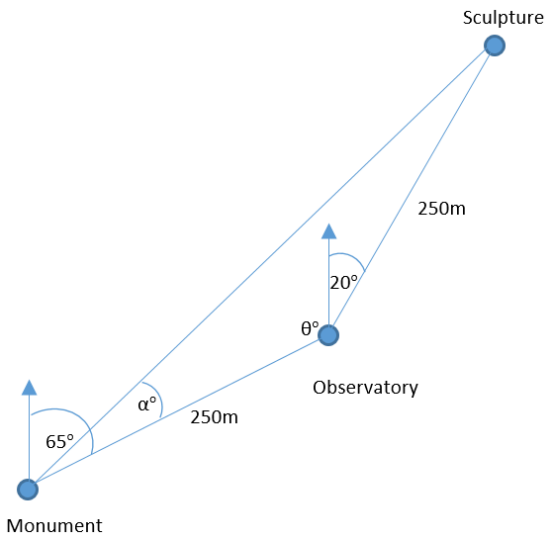
$k^3 = \frac{3}{10}$

$k = \sqrt[3]{\frac{3}{10}} = 0.6694$ [1]

The height *h* can be calculated as

$h = k \times 96.59 = 65\, m$ [1]

Question 1d



Student needs to acknowledge that in order to make an isosceles, the distance from the sculpture to the observatory has to be the same as between the observatory and the monument. This is the **only** way to construct an isosceles.

$\boldsymbol{\theta} = 180 - 65 = 115^o$
$Angle\ MOS = 20 + 115 = 135^o$

$\alpha = \frac{180-135}{2} = 22.5$ [0.5]
Bearing = 65 – 22.5 = 42.5° [0.5].

Distance can be obtained from using cosine rule

$d = \sqrt{250^2 \times 2 - 2 \times 250 \times 250 \times \cos(135)} = 461.9\ m$ [1]

Question 1e i
$radius = 6400 \times \cos(43) = 4680.664\ km$
$distance = 4680.664 \times 2 \times \frac{\pi}{360} \times (26 + 15) = 3349\ km$ [1]

Question 1e ii
Time difference = 4 minutes (per degree) x (26+15) = 164 minutes [1].

Module 4 – Graphs and Relations

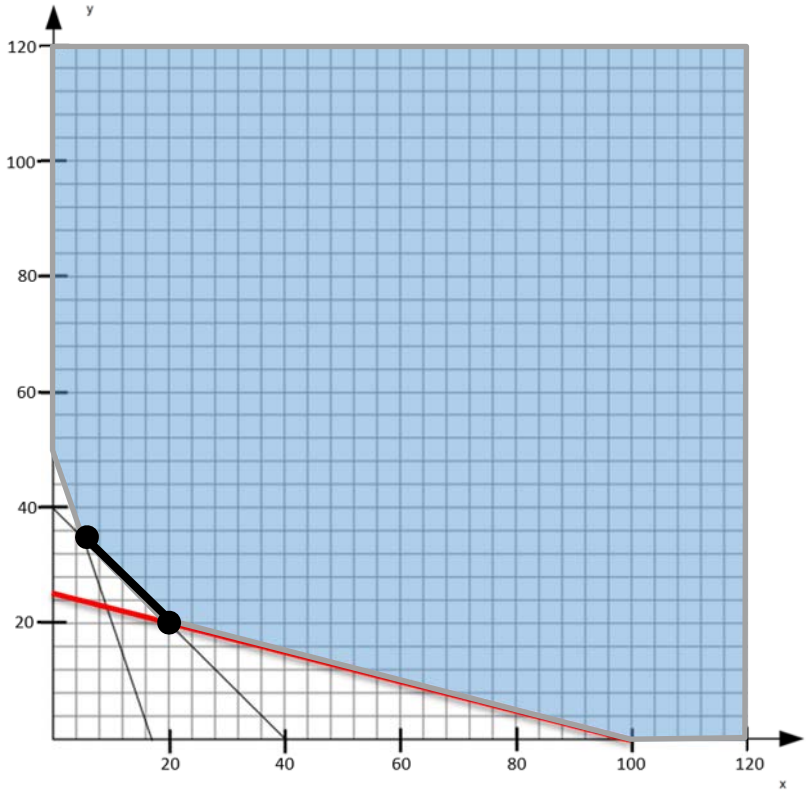
Question 1a
20 x 3.5 = 70 mg [1].

Question 1b
3.5 x A + 3.5 x B = 10.
Any combinations that satisfy this will be awarded [1].

Question 1c i
$9x + 3y \geq 150$ [1].

Question 1d i

The missing inequality is Inequality 4, shown on the graph in red.



The boundary equation is $x + 4y = 100$ [1].

Correct drawing of the boundary is [1].

Question 1d ii

Done on the graph above [1].

Question 1e i

The objective function is D = x + y.
This has the same gradient as inequality 3 [1]. And hence, solution if found in the line will occur all along this line.
Whatever x and y can be, they add up to 40 mL [1].

Question 1e ii

Done on the graph [1]. A line with closed circles at both ends is required for the mark.

Question 2a

For 20GB, DataNet charges him $20 while DeltaCom charges $30 anyway. The cost difference is $10 with DataNet being cheaper.

Question 2b

Substitute x = 80 into $1.5x - 30 = 1.5 \times 80 - 30 = 90$.

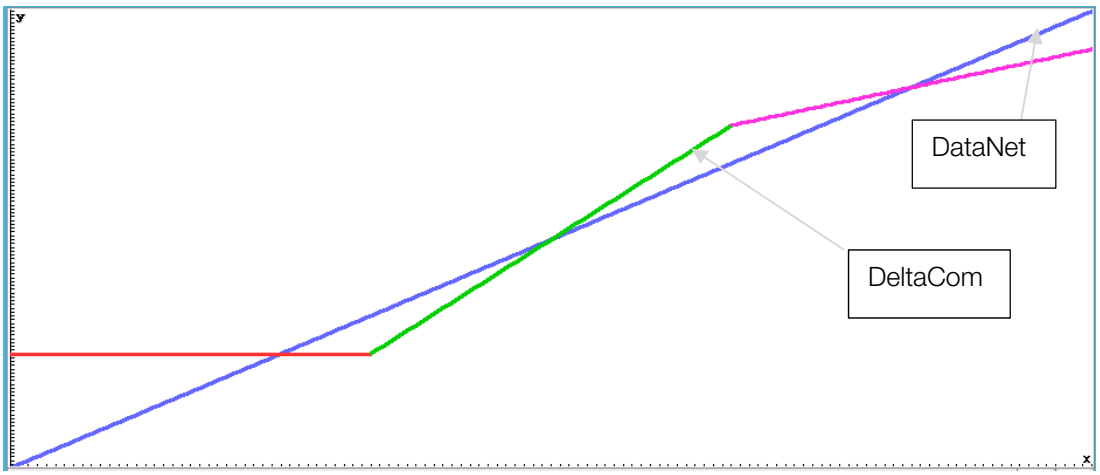
$$0.5 \times 80 + b = 90$$
$$40 + b = 90$$

$$b = 50$$

Correct proof is needed for the mark.

Question 2c

The graphs of both companies can be found by using the CAS. It appears as follows.



To find the ranges, we first need to find the intersections.
1st intersection is when x = 30.

2nd intersection can be found by solving: $1.5x - 30 = x \ \therefore x = 60$

3rd intersection can be found by solving: $0.5x + 50 = x \ \therefore x = 100$

So DeltaCom is more expensive than DataNet when the data usage is:

Smaller than 30GB [1].

Larger than 60 and smaller than 100GB [1].