Neap:

Trial Examination 2015

VCE Further Mathematics Units 3&4

Written Examination 1

Multiple-choice Question Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question booklet of 33 pages and a sheet of miscellaneous formulas.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** on your answer sheet for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

You may keep this question booklet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Further Mathematics Units 3&4 Written Examination 1.

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SECTION A

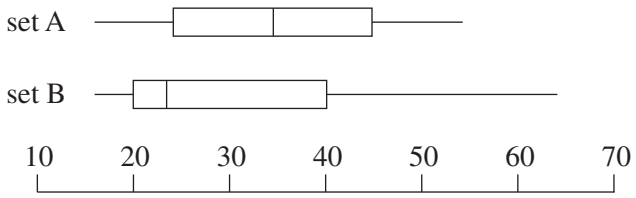
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core: Data analysis

Question 1

Data for two different population sets are shown in the parallel boxplot below.



Which of the following is true?

A. The range for set A is approximately 18.
B. The IQR for set B is approximately 42.
C. The mean of set A is 33.
D. The IQR for both sets is about the same.
E. The mode for set B is 23.

Question 2

The IQR of the data set 24, 32, 14, 28, 49, 35, 15 is

A. 3
B. 15
C. 20
D. 28
E. 28.1

Question 3

Over a three-year period, statistics are kept on the average monthly water level in a dam.

The most appropriate process to analyse this data would be to

A. find the regression equation.
B. deseasonalise the data.
C. use a scatterplot.
D. use a residual plot.
E. use a time series graph.

Use the following information to answer Questions 4 and 5.

The age of each of the 150 people attending a demonstration on changes to university fees is summarised below.

Age	Frequency
10–<20	40
20–<30	60
30–<40	20
40–<50	20
50–<60	10

Question 4

Which of the following is true of the above data set?

A. The median is 20.
B. The range is 50.
C. The modal class is 10–<20.
D. The mean is 28.3.
E. The mean is 30.

Question 5

Checking the original data, a mistake is found: five of the ten people recorded in the 50–<60 group are actually in their sixties and should have been in a new group, 60–<70.

The effect of correcting this mistake will be to increase

A. both the range and IQR.
B. the range and standard deviation, but the IQR will stay the same.
C. both the range and the median.
D. the mean, median and mode.
E. the mean and median, but the mode remains the same.

Question 6

Consider the following table:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal index	0.9	0.95	x	1.4	1.25	1.1	0.75	0.6	0.85	1.05	1.05	1.1

The missing seasonal index to complete this table is

A. 0.9
B. 1.0
C. 1.1
D. 1.15
E. 1.2

Question 7

For the relationship between x and y, where x is the independent variable, the correlation coefficient is 0.8.

Which of the following is true?

A. 80% of the variation in x is due to a variation in y.

B. 80% of the variation in y is due to a variation in x.

C. The relationship is moderately strong, linear and negative.

D. 64% of the variation in x is due to a variation in y.

E. 64% of the variation in y is due to a variation in x.

Question 8

A set of ten bivariate results has the statistics $s_x = 8$, $s_y = 5$, $r = -0.8$, $\bar{x} = 4.2$ and $\bar{y} = 2.1$.

The least squares regression equation is

A. $y = 0.5x$

B. $y = 7.48 - 1.28x$

C. $y = 4.2 - 0.8x$

D. $y = -0.5x$

E. $y = 4.2 - 0.5x$

Use the following information to answer Questions 9 and 10.

x	10	20	30	40	50
y	134	83	34	32	30

Question 9

The least squares regression equation and coefficient of determination for the data set above are closest to

A. $y = -2.6 + 140.3x,\ r = -0.89$

B. $y = -2.6 + 140.3x,\ r^2 = 0.81$

C. $y = 140.3 - 2.6x,\ r = -0.89$

D. $y = 140.3 - 2.6x,\ r^2 = 0.81$

E. $y = -2.6 + 140.3x,\ r^2 = -0.81$

Question 10

After performing a $\frac{1}{x}$ transformation, the equation of the line of best fit for the data set above is

A. $y = \frac{1375}{x} - 0.22$

B. $y = 1375x - 0.22$

C. $y = -0.22x + 1375$

D. $y = \frac{-1375}{x} + 0.22$

E. $y = \frac{-1375}{x} - 0.22$

Question 11

The ages of men and women competing in a local bridge tournament are recorded in the table below.

Women	45, 56, 23, 26, 32, 38, 48, 64, 16, 55, 63, 38, 45, 46, 47, 52, 21
Men	28, 35, 65, 45, 38, 48, 46, 29, 59, 64, 72, 43, 56, 58, 31

To enable a comparison of the spread of data, the above information could best be shown using a

A. frequency table.
B. segmented bar chart.
C. back-to-back stemplot.
D. pie chart.
E. scatterplot.

Question 12

The number of nails contained in fifty boxes of the same size has a mean of 240 and a standard deviation of 6.

How many boxes could be expected to contain more than 234 nails?

A. 8
B. 16
C. 25
D. 42
E. 45

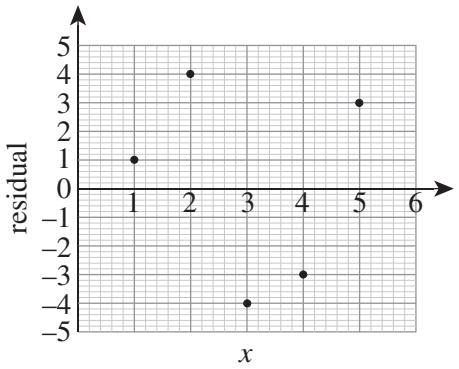
Question 13

Consider the table below.

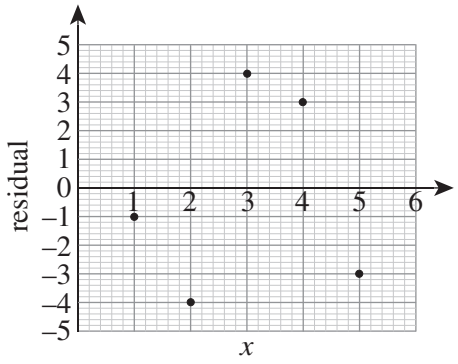
x	1	2	3	4	5
y	24	31	49	58	62
Predicted					
Residual					

Given the equation to the line of best fit is $y = 10x + 15$, which of the graphs below best represents the residuals for the above data set?

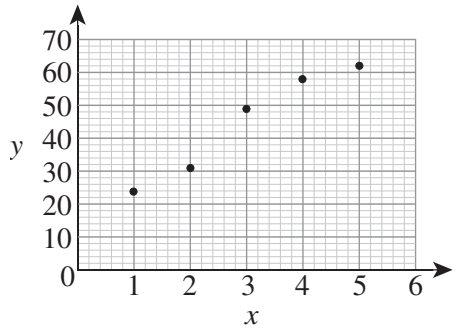
A.



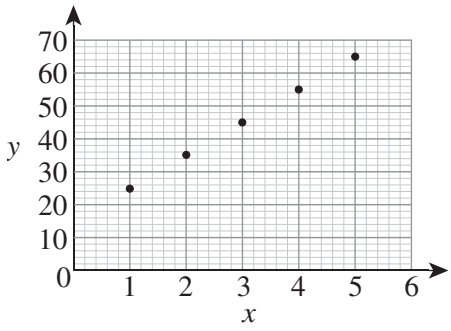
B.



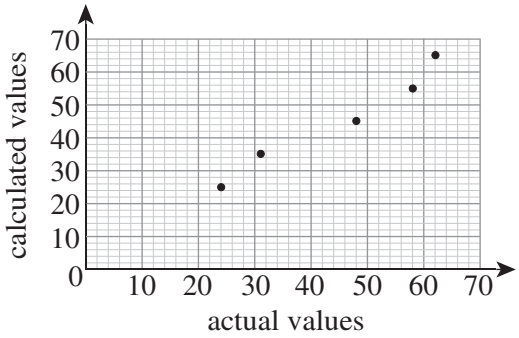
C.



D.



E.



END OF SECTION A

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.
Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Module **Page**

Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

An arithmetic sequence has first term 4 and third term 1.

The second term must be

A. –2
B. 2
C. 2.5
D. 3
E. 8

Question 2

A certain sequence is governed by the difference equation $t_{n+1} = 2t_n - 1$.

The sequence generated is

A. always geometric.
B. always arithmetic.
C. sometimes geometric depending on the value of t_1.
D. sometimes arithmetic depending on the value of t_1.
E. never arithmetic nor geometric.

Question 3

Summer Beach Road is in many sections. The first section is 10 km long. The last section is 3 km long. Each section is 700 m shorter than the previous section.

The entire length of the road is

A. 13 km
B. 65 km
C. 68.5 km
D. 71.5 km
E. 78 km

Question 4

A ball is dropped from a height of 10 m. After each bounce it rises to 70% of its previous height.

After five bounces, the ball will be at height

A. 7.0 m
B. $10(0.7)^4$ m
C. $10(0.7)^5$ m
D. $10(0.3)^4$ m
E. $10(0.3)^5$ m

Question 5

Clive puts $10 into a bank account at the start of every year. Interest is added at the end of the year at such a rate that the balance is multiplied by 1.10.

A difference equation for this, where B_n is the balance after n years, would be

A. $B_{n+1} = 1.1B_n + 10 \qquad B_1 = 0$

B. $B_{n+1} = 1.1(B_n + 10) \qquad B_1 = 0$

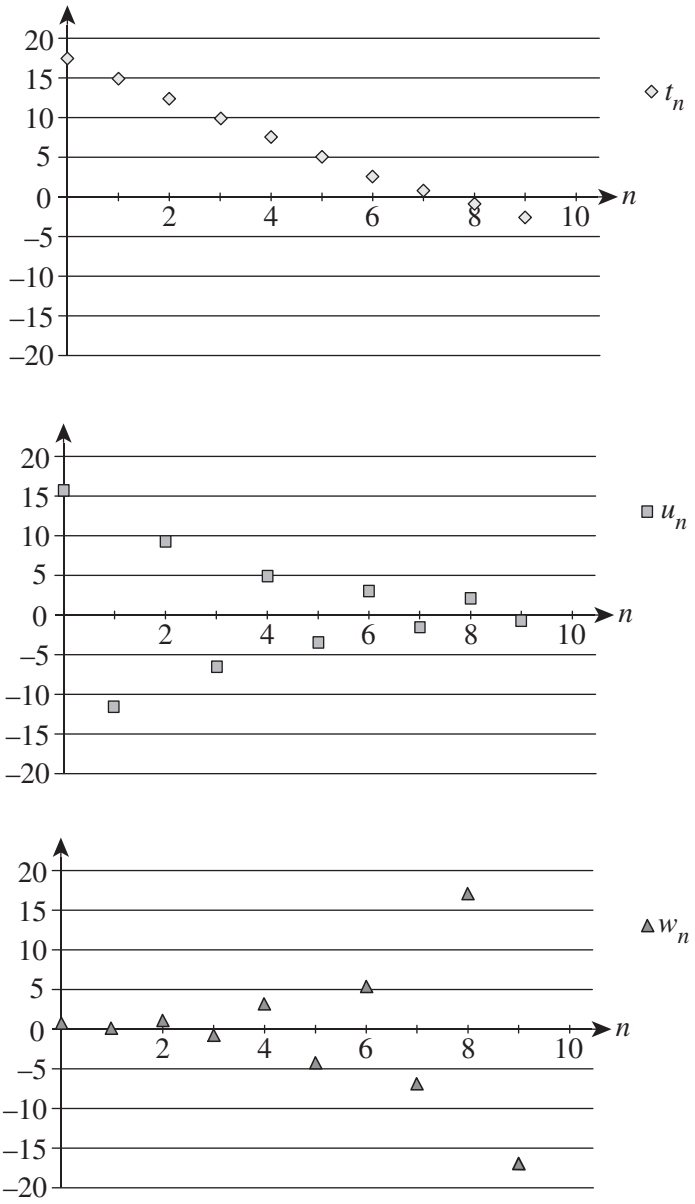
C. $B_{n+1} = 1.1B_n + 10 \qquad B_0 = 0$

D. $B_{n+1} = 1.1(B_n + 10) \qquad B_0 = 0$

E. $B_{n+1} = 1.1(B_n + 10) + 10 \qquad B_1 = 0$

Question 6

Consider the following graphs:



The sequences shown in the graphs above could be either arithmetic or geometric, except for

A. t_n

B. u_n

C. w_n

D. both t_n and w_n

E. both u_n and w_n

Question 7

Consider the difference equation $T_{n+1} = 9 - T_n - T_{n-1}$.

This will produce a sequence of positive numbers

A. only if $t_1 = 2$ and $t_2 = 3$.

B. only if $t_1 = 4$ and $t_2 = 3$.

C. if $t_1 > 0$ and $t_2 > 0$.

D. if $t_1 > 0$, $t_2 > 0$ and $t_1 + t_2 < 9$.

E. only if all values are 3.

Question 8

The sixth term of a certain geometric sequence is 4. The second term is 20.25.

The first term is

A. $\frac{2}{3}$

B. $\frac{8}{3}$

C. 13.5

D. 30.375

E. 40.5

Question 9

Miriam is offered a new job with the hourly pay rate changing with experience. She is told that the difference formula $R_{n+1} = 0.4R_n + 12$, $R_1 = 8$ gives the rate of pay in her nth month of work.
Miriam has decided that she wants her rate of pay to be \$20, but will accept a reduction of D_n in month n, where D_n is a geometric sequence with first term 12 and common ratio 0.4.

If she will accept nothing less than this arrangement then Miriam will

A. accept the offer, as it is identical to her plan.

B. accept the offer, as it is always superior to her plan.

C. reject the offer, as it is initially better than her plan but ultimately worse.

D. reject the offer, as it is initially worse than her plan.

E. reject the offer, as it is always worse than her plan.

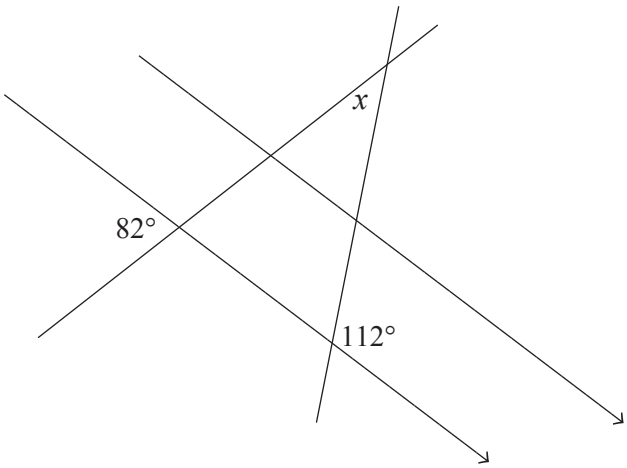
END OF MODULE 1

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

Question 1

Consider the diagram below.

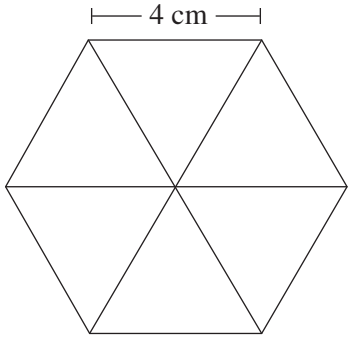


The size of angle x is

A. 30°
B. 60°
C. 68°
D. 82°
E. 98°

Question 2

Carol draws a hexagon with a side length of 4 cm as shown below. All triangles within the hexagon are equilateral.

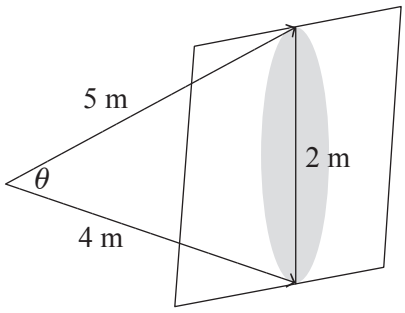


The longest line that can be drawn within the hexagon is

A. 4 cm
B. $4\sqrt{2}$ cm
C. 6 cm
D. 8 cm
E. 24 cm

Question 3

When Clark shines his torch on a 2 m wall, it illuminates an elliptical section extending from the base of the wall to the exact top. The torch is 4 m from the base of the wall and 5 m from the top. This is shown in the diagram below.



What is the angle θ between the highest and lowest rays?

A. $\sin^{-1}\frac{1}{5}$

B. $\sin^{-1}\frac{1}{4}$

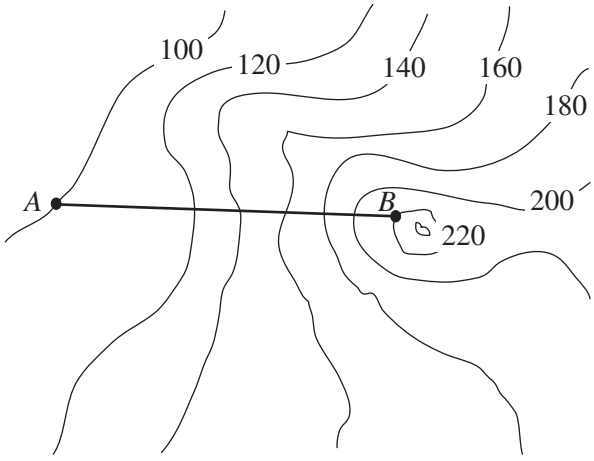
C. $\cos^{-1}\left(\frac{5^2+4^2-2^2}{2\times4\times5}\right)$

D. $\frac{1}{2}\times4\times5\times2$

E. $\sqrt{4^2+5^2-2^2}$

Question 4

The map below shows the proposed line of a new chairlift ride that will stretch from point A to B in a straight line. The map scale is 1 : 6000 and points A and B are 10 cm apart on the map. All contours are in metres.



The length of the line required is

A. 120 m

B. 134 m

C. 600 m

D. 612 m

E. 639 m

Question 5

Before a new model of car is built, a scale model is produced. The real car has a storage volume of 1.6 m^3. The scale model has a storage volume of 200 cm^3.

The exteriors of both the scale model and real cars are painted using the same paint, of the same thickness. The model requires one small tin of paint.

How many tins would the real car require?

A. 5
B. 20
C. 125
D. 400
E. 8000

Question 6

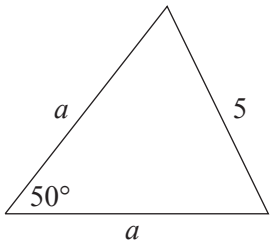
Jasmine prints out an isosceles triangle of area 100 cm^2. Two of the sides are 20 cm in length.

The smallest angle must be

A. 14.5°
B. 30.0°
C. 60.0°
D. 75.5°
E. 90.0°

Question 7

Consider the triangle below.



The length of side *a* is

A. $\frac{5\sin 50}{\sin 65}$

B. $\frac{5\sin 65}{\sin 50}$

C. $\sqrt{5^2} - 2 \times 5\cos 50$

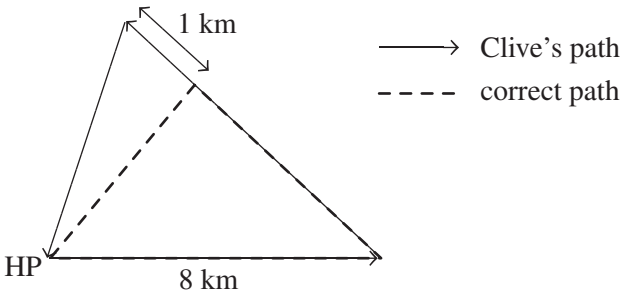
D. $\sqrt{\frac{5^2}{2}}$

E. $\frac{5}{\sqrt{2}\sqrt{1+\cos 50}}$

Question 8

Clive takes part in a yacht race starting and ending at Harper Point (HP). The first leg is 8 km due east. The second leg is on a bearing of 310° true and extends to the point on this path closest to Harper Point. The third leg heads directly back to where the race started.

On the second leg, Clive misses the buoy marking the end of that leg and extends 1 km too far. At this point, Clive heads directly to Harper Point. His journey is shown in the diagram below.



The total distance that Clive travels is

A. 5.14 km
B. 19.27 km
C. 20.27 km
D. 20.37 km
E. 21.27 km

Question 9

A square-based pyramid has four slanted faces as well as its square base. The triangular faces have two sides of length 13 cm and one side of length 10 cm.

The height of the pyramid is exactly

A. $\sqrt{44}$
B. 10
C. $\sqrt{119}$
D. 12
E. 13

END OF MODULE 2

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1

Which of the following lines contains the point (3, 4)?

A. $3x - 4y = 0$
B. $3x + 4y = 0$
C. $3x + 4y = 24$
D. $3x - y = 5$
E. $3x + y = 12$

Question 2

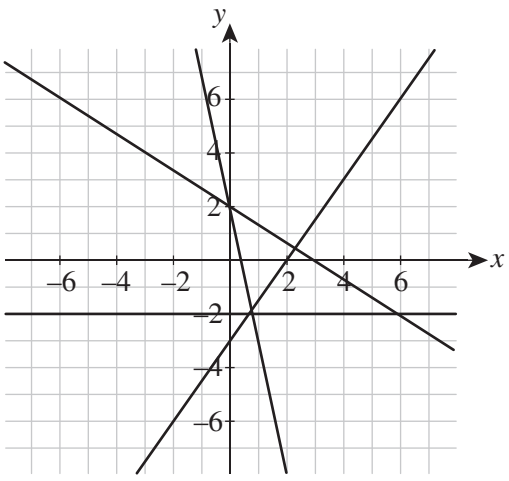
The number of cats in Cambellbury is c and the number of dogs is d.

If the council decides that cat registration costs \$20 and dog registration costs \$50, then the total animal registration revenue is

A. \$70
B. $20c + 50d$
C. $1000cd$
D. $50c + 20d$
E. $70 + c + d$

Question 3

Consider the following graph:

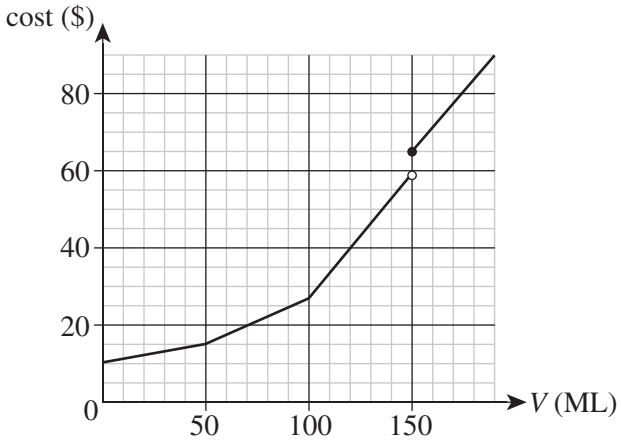


Which of the equations listed below is **not** that of any of the lines shown on the graph?

A. $2x + 3y = 6$
B. $3x - 2y = 6$
C. $y = 1.5x - 3$
D. $x = -2$
E. $y = 2 - \frac{2}{3}x$

Use the following information to answer Questions 4 and 5.

Water usage charges in the city of Sheldonville consist of a supply charge – which is a fixed cost regardless of water usage – and a water charge that depends on the volume, V, of water consumed in megalitres. The charges are shown in the graph below.



Question 4

The supply charge for the water is

A. \$0.10
B. \$5
C. \$10
D. \$20
E. \$50

Question 5

The equation that governs the total cost when consumption is between 100 and 150 ML is closest to

A. $C = 0.6V + 27.5$

B. $C = 0.6V - 32.5$

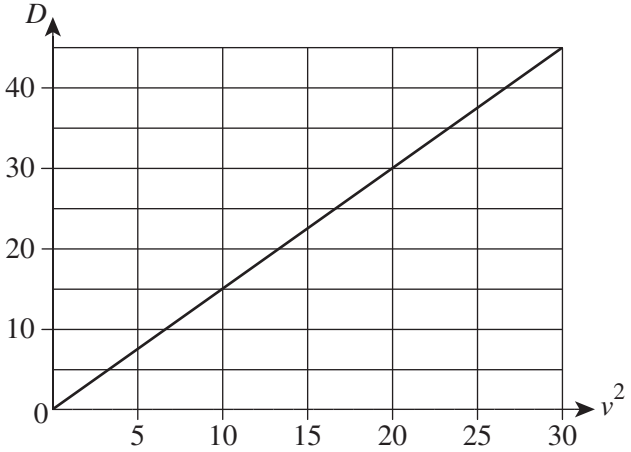
C. $C = 1.6V + 27.5$

D. $C = 1.6V - 132.5$

E. $C = \frac{V}{10} + 10$

Question 6

The graph below relates to the expected damage resulting from the strong winds of a tropical storm in a certain location. The damage (D in \$1000s) is graphed against the square of the wind speed, v.



The correct equation relating damage (D) to wind speed (v) is

A. $D = 1.5v$

B. $D = \frac{2}{3}v$

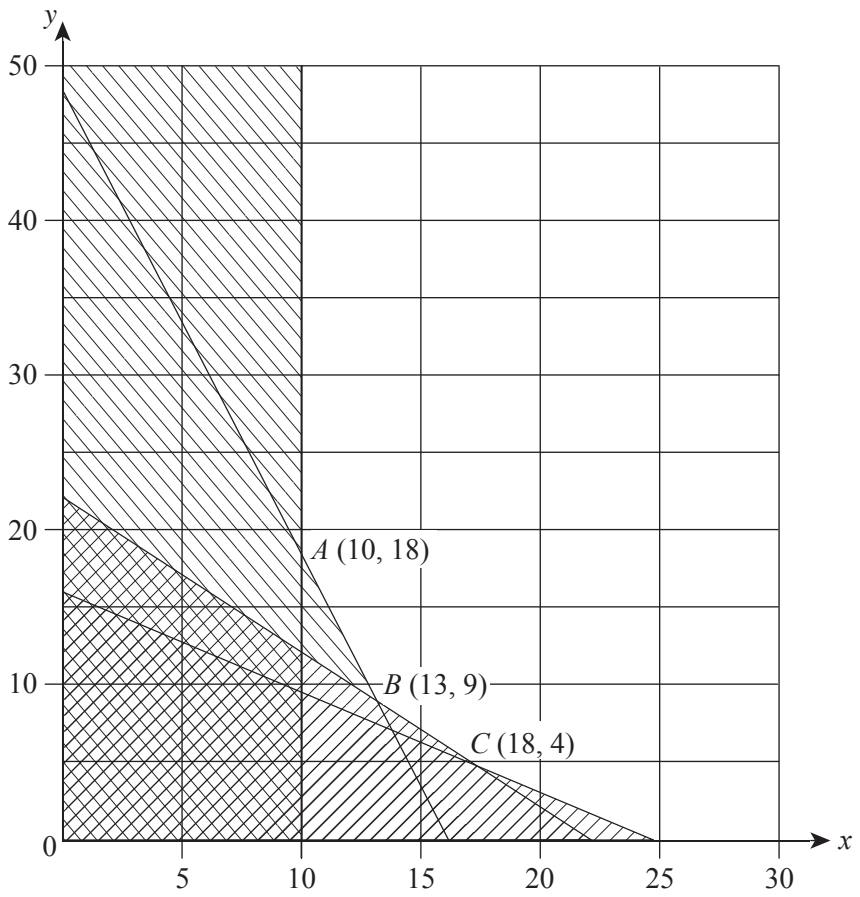
C. $D = v^2$

D. $D = 1.5v^2$

E. $D = \frac{2}{3}v^2$

Use the following information to answer Questions 7 and 8.

A set of constraints is illustrated in the diagram below. The feasible region is unshaded.



Question 7

An inequation that is **not** shown in the above graph is

A. $x \geq 10$

B. $2x + 3y \geq 48$

C. $x + y \leq 22$

D. $3x + y \geq 48$

E. $y \geq 22 - x$

Question 8

Which of these possible objective functions has a minimum value at point B?

A. $P = 4x + y$

B. $P = 2x + y$

C. $P = x + 2y$

D. $P = 2x + 3y$

E. $P = x + 3y$

Question 9

A company sells rocket fuel cartridges at a cost of $200 each. A discount is given for any purchases of more than ten cartridges. The first ten cartridges will be at the normal price, but all cartridges after the first ten are sold at $120 each. The cost is given by a continuous hybrid function of the form

$$C = \begin{cases} 200n & n \leq 10 \\ an + b & n > 10 \end{cases}$$

Which of the following is correct?

A. $a = 120$ and $b = 0$

B. $a = 120$ and $b = 2000$

C. $a = 120$ and $b = 800$

D. $a = 200$ and $b = 120$

E. $a = 200$ and $b = 2000$

END OF MODULE 3

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

$1200 is invested for three years at a simple interest rate of 5% per annum.

The total amount of interest earned during the three years is

A. $60
B. $180
C. $190
D. $332
E. $1380

Question 2

$1200 is invested for two years at a compound interest rate of 4% per annum paid annually.

The total amount of interest earned during the two years is closest to

A. $24
B. $48
C. $96
D. $98
E. $1298

Question 3

A new tram was purchased for $375 000. The tram depreciates in value by 39 cents for every kilometre travelled.

The value of the tram after it has travelled 16 000 kilometres will be

A. $6240
B. $336 000
C. $359 000
D. $368 760
E. $381 240

Question 4

All transactions on a bank account for one month are shown below.

Date	Debit ($)	Credit ($)
7 March		$45
10 March	$305	
18 March		$294
27 March		$102
31 March	$305	

The opening balance on 1 March was $400. Interest is calculated and paid monthly on the minimum monthly balance. The interest rate is 2.8% per annum.

The amount of interest earned for the month of March is

A. $0.33
B. $0.54
C. $0.93
D. $1.01
E. $1.04

Use the following information to answer Questions 5 and 6.

Danielle purchased a computer for $1800 using hire-purchase. She paid a $300 deposit and agreed to pay the balance in equal monthly repayments for one year. A flat interest rate of 4% per annum is charged.

Question 5

The size of each monthly repayment is

A. $130
B. $131
C. $156
D. $175
E. $182

Question 6

The effective interest rate is closest to

A. 4.0%
B. 7.4%
C. 8.0%
D. 16.7%
E. 20.0%

Question 7

Ken's salary is based on hours worked and varies each year.

- In 2011, his salary was $55 000.
- His salary was 8% greater in 2012 than it was in 2011.
- His salary was 3% less in 2013 than it was in 2012.
- His salary was 5% greater in 2014 than it was in 2013.

In 2014, his salary was closest to

A. $55 010
B. $57 750
C. $59 400
D. $60 499
E. $64 241

Question 8

Helena invested $12 500 at an annual interest rate of 3.5% with interest paid quarterly. She adds $400 to her investment at the end of each quarter.

The amount in her account at the beginning of the fourth quarter is closest to

A. $12 831
B. $13 700
C. $14 042
D. $14 164
E. $14 564

Question 9

Brian invested $8000 in an account with a constant interest rate for the first three years. The investment earned $r\%$ interest per annum, compounding monthly during the first three years. Brian agreed that the interest rate in the fourth year would be halved.

The total amount of interest earned at the end of four years is given by

A. $8000 \times \left(1 + \frac{r}{100}\right) \times 3 + 8000 \times \left(1 + \frac{r}{100}\right) \times 1$

B. $8000 \times \left(1 + \frac{r}{100}\right)^3 + 8000 \times \left(1 + \frac{r}{100}\right)^1$

C. $8000 \times \left(1 + \frac{r}{1200}\right)^3 + 8000 \times \left(1 + \frac{r}{2400}\right)^1$

D. $8000 \times \left(1 + \frac{r}{1200}\right)^{36} + 8000 \times \left(1 + \frac{r}{1200}\right)^{12}$

E. $8000 \times \left(1 + \frac{r}{1200}\right)^{36} + 8000 \times \left(1 + \frac{r}{2400}\right)^{12}$

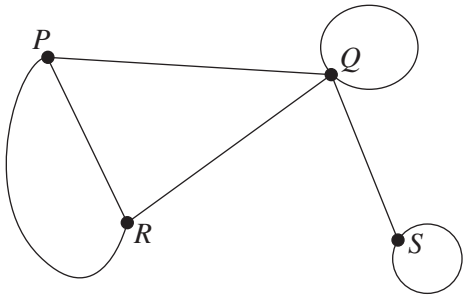
END OF MODULE 4

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

Question 1

Consider the graph below.



Which of the following represents its associated adjacency matrix?

A.
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

D.
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

E.
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Question 2

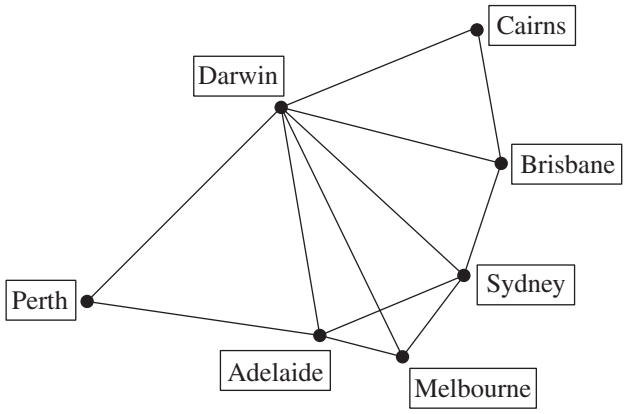
A complete planar graph has 4 faces.

How many edges does it have?

A. 6
B. 7
C. 8
D. 9
E. 10

Question 3

The diagram below illustrates the available routes on a bus company's network throughout Australia.



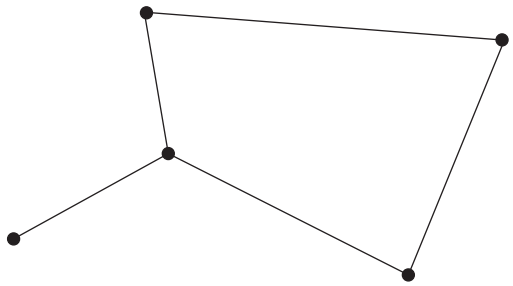
Jane buys a bus ticket that allows travel on any route once. (Note that she cannot travel both directions on the same route.)

If Jane begins her journey in Melbourne and wishes to travel every route, where will her trip finish?

A. Brisbane
B. Melbourne
C. Darwin
D. Perth
E. none of the above

Question 4

Consider the graph below.



How many spanning trees can be found for the graph?

A. 1
B. 2
C. 3
D. 4
E. 5

Question 5

A mountain region contains a settlement at *A* and road junctions at *B*, *C* and *D*. All the roads are one-way, single-lane mountain tracks. The capacity of the tracks in the region is shown below.

Region	Capacity
from *A* to *B*	100 vehicles per hour
from *C* to *D*	100 vehicles per hour
from *B* to *D*	160 vehicles per hour
from *B* to *C*	100 vehicles per hour
from *A* to *C*	140 vehicles per hour

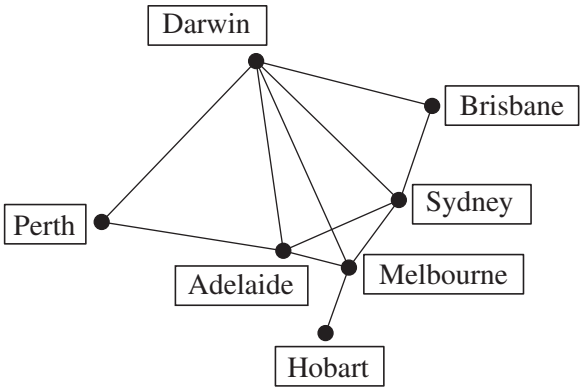
In the event of a bushfire, regions *A*, *B* and *C* are unsafe, but location *D* is considered safe.

What is the maximum number of vehicles that can escape per hour from *A* and arrive safely at *D* in the event of an emergency?

A. 200
B. 240
C. 260
D. 300
E. 400

Question 6

The available flights for Panther Airlines are indicated on the network diagram below, with the edges denoting these.



A traveller has asked his travel agent to design a tour that visits each city in the network only once.

What mathematical term is used to describe such a tour?

A. Newtonian path
B. Hamiltonian path
C. Eulerian path
D. critical path
E. minimal spanning tree

Question 7

The matrix below shows the results of a tug-of-war competition between five teams: *A*, *B*, *C*, *D* and *E*.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

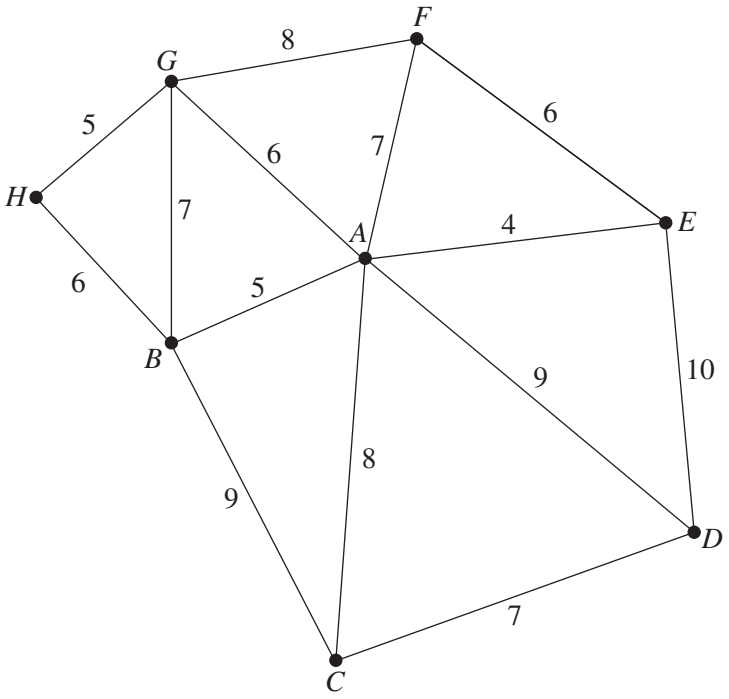
The first row indicates team *A* defeated teams *B*, *C* and *D*. The judges needed to choose a winning team or teams as fairly as possible, taking into account both the quality and quantity of the teams defeated by each other.

On this basis, the most reasonable decision of the judges would be to pick team/s

A. *A* as the winner.
B. *B* as the winner.
C. *C* as the winner.
D. *A* and *E* as equal first.
E. *E* as the winner.

Question 8

A group of engineers are planning an underground railway system for a large city. The estimated costs (in millions of dollars) of tunnelling between the sites of the proposed stations at *A*, *B*, *C*, *D*, *E*, *F* and *G* are shown in the diagram below.



The project will be viable, provided that each location is reachable – either directly or via other stations – from any other location. Therefore, not all the proposed tunnels need to be built. The engineers are proposing a tree-shaped network.

What is the minimum cost for the necessary tunnels?

A. \$41 000 000
B. \$43 000 000
C. \$52 000 000
D. \$54 000 000
E. \$55 000 000

Question 9

A rival group of engineers is considering an alternative proposal to link all the stations in an underground rail loop so trains visit each station just once, and then return to the starting point. The engineers argue that this would be considerably cheaper to run.

They find that the minimum cost for building such a loop will be

A. \$41 000 000
B. \$43 000 000
C. \$52 000 000
D. \$54 000 000
E. \$55 000 000

END OF MODULE 5

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions.

Question 1

The matrix $\begin{bmatrix} 3 & 2 & 4 \\ 1 & -3 & -2 \\ 3 & 6 & 8 \\ 2 & 7 & 8 \end{bmatrix}$ has order

A. 12
B. 3×4
C. 4×3
D. 3, 2, 4
E. 3, 1, 3, 2

Question 2

Matrix P has 3 rows and 2 columns, Q has 4 rows and 3 columns and R has 2 rows and 3 columns.

Of the matrix products,

A. PQ, PR and QR are all defined.
B. PR and QR are both defined, but PQ is not.
C. PQ is defined, but PR and QR are not.
D. PR is defined, but QR and PQ are not.
E. QR is defined, but PQ and PR are not.

Use the following information to answer Questions 3 and 4.

Crimean butterflies have a life cycle over three stages – larvae, caterpillar and butterfly. The first two stages last exactly one month each and the third stage lasts as long as the butterfly lives.

The transition between these three stages is governed by a transition matrix as shown below.

current state

	L	C	B	
	0.00	0.00	0.40	L
	0.75	0.00	0.00	C
	0.00	0.80	0.95	B

next month

On 1 January 2015 there were 1000 larvae, 900 caterpillars and 2000 butterflies.

Question 3

The proportion of larvae that die before becoming caterpillars is

A. 15%
B. 20%
C. 25%
D. 75%
E. 80%

Question 4

After the first four months, the total number of larvae, caterpillars and butterflies is expected to be closest to

A. 3873
B. 3900
C. 4170
D. 4737
E. 6165

Question 5

Karl, Andrea, Thanh, Hilda and Magnus are asked to state their interest in various university courses. The results are shown in the table below.

	Medicine	**Law**	**Science**	**Arts**	**Engineering**
Karl	yes	yes	no	no	no
Andrea	yes	no	yes	no	no
Thanh	no	no	yes	yes	yes
Magnus	yes	no	yes	no	yes
Hilda	no	yes	no	yes	no

Their responses are placed in a matrix, with 1 indicating 'yes' and 0 indicating 'no'.

The resulting matrix could be

A.
$$\begin{array}{c} \\ \text{Med} \\ \text{Law} \\ \text{Sci} \\ \text{Arts} \\ \text{Eng} \end{array}\begin{array}{c}\begin{array}{ccccc}\text{K} & \text{A} & \text{T} & \text{M} & \text{H}\end{array}\\ \begin{bmatrix}1&1&0&0&0\\1&0&1&0&0\\0&0&1&1&1\\1&0&1&0&1\\0&1&0&1&0\end{bmatrix}\end{array}$$

B.
$$\begin{array}{c} \\ \text{Med} \\ \text{Law} \\ \text{Sci} \\ \text{Arts} \\ \text{Eng} \end{array}\begin{array}{c}\begin{array}{ccccc}\text{K} & \text{A} & \text{T} & \text{M} & \text{H}\end{array}\\ \begin{bmatrix}1&1&0&1&0\\1&0&0&0&1\\0&1&1&1&0\\0&0&1&0&1\\0&0&1&1&0\end{bmatrix}\end{array}$$

C.
$$\begin{array}{c} \\ \text{Karl} \\ \text{Andrea} \\ \text{Hilda} \\ \text{Magnus} \\ \text{Thanh} \end{array}\begin{array}{c}\begin{array}{ccccc}\text{M} & \text{L} & \text{S} & \text{A} & \text{E}\end{array}\\ \begin{bmatrix}1&1&0&0&0\\1&0&1&0&0\\0&0&1&1&1\\1&0&1&0&1\\0&1&0&1&0\end{bmatrix}\end{array}$$

D.
$$\begin{array}{c} \\ \text{Karl} \\ \text{Andrea} \\ \text{Hilda} \\ \text{Magnus} \\ \text{Thanh} \end{array}\begin{array}{c}\begin{array}{ccccc}\text{M} & \text{L} & \text{S} & \text{A} & \text{E}\end{array}\\ \begin{bmatrix}1&1&0&0&0\\1&0&1&0&0\\0&1&1&1&1\\1&0&1&0&1\\0&1&0&1&0\end{bmatrix}\end{array}$$

E.
$$\begin{array}{c} \\ \text{Karl} \\ \text{Andrea} \\ \text{Hilda} \\ \text{Magnus} \\ \text{Thanh} \end{array}\begin{array}{c}\begin{array}{ccccc}\text{M} & \text{L} & \text{S} & \text{A} & \text{E}\end{array}\\ \begin{bmatrix}1&1&0&0&0\\1&0&1&0&0\\0&0&1&0&1\\1&0&1&0&1\\0&1&0&1&0\end{bmatrix}\end{array}$$

Question 6

If Jason brings $20 to school he can buy exactly 4 hot dogs and 3 pies. If he brings an extra $5, he can purchase 3 hot dogs and 6 pies.

If each hot dog costs h and each pie costs p, then the matrix $\begin{bmatrix} h \\ p \end{bmatrix}$ will be given by

A. $\begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 20 \\ 25 \end{bmatrix}$

B. $\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix} \begin{bmatrix} 20 \\ 25 \end{bmatrix}$

C. $\begin{bmatrix} 6 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 20 \\ 25 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 20 \\ 5 \end{bmatrix}$

E. $\begin{bmatrix} 6 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 20 \\ 25 \end{bmatrix}$

Question 7

Four clubs compete at a Little Athletics event. Each club has multiple teams. The details of these clubs' memberships and teams are shown in the table below.

	Razorbacks	**Feathertops**	**Alpines**	**Buffalos**
Number of teams	2	4	3	5
Members per team	12	8	7	9

Members of the Buffalos and Feathertops can compete in two events each. Alpines may compete three times and Razorbacks four times.

This information is incorporated into a series of matrices.

$$P = \begin{bmatrix} 2 & 4 & 3 & 5 \end{bmatrix} \quad Q = \begin{bmatrix} 12 \\ 8 \\ 7 \\ 9 \end{bmatrix} \quad R = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

If a member can be counted once for each event they compete in, then the **total** number of members of all clubs counted can be found by

A. $P \times (Q \times R)$

B. $P \times (R \times Q)$

C. $Q \times (R \times P)$

D. $R \times (Q \times P)$

E. $R \times (P \times Q)$

Question 8

Romeo Dance Club members can choose between two locations for their private practice. They can dance either at the Stranger Hall or the Commerce Club.

A transition matrix below governs their choices from week to week.

$$T = \begin{array}{c} \begin{matrix} S \text{ now} & C \text{ now} \end{matrix} \\ \begin{bmatrix} 0.75 & 0.10 \\ 0.25 & 0.90 \end{bmatrix} \begin{matrix} S \text{ next} \\ C \text{ next} \end{matrix} \end{array}$$

The long-term proportion of dancers who choose Stranger Hall is closest to

A. 0.25
B. 0.286
C. 0.5
D. 0.714
E. 0.75

Question 9

The matrix equation $\begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ p \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} q \\ 5 \end{bmatrix}$ can be rewritten as

A. $\begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

B. $\begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} = \begin{bmatrix} 3 & -q \\ p & -5 \end{bmatrix}$

C. $\begin{bmatrix} 2 & -3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

E. $\begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

END OF MULTIPLE-CHOICE QUESTION BOOKLET