Neap:

Trial Examination 2015

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – Core: Data analysis

Question 1 D

The IQR for both sets is 20.

Question 2 C

Order the data first: 14, **15**, 24, 28, 32, **35**, 49

The median is 28, so $Q_3 = 35$ and $Q_1 = 15$.

IQR = Q3 – Q_1

Question 3 E

As one of the variables is time, a time series is the best option.

Question 4 D

Use the average figure for each group to calculate the mean:

$$\frac{(15 \times 40 + 25 \times 60 + 35 \times 20 + 45 \times 20 + 55 \times 10)}{150} = 28.3$$

Question 5 B

Only the right-hand extreme of the figures is affected, so median, mode and IQR will be the same. The range, standard deviation and mean will all increase.

Question 6 B

The figures must add to 12.

Question 7 E

The coefficient of determination, r^2, is 0.64. This means that 64% of the variation in y is due to a change in x.

Question 8 E

Use the formulas $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$.

Question 9 D

Enter figures into lists on your CAS and calculate the linear regression.

Question 10 A

Create a new column on your CAS by changing the x list to $\dfrac{1}{x}$, then recalculate the linear regression equation. Remember that the value of x is now $\dfrac{1}{x}$.

Question 11 C

The back-to-back stemplot orders the data. Comparisons can easily be made, as the figures are next to each other.

Question 12 D

234 is one standard deviation below the mean, therefore only 16%, or 8, of the boxes have less than 234 nails. This means 84%, or 42, of the boxes have at least 234.

Question 13 B

The residual is calculated by subtracting the predicted value from the actual value.

SECTION B

Module 1: Number patterns

Question 1 **C**

$a = 4$

$a + 2d = 1$

$2d = -3$

$d = -1.5$

$t_2 = a + d$

$= 2.5$

Question 2 **E**

The sequence involves multiplication by 2 and then 1 is subtracted.

If the process involved multiplication only, it would be geometric. If it involved only addition/subtraction, it would be arithmetic. As both multiplication and addition/subtraction are involved, the sequence is neither arithmetic nor geometric.

Question 3 **D**

This is an arithmetic series question. It involves adding terms. The first term is clearly 10 and the common difference is –0.7. The last term is 3.

What we are not given in the question is the number of terms. We need to calculate this:

$3 = 10 - 0.7(n - 1)$

$n - 1 = 10$

$n = 11$

Now we can find the sum:

$S_{11} = \frac{11}{2}[2 \times 10 + 10 \times -0.7]$

$= 71.5 \text{ km}$

Question 4 **B**

This is a basic geometric sequence.

$a = 10$

$r = 0.7$

$t_5 = ar^4$

$= 10(0.7)^4$

Question 5 D

There are two issues in this question. The possible solutions differ in that:

- some start with term zero being 0, while others start with term one being 1.
- some add the $10 before interest is calculated, while others reverse this order.

In both of these cases, the first option is correct. It is term zero that is value 0. The balance is 0 after zero years, not after one year. The $10 is added at the start of the year, while the interest is added at the end of the year. Thus the $10 is added first. The equation in **D** does this, as the 10 is added in brackets before the multiplication occurs.

Question 6 C

- Sequence t_n seems to form a straight line. Thus this appears to be an arithmetic sequence.
- Sequence u_n forms an alternating positive, then negative sequence. Is it geometric, however? An alternating sequence can be geometric if the ratios are all the same. The ratios in the case of the square points are approximately $\frac{-12}{16}$, then $\frac{9}{-12}$, then $\frac{-7}{9}$ and then $\frac{5}{-7}$. These values, to three decimal places, work out to be –0.750, –0.750, –0.778 and –0.714 respectively. These values are not quite the same as each other, but they are similar enough to make it impossible for us to state that it is not geometric.
- Sequence w_n is neither arithmetic nor geometric. That it is not arithmetic is obvious, as it does not form a line. That it is not geometric is clear from term one; it is zero. No term in a geometric can be zero unless all others are zero.

Question 7 D

The first point to note about this sequence is that it consists of three numbers repeating, regardless of the first two terms. Thus if positive values are chosen for terms one and two, we only require that term three is also positive. This will be the case so long as the sum of the first two terms is below 9.

Question 8 D

The sixth term is 4 and thus $ar^5 = 4$.

The second term is 20.25 and thus $ar = 20.25$.

Divide the terms:

$$r^4 = \frac{4}{20.5}$$
$$= \frac{16}{81}$$
$$r = \frac{2}{3}$$
$$a = \frac{20.25}{r}$$
$$= 3 \times \frac{20.25}{2}$$
$$= 30.375$$

Question 9 A

It is not necessary for students to prove that both schemes are identical. They merely need to see for themselves that this is the case.

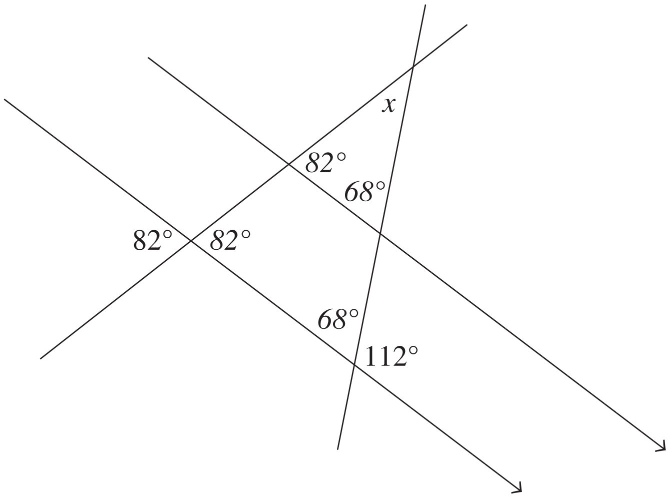
The company offer is \$8 in the first month, but \$15.20 in the second month. The third, fourth and fifth months have salaries of \$18.08, \$19.23 and \$19.69 to the nearest cent respectively.

Now determine the sequence D_n. The first five terms are 12, 4.8, 1.92, 0.768 and 0.3072. Thus the minimum rates of pay that Miriam would accept are \$8, \$15.20, \$18.08, \$19.23 and \$19.69. They are identical.

Module 2: Geometry and trigonometry

Question 1 A

Using knowledge of supplementary and vertically opposite angles, the correct values are shown in the diagram below.



We also know that every triangle has an angle sum of 180° and thus:

$x = 180 - 82 - 68$

$= 30°$

Question 2 D

Each of the internal triangles is equilateral. Thus all lengths are 4 cm. The longest length is exactly two of these side lengths, thus 8 cm.

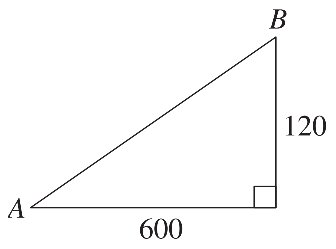
Question 3 C

This is a straight cosine rule scenario.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Question 4 D

The horizontal distance is 6000 × 10 cm = 600 m. This is shown in the diagram below.



The vertical distance is the difference between the contour values.

$$\text{vertical distance} = 220 - 100$$
$$= 120$$

The actual distance from A to B can be found using Pythagoras' theorem.

$$d = \sqrt{600^2 + 120^2}$$
$$\cong 612 \text{ m}$$

Question 5 D

Firstly, students need to convert all volumes to the same units: 1 $m^3 = 100^3$ $cm^3 = 10^6$ cm^3

$$\text{ratio of volumes} = \frac{1\ 600\ 000}{200}$$
$$= 8000$$

$$\text{length ratio} = \sqrt[3]{8000}$$
$$= 20$$

Paint used will depend on the surface areas of both vehicles. Thus the area ratio is the required measure.

$$\text{area ratio} = 20^2$$
$$= 400$$

Thus 400 tins of the paint will be required for the real car.

Question 6 B

$$A = \frac{1}{2}ab\sin C$$

$$100 = \frac{1}{2} \times 20 \times 20 \sin\theta$$

$$\frac{1}{2} = \sin\theta$$

$$\theta = 30°$$

Question 7 B

The other two angles are both 65°.

$$\text{Thus } \frac{a}{\sin 65} = \frac{5}{\sin 50}$$

$$a = \frac{5\sin 65}{\sin 50}$$

Question 8 D

The path actually taken by Clive is a triangle, with side lengths of 8 km and 7.1284 km ($8\cos 40 + 1$) and a 40° angle between them.

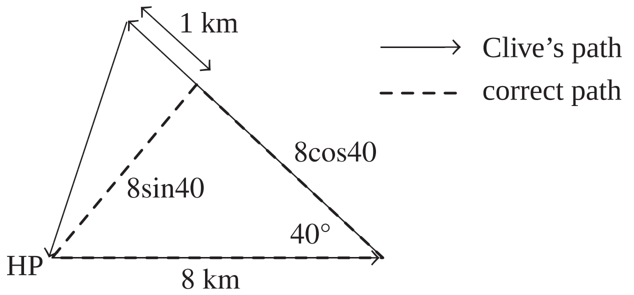
Students can use the cosine rule:

$c^2 = 8^2 + 7.1284^2 - 2 \times 8 \times 7.1284 \times \cos 40$

$c = 5.24$

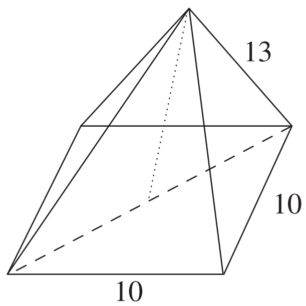
total = 8 + 5.24 + 7.13

= 20.37



Question 9 C

The pyramid must have a base length of 10 cm, as the bottom of the triangles connect to the base. Students must then find the length of the base diagonally. This is shown in the diagram below.



$l = \sqrt{10^2 + 10^2}$

$= 10\sqrt{2}$

$h = \sqrt{13^2 - (5\sqrt{2})^2}$

$= \sqrt{119}$

Module 3: Graphs and relations

Question 1 D

It is necessary to employ trial and error in a question like this, but it is also possible to determine the less-likely options without calculation.

Option **B** can be ignored as it requires the sum of two positive numbers to be zero – an impossibility. For the remaining options, students should calculate the left side and determine if the result matches the right side. This is true for option **D** only.

Question 2 B

Firstly determine the total cat and dog revenue separately.

The revenue from cats is $20c$ and the revenue from dogs is $50d$. Thus, the total revenue is $20c + 50d$.

Question 3 D

For students with a clear understanding of the relationship between linear graphs and their equations, this is easily achieved.

There is a vertical line equation listed and yet no vertical line exists on the graph. Every equation of the form $x = c$ for some constant value c will be a vertical line. A horizontal line will have an equation of the form $y = c$.

Question 4 C

The result can be read off the graph.

When zero water is consumed, the cost is $10. This must be the supply charge. Various linear sections of the graph exist, and the gradient of each gives a value of the rate of the variable usage charge. These are irrelevant, however, as it is the supply charge that is sought by the question.

Question 5 B

Firstly, students need to determine the gradient of the line segment. The line segment referred to in this question has endpoints (100, 27.5) and (150, 57.5) or similar, depending on the reading of the graph.

Thus the gradient is $\frac{57.5 - 27.5}{150 - 100} = 0.6$.

This narrows options to **A** and **B**. It is clear that the correct y-intercept of the line segment is negative and thus **B** must be true. Students who wish to be certain, however, can determine the value themselves.

The equation is known to be of the form $C = 0.6V + p$, where p is the vertical intercept.

Use point (100, 27.5). Thus:

$$27.5 = 100 \times 0.6 + p$$
$$p = -32.5$$

Question 6 D

Students must first obtain the gradient of the graph, which is 1.5. Accordingly, the equation is $D = 1.5v^2$.

Question 7 C

One of the possible constraints stands out as different. The third option has a ≤ sign and thus the required region would be below and left of the boundary. The graph shows that all constraints are above and to the right. Thus there is a problem with this option, and it is not shown in the diagram.

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Question 8 B

It is possible to obtain an answer by finding the values of each of the objective functions at the points *A*, *B* and *C*. This process could be long and tedious. A better method is to compare the gradients of the lines that meet at *B* with that of the objective functions. The lines that meet at *B* have gradients –1 and –3.

Now consider the objective functions. We will treat them as if each function is a constant value and then get the gradient of each.

Thus **A** has gradient –4, **B** has gradient –2, **C** has gradient $-\frac{1}{2}$, **D** has gradient $-\frac{2}{3}$ and **E** has gradient $-\frac{1}{3}$.

We want the gradient to be between –1 and –3. If the gradient is steeper than –3, then point **A** will be best instead of **B**. If the gradient is more shallow than –1, then **C** will be optimal instead of **B**. However, the only option with a gradient between –3 and –1 is **B,** making it the correct answer.

Question 9 C

The line must have a gradient of 120, as each new cartridge after the first ten costs \$120, which shows a price increase of \$120 per cartridge. Thus we have a line of the form $C = 120n + b$.

The final information required is a point on the line. The point is (10, 2000), as ten cartridges cost \$2000. This point gives the result $2000 = 120 \times 10 + b$.

Thus $b = 800$.

Module 4: Business-related mathematics

Question 1 **B**

$$SI = \frac{\$1200 \times 5 \times 3}{100}$$
$$= \$180$$

Question 2 **D**

$$A = \$1200 \times 1.04^2$$
$$= \$1297.92$$
interest = \$97.92

Question 3 **D**

$375 000 – 16 000 × 0.39 = $368 760

Question 4 **A**

$400 + $45 – $305 = $140

$$\$140 \times \frac{2.8}{12 \times 100} = \$0.33$$

Question 5 **A**

$1800 – $300 = $1500

$$\frac{\$1500 \times 1.04}{12} = \$130$$

Question 6 **B**

$$\text{interest} = \frac{4}{100} \times \$1500$$
$$= \$60$$
$$r_{\text{e}} = \frac{100 \times 60}{\$1500 \times 1} \times \frac{2 \times 12}{12 + 1}$$
$$= 7.3846\ldots\%$$

Question 7 **D**

$55 000 × 1.08 × 0.97 × 1.05 = $60 498.90

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Question 8 C

Use the finance solver.

$N = 3$

$I = 3.5$

$PV = -12\ 500$

$Pmt = -400$

$FV = 14\ 041.535\ldots$

$P_PY = 4$

$C_PY = 4$

Question 9 E

First three years:

$$A = 8000 \times \left(1 + \frac{\frac{r}{12}}{100}\right)^{3 \times 12}$$

$$= 8000 \times \left(1 + \frac{r}{1200}\right)^{36}$$

Final year:

$$A = 8000 \times \left(1 + \frac{\left(\frac{r}{2}\right)}{\frac{12}{100}}\right)^{1 \times 12}$$

$$= 8000 \times \left(1 + \frac{r}{2400}\right)^{12}$$

Module 5: Networks and decision mathematics

Question 1 B

A is incorrect because the matrix indicates that R has a loop (a connection to itself) that is not present in the graph. **C** and **D** are incorrect because they are not symmetrical. **E** is incorrect as the matrix does not indicate that S has a loop, leaving **B** as the correct answer.

Question 2 A

Since the graph is planar, Euler's formula, $V + F - E = 2$, can be used to deduce the number of vertices in each case from A to E.

	Faces	Edges	$V = E + 2 - F$	$E = \frac{V(V-1)}{2}$
A	4	6	4	6
B	4	7	5	10
C	4	8	6	15
D	4	9	7	21
E	4	10	8	28

Since the graph is complete, the number of edges must also satisfy $E = \frac{V(V-1)}{2}$.

Hence, the correct answer is **A**.

Question 3 A

To solve this problem, an Eulerian path is required that traverses all edges just once.

The graph has two vertices of odd degrees – Melbourne and Brisbane. The Eulerian path must start at one of these odd-degree vertices and terminate at the other. Since the path starts in Melbourne, it must terminate in Brisbane, making the correct answer **A**.

Question 4 D

The graph contains five edges. If we remove each one in turn we can observe that in four out of the five cases, a spanning tree is left. In the remaining case, the resulting graph becomes disconnected, hence the graph has 4 spanning trees.

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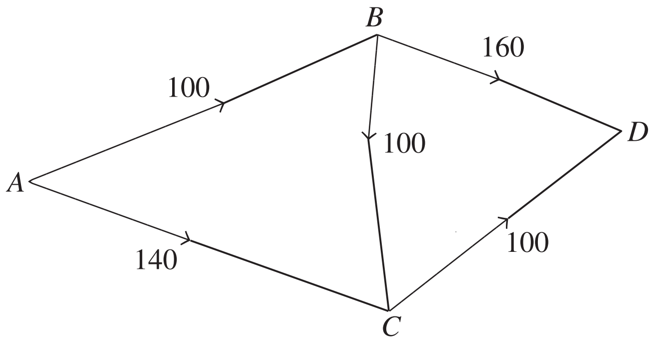
Question 5 A



The above network diagram can be drawn based on the information given. The possible cuts that separate the source from the sink are described in the table below, where their values are calculated.

Cut	Value
through *AB* and *AC*	100 + 140 = 240
through *AB*, *BC* and *CD*	100 + 100 = 200
through *AC*, *BC* and *BD*	140 + 100 + 160 = 400
through *BD* and *CD*	100 + 160 = 260

The minimal cut is through *AB*, *BC* and *CD*, which is 200. Hence the maximum number of vehicles per hour that can escape from *A* to *D* is 200.

Question 6 B

A path that visits each node of a network just once is known as a Hamiltonian path.

Question 7 E

Define the matrix M:

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Use technology to calculate $M + M^2$:

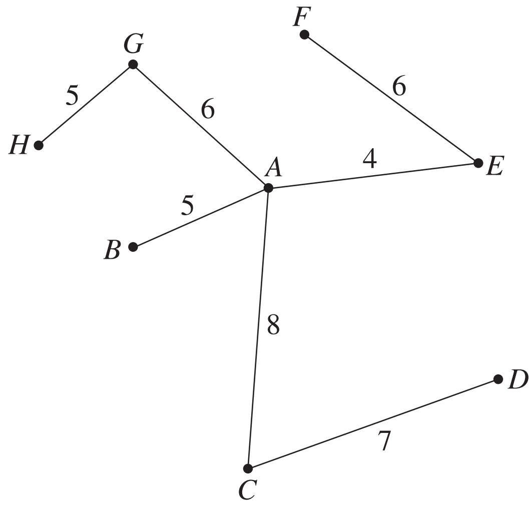
$$M + M^2 = \begin{bmatrix} 0 & 1 & 3 & 2 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 1 & 1 & 3 & 3 & 0 \end{bmatrix}$$

Team	Row total
A	1 + 3 + 2 + 1 = 7
B	1 + 2 + 1 + 1 = 5
C	0
D	2 + 2 = 4
E	1 + 1 + 3 + 3 = 8

We can conclude that team *E* is the strongest, making **E** the correct response.

Question 8 A

Use Prim's algorithm to find the minimal spanning tree.



Now add the cost of each edge.

Edge	**Cost ($ millions)**
AE	4
EF	6
HG	5
GA	6
BA	5
AC	8
CD	7

The total cost of the edges is 41, making **A** the correct response.

Question 9 D

A circuit running around the perimeter connects all stations except *A*. In order to connect *A,* it is necessary to omit one of the edges on the perimeter, make a detour to *A,* and then return to the perimeter at the end of the omitted edge.

The problem reduces to deciding which edge to omit. This is done by choosing to omit the edge and add the two new edges to and from *A* in such a way that the increase in cost is least.

Replacing edge *DE* with *DA,* followed by *AE,* reduces the total cost by 10 and increases it by 9 and 4. This is a net increase of 3, which is less than the alternatives.

Therefore, the required circuit is:

HBCDAFGH

Now add the cost of each edge.

Edge	**Cost**
HB	6
BC	9
CD	7
DA	9
AE	4
EF	6
FG	8
GH	5

The total cost of the edges is 54, making **D** the correct response.

Module 6: Matrices

Question 1 C

There are 4 rows and 3 columns and thus the order is 4×3.

Question 2 D

In order for a matrix product to be defined, we require that the number of columns of the left matrix matches the number of rows of the right matrix.

For PQ this is not true, as P has 2 columns and Q has 4 rows.

For PR, the columns of P and rows of R do match in number and the result is a 3×3 matrix.

QR is also a failed multiplication, as Q has 3 columns while R has 2 rows.

Question 3 C

We are interested in the matrix element that applies to current state L, next month C.

This is row 2, column 1, and the value is 0.75. Therefore 75% of larvae survive to become caterpillars, and 25% die.

Question 4 E

$$T^4 S_0 = \begin{bmatrix} 0.00 & 0.00 & 0.40 \\ 0.75 & 0.00 & 0.00 \\ 0.00 & 0.80 & 0.95 \end{bmatrix}^4 \begin{bmatrix} 1000 \\ 900 \\ 2000 \end{bmatrix}$$

$$= \begin{bmatrix} 1365.8 \\ 926.7 \\ 3872.6 \end{bmatrix}$$

$$\text{total insects} = 1365.8 + 926.7 + 3872.6$$
$$= 6165$$

Question 5 B

This is largely a case of redrawing the table as a matrix and replacing yes responses with 1 and no responses with 0. However, some of the matrices have the rows and columns swapped compared to the arrangement in the table and students need to be aware of this. **B** is one such matrix. If the rows and columns in **B** are swapped (not just the headings) we do obtain the required matrix.

Question 6 **B**

This is just a set of simultaneous equations:

$4x + 3y = 20$

$3x + 6y = 25$

Note that the cost in the second scenario is \$25, an additional \$5. In matrix format this is $\begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 25 \end{bmatrix}$.

To solve this, students need the inverse matrix.

determinant = 24 – 9 = 15

$$\text{inverse} = \frac{1}{15}\begin{bmatrix} 6 & -3 \\ -3 & 4 \end{bmatrix}$$

$$\text{Thus } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix}\begin{bmatrix} 20 \\ 25 \end{bmatrix}.$$

Question 7 **B**

Firstly, we need to determine which products are possible and the order of the result. This rules out **A**, as Q is 4×1 and R is 4×4; thus QR is not a possible product.

RQ can exist and will have order 4×1. P is of order 1×4 and thus $P \times (R \times Q)$ does exist and is of order 1×1. **B** is therefore possible.

RP exists and is of order 1×4. $Q \times (R \times P)$ will exist and be ordered 4×4. As we want a single result, not a 4×4 matrix of results, **C** has an inappropriate format and is therefore incorrect.

QP exists and has order 4×4. Multiplying by another 4×4 produces a 4×4 result, and thus **D** can be ruled out for the same reason as **C**.

PQ exists and is 1×1. We are unable to multiply a 1×1 by a 4×4 and thus **E** does not make sense and is incorrect.

Question 8 **B**

This is simply a technology question. If we raise the matrix T to a sufficiently large power, we will be able to obtain a consistent result.

The long-term proportions are $T^{100} = \begin{bmatrix} 0.286 & 0.286 \\ 0.714 & 0.714 \end{bmatrix}$.

Question 9 **C**

Students must first expand the existing matrix equation:

$15 + 2p = 3q + 15$

$12 + p = 4q + 10$

Thus $2p - 3q = 0$

$p - 4q = -2$

Thus the correction option is **C**.

