The Mathematical Association of Victoria
FURTHER MATHEMATICS
SOLUTIONS: Trial Exam 2015

Written Examination 1

SECTION A: CORE- Data Analysis

1. D	**2.** C	**3.** C	**4.** E	**5.** C
6. A	**7.** A	**8.** D	**9.** E	**10.** B
11. D	**12.** C	**13.** D		

SECTION B: MODULES

Module 1: Number Patterns

1. C	**2.** A	**3.** B	**4.** D	**5.** D
6. C	**7.** B	**8.** C	**9.** E	

Module 2: Geometry and trigonometry

1. C	**2.** B	**3. A**	**4. A**	**5. B**
6. B	**7.** D	**8.** C	**9.** D	

Module 3: Graphs and relations

1. C	**2.** E	**3.** B	**4.** A	**5.** C
6. B	**7.** C	**8.** A	**9.** E	

Module 4: Business- related mathematics

1. B	**2.** B	**3.** C	**4.** C	**5.** B
6. B	**7.** C	**8.** C	**9.** D	

Module 5: Networks and decision mathematics

1. B	**2.** C	**3.** D	**4.** A	**5.** E
6. C	**7.** B	**8.** E	**9.** C	

Module 6: Matrices

1. B	**2.** B	**3.** A	**4.** C	**5.** E
6. D	**7.** B	**8.** E	**9.** D	

Worked Solutions Core – Data Analysis

Question 1
The mean number is given by using the formula

$$\bar{x} = \frac{0\times2+1\times4+2\times8+3\times7+4\times25+5\times30+6\times20+7\times4}{100} \approx 4.4$$

Answer D.

Question 2
The data is clustered in the upper region of the range and as such it is negatively skewed.

Answer C.

Question 3
There are 20 values so that the median will lie between the 10th and 11th values.
10th value : 69.8 11th value : 70.7

$$\text{Median value} = \frac{69.8+70.7}{2} = 70.25 \approx 70.3$$

Answer C

Question 4
82.3 = 78.4 + 3.9, i.e. it is one standard deviation ABOVE the mean.
Mean ± 1 SD encompasses 68% of the data with 16% above (and 16% below)
If there are 16% of countries above this value, then (100 – 16) = 84% are below.

Answer E

Question 5
The five figure summary for this data is :
Min : 1Q1 : 10 Median : 13 Q3 : 16 Max : 26
IQR = Q3 – Q1 = 16 – 10 = 6
Lower fence : Q1 – 1.5 × IQR = 10 – 1.5 × 6 = 10 – 9 = 1
Upper fence : Q3 + 1.5 × IQR = 16 + 1.5 × 6 = 16 + 9 = 25
Lower fence = minimum, hence minimum value is NOT outlier
Upper fence < maximum, hence maximum value IS outlier.
The only boxplot that matches all the above data is C

Answer C

Question 6
Both of the variables "gender" and "regularity of homework" are categorical variables. The only appropriate display for two categorical variables listed here is option A.
A histogram is for univariate numerical data. Parallel boxplots and back to back stem and leaf plots are used when there is one categorical and one numerical variable. A scatterplot would be used for two numerical variables.

Answer A.

Question 7
The predicted income is: *Income* = 7.58 – 0.60 × 6 = \$3.98 million
The residual is the actual income – predicted income = 2.6 – 3.98 = –1.38 ≈ –1.4.

Answer A.

Question 8

The transformed equation is $Income = \frac{7.37}{ranking} + 2.14$. The ranking of 3 can be substituted into the equation as follows $Income = \frac{7.37}{3} + 2.14 = 4.5966... \approx 4.6$

so the income is predicted to be $4.6 million.

Answer D.

Question 9

The research highlights an **association** between the two variables, not any cause/effect situation. There is only one option that correctly states this.

Answer E

Question 10

For the equation $y = a + bx$

$$b = r\frac{S_y}{S_x} = 0.965 \times \frac{10.59}{6.08} = 1.68$$

and thus for a: $31.86 = a + 1.68 \times 20.57 \Rightarrow a = 31.86 - 1.68 \times 20.57 = -2.70$

Answer B

Question 11

Twelve months for the seasons means that the sum of the seasonal indices will be 12.00.

Sum existing $= 0.47 + 0.88 + 0.99 + 1.08 + 1.31 + 1.48 + 1.32 + 1.23 + 0.77 + 0.53$
$= 10.06$

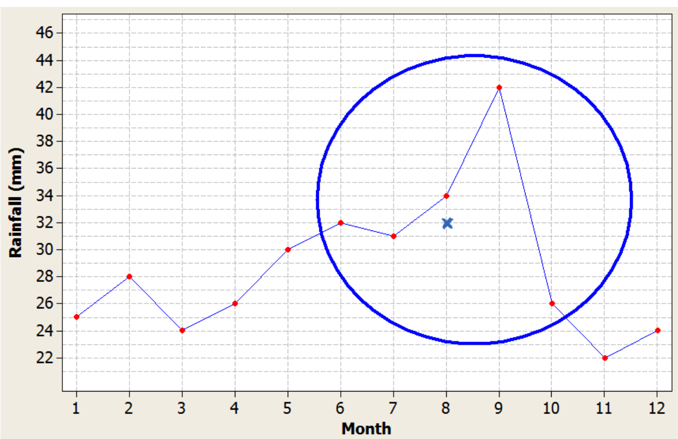
Mar + Oct $= 12.00 - 10.06 = 1.94$

Mar (or Oct) $= \frac{1.94}{2} = 0.97$

Answer D

Question 12

The five points under consideration have been circled on the diagram below. The point marked X is the median of these five points. it is at the point (8, 32).



Answer C.

Question 13
Which of these index values satisfies the following equation?

$$\text{Index} - \frac{1}{4} \times \text{index} = 1.00$$

Checking the options :

A.	$1.00 - 0.25 \times 1.00 = 0.75$	TOO SMALL
B.	$1.20 - 0.25 \times 1.20 = 0.90$	TOO SMALL
C.	$1.25 - 0.25 \times 1.25 = 0.9375$	TOO SMALL
D.	$1.33 - 0.25 \times 1.33 = 0.9975$	CLOSEST
E.	$1.50 - 0.25 \times 1.50 = 1.125$	TOO LARGE

Answer D

Worked Solutions Modules
Module 1: Number Patterns

Question 1
The common difference is $16 - 13 = 19 - 16 = 3$
Tracking backwards: 19, 16, 13, 10, 7, 4, 1, …
Given that the first term is greater than 1, the most likely first term is 4.

Answer C

Question 2
$a = 5$

$$S(3) = \frac{3}{2}\left(2\times 5+\left(3-1\right)d\right) = 3$$

So
$$\begin{aligned} 3(10 + 2d) &= 6 \\ 30 + 6d &= 6 \\ 6d &= -24 \\ d &= -4 \end{aligned}$$

Answer A

Question 3
Using $t_n = ar^{n-1}$

Fifth term : $ar^4 = 405$
Second term : $ar = -15$

Dividing the terms : $\dfrac{ar^4}{ar} = \dfrac{405}{-15}$, gives $r^3 = -27$ and hence $r = \sqrt[3]{27} = -3$

Answer B

Question 4

$t_3 = 2 \times t_2 - 3$	$t_2 = 2 \times t_1 - 3$
$27 = 2 \times t_2 - 3$	$15 = 2 \times t_1 - 3$
$30 = 2 \times t_2$	$18 = 2 \times t_1$
$15 = t_2$	$9 = t_1$

The first term (t_1) is 9

Answer D

Question 5

Ratio $= \dfrac{19}{20} = \dfrac{18.05}{19} = 0.95$

This is a sum to infinity with first term 20 and common ratio (r) of 0.95.

From $S_\infty = \dfrac{a}{1-r}$, area $= \dfrac{20}{1-0.95} = 400$

Answer D

Question 6

The first six terms of this Fibonacci sequence can be written as :

$p,\ p,\ p+p,\ p+(p+p),\ (p+p)+p+(p+p),\ p+(p+p)+(p+p)+p+(p+p), \ldots$

which can be simplified to :

$p,\ p,\ 2p,\ 3p,\ 5p,\ 8p,\ \ldots$

If $8p = 48$

Then $p = 6$

Answer C

Question 7

The first four terms of a geometric sequence can be written $a, ar, ar^2, ar^3, \ldots$

So, $a + ar^2 = 30$

And, $a + ar + ar^2 + ar^3 = -30$

Use the SOLVE function

SOLVE ($a + ar^2 = 30$ and $a + ar + ar^2 + ar^3 = -30$, {a,r}) giving $r = -2$

Answer B

Question 8

Percentage increase implies a geometric sequence

For this sequence : $a = 15$, $r = 1.35$

So $135 = 15 \times (1.35)^{n-1}$

Using SOLVE function in calculator, $n = 8.321\ldots \approx 9$ days

Answer C

Question 9

If 20% is lost each day, then (100 – 20) = 80% will remain

A difference equation can be written for this situation :

$t_{n+1} = 0.8 \times t_n + 500$ where $t_1 = 4500$

Model in sequence generator function of calculator, evaluating AT LEAST 60 days (to cover to the end of March for option E).

Examine each option

A.	At day 28, there is still 2505 litres in the dam	NOT TRUE
B.	The values have decreased to 2505 by the end of February	NOT TRUE
C.	Day 13 is 2637, day 14 is 2609, day 15 is 2588	NOT TRUE
D.	Day 27 is 2506, day 28 is 2505, day 29 is 2504	NOT TRUE
E.	Day 39 (11th March) is 2500, day 40 is 2500, day 41 is 2500	TRUE

Answer E

Module 2: Geometry and Trigonometry

Question 1
Base angle adjacent to 125º = 180º – 125º = 55º
Other base angle = 55º (Isosceles triangle)
Hence θ = 180º – 55º – 55º = 70º

Answer C

Question 2
Radius inner circle = 1.5 ÷ 2 = 0.75 m
Radius outer circle = 1 + 0.75 = 1.75 m
Area path = area outer circle – area inner circle
= $\pi(1.75)^2 - \pi(0.75)^2$
= 7.854
≈ 8 m2

Answer B

Question 3
Bearing of main jetty from boat is 048º,
Bearing of boat from main jetty is 180º + 48º = 228º
Bearing of navigational marker from main jetty = 228º ÷ 2 = 114º

Answer A

Question 4
Reduced length of straw in can = 160 ÷ 2 = 80 mm
Distance up side = $\sqrt{80^2 - 60^2}$ = 52.915… ≈ 53 mm

Answer A

Question 5
Area wall covered = 1.5 × 8 = 12 m^2
If *A* is the area of the walls of the house
$\frac{12}{A} = \frac{2^2}{5^2}$ (comparing areas !) So $A = 12 \times \frac{5^2}{2^2} = 75\ m^2$
Paint required = 75 ÷ 8 = 9.375 L ≈ 10L

Answer B

Question 6
∠*BAC* = 105º – 37º = 68º
Use Cosine Rule to find *BC*.
BC = $\sqrt{2500^2 + 2750^2 - 2 \times 2500 \times 2750 \times \cos 68^\circ}$ = 2943.0697 ≈ 2943 m

Answer B

Question 7
Find ∠*ABC* using Sine Rule
$\frac{2750}{\sin ABC} = \frac{2943}{\sin 68^\circ}$, giving ∠ABC = 60.04º
Angle east of south for *BC* = 60.04º – 37º = 23.04º
Bearing of marker *C* from marker *B* = 180º – 23.04º = 156.96º ≈ 157º

Answer D

Question 8
If point A is on the 150 m contour, then point B is on the 400 m contour
Height difference between points A and B = 400 – 150 = 250 m

$\frac{250}{AB} = \sin 24^{\circ}$, giving AB = 614.648…

Total cable length = 2 × 614.65 + 20% of 2 × 614.65 = 1475.16 ≈ 1475 m

Answer C

Question 9
Let D be depth of well
Use similar triangles

$\frac{D}{1000} = \frac{750}{200}$, giving D = 3750 mm = 3.75 m

Answer D

Module 3 Graphs and Relations

Question 1
A horizontal line has the equation y = *constant* where every y value on the line is the same. This line passes through the point (5, 4) and has a y-coordinate of 4, so the equation is $y = 4$.

Answer C.

Question 2
The first part of the journey is the bike ride. He travels 40 km over a four hour period, so his average speed is given by:

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{40}{4} = 10 \text{ km/h}$

Answer E.

Question 3
The speed when Mitch rides his bike is 10 km/h so the gradient of the line is 10. The y intercept is zero and this part of the journey is from 0 hours to 4 hours, so the equation for the bike segment is $d = 10t \quad 0 < t \le 4$.

After the bike ride he waits for 2 hours from t = 4 to t = 6 and his distance is constant at 40 km. Therefore the equation for the waiting segment is $d = 40 \quad 4 < t \le 6$.

The last car segment is from the point (6, 40) to (8, 120). The equation of this segment is calculated as follows:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 40 = \frac{120 - 40}{8 - 6}(x - 6)$$

$$y - 40 = 40(x - 6)$$

$$y - 40 = 40x - 240$$

$$y = 40x - 200$$

Therefore it can be written as $d = 40t - 200 \quad 6 < t \le 8$.

Answer B.

Question 4
The first line has the equation $y = 2x - 6$ and the second line has the equation $y = -3x + 9$ as shown below:

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$	$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$y - -10 = \dfrac{4 - -10}{5 - -2}(x - -2)$	$y - -3 = \dfrac{15 - -3}{-2 - 4}(x - 4)$
$y + 10 = 2(x + 2)$	$y + 3 = -3(x - 4)$
$y + 10 = 2x + 4$	$y + 3 = -3x + 12$
$y = 2x - 6$	$y = -3x + 9$

The point both lines have in common is where they intersect. This can be calculated using the simultaneous equation as shown on the ClassPad below left, or SOLVER functions in TInspire below right. This is the point $(3,0)$:

$$\left\{\begin{array}{l} y=2x-6 \\ y=-3x+9 \end{array}\right|_{x,y} \quad \{x=3, y=0\}$$

$$\text{linSolve}\left(\left\{\begin{array}{l} y=2\cdot x-6 \\ y=-3\cdot x+9 \end{array}\right., \{x,y\}\right) \quad \{3,0\}$$

$$\text{solve}(y=2\cdot x-6 \text{ and } y=-3\cdot x+9, \{x,y\}) \quad x=3 \text{ and } y=0$$

Answer A.

Question 5
The axes are labelled y and x^2 and there is a straight line through the origin, so the relationship is of the form $y = kx^2$ where $k = \dfrac{y}{x^2}$. The coordinate of any point on the line is (x^2, y) so using the point $(1,5)$ the value of $k = \dfrac{5}{1} = 5$. Therefore the relationship is $y = 5x^2$.

Answer C.

Question 6
The number of chairs (x) must be at least six times the number of tables (y). Therefore $x \geq 6 \times y$. This can be rewritten as follows:

$$x \geq 6 \times y$$

$$\frac{1}{6}x \geq y$$

$$y \leq \frac{1}{6}x$$

Answer B.

Question 7

The equation for the cost relationship is $C = 150n + 2500$ as shown alongside using the points (10, 4000) and (20, 5500):

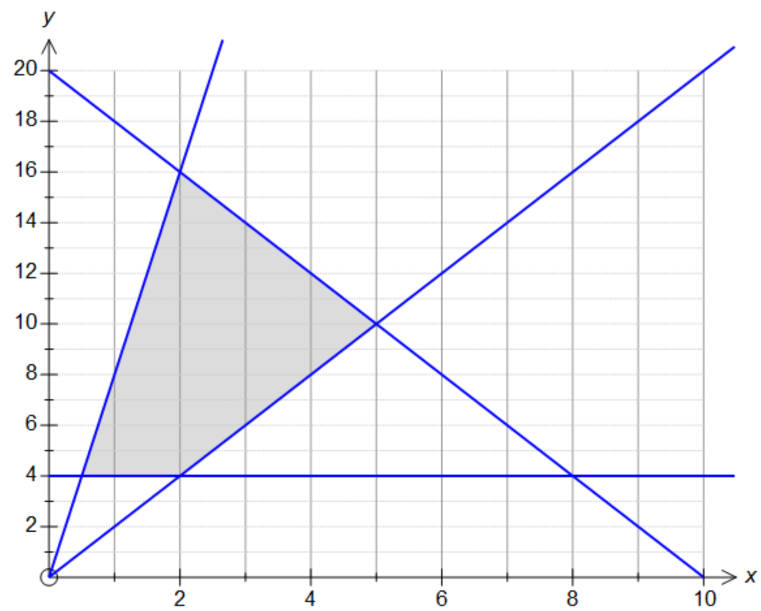
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 4000 = \frac{5500 - 4000}{20 - 10}(x - 10)$$
$$y - 4000 = 150(x - 10)$$
$$y - 4000 = 150x - 1500$$
$$y = 150x + 2500$$

The cost of manufacturing 120 washing machines is $C = 120 \times 150 + 2500 = \$20500$.
As there is a profit of \$13500, the total revenue is $20500 + 13500 = \$34000$.
Therefore the selling price per washing machine is 34000 ÷ 120 = \$283.33.

Answer C.

Question 8

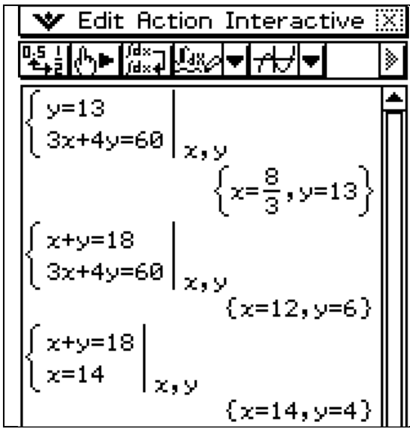
The required region is shown shaded below and it can be seen that the corner points are (0.5, 4), (2, 16), (5, 10) and (2, 4).

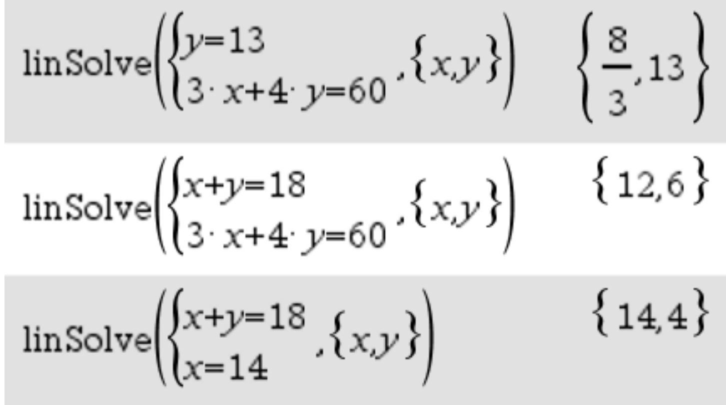


Answer A.

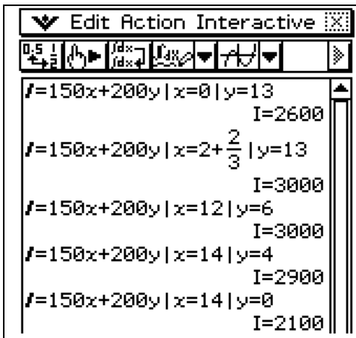
Question 9

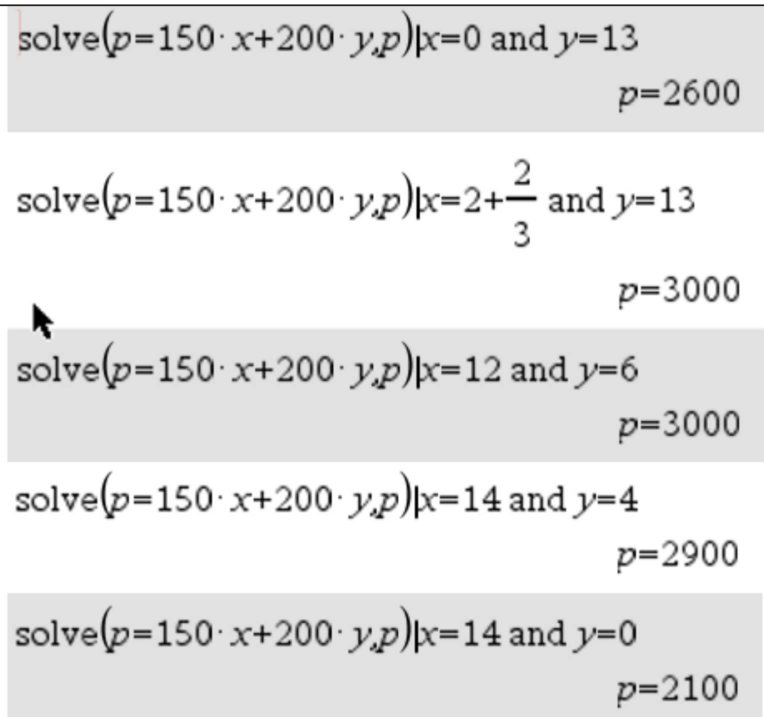
The region is represented by the inequations $0 \le x \le 14$, $0 \le y \le 13$, $x + y \le 18$ and $3x + 4y \le 60$. The corner points of the feasible region shown are $(0, 13)$, $\left(2\frac{2}{3}, 13\right)$, $(12, 6)$, $(14, 4)$ and $(14, 0)$ as shown by using a simultaneous equation solver below:



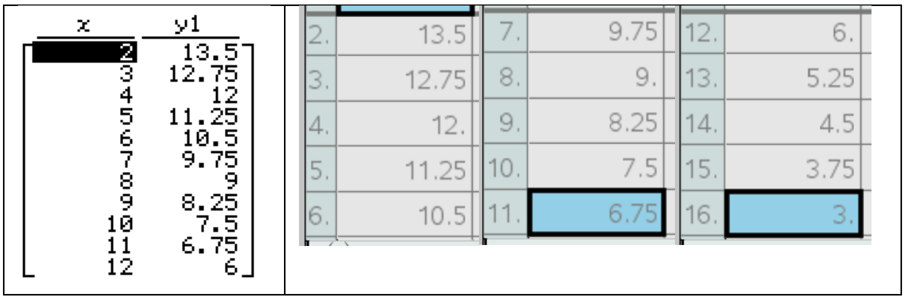


The income is $I = 150x + 200y$. The income can be maximised as shown below:

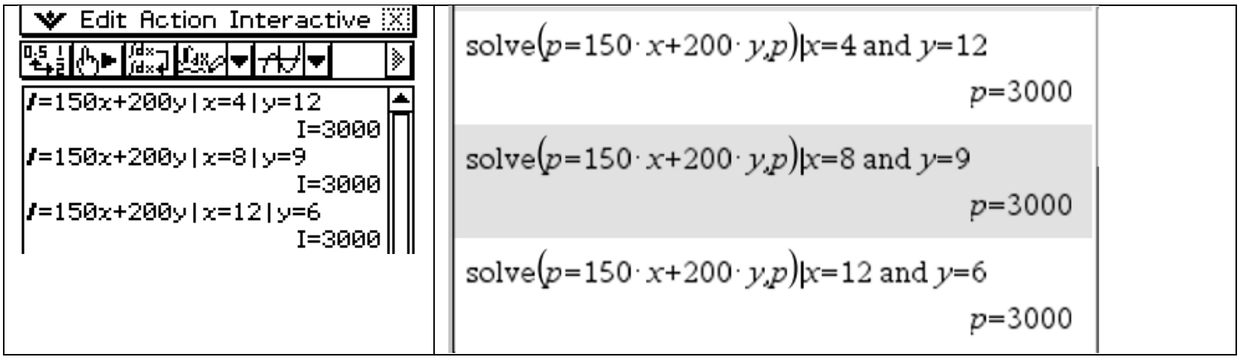




The points $\left(2\frac{2}{3}, 13\right)$ and $(12, 6)$ are a maximum, so the maximum is any point on the line segment between these two points. The variables are both people so the only available solutions are the integer points on the line segment of $3x + 4y = 60$. These points can be found using a table menu by first transposing $3x + 4y = 60$ to $y = 15 - 0.75x$:



Therefore the integer points in this segment are $(4,12)$, $(8,9)$ and $(12,6)$ where the income is \$3000 as shown below:



Answer E.

Module 4: Business Maths

Question 1

EITHER	New price	=	(100 – 7.5)% of \$25 000
		=	92.5% of \$25 000
		=	\$23 125
OR	Discount	=	7.5% of \$25 000
		=	\$1875
	New price	=	\$25 000 – \$1875
		=	\$23 125

Answer B

Question 2

The GST is $\frac{1}{11}$ of the final GST-inclusive price.

GST payable $= \frac{1}{11} \times \$2695 = \$245$

Answer B

Question 3

Total value investment = \$17 000 + interest

$= \$17\,000 + \dfrac{17000 \times 4.2 \times 5}{100 \times 12}$

= \$17 000 + \$297.50

= \$17 297.50

Answer C

Question 4

$\text{Value} = \text{Price}\left(1 - \dfrac{7.5}{100}\right)$ per month

$325 = 650\left(1 - \dfrac{7.5}{100}\right)^n$ where n = number of months

Find time, using Finance Solver

$I\% = -7.50$, $PV = -650$, $Pmt = 0$, $FV = \$350$, $PpY = CpY = 1$,

Solve for N = 8.89… ≈ 9 months

Answer C

Question 5

Find time to repay fully, using Finance Solver

$I\% = 6.75$, $PV = 300\,000$, $Pmt = -2400$, $FV = 0$, $PpY = CpY = 12$,

Solve for N = 216.507… ≈ 217 months

217 ÷ 12 = 18 years and 1 month

Answer B

Question 6

Value depreciated = \$32 000 – \$20 000 = \$12 000

Kilometres travelled $= \dfrac{12000}{0.425}$ = 28 235.29 ≈ 28 235

Answer B

Question 7

Total repaid on loan = 24 × \$65 = \$1560

Interest paid = \$1560 – (\$1350 – \$100) = \$310

Interest rate $= \dfrac{100 \times 310}{(1350 - 100) \times 2}$ = 12.4%

Effective interest rate $= \dfrac{2 \times 12.4 \times 24}{24 + 1}$ = 23.808… ≈ 23.8%

Answer C

Question 8

Find amount paid monthly, using Finance Solver

$N = 6$, $I\% = 4.00$, $PV = -3000$, $FV = \$5480.59$, $PpY = CpY = 12$,

Solve for Pmt = 399.999… ≈ \$400

Answer C

Question 9
Find length of loan, using Finance Solver
$I\% = 6.45$, $PV = -350\,000$, $Pmt = -2600$, $FV = 0$, $PpY = CpY = 12$,
Solve for $N = 239.852\ldots \approx 240$ months (20 years)

5 years = 5 × 12 = 60 month
Find amount owed after 5 years, using Finance Solver
$N = 60$, $I\% = 6.45$, $PV = 350\,000$, $Pmt = -2600$, $PpY = CpY = 12$,
Solve for $FV = 299\,268.538\ldots \approx \$299\,268.54$

Time remaining = 240 – 60 = 180 months
Find new amount paid monthly, using Finance Solver
$N = 180$, $I\% = 6.95$, $PV = 299268.54$, $FV = 0$, $PpY = CpY = 12$,
Solve for $Pmt = 2681.551\ldots \approx \$2681.55$

Answer D

Module 5: Networks and Decision Mathematics

Question 1
There are 7 vertices. Vertex A has a degree of 2, Vertex B has a degree of 4, Vertex C has a degree of 3, Vertex D has a degree of 1, Vertex E has a degree of 3, Vertex F has a degree of 3 and Vertex G has a degree of 2. There are 3 vertices of even degree and 4 vertices of odd degree, so neither an Eulerian circuit or an Eulerian path exists.
There is a Hamiltonian path AGFBECD (others are available) but because vertex D has only one edge there is no Hamiltonian circuit.

Answer B.

Question 2
A complete graph with 4 vertices has $\frac{4\times 3}{2}=6$ edges. Any network with only 4 vertices is planar so Euler's rule for planar graphs can be used:

$$V+R=E+2$$
$$4+R=6+2$$
$$R=6+2-4=4$$

Answer C.

Question 3

The matrix of two-step dominances is $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$.

The element in the first row and third column is Andrea's two two-step dominances over Charlie. From the original matrix it can be seen that Andrea defeated Brett and Emmett and both Brett and Emmett defeated Charlie.
A is incorrect as Andrea has two-step dominances over Charlie and Emmett.
B is incorrect as Charlie has two-step dominances over Andrea, Brett and Emmett.
C is incorrect as Emmett has two-step dominances over Andrea and Digby
E is incorrect because Digby has a total of 5 two-step dominances.

Answer D.

Question 4

Ari must dust because he is the only person and it follows that Greta must do dishes because only she and Ari would do dishes. Leonard can do any of the 3 remaining tasks whereas Susan can only vacuum or do rubbish bins and Shia can cook or do rubbish bins. There are a few ways to allocate the last 3 tasks:

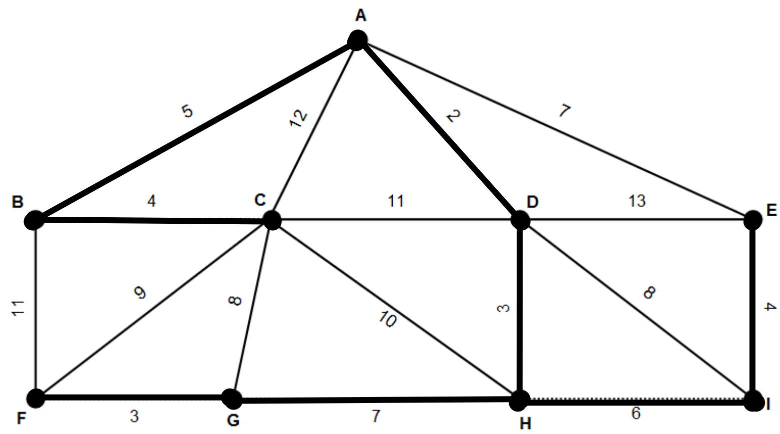
	Vacuuming	Rubbish bins	Cooking
Option 1	Susan	Leonard	Shia
Option 2	Leonard	Susan	Shia
Option 3	Susan	Shia	Leonard

The only one of these available is option A.

Answer A.

Question 5

The minimum spanning tree is shown below:



The edges would be added in order AD – DH – AB – BC – IH – IE – HG – GF.

Answer E.

Question 6

Line A is not a cut as it does not cut off both sources from the sink.
The capacity of Cut B is 9 + 0 + 4 + 0 + 7 = 20.
The capacity of Cut C is 5 + 4 + 0 + 7 = 16.
The capacity of Cut D is 5 + 3 + 10 + 7 = 25.
The capacity of Cut E is 7 + 10 + 5 = 22.
The minimum cut is 16 at cut C.

Answer C.

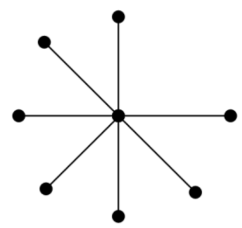
Question 7

Each statement is investigated below:
The network must be a tree – True.
A tree has one more vertex than edge and has one region. Using Euler's rule it can be seen that there is one region:

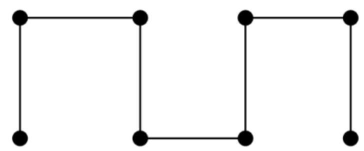
$$V + R = E + 2$$
$$8 + R = 7 + 2$$
$$R = 7 + 2 - 8 = 1$$

The network could have an Eulerian circuit – False.
It is a tree so there are no circuits by definition.

The network must have an Eulerian path – False.
It is possible that an Eulerian path exists but not necessarily so. An example of an appropriate network without an Eulerian path is shown alongside.

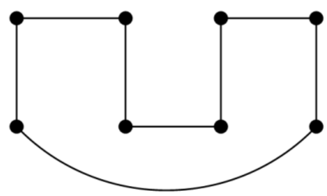


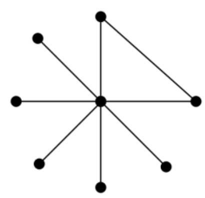
The network could have a Hamiltonian path – True.
An example of a network with a Hamiltonian path is shown alongside:



Adding an edge to the network will make sure that an Eulerian circuit exists – False.
Adding an edge may produce an Eulerian circuit as shown in the left figure.

Adding an edge may not mean an Eulerian circuit as shown in the right figure





The network is complete – False.

A complete network with 7 vertices would have $\frac{7 \times 6}{2} = 21$ edges and would not be planar.

There are two (2) True statements.

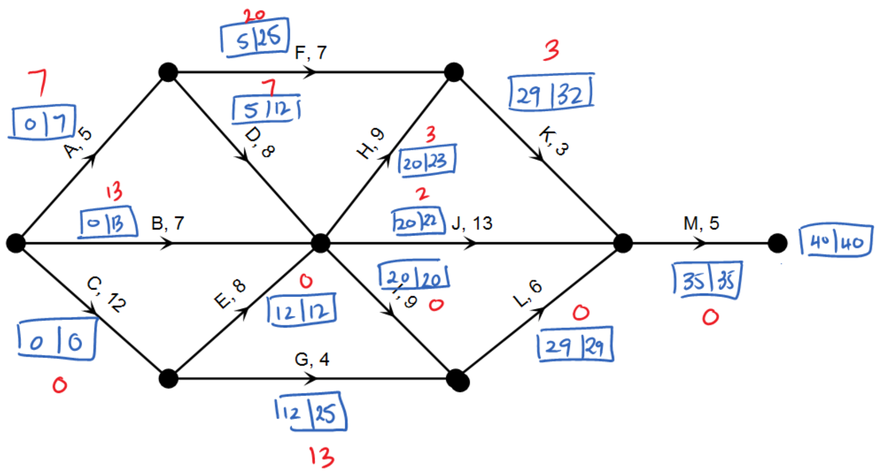
Answer B.

Question 8

The critical path is the longest path through the network.

A forward and backward scan is shown below:

Activity	Time	EST	LST	Float
A	5	0	7	7
B	7	0	13	13
C	12	0	0	0
D	8	5	12	7
E	8	12	12	0
F	7	5	25	20
G	4	12	25	13
H	9	20	23	3
I	9	20	20	0
J	13	20	22	2
K	3	29	32	3
L	6	29	29	0
M	5	35	35	0



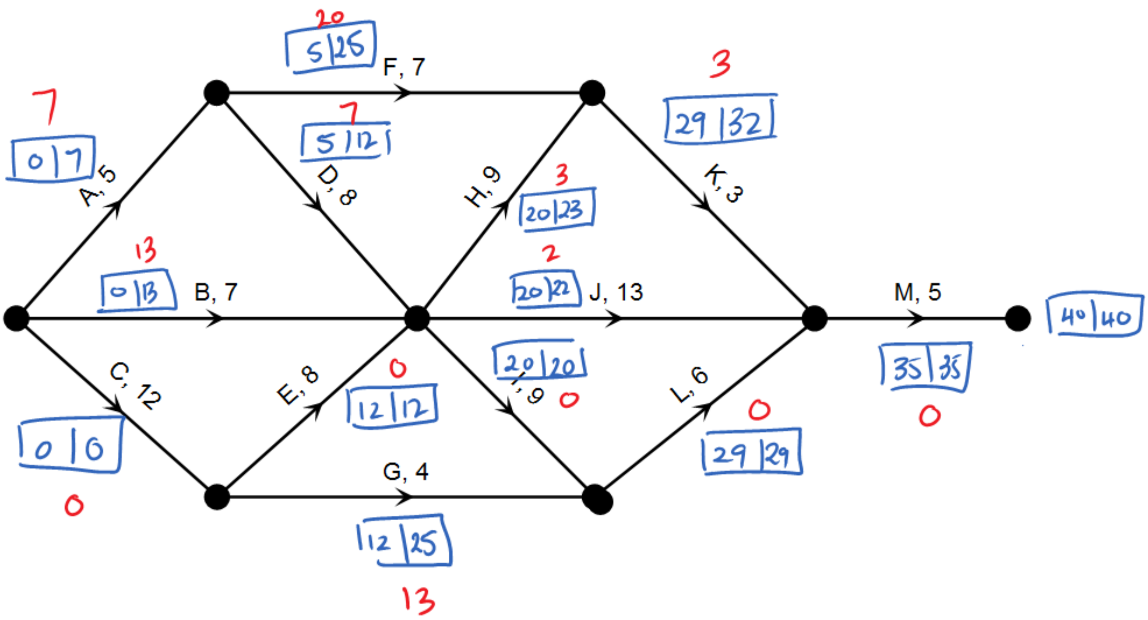
It can be seen that the activities with zero float time (and hence on the critical path) are CEILM.

Answer E.

Question 9

The forward and backward scans are shown below:

Activity	Time	EST	LST	Float
A	5	0	7	7
B	7	0	13	13
C	12	0	0	0
D	8	5	12	7
E	8	12	12	0
F	7	5	25	20
G	4	12	25	13
H	9	20	23	3
I	9	20	20	0
J	13	20	22	2
K	3	29	32	3
L	6	29	29	0
M	5	35	35	0



From this scan it can be seen that 5 activities have 5 or more hours float time A – 7 hours, B – 13 hours, D – 7 hours, F – 20 hours and G – 13 hours.

Answer C.

Module 6 Matrices

Question 1

The matrix multiple QP exists because the number of columns in Q is equal to the number of rows in P. The order of matrix QP is 5 × 4. QP^2 is not defined because only square matrices can be raised to a power and the number of rows in QP is 5 which is not equal to the number of columns in QP (4).

Answer B.

Question 2

The product matrix is a 3 × 3 matrix, so the first matrix must have 3 rows and the second matrix must have 3 columns. All of the available options would result in a 3 × 3 matrix. The resultant matrix has zero elements in all positions except x_{23}. Therefore the multiple of row 2 from the first matrix by row 3 of the second matrix must be 20. Here this multiple is 3 × 3 + 4 × 2 + 1 × 3 = 20. The other multiples will all result in zero because every multiple has zero as one of the numbers being multiplied.

Answer B.

Question 3

The determinant of a 2 × 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$, which in this case is

$1 \times 7 - 5 \times -2$.

Answer A.

Question 4

The total cost of each size of plant and pot is given by $\begin{bmatrix} 2.5 & 4.7 & 9.8 \end{bmatrix} + \begin{bmatrix} 1.3 & 2.5 & 5.1 \end{bmatrix}$.

The profit from the sales is the additional 40% added to the price and is therefore given by multiplying the total cost by 1.4 so it would be $1.4 \times \left(\begin{bmatrix} 2.5 & 4.7 & 9.8 \end{bmatrix} + \begin{bmatrix} 1.3 & 2.5 & 5.1 \end{bmatrix} \right)$.

This will give a row matrix that has the profit from each size plant sold. Multiplying this row matrix by a column matrix containing the number of each size sold will give the required total profit from all sales. This would result in the matrix equation

$$P = 1.4 \times \left(\begin{bmatrix} 2.5 & 4.7 & 9.8 \end{bmatrix} + \begin{bmatrix} 1.3 & 2.5 & 5.1 \end{bmatrix} \right) \times \begin{bmatrix} 35 \\ 40 \\ 24 \end{bmatrix}$$

Answer C.

Question 5
The use of a table would assist by placing the number of each item bought in the order in the matrix $\begin{bmatrix} U \\ F \\ H \end{bmatrix}$ as shown below:

	USB drives	Folders	Highlighters	Cost
John	2	0	3	\$29.90
Nicholas	4	5	0	\$44.30
Maria	0	2	1	\$9.10

The rows can be interchanged so the matrix of coefficients could be any 3 × 3 matrix with the rows $\begin{bmatrix} 2 & 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 4 & 5 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 2 & 1 \end{bmatrix}$, so long as the values 29.9, 44.3 and 9.1 are in the same order as their respective rows. Therefore options A, B, C and D are all appropriate, but option E uses the row values in columns instead and could not be used to solve this problem.

Answer E.

Question 6
The transition matrix that is equivalent to the transition diagram is

$$\begin{matrix} & C & H & \\ & \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} & & \begin{matrix} C \\ H \end{matrix} \end{matrix}$$

The number of customers buying each drink the next day will be given by the transition matrix multiplied by the state matrix for this day. The state matrix for this day is

$$\begin{bmatrix} 35 \\ 45 \end{bmatrix} \begin{matrix} C \\ H \end{matrix} \quad .$$

Answer D.

Question 7
The previous day would be given by using

$$\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^{-1} \times \begin{bmatrix} 35 \\ 45 \end{bmatrix} = \begin{bmatrix} 25 \\ 55 \end{bmatrix} \begin{matrix} C \\ H \end{matrix}$$

Answer B.

Question 8

Option A is true because the steady state is $\begin{bmatrix} 50 \\ 30 \end{bmatrix} \begin{matrix} C \\ M \end{matrix}$ so 20 more cappuccinos are sold each day
Option B is true because the steady state is independent of the starting state matrix, it may just reach the steady state in more or less transitions.
Option C is true because the steady state is a dynamic situation where the movement is continuing but each movement in one direction is counteracted by equal and opposite movement.
Option D is true because the steady state is reached after 7 transitions.
Option E is false because while there are 15% of cappuccino drinkers who change to hot chocolate each day, there are 85% who have a cappuccino the next day. There is no guarantee that an individual will move into the other group at any point.

Answer E.

Question 9

The number of sheep is 2015 can be calculated as follows :

$$S_{2015} = \begin{bmatrix} 0.82 & 0.26 \\ 0.18 & 0.74 \end{bmatrix} \times \begin{bmatrix} 2000 \\ 3000 \end{bmatrix} + \begin{bmatrix} 120 \\ 30 \end{bmatrix} = \begin{bmatrix} 2540 \\ 2610 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

It can be seen that farm A gains 2540 – 2000 = 540 sheep, so a total of 540 sheep must leave farm A.
Farm B loses 390 sheep (2610 – 3000 = -390), so 390 sheep must go over the farm B.
If 390 of the sheep from farm A go to farm B, then farm B has kept its numbers, but there are still 540 – 390 = 150 extra sheep at farm A, so these must be sold.

Answer D.