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2014 Trial Examination

STUDENT NUMBER

Figures										Letter
Words										

FURTHER MATHEMATICS

Units 3 & 4 - Written Examination 2

Reading Time:15 minutes
Writing Time: 1 Hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 28 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This paper consists of a core and six modules. Students are to answer all questions in the core and then select **three** modules and answer all questions within those modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve e.g. π, surds, fractions.
Diagrams are not drawn to scale unless specified otherwise.

	Page
Core	4
Module	
Module 1: Number patterns.....	9
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Module 3: Graphs and relations.....	15
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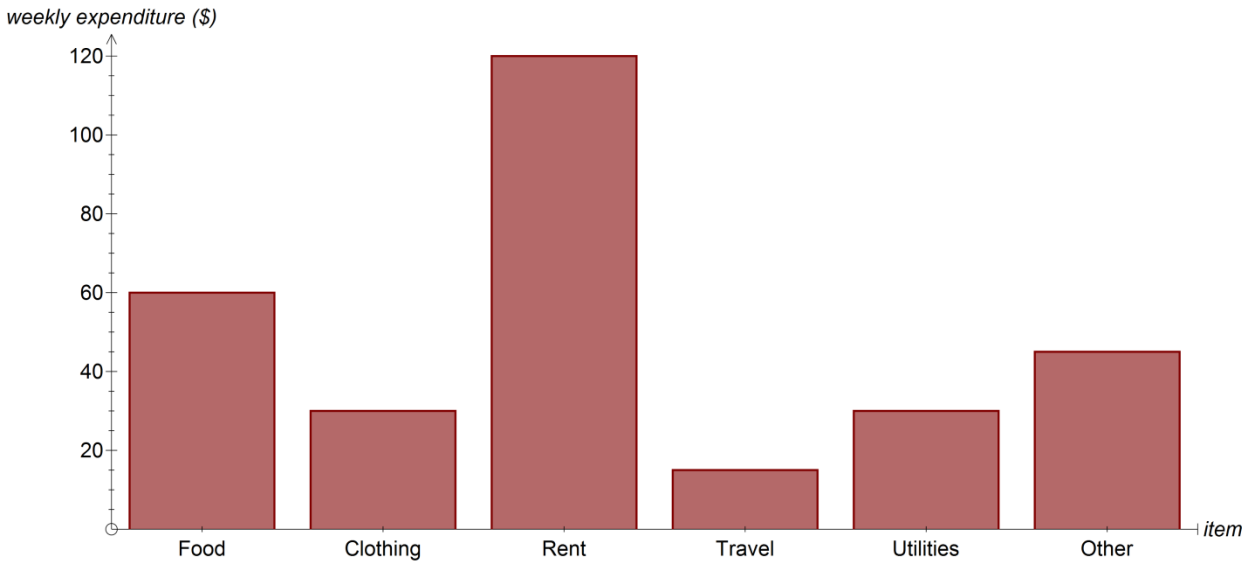
Module 5: Networks and decision mathematics.....	22
Module 6: Matrices.....	26

TURN OVER

Core

Question 1(7 marks)

The following bar graph shows the weekly expenditure of a household on different items.



a. How much is spent per week on rent and utilities? 1 mark

_____

b. What percentage of total expenditure is spent on food, travel and clothing? Write your answer to the nearest percentage. 2 marks

_____

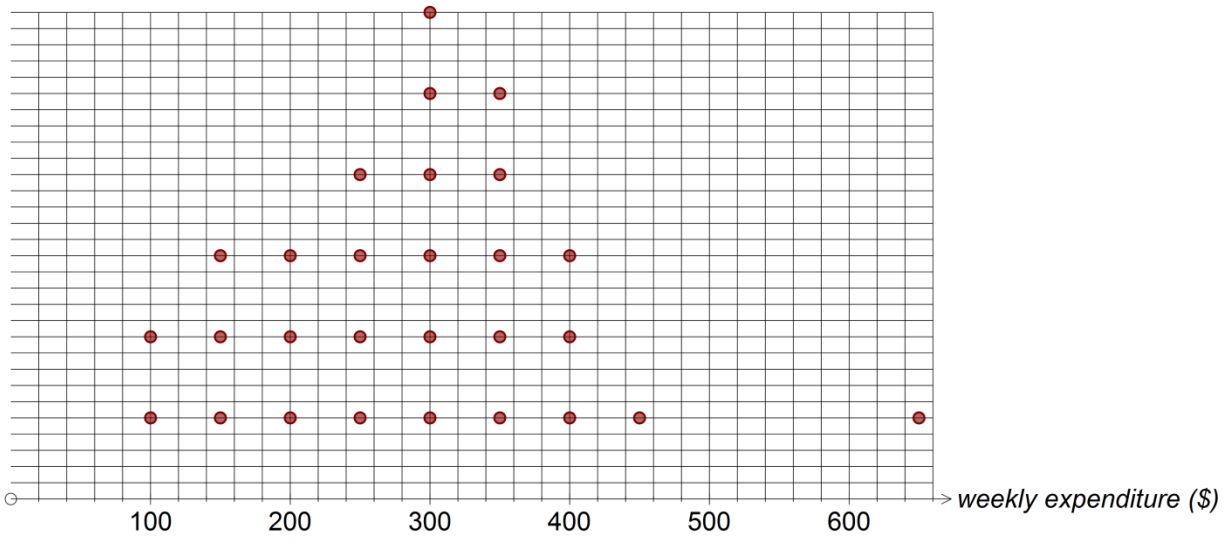
_____

_____

_____

Core - continued

The weekly expenditure of 28 households is displayed in the following dot plot.



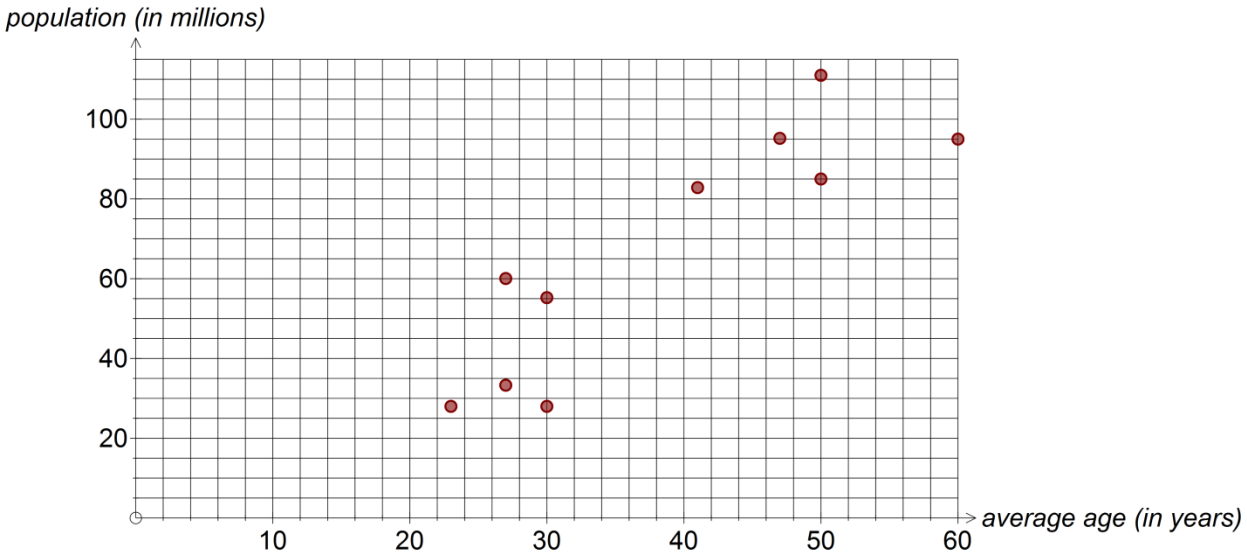
c. Find the median and the range of the above data. 2 marks

d. Use mathematical calculations to show that weekly expenditure of $650 is an outlier for this group of 28 households. 2 marks

Core - continued
TURN OVER

Question 2 (4 marks)

The following scatterplot shows the association between a country's population (in millions) and the average age (in years) of a country's population.



It is also known that

- mean of average age is 38.5 and standard deviation of average age is 12.7126
- mean of population is 67.4 and standard deviation of population is 30.6471
- $r = 0.8834$

a. Find the least squares regression line to describe the relationship population and average age correct to 4 decimal places. 2 marks

_____

_____

_____

_____

Core - continued

b. The population of a country with an average age of 30 years is 22 million. Find the residual value of this country's population using the regression line found in part **a.** Write the residual value to the nearest whole number. 2 marks

_____

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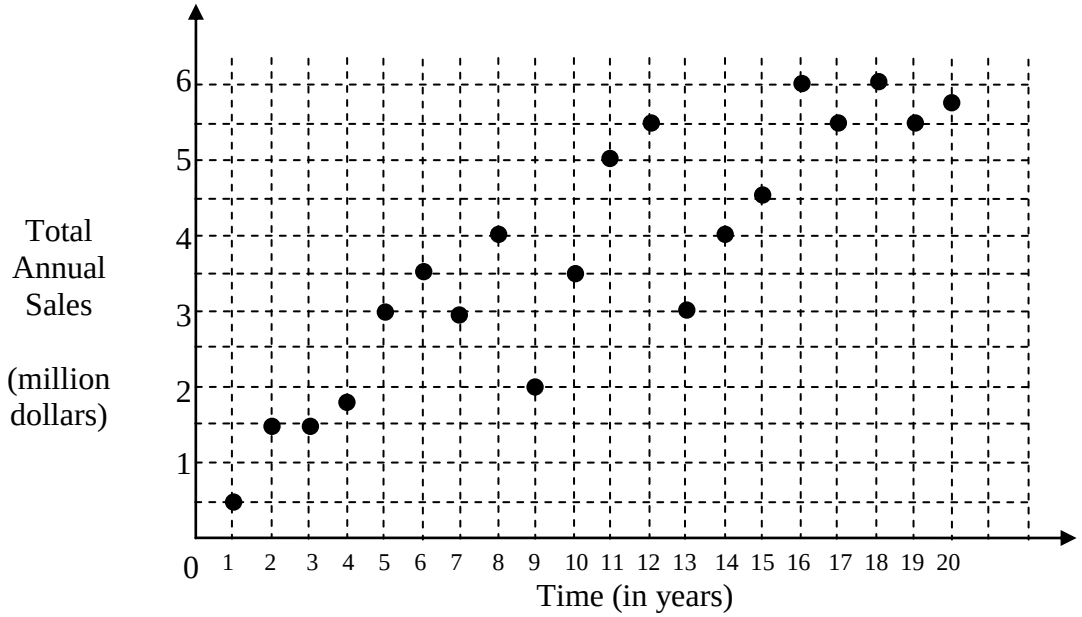
_____

Question 3 (4 marks)

The time series plot shown below describes the total annual sales of a manufacturer over a 20 year period



A three median regression line method is used to fit the regression line above.

a. Plot the three medians on the graph above. 2 marks

Core - Question 3 - continued
TURN OVER

b. Find the three-median regression line, written to 4 decimal places. 1 mark

_____

_____

_____

_____

c. Interpret the slope of the three median regression line in terms of the Annual Sales and Time. 1 mark

_____

_____

_____

_____

Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1 (4 marks)

The population of species *A* on an island is increasing each year by a constant number. Its population in the first year is 12 000 and in its third year is 13400.

a. What is the population of species *A* in the second year? 1 mark

_____

_____

b. If the population growth continues to follow the same pattern, after how many years will the population of species *A* become 17000? 2 marks

_____

_____

c. Write down the expression to find the population of species *A* after *n* years, in the form $t_n = an + b$. 1 mark

_____

Question 2 (7 marks)

Another species *B* on the same island is decreasing each year at the rate of 7% per annum.

a. If the population of species *B* in its second year is 18600, find its population in the first year. 2 marks

_____

_____

MODULE 1 - Question 2 - continued
TURN OVER

b. The population decay of species B can be written in the form $P_n = aP_{n-1} + b,\ where\ P_0 = c$, where *a*, *b* and *c* are real numbers. 3 marks
Write down the values of *a*, *b* and *c*

_____

_____

c. When will the population of species *A* first be more than the population of species *B*? 2 marks

_____

_____

_____

_____

Question 3 (4 marks)

For another species *C* on the same island a scientist uses the following growth model:

$p_n = 1.13p_{n-1} - 400,\ where\ p_0 = 14000$

a. What does the number 1.13 in the above model mean in terms of the population of species *C*? 1 mark

_____

b. Find the population of species *C* after 9 years, according to this model. 2 marks

_____

_____

MODULE 1 - Question 3 - continued

Instead of the above model for growth of species *C*, the scientist decides to use the following model to measure growth:

$$t_{n+2} = t_{n+1} + t_n, \qquad where\ t_1 = 14000\ \ and\ t_2 = 14200$$

c. Find the population of species *C* after 3 years, according to this model. 1 mark

_____

_____

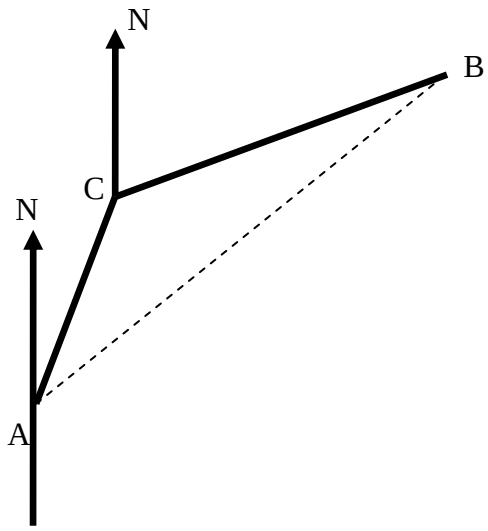
Total 15 marks

END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1 (8 marks)
There are only two ways to go around a bushland from point A to point B.

A hiker walks from point *A* on a bearing 010°T for 5 km and then on a bearing of 075°T for 7km to reach point *B*. The diagram showing his journey is drawn below.



a. Find angle ACB. 1 mark

_____

_____

b. Find the length AB, correct to one decimal place. 1 mark

_____

_____

c. Find the bearing, to the nearest degree, of B from the start point A. 2 marks

_____

_____

MODULE 2 - Question 1 - continued

A second hiker starts from the point A and travels 4km on a bearing of 080°T to a point P, and then travels in a straight line to point B.

d. Find the total distance travelled by the second hiker, correct to one decimal place. 2 marks

_____

_____

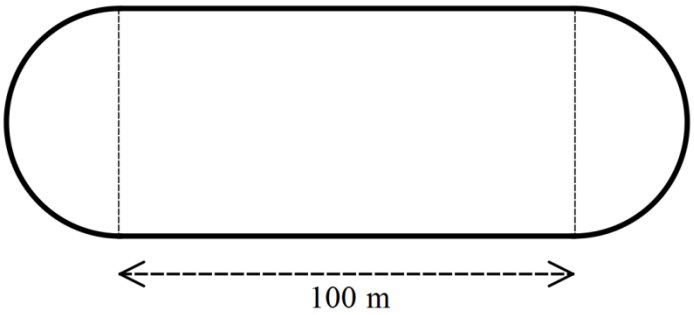
e. Find the area of the bushland, to the nearest km^2. 2 marks

_____

_____

Question 2 (4 marks)

After the hikers finish their journey at point B, they take a break and relax in a swimming pool. The pool is in the shape of a rectangle with semi-circular ends on the sides.



The perimeter of the pool is 420m.

a. Find the radius of the semi-circular ends, giving your answer correct to 3 decimal places.
1 mark

_____

_____

MODULE 2 - Question 2 - continued
TURN OVER

b. Find the area, correct to one decimal place, of the swimming pool. 2 marks

_____

_____

_____

c. If the depth of the water in the pool is 150cm, find the volume of water in the pool to the nearest litre. 1 mark

_____

_____

_____

Question 3 (3 marks)
Each of the hikers has a similar tent. The first hiker's tent has a base length of half a metre and the second hiker's tent has a base length of 0.75 m.

a. Find the ratio of the heights of the tents. 1 mark

_____

_____

b. The smaller tent covers an area of 15 m^2, Find the area of the larger tent to 2 decimal places. 2 marks

_____

_____

_____

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1 (6 marks)
Mike has been given two quotes for internet connection by two internet providers *A* and *B*. Provider *A* offers a monthly rental rate of $52 plus 15 cents for each hour, or part thereof, of internet usage.

a. Write down a cost function, $C_A(n)$, to calculate the cost per month after using *n* hours of internet. 1 mark

_____

_____

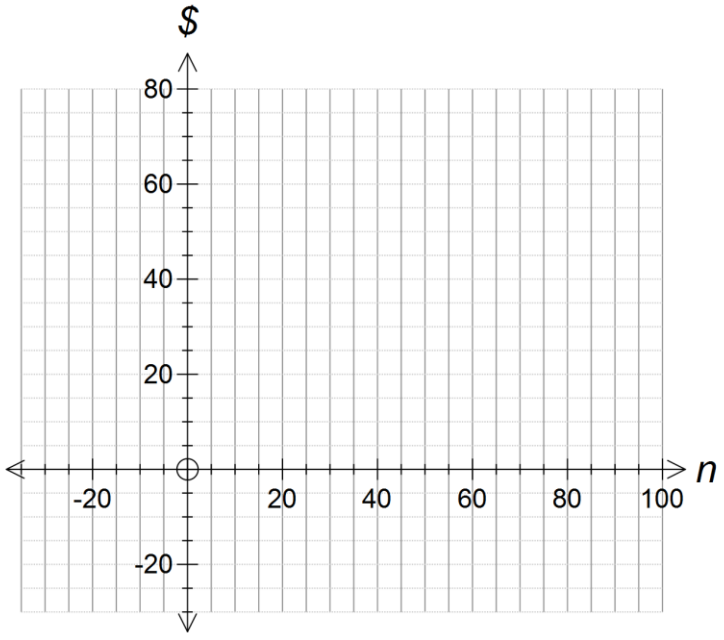
Provider *B* offers a fixed fee of $64 per month for unlimited use of internet.

b. Write an equation for the cost, $C_B(n)$, in dollars. 1 mark

_____

_____

c. Sketch the cost function graphs of both quotes for the first 4 days on the axes below.
2 marks



d. If Mike goes with the first quote, how many hours can he work on the internet so that it is cheaper than the second quote? 2 marks

_____

_____

Question 2 (4 marks)
Mike decides to go with the second internet provider as his job requires long hours of work from home. He spends some time in his office and at other times works from home.

He works for a maximum of 49 hours per week. The following constraints define his preference for splitting working hours between office and home.

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 49, \quad y \geq 2x$$

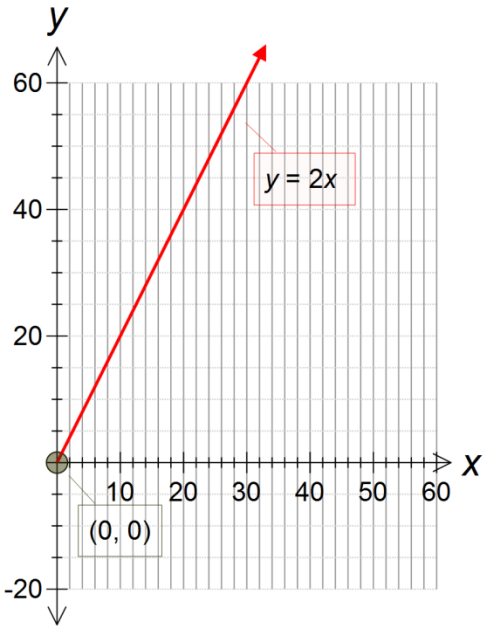
where x denotes the number of hours Mike works in the office and y denotes the number of hours he works at home.

a. Explain the constraint $y \geq 2x$ in relation to his working hours. 1 mark

_____

_____

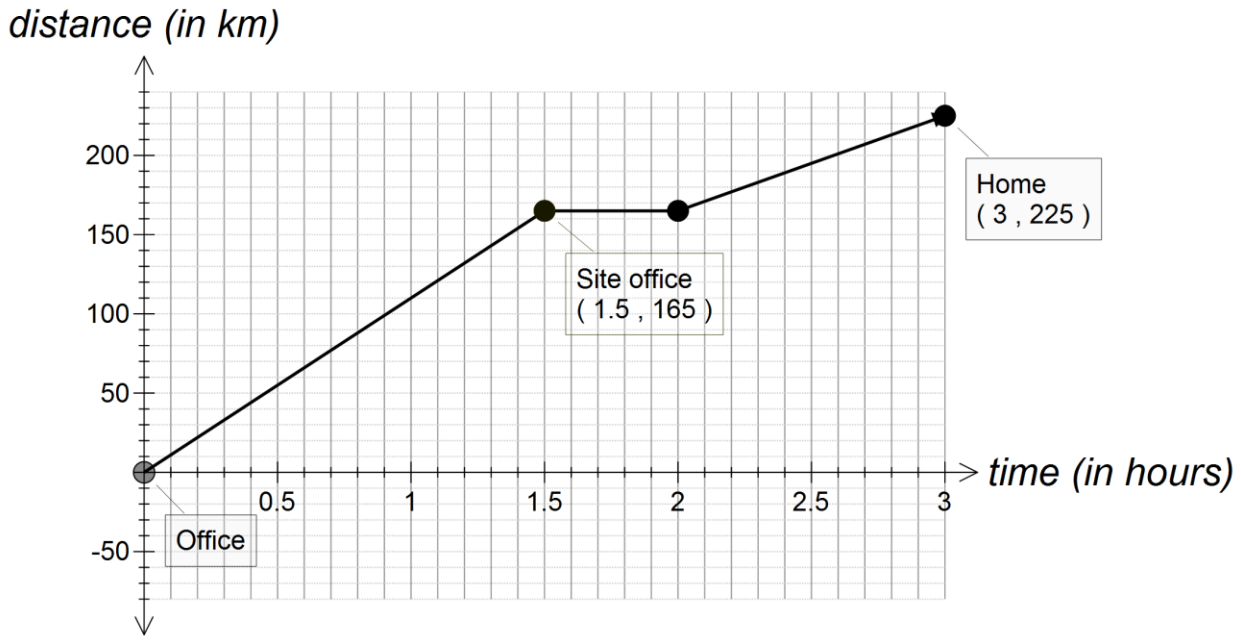
The following graph shows the line $y = 2x$.



MODULE 3 - Question 2 - continued

b. Sketch the other constraints on the axes above and shade the feasible region. Label all corner points of the feasible region. 3 marks

Question 3 (5 marks)
Mike travels from his office to the work site and then to his home as shown in the graph below.



a. At what constant speed does Mike travel from his office to the work site? 1 mark

b. For how much time did Mike stay at the site office? 1 mark

MODULE 3 - Question 3 - continued
TURN OVER

c. Find the average speed of the car over the three hours. 1 mark

_____

_____

d. Write down the function that defines Mike's journey for the three hours. 2 marks

_____

_____

_____

_____

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Mike needs to purchase a new computer for his site office and gets a quote of $4575, including 10% GST amount.

Question 1 (5 marks)

a. What amount of GST is Mike paying on this purchase? 1 mark

_____

_____

The retailer offers a 12% discount (including the GST).

b. How much will Mike have to pay after the discount? 2 marks

_____

_____

The computer depreciates using reduced balance depreciation at a rate of 18% per year.

c. When will the computer be worth less than half its discounted purchase price? 2 marks

_____

_____

_____

Question 2 (3 marks)

Mike also purchases a printer for $730 that depreciates at the rate of $0.03 per A4 print. The printer makes 6800 prints in its first 2 years of purchase.

a. What is its book value at the end of two years? 1 mark

_____

_____

b. If the scrap value of the printer is $100, how many prints can the printer make in its useful life? 2 marks

_____

_____

_____

Question 3 (7 marks)

Mike's company decides to purchase a larger office space. They find a new space selling for $750 000, but the business will need to take out a loan of 78% of the total value.

a. How much money does the company need to borrow? 1 mark

_____

_____

Mike investigates taking out a reducing balance loan for a term of 20 years, with monthly repayments and an interest rate of 8.2% calculated and charged monthly.

b. What would the monthly repayments be on this loan? 2 marks

_____

_____

MODULE 4 - Question 3 - continued

c. How much would still be owed on the loan after 5 years? 2 marks

_____

_____

After 5 years, the company decides to increase their monthly payment to $7000.

d. How many more years will it take to pay off the loan? Give your answer to the nearest year. 2 marks

_____

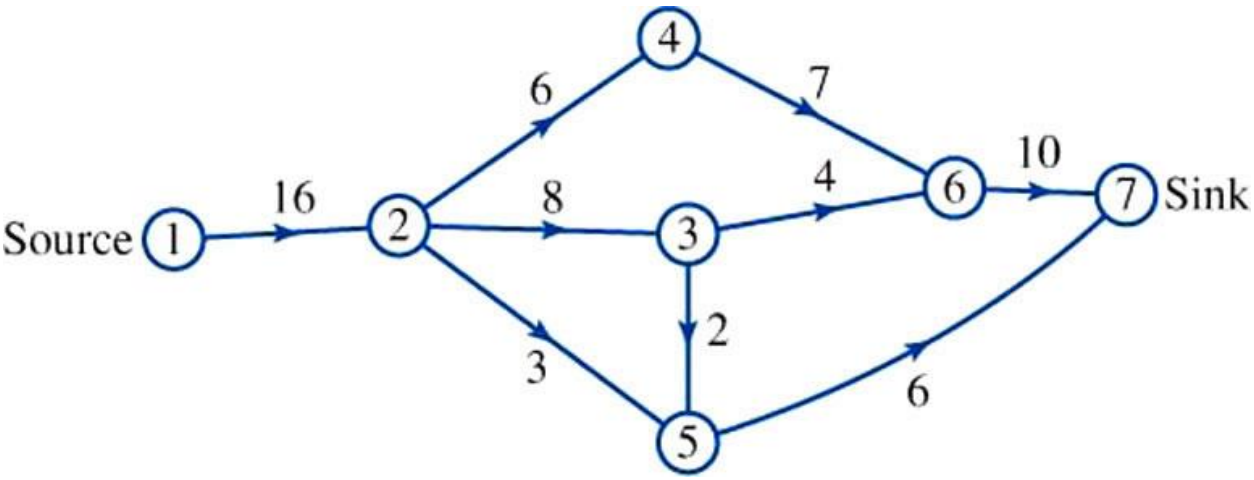
_____

Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1 (8 marks)
The network below shows the capacity (distances in kilometres) of the road network connecting small towns.



a. What is the maximum capacity of the network? Show the minimum cut. 2 marks

_____

_____

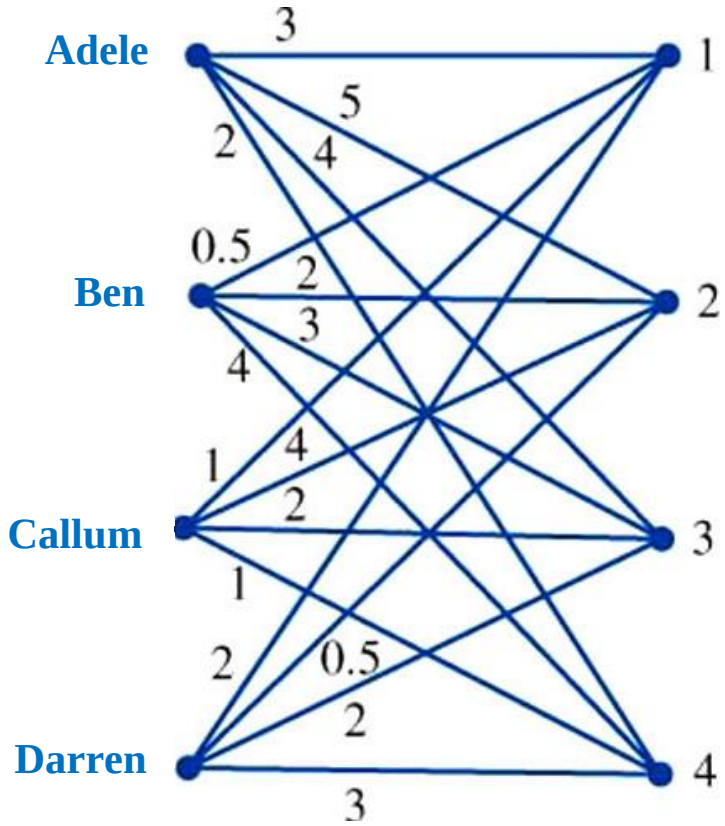
Due to cutbacks, the edge between vertices 2 and 3 is removed from the network.

b. What is the new maximum capacity? 1 mark

_____

MODULE 5 - Question 1 - continued

The bipartite graph below shows four bus drivers and their distances, in kilometres, from four points 1, 2, 3 and 4.



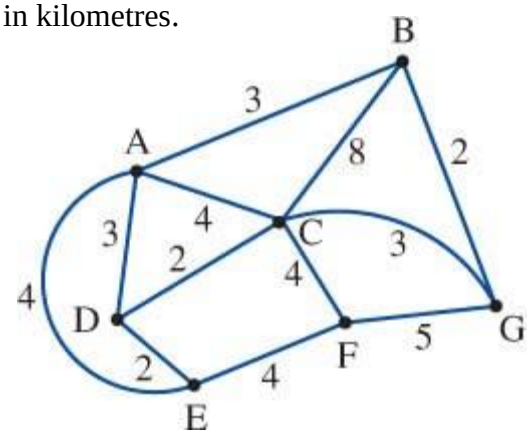
c. Draw an allocation matrix depicting this information. 2 marks

MODULE 5 - Question 1 - continued
TURN OVER

d. Allocate each of the bus drivers to one route so that the total distance travelled to begin the routes will be minimized. 2 marks

e. What is the longest distance any of the four need to travel home at the end of their route? 1 mark

Question 2 (3 marks)
Darren's mail route has him visiting 7 major destinations, represented by the vertices in the graph below. Distances are in kilometres.



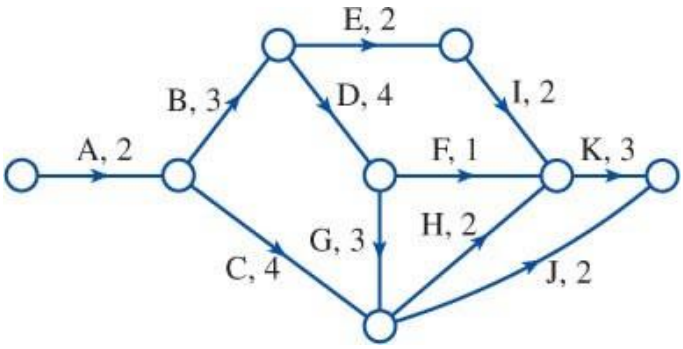
a. Find the shortest circuit, starting at G that has Darren visiting each destination exactly once. What is the length of this circuit? 2 marks

MODULE 5 - Question 2 - continued

b. What is the name of this type of circuit? 1 mark

_____

Question 3 (4 marks)
The network below shows the activities required before buses can leave the bus depot for their respective destinations. Times are in minutes.



a. List the immediate predecessors for activity H. 1 mark

_____

b. Find the earliest start times for activities E and H. 2 marks

_____

_____

c. The critical path is A-B-D-G-H-K. What is the minimum amount of time processing the mail can take? 1 mark

_____

_____

Total 15 marks

END OF MODULE 5
TURN OVER

Module 6: Matrices

Question 1 (5 marks)
The following system of equations can be used to estimate the number of three species *A*, *B* and *C* on an island.

$$2x + y + 7z = 9556$$
$$3x + y + 4z = 5899$$
$$5x + 2y + z = 3155$$

a. Write this system of simultaneous linear equations in matrix form. 2 marks

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

b. Write down the inverse matrix that can be used to solve this system of simultaneous linear equations. 1 mark

c. Solve the system of simultaneous linear equations and hence estimate the number of the three species on the island. 2 marks

MODULE 6 - continued

Question 2 (6 marks)
Each year migratory birds nest at one of three sites, *P*, *Q* or *R*, on the island. While the birds generally attempt to nest at the same site each year, this does not always happen.
The following transition matrix describes the movement of birds from one year to the next year.

$$T = \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0.85 & 0.15 & 0.05 \\ 0.05 & 0.8 & 0.05 \\ 0.1 & 0.05 & 0.9 \end{bmatrix} \end{array}$$

a. Explain what the element in the third row and second column represents. 2 marks

In 2012, 4000 birds nested at Site *P*, 2500 nested at Site *Q* and 3300 nested at Site *R*

b. Using the matrix *T* and above information in a matrix, write down an equation that can be used to determine the number of birds at each site in 2014. 1 mark

c. Determine the number of birds at each site in 2014 as a 3 × 1 matrix. 1 mark

d. If this pattern continues, find the number of birds at each site in the long run. 2 marks

Question 3 (4 marks)
There are four types of pets on the island that families can keep: cats, dogs, parrots and rabbits. The matrix F shows the proportion of families owning these pets.

$$F = \begin{matrix} \textit{cats} & \textit{dogs} & \textit{parrots} & \textit{rabbits} \end{matrix} \begin{bmatrix} 0.015 & 0.01 & 0.005 & 0.018 \end{bmatrix}$$

The number of families at each of the three sites is given by the following matrix

$$R = \begin{matrix} P \\ Q \\ R \end{matrix} \begin{bmatrix} 10000 \\ 6500 \\ 9750 \end{bmatrix}$$

a. Which of the matrix products RF or FR is defined? Explain why. 2 marks

b. Find the product which is defined above and explain what that represents. 2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK