FURTHER MATHEMATICS

Units 3&4 - Written Examination 2

2014 Trial Examination

SOLUTIONS

Core

Question 1

a. $120 + 30 = \$150$

1 mark

b. $\dfrac{60+15+30}{300} \times 100 = 35\%$

2 marks

c. $Median = \$300$
$Range = 650 - 100 = \$550$

2 marks

d. $Q_3 + 1.5 \times IQR = 350 + 1.5 \times (350 - 200) = 575$
Since 650 is more than 575, it is an outlier.

2 marks

Question 2

a.

$$m = 0.8834 \times \frac{30.6471}{12.7126} = 2.1297$$

$$c = 67.4 - 2.1297 \times 38.5 = -14.59234$$

$$Population = 2.1297 \times average\ \ age - 14.5935$$

2 marks

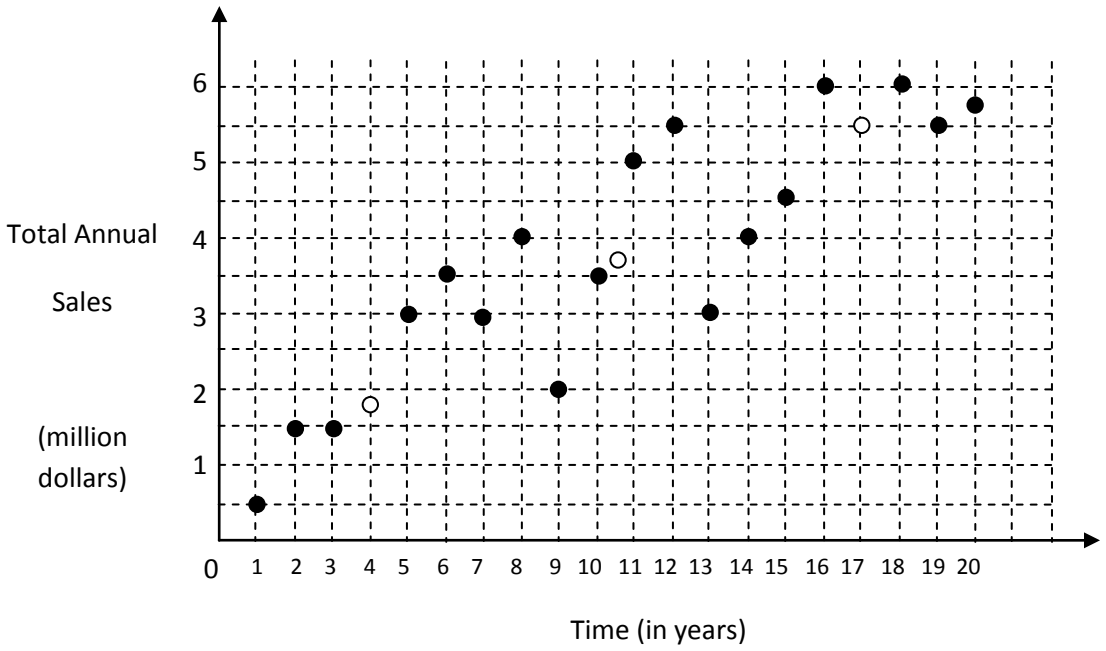
b. $\text{Residual} = 22 - (2.1297 \times 30 - 14.5935) = -27.2975$

Residual = -27 297 500

2 marks

Question 3

a.



Medians marked as open circles.

2 marks

b. CAS inputs of medians Annual Sales = 0.6378 + 0.2885 × Time

1 mark

c. The Annual Sales increases by $288500 each year.

1 mark

Module 1: Number patterns

Question 1

a. $13400 = 12000 + (3-1) \times d \Rightarrow d = 700$

$t_2 = 12\ 700$

1 mark

b.

$17000 = 12000 + (n-1) \times 700 \Rightarrow n \approx 8.14286$

The population will become 17000 during the 9^{th} year.

2 marks

c. $t_n = 12000 + 700(n-1) = 700n + 11300$

1 mark

Question 2

a. $18600 = a \times 0.93 \Rightarrow a = 20000$

2 marks

b. $a = 0.93,\ b = 0,\ c = 20000$

3 marks

c. $700n + 11300 = 20000 \times 0.93^{n-1} \Rightarrow n \approx 5.09$

The population of species *A* will be more than species *B* during the 6^{th} year.

2 marks

Question 3

a. Species *C* increases at the rate of 13% each year.

1 mark

b. Generate list on CAS.

Read the 10^{th} term = 35890

2 marks

c. 14000, 14200, 28200, 42400

42400 after 3 years

1 mark

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a. $10° + 105° = 115°$

1 mark

b. $AB = \sqrt{25 + 49 - 70 \times \cos(115°)} = 10.2 \text{ km}$

1 mark

c. $\frac{7}{\sin(\theta)} = \frac{10.2}{\sin(115°)} \rightarrow \theta = 38°28'$ where $\theta = \angle CAB$

$Bearing = 38°28' + 10° = 048°\text{ T}$

2 marks

d. $PB = \sqrt{10.2^2 + 4^2 - 8 \times 10.2 \times \cos(32°)} = 7.13017 \text{ km}$

$Total\ distance = 4 + 7.13017 = 11.1 \text{ km}$

2 marks

e. $Area = \frac{1}{2} \times 5 \times 7 \times \sin(115°) + \sqrt{10.7(10.7 - 10.2)(10.7 - 4)(10.7 - 7.13)}$

$Area = 15.86 + 11.31 = 27 \text{ km}^2$

2 marks

Question 2

a. $200 + 2\pi r = 420 \Rightarrow r = \dfrac{110}{\pi} = 35.014 \text{ m}$

1 mark

b. $Area = 100 \times \dfrac{220}{\pi} + \pi \times \left(\dfrac{110}{\pi}\right)^2 = 10854.4 \text{ m}^2$

2 marks

c. $Volume = 10854.4 \times 0.15 \times 1000 = 1628160 \text{ L}$

1 mark

Question 3

a. $\left(\frac{0.75}{0.5}\right)=\frac{3}{2} \quad \left(\text{or} \quad \frac{2}{3}\right)$

1 mark

b. $Area=\frac{9}{4}\times 15=33.75\ m^2$

2 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

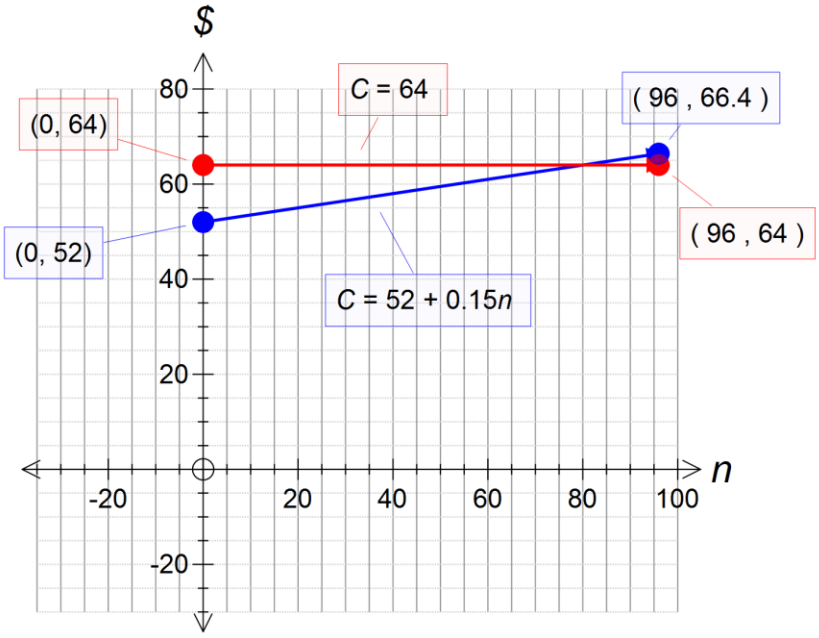
a. $C_A(n) = 52 + 0.15n$

1 mark

b. $C_B(n) = 64$

1 mark

c.



2 marks

d.

$52 + 0.15n = 64 \Rightarrow n = 80$

< 80

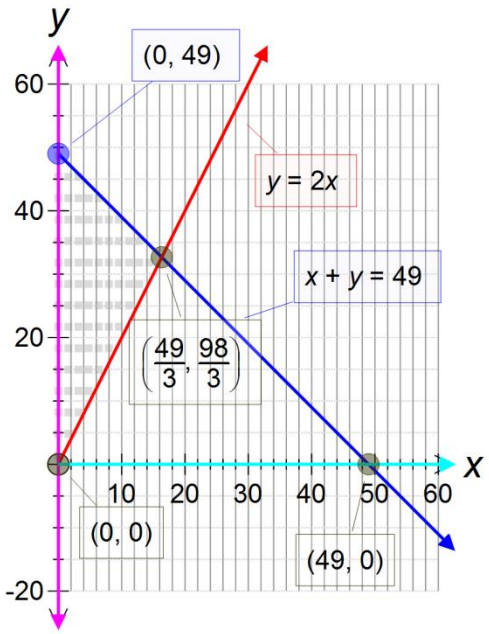
2 marks

Question 2

a. Mike prefers to work at least twice the number of hours in office as at home.

1 mark

b.



1 mark for the line, 1 mark for shading, 1 mark for the intersection point.

Question 3

a. 110 km/hr

1 mark

b. Half-an hour or 30 minutes

1 mark

c. 75 km/hr

1 mark

d. $D(t) = \begin{cases} 110t\,, & 0 \le t \le 1.5 \\ 165\,, & 1.5 < t \le 2 \\ 60t + 45\,, & 2 < t \le 3 \end{cases}$

2 marks
Total 15 marks

Module 4: Business-related mathematics

Question 1

a. $4575 - \dfrac{4575}{1.1} = \$415.91$

1 mark

b. $0.88 \times 4575 = \$4026$

2 marks

c. $2013 = 4026\left(1 - \dfrac{18}{100}\right)^n \Rightarrow n = 3.49$

After 4 years.

2 marks

Question 2

a. $730 - 6800 \times 0.03 = \$526$

1 mark

b. $100 = 730 - 0.03n \Rightarrow n = 21000$ copies

2 marks

Question 3

a. $\dfrac{78}{100} \times 750000 = \$585000$

1 mark

b. Use TVM Solver: \$4966.24

2 marks

c. Use TVM solver to get \$513447.73

2 marks

d. Use TVM solver to get 9 years.

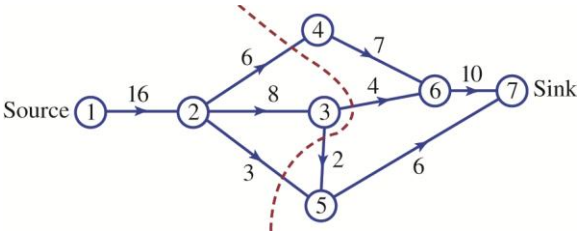
2 marks

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

a.



Maximum capacity is given by the minimum cut.

Maximum capacity = 6 + 4 + 3 + 2 = 15 km

2 marks

b. The new maximum capacity = 6 + 3 = 9 km

1 mark

c.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & 5 & 4 & 2 \\ 0.5 & 2 & 3 & 4 \\ 1 & 4 & 2 & 1 \\ 2 & 0.5 & 2 & 3 \end{bmatrix} \end{array}$$

2 marks

d. The optimal allocation is:

Adel- route 4; Ben- route 1; Callum- route 3; Darren- route 2

2 marks

e. Adel and Callum both need to travel 2 km

1 mark

Question 2

a. GBACDEFG
Length = 22km

2 marks

b. Hamiltonian Circuit

1 mark

Question 3

a. Activity H is immediately preceded by activities C and G

1 mark

b. E follows activities A and B. Earliest start time for E = 2 + 3 = 5 hours.

H follows ABDG and AC. Its earliest start time is the longer of these two paths.

Length ABDG = 2 + 3 + 4 + 3 = 12 hours.

Length AC = 2 + 4 = 6 hours.

Earliest start time for H is 12 hours

2 marks

c. The length of the critical path is 2 + 3 + 4 + 3 + 2 + 3 = 17 hours

1 mark

Total 15 marks

Module 6: Matrices

Question 1

a.

$$\begin{bmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9556 \\ 5899 \\ 3155 \end{bmatrix}$$

2 marks

b. $$\begin{bmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -0.7 & 1.3 & -0.3 \\ 1.7 & -3.3 & 1.3 \\ 0.1 & 0.1 & -0.1 \end{bmatrix}$$

1 mark

c. $$\begin{bmatrix} -0.7 & 1.3 & -0.3 \\ 1.7 & -3.3 & 1.3 \\ 0.1 & 0.1 & -0.1 \end{bmatrix} \times \begin{bmatrix} 9556 \\ 5899 \\ 3155 \end{bmatrix} = \begin{bmatrix} 33 \\ 880 \\ 1230 \end{bmatrix}$$

Species A- 33, Species B- 880 and Species C- 1230

2 marks

Question 2

a. 5% of birds who nest in site *Q* migrate to site *R* the following year.

2 marks

b.

$$S_2 = T^2 \times S_0$$

$$S_2 = \begin{bmatrix} 0.85 & 0.15 & 0.05 \\ 0.05 & 0.8 & 0.05 \\ 0.1 & 0.05 & 0.9 \end{bmatrix}^2 \times \begin{bmatrix} 4000 \\ 2500 \\ 3300 \end{bmatrix}$$

1 mark

c.

$$S_2 = \begin{bmatrix} 0.85 & 0.15 & 0.05 \\ 0.05 & 0.8 & 0.05 \\ 0.1 & 0.05 & 0.9 \end{bmatrix}^2 \times \begin{bmatrix} 4000 \\ 2500 \\ 3300 \end{bmatrix} = \begin{bmatrix} 3878.5 \\ 2263.75 \\ 3657.75 \end{bmatrix}$$

1 mark

d.

$$S_{100} = \begin{bmatrix} 3430 \\ 1960 \\ 4410 \end{bmatrix} = S_{101}$$

Species A- 3430, Species B -1960 and Species C- 4410

2 marks

Question 3

a. *RF* is defined as the number of columns in *R* equals the number of rows in *F*.

2 marks

b.

$$RF = \begin{bmatrix} 150 & 100 & 50 & 180 \\ 97.5 & 65 & 32.5 & 117 \\ 146.25 & 97.5 & 48.75 & 175.5 \end{bmatrix}$$

This matrix represents the number of families who have the four different kinds of pets at each of the sites.

2 marks

Total 15 marks