

insight™

YEAR 12 *Trial Exam Paper*

2014

FURTHER MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core			
	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	2	2	15
Module			
	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	6	3	45
		Total	60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference book, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring blank sheets of paper and/or white out liquid/tape into the examination.

Materials provided

- The question book of 43 pages.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Remove the formula sheet during reading time.
- Write your **name** in the box provided above on this page.
- All responses must be written in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

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Instructions

This examination contains a core and six modules. Students should answer **all** questions in the core and then choose **three** modules and answer **all** questions within the modules chosen.

You don't need to give numerical answers as decimals unless instructed to do so. Other forms may involve, for example, π , surds or fractions.

Diagrams are not to scale unless specified otherwise.

Section	Page
Core	3
Module	
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Module 6: Matrices.....	37

b. Complete the table below.

1 mark

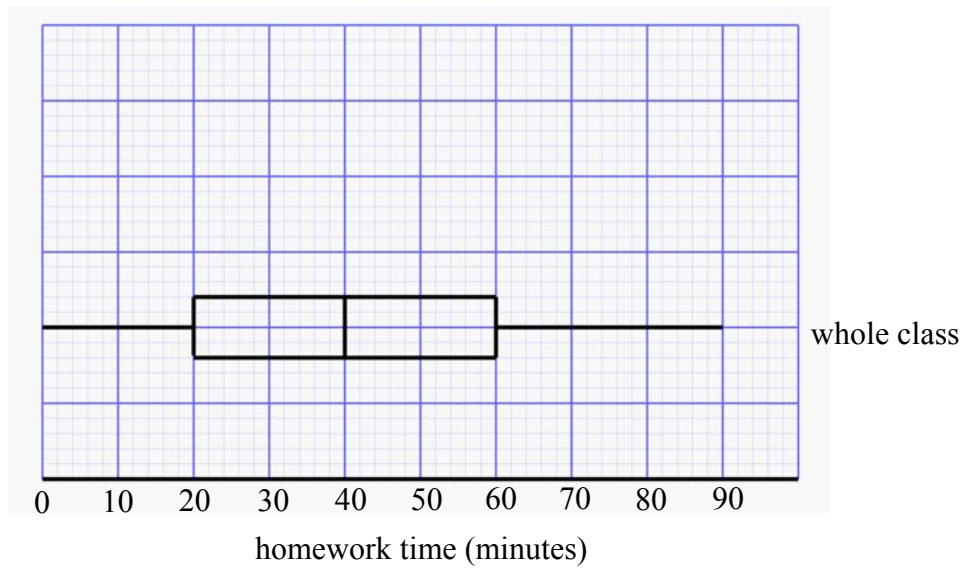
Summary statistic	Value
minimum	
Q_1	
median	
Q_3	
maximum	
mean	
standard deviation	

c. Calculate the upper and lower fences and state any outliers if they exist.

2 marks

- d. The boxplot of homework times for a class of Year 12 students on that particular Wednesday is shown below. Using the same scale, draw the boxplot of the homework times for the sample of 20 students.

2 marks



- e. State two differences between the distributions of the sample of 20 and the whole class data.

2 marks

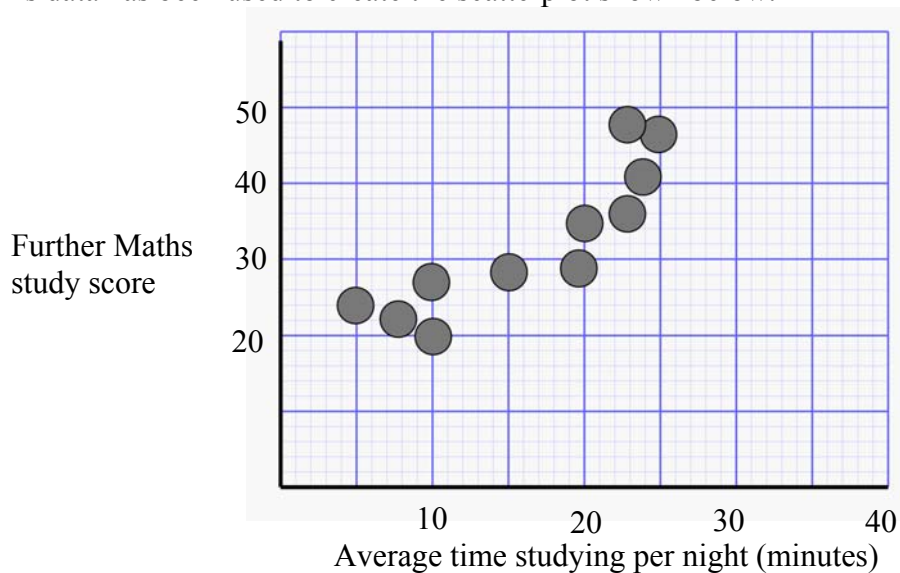
Core – continued
TURN OVER

Question 2 (7 marks)

The table below shows the Further Maths study scores for 12 students and the average number of minutes that they studied for each night during the year.

Average time studying per night (minutes)	Further Maths study score
5	24
8	22
10	20
10	27
15	28
17	32
19	29
20	35
23	36
24	41
25	46
23	48

This data has been used to create the scatterplot shown below.



- a. The student who scored 32 is missing from the scatterplot. Draw this point on the scatterplot with an X.

1 mark

- b. Find the value of the coefficient of determination (r^2). Express your answer correct to 3 decimal places. Interpret this value in terms of the variables.

2 marks

- c. The scatterplot of study score versus average time spent studying appears non-linear. To linearise the scatterplot, a log y transformation is performed. Find the value of the coefficient of determination (r^2) for the log y transformation.

1 mark

- d. The transformed equation is found to be
 $\log(\text{study score}) = 0.016(\text{average study time}) + 1.23$

Use this equation to predict the Further Maths study score of a student who spends an average of 30 minutes studying each night and comment on the validity of this prediction.

2 marks

- e. Which 2 other types of transformations would also be likely to linearise the data?

1 mark

**End of Core
TURN OVER**

Module 1: Number patterns

Question 1 (6 marks)

Steve is training with the aim of making the World Athletics Junior Games in Oregon. On Monday 1 February, he ran 10 laps of a 400 m track. On Friday 5 February, he ran 22 laps.

- a. If Steve increased the distance he ran by the same amount each day, how many laps did he run on Wednesday 3 February?

1 mark

- b. If Steve continues to increase the number of laps he runs by the same amount and trains every day of the week in February, on what date will the sum of the total number of laps he has run from the start of training exceed 250 laps?

2 marks

Sally is also training for the Junior Games in Oregon. She plans to run a total of 480 laps during her training regime. She plans to run 30 laps each day beginning on 1 February.

- c. How many laps will she have left to run after 5 days?

1 mark

- d. On what day will Sally complete her training regime?

1 mark

- e. The number of laps Sally has left to run after 8 days is T_8 , where $T_8 = 480 + (8 \times m)$.
What is the value of m ?

1 mark

Module 1 – continued
TURN OVER

Question 2 (5 marks)

Paul is also training to qualify for the Junior Games in Oregon and is hoping to make the team as a shot putter. His training involves repetitious weight lifting. He does 100 repetitions on 1 February, 120 repetitions on 2 February and 144 repetitions on 3 February. The number of repetitions Paul does each day follows a geometric sequence.

- a. Show that the common ratio, r , is 1.2. 1 mark

- b. How many repetitions does Paul do on 7 February? 1 mark

- c. Write an expression for the number of repetitions Paul does on the n th day of February. 1 mark

- d. If Paul continues to increase his repetitions according to this geometric sequence, how many more repetitions will he do in the second week of February compared with the first week of February? 2 marks

Question 3 (4 marks)

Sebastian has also been training hard. His times for the 800 m are coming down. He is able to predict his next time for the 800 m by calculating the mean of his previous 2 times and then subtracting 2 seconds.

- a. Determine a difference equation for his time in $n + 2$ races, T_{n+2} .

1 mark

- b. Sebastian completed his first 800 m in 2 minutes and 20 seconds and his second 800 m in 2 minutes and 16 seconds. Calculate his third 800 m time, and give your answer to the nearest second.

1 mark

- c. To qualify for the team going to the Junior Games in Oregon, Sebastian needs to run the 800 m in 2 minutes and 9 seconds or less by 8 February. Assuming he ran 2 minutes and 20 seconds on 1 February and then 2 minutes and 16 seconds on 2 February and his times continue to follow the difference equation above, calculate whether or not he is on track to qualify.

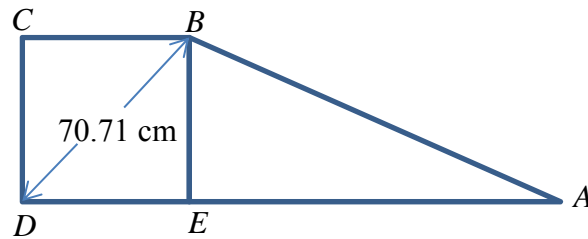
2 marks

End of Module 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1 (3 marks)

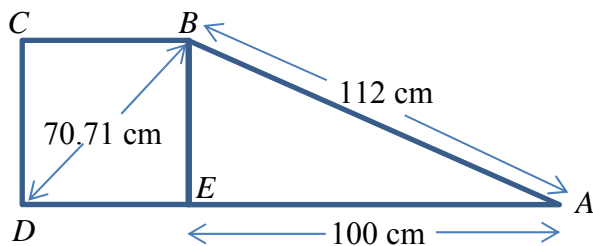
Annie has joined a gym and undertaken a cross-training program to get fit. The diagram below is a cross-sectional diagram of a sloping ramp, AB , with a horizontal platform, CB , at the end of the ramp. The base, AD , is on flat ground.



Annie’s first exercise (step-ups) requires her to step from ground level, D , up on to the platform, CB , and down again. Annie will repeat this exercise 100 times on each leg.

- a. Calculate the height of the platform above the ground, given that $CBDE$ is a square and the length of the diagonal, DB , is 70.71 cm. Give your answer correct to the nearest centimetre.

2 marks

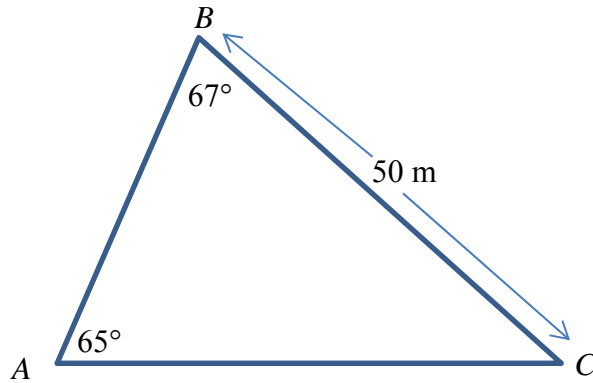


- b. Find the angle that the ramp, AB , makes with the ground, AE , given that the length of AB is 112 cm and the length of AE is 100 cm. Give your answer correct to the nearest degree.

1 mark

Question 2 (8 marks)

Another of Annie's activities requires her to run-walk-run between 3 points that make up the vertices of a triangle. She runs from A to B , walks from B to C (50 m) and then runs from C to A . She then rests for 30 seconds and repeats the process. She does this routine 20 times.



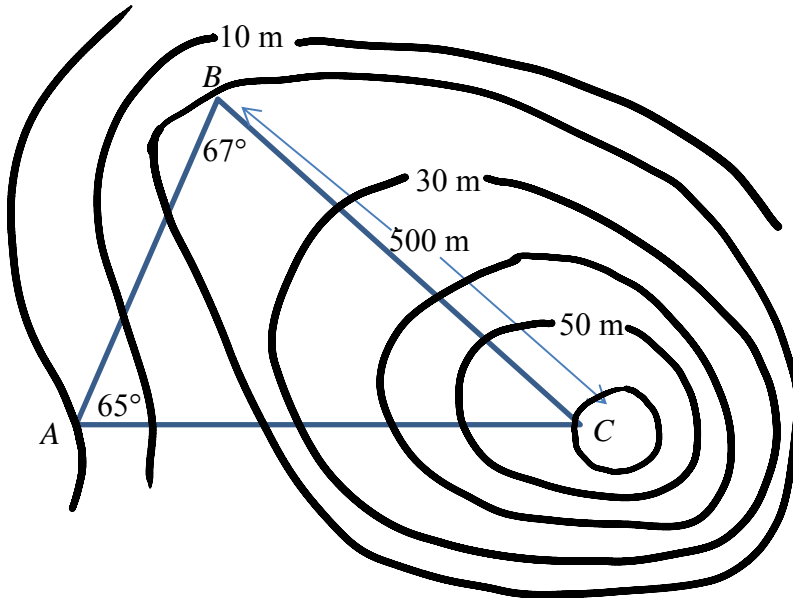
- a. Find the angle at vertex C .

1 mark

- b. Find the distance Annie runs from C to A . Give your answer to the nearest metre.

2 marks

Annie’s trainer, Arnie, ups the ante on her training and moves the run–walk–run course onto a cross-country course that is not flat. All distances are increased by a factor of 10. Vertex C is close to the top of a hill.



- c. What is the vertical drop from C to A ? 1 mark

- d. What is the angle of elevation from point B to point C ? 2 marks

To make Annie's training even tougher, her trainer, Arnie, now gets her to carry a bucket of water (the bucket is cylindrical in shape) in each hand on the walk between B and C .



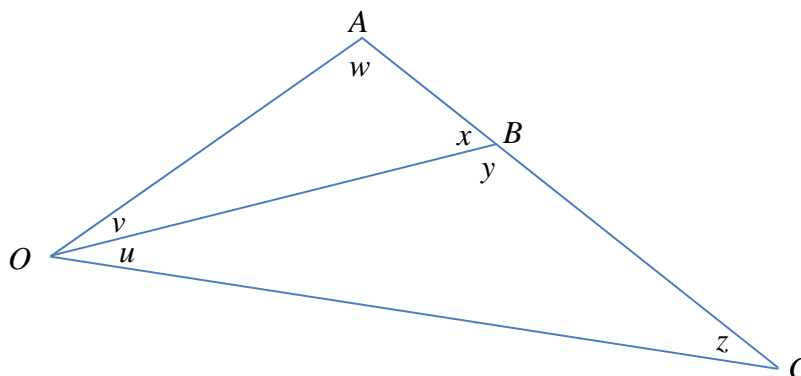
After the first circuit, Annie's trainer increases the size of the buckets. The height of the buckets remains the same (30 cm) but the diameter of the buckets doubles.

- e. If a smaller bucket held 2 litres, what is the volume of a larger bucket?

2 marks

Question 3 (4 marks)

The last training session requires Arnie’s athletes to complete an orienteering course.



The starting point on the course is O and there are 3 other checkpoints A , B and C that are in a straight line. The bearing of C from A is 130°T . The bearings of A from O and B from O are 060°T and 080°T , respectively. The bearing of C from O is 110°T .

a. The magnitude of the angles are

2 marks

$u =$

$v =$

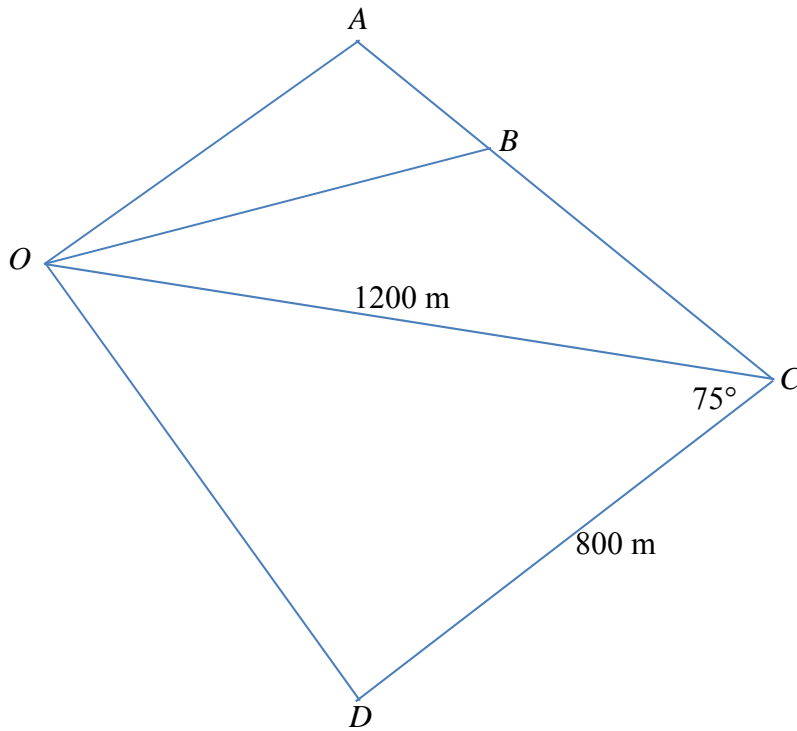
$w =$

$x =$

$y =$

$z =$

Arnie adds another checkpoint to the course at D . The distance of OC is 1200 m, the distance CD is 800 m and the angle OCD is 75° .



- b.** Calculate the distance from D to O . Give your answer correct to the nearest metre.

2 marks

**End of Module 2
TURN OVER**

Module 3: Graphs and relations

Question 1 (6 marks)

The Runfast shoe shop manufactures and sells its running shoes directly to the public. The latest shoe, the Nikedas, requires \$50 worth of raw materials per pair to make. It also costs \$550 per month to provide the manufacturing facilities to make Nikedas shoes, regardless of the number of pairs made. It is possible for the Runfast shoe shop to make up to 30 pairs of Nikedas shoes per month.

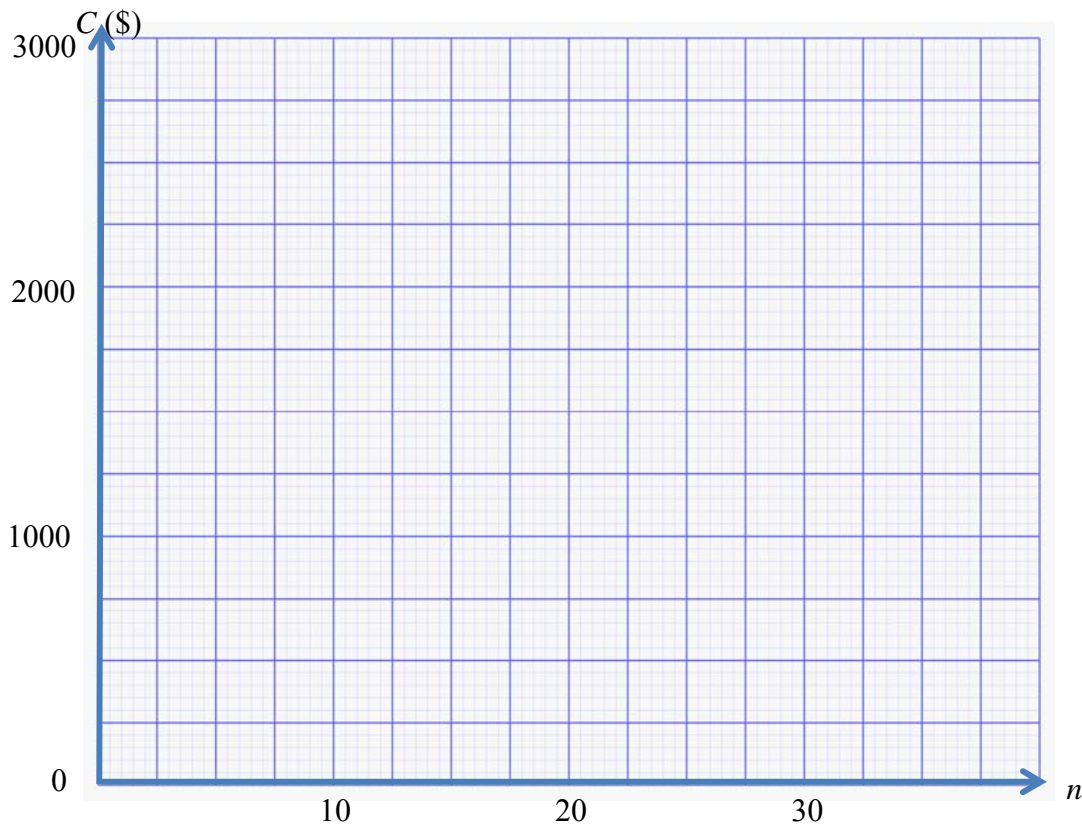
- a. If n is the number of pairs of Nikedas shoes made in one month, the equation for the total cost (C) of manufacture would be

$C =$ $0 \leq n \leq 30$

1 mark

- b. Sketch the graph of the cost equation on the axes below.

1 mark



The Runfast shoe shop sells the first 10 pairs of Nikedas for \$80 to get stock moving.
The next 20 pairs are sold for \$100.

- c. Complete the equation for revenue in terms of n .

2 marks

$$R = \begin{cases} & 0 \leq n \leq 10 \\ & 10 < n \leq 30 \end{cases}$$

- d. Sketch the revenue equation on the axes opposite.

1 mark

- e. How many pairs of Nikedas shoes does the Runfast shoe shop need to sell to break even?

1 mark

Question 2 (9 marks)

Zola is training for the Olympic 1500 m race. Her trainer, Fritz, has her on a strict diet that includes taking two supplements. One supplement is called Xanabola and the other supplement is called Yesterola. Every Xanabola tablet contains 40 g of protein A and 80 g of protein B. Every Yesterola tablet contains 50 g of protein A and 30 g of protein B. Zola cannot take more than 200 g of protein A and no more than 240 g of protein B. Let x represent the number of Xanabola tablets she can take and y represent the number of Yesterola tablets she can take.

- a. Write the two constraints described above.

2 marks

Another athlete, Marianna, is also training for the Olympic 1500 m race and takes the same two supplements as Zola. x represents the number of Xanabola tablets she takes and y represents the number of Yesterola tablets she takes. The inequalities below define the constraints on Marianna's supplement regime.

$$x \geq 0$$

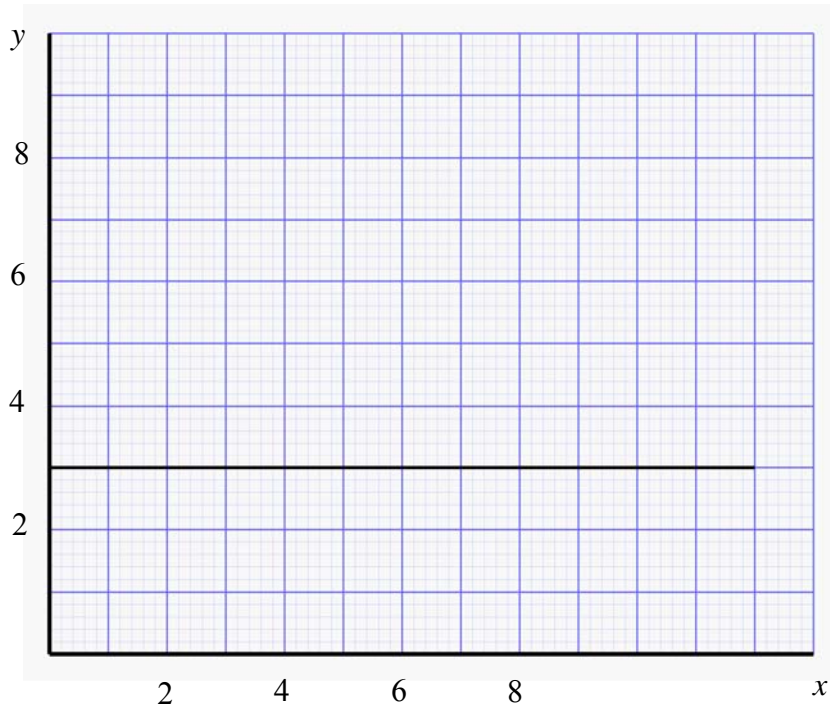
$$y \geq 0$$

$$y \leq 3$$

$$20x + 40y \leq 160$$

$$40x + 30y \leq 240$$

The lines $x = 0$, $y = 0$ and $y = 3$ are already sketched on the axes opposite.



- b.** On the set of axes above, sketch the lines
 $20x + 40y = 160$ and $40x + 30y = 240$

2 marks

- c.** Shade in the feasible region defined by these constraints.

1 mark

The performance benefit, P , of taking Xanabola tablets is twice the performance benefit of taking Yesterola tablets.

- d.** Write an objective function for the performance benefit, P , in terms of x and y .

1 mark

- e. How many whole tablets of Xanabola and whole tablets of Yesterola should Marianna take to maximise her performance benefit?

3 marks

Module 4: Business-related mathematics

Question 1 (3 marks)

Jenny wants to purchase the latest smartphone that has just come on to the market. The retail price is \$825.00

- a. If the retailer has already added the 10% goods and services tax (GST) to the \$825 price, how much GST will Jenny pay if she buys the new smartphone?

1 mark

Jenny only has \$600 and cannot afford the new smartphone. Jenny has noticed, however, that the new model smartphones usually become cheaper with time. She estimates that the price of the new smartphone will be reduced by 5% of the original price of \$825 each month from the release date.

- b. How many months will Jenny have to wait until she can afford the new smartphone?

1 mark

Jenny decides not to buy the new smartphone because she knows that in less than a year there will be a new model on the market. Instead, she puts her \$600 into a simple interest account that pays 9% per annum. Her interest is paid at the time she closes her account.

- c. If she closes her account after 8 months, how much will she receive?

1 mark

Module 4 – continued
TURN OVER

Question 2 (4 marks)

Chloe also wants to buy the new smartphone. She finds one advertised for \$850 at Rick Smyth Electrics, and although they can be purchased for less elsewhere, at Rick Smyth Electrics she is only required to pay \$50 at the time of purchase. The balance owing is to be paid in monthly repayments of \$45 over 2 years.

- a. Calculate the total amount of interest that Chloe pays. 1 mark

- b. Calculate the flat rate of interest that Chloe is being charged by Rick Smyth Electrics. 1 mark

- c. Calculate the effective interest rate that Chloe pays. 1 mark

- d. Explain why the advertised flat interest rate may be misleading and why the effective interest rate is a more accurate measure of the interest rate being charged to customers. 1 mark

Question 3 (4 marks)

Lucky has won \$355 000 in a lottery. He wants to secure his future and retire in 5 years so he is considering a number of financial options.

- a. Lucky considers putting the \$355 000 into a compound interest account earning 4.25% per annum compounding monthly. If he chooses this option, what will be the balance of his account after 5 years?

1 mark

- b. Lucky knows that the compound interest option won't give him enough to retire. He wants to have \$1 000 000 in 5 years. If he keeps working and invests the \$355 000 into the compound interest account earning 4.25% per annum compounding monthly how much will he need to add to the account at the end of every month so that he has \$1 000 000 in 5 years?

1 mark

Lucky cannot afford the amount needed to add to the account each month to get him to \$1 000 000 in 5 years. Instead, he invests the \$355 000 into a compound interest account earning 4.25% per annum compounding monthly and adds \$1200 each month. After 2 years, the interest rate changes to 5.5% per annum and at the same time he is able to increase his monthly payment to \$1400.

- c. How much will he have at the end of the 5 years?

2 marks

Module 4 – continued
TURN OVER

Question 4 (4 marks)

Bruce has just bought a brand new truck and plans to work for himself doing contract transport. The truck cost him \$320 000 and he plans to offset the cost by using the depreciation of the truck as a tax deduction.

- a. One option is to calculate the depreciation using unit cost depreciation. If the truck can be depreciated by 10 cents for every kilometre he travels, and if he travels 100 000 km in the first year, what will be the amount of depreciation for the first year?

1 mark

- b. Bruce wants the depreciation in the first year to be \$16 000, using the unit cost method. How many kilometres would he need to travel if the truck depreciates by 8 cents per kilometre?

1 mark

- c. Using flat rate depreciation Bruce is able to depreciate the truck by \$30 000 in the first year. Calculate the flat rate used to achieve this amount of depreciation. Give your answer correct to one decimal place.

1 mark

Bruce's accountant, Cyril, advises him to use reducing balance depreciation. Cyril says that once the truck's value reaches \$30 000 (scrap value), they will trade the truck in for a new one. Cyril claims that, because the taxation office allows a higher depreciation rate using the reducing balance method, the truck can be written off as scrap sooner.

- d. If Cyril uses a reducing balance rate of 18%, how many years will it take before the value of the truck is below scrap value?

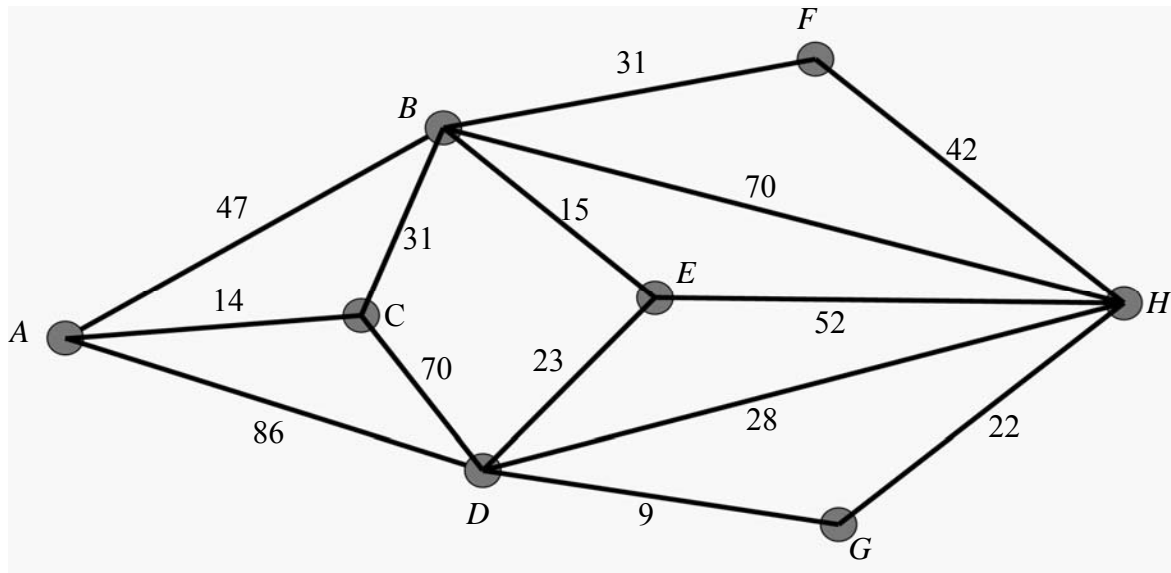
1 mark

End of Module 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1 (4 marks)

The diagram below represents a draft plan for recycled water plumbing at part of the Armstrong Creek Development. Most houses are expected to have one recycled water outlet. The vertices (*A* to *H*) represent outlets (taps) for recycled water. The distances between the outlets are shown as weighted edges. Some, but not all of the edges in the final plan will represent pipes. All distances are in metres and the diagram is NOT to scale.

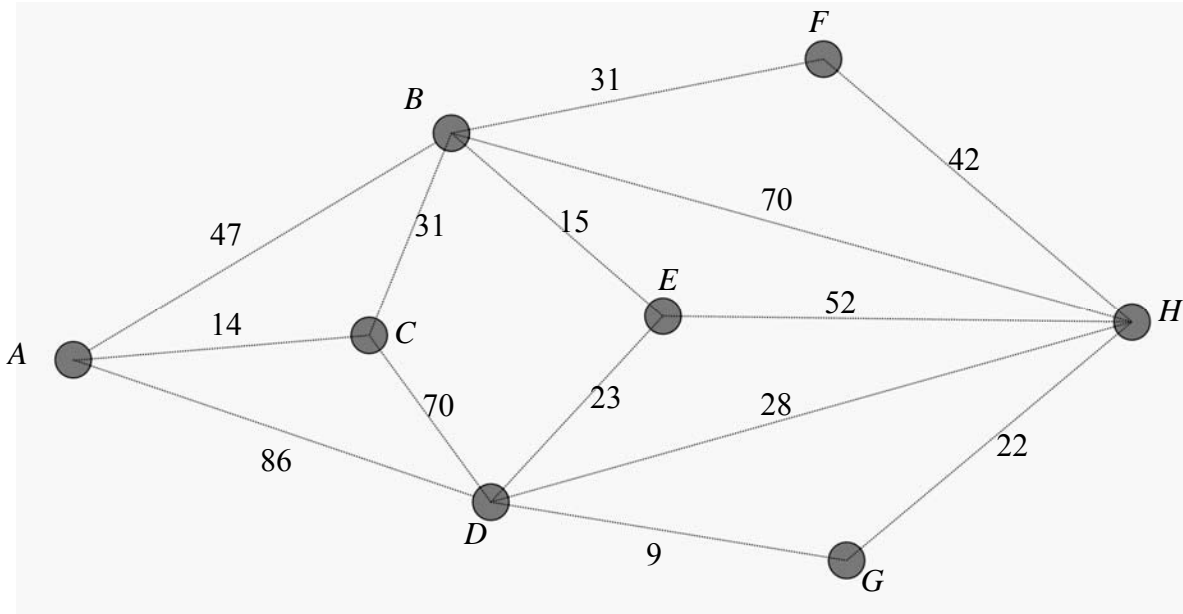


- a. What is the length of the shortest distance from *A* to *H* by travelling along edges?

1 mark

- b.** The architect's final plan will only show pipes that allow the minimum length of pipe to be used that still allows all outlets to be connected to the network.
 Draw the architect's final plan on the diagram below.

1 mark



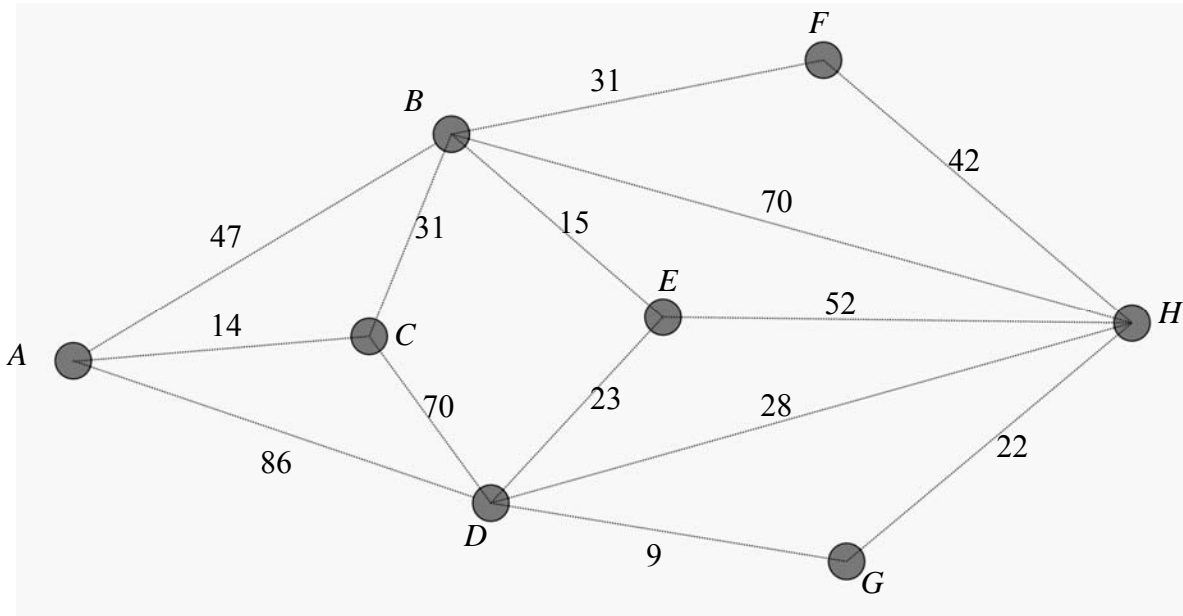
- c.** What is the mathematical term used to describe the network drawn in part **b**?

1 mark

- d.** Construction workers have encountered underground rock that prevents them from laying water pipes between vertices D and G , vertices A and C and vertices B and F . What is the minimum length of pipe they will now need to use so that all outlets have water?

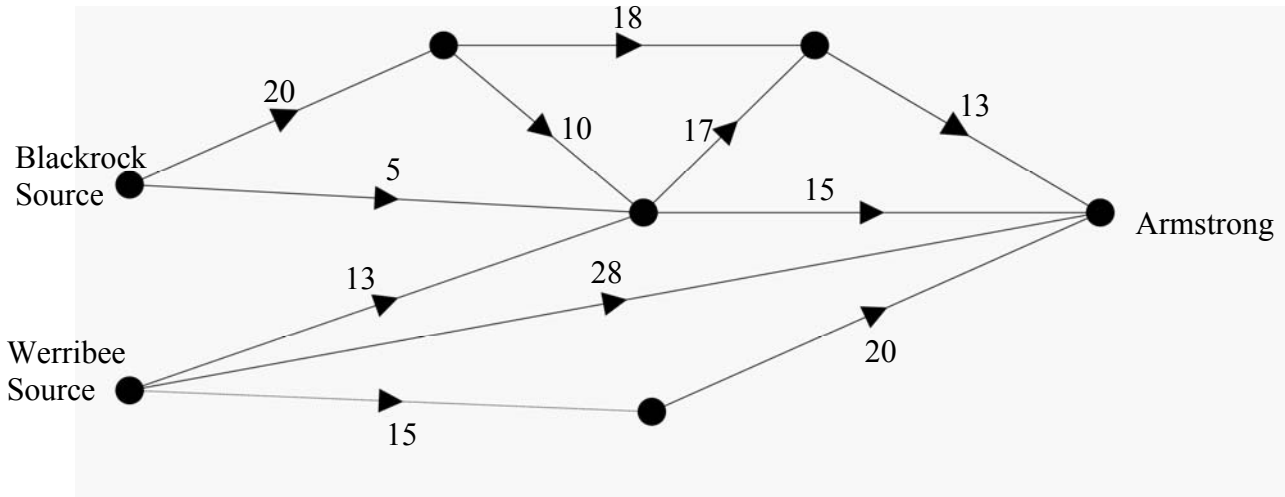
The diagram below may be used for working.

1 mark



Question 2 (3 marks)

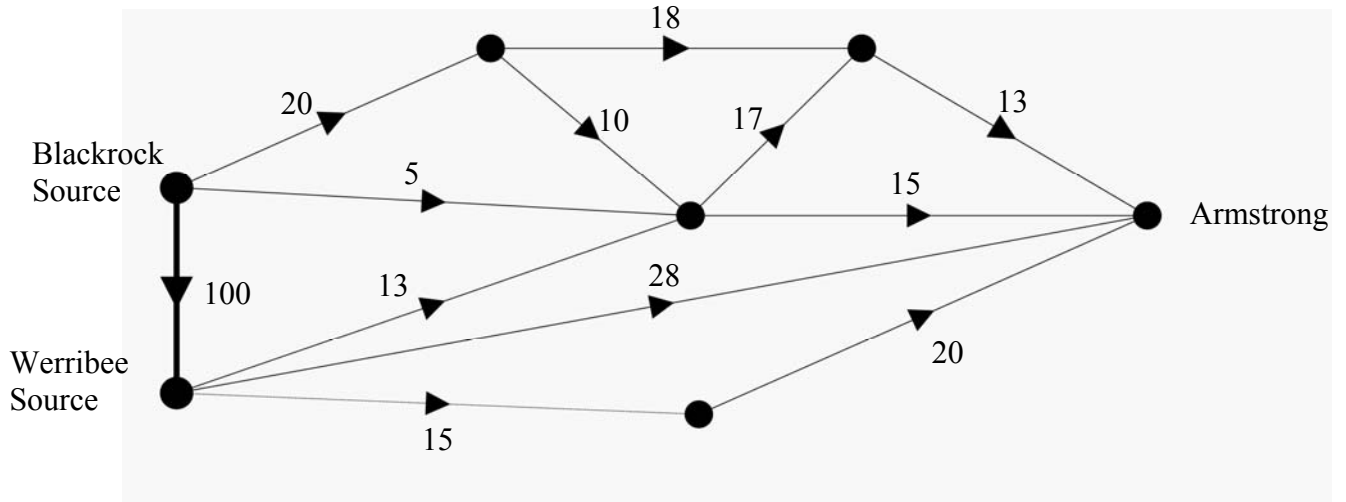
The recycled water for the Armstrong Creek Development is available from 2 sources, Blackrock and Werribee. When the pipes were laid for the supply, the government tried to save money by using second-hand materials. As a result, the pipes are not the same size and the supply is not direct. The network below shows the council diagram of pipes able to supply recycled water to Armstrong Creek. Numbers along edges show the flow capacity of the pipes in megalitres per hour.



- a. What is the maximum flow capacity (in megalitres per hour) from the Blackrock and Werribee sources to Armstrong Creek?

1 mark

A miscalculation of the amount of available water means that the Blackrock plant now has more water than it knows what to do with, but the Werribee plant can no longer produce its own water. A direct pipe to Werribee that can carry 100 megalitres per hour is built to transport excess water from Blackrock to Werribee.

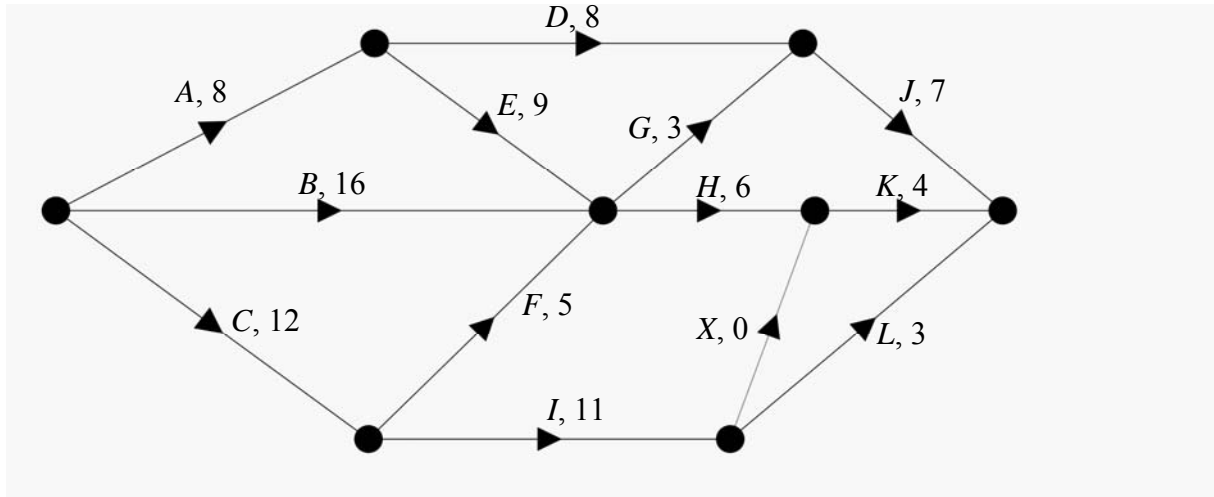


- b. Werribee water is now more expensive and water that can be sourced from Blackrock must be used before water from Werribee is used. What is the maximum number of megalitres per hour available to Armstrong Creek from each plant under these conditions?

2 marks

Question 3 (6 marks)

The government decides to demolish the Werribee recycled water plant and build a desalination plant. The network diagram below shows the number of weeks it will take to complete each of 12 activities, A to L, to build the desalination plant.



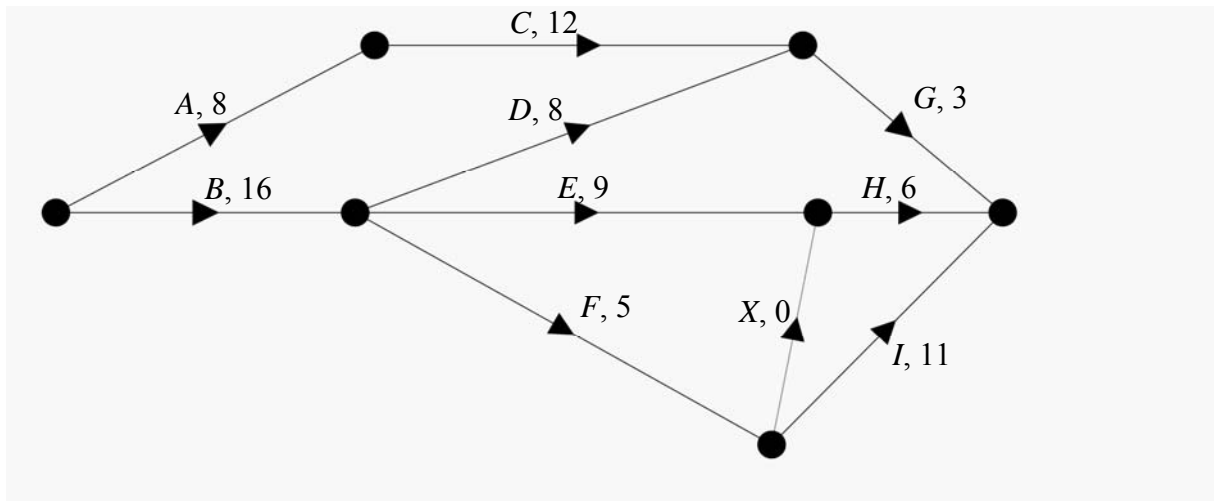
- a. What is the shortest time that it could take for the desalination plant to be built? 1 mark

- b. Work has stopped on activity L due to an accident. For how many weeks can work on activity L be stopped before the whole project is delayed? 1 mark

Activities A, B and C can all be completed faster if more money is spent. Activity A completion time can be reduced by 3 weeks down to 5 weeks, but each week of time reduction will cost \$15 000. Activity B can be reduced by 1 week down to 15 weeks but this will cost \$20 000. Activity C can be reduced by 4 weeks down to 8 weeks, but each week of time reduction will cost \$25 000.

- c. What is the shortest time that it could now take to have the desalination plant built and what is the total minimum cost to achieve this? 2 marks

Unexpected problems arise with a number of activities and the contractor goes bankrupt. After sustaining considerable losses, the government employs another contractor with a simpler plan to complete construction. The network is shown below.



- d. If all activities go to plan without delays, and activities A and B start equal 1st, with activity C starting 3rd etc., which of the 9 activities will start 7th?

2 marks

Module 6: Matrices**Question 1** (4 marks)

The table below gives the personal best times of 3 athletes in 3 different track events.

Athlete	Event	100 m	200 m	400 m
Abe		12.3	25.6	48.2
Ben		10.5	23.3	52.3
Chris		10.7	23.2	49.6

This data has been put into matrix T .

$$T = \begin{bmatrix} 12.3 & 25.6 & 48.2 \\ 10.5 & 23.3 & 52.3 \\ 10.7 & 23.2 & 49.6 \end{bmatrix}$$

- a. Matrix $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Calculate the product matrix TA .

1 mark

b. What do the elements of the product matrix TA represent? 1 mark

c. With training, all 3 athletes can take 5% off their personal best 100 m times, 10% off their personal best 200 m times and 15% off their personal best 400 m times. Construct matrix B so that the product TB results in a matrix that shows these new times. 1 mark

d. Calculate the product TB . Round off to 1 decimal place. 1 mark

Question 2 (5 marks)

The state athletics teams of Victoria, New South Wales and Queensland have booked their accommodation at the Darwin International Hotel for the National Athletics championships. The total accommodation costs for each of the teams and the numbers booked into each type of accommodation are shown in the table below.

Team	Acc type	Economy (x)	Ensuite (y)	Deluxe (z)	Total cost
Victoria		10	5	2	2400
New South Wales		25	2	0	2320
Queensland		10	0	4	2400

- a. Set up 3 simultaneous equations to find the prices of economy, ensuite and deluxe rooms using the following variables

$$x = \text{economy price} \quad y = \text{ensuite price} \quad z = \text{deluxe price}$$

2 marks

The simultaneous equations for the accommodation costs of the Tasmanian team, the South Australian team and the Western Australian team at the Pacific Hotel, where a represents the price of a standard room, b represents the price of view rooms and c represents the price of pool rooms, can be represented by the 3 equations

$$10a + 3b + 4c = 2675$$

$$15a + 2c = 1885$$

$$12a + 8b = 1980$$

- b.** Represent these equations in a matrix equation in the form of $AX = B$.

1 mark

- c.** Use the matrix equation to find the cost of a , standard rooms, b , view rooms and c , pool rooms.

2 marks

Question 3 (6 marks)

All 260 athletes at the 2001 National Championships were asked to nominate their favourite track event. The numbers of athletes who nominated the 800 m, the 1500 m and the 5000 m events as their favourite event are given by the initial state matrix S_0 where

$$S_0 = \begin{bmatrix} 120 \\ 80 \\ 60 \end{bmatrix} \begin{array}{l} 800 \text{ m} \\ 1500 \text{ m} \\ 5000 \text{ m} \end{array}$$

- a. How many athletes nominated the 1500 m event as their favourite in the year 2001?

1 mark

The 260 athletes were surveyed each subsequent year if they were still competing, and asked to choose their favourite event from the 3 track events.

	this year			
	800	1500	5000	
$T =$	$\begin{bmatrix} 0.5 & 0.0 & 0.1 \\ 0.3 & 0.6 & 0.2 \\ 0.2 & 0.4 & 0.5 \end{bmatrix}$	$\begin{array}{l} 800 \\ 1500 \\ 5000 \end{array}$	next year	

- b. In the transition matrix, T , above, the 3rd column does not add up to 1. What is the significance of this in terms of the numbers of athletes nominating their favourite event in each subsequent year?

1 mark

- c.** How many of the original 120 athletes who nominated the 800 m as their favourite event in the year 2001 also nominated the 800 m as their favourite event in the year 2003?

1 mark

- d.** If $S_1 = TS_0$, find the matrix S_1 .

1 mark

- e.** How many athletes chose the 1500 m as their favourite event in 2005?

1 mark

By 2009, the number of athletes left from the original 260 who took part in the survey in 2001 is down to 131. In 2009, their favourite events are given by S_8 , where

$$S_8 = \begin{bmatrix} 15 \\ 55 \\ 61 \end{bmatrix}$$

The surveyor decides to try to account for new athletes entering the championships each year. To predict an athlete's favourite event, she uses the following model

$$S_{n+1} = TS_n + B \quad \text{where } B = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

- f. By using matrices T , S_8 and B , find the number of athletes choosing the 1500 m event as their favourite in 2011.

1 mark

END OF QUESTION AND ANSWER BOOK