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# ***Further Mathematics***

## ***2012***

### ***Trial Examination 2***

***Core – Data analysis***

***Module 2 – Geometry and trigonometry***

***Module 3 – Graphs and relations***

***Module 4 – Business-related mathematics***

**Instructions:**

Answer all questions in the core and the three modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example,  $\pi$ , surds or fractions.

**Core – Data analysis**

From colonial times, patterns of migration to Australia have been shaped by historical events and policies both in Australia and other parts of the world.

**Question 1** In 2011, the Census revealed that over a quarter (26%) of Australia's population was born overseas and a further one fifth (20%) had at least one overseas-born parent.

- a. In 2011, what percentage of Australia's population had both parents born in Australia? 1 mark

**TOP 10 COUNTRIES OF BIRTH FOR THE OVERSEAS-BORN POPULATION**

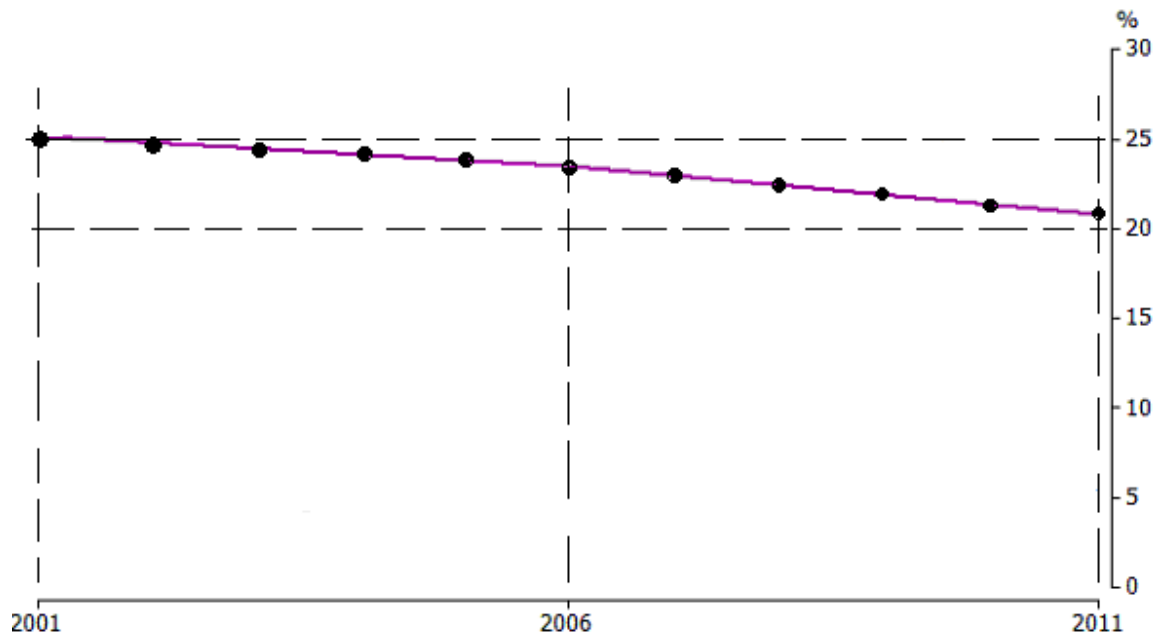
Country of birth	Persons (thousands)	Proportion of all overseas-born (%)	Median age (years)	Sex ratio i.e. Number of males per 100 females
United Kingdom	1 101.1	20.8	54	101.7
New Zealand	483.4	9.1	40	102.8
China	319.0	6.0	35	79.8
India	295.4	5.6	31	125.2
Italy	185.4	3.5	68	104.7
Vietnam	185.0	3.5	43	84.6
Philippines	171.2	3.2	39	60.6
South Africa	145.7	2.8	39	96.9
Malaysia	116.2	2.2	39	83.5
Germany	108.0	2.0	62	90.6
Born elsewhere overseas	2 183.8	41.2	44	95.6
<b>Total overseas-born</b>	<b>5 294.2</b>	<b>100</b>	<b>49</b>	<b>96.1</b>

Source: abs

- b. Refer to the above table. In 2011, what percentage of the overseas-born population were under 49 years old? 1 mark

- c. Refer to the above table. In 2011, what percentage of the overseas-born population were females? 1 mark

**Question 2** In 2011, the United Kingdom was the leading country of birth for the overseas-born population (21%). The percentage has been decreasing since 2001 as shown in the graph below.



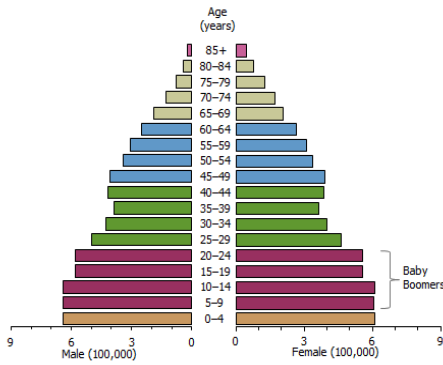
a. Draw a line of best fit for the annual data points on the above graph. 1 mark

b. Let  $P$  be the percentage and  $t$  be the year, and year 2000 be  $t = 0$ . Find the equation of your line of best fit. 2 marks

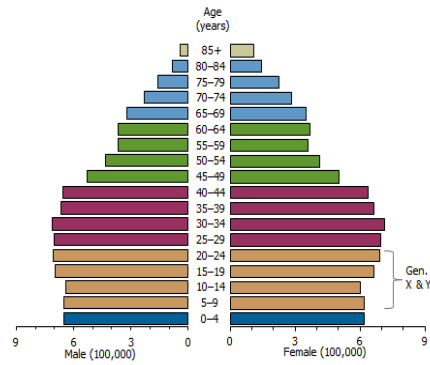
c. For the overseas-born population predict the percentage are from United Kingdom in 2015. 1 mark

**Question 3**

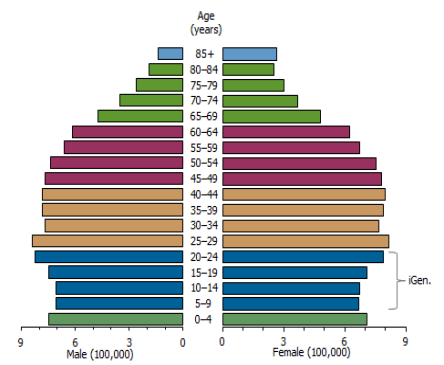
**Population structure 1971**



**Population structure 1991**



**Population structure 2011**



a. Based on the population structures shown above, estimate the increase in the population of 45-49 age group from 1971 to 2011. 1 mark

b. Explain the large difference between the population of the 5-9 age group in 1991 and the population of the 25-29 age group in 2011. 1 mark

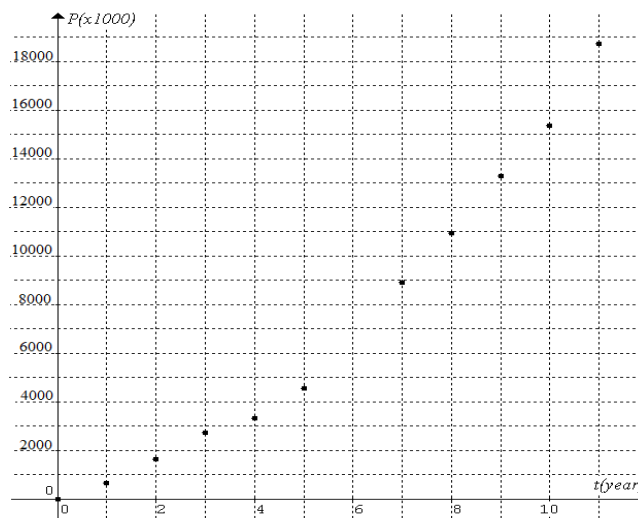
c. The population structures have different shapes. Describe the change from one population structure to another structure, which indicates the process of population ageing in Australia. 1 mark

d. Consider the 2011 population structure shown above. Give the modal age group of the distribution. 1 mark

**Question 4**

Year	Time $t$	Australian population (thousands)	Increase in population since 1901 $P$ (thousands)
1901	0	3770	0
1911	1	4430	660
1921	2	5410	1640
1931	3	6500	2730
1941	4	7080	3310
1951	5	8310	4540
1961	6	10390	6620
1971	7	12660	8890
1981	8	14700	10930
1991	9	17070	13300
2001	10	19150	15380
2011	11	22490	18720

a. The increase in Australian population  $P$  (thousands) since 1901 is plotted against time  $t$  (years) in the following graph with one data point missing. Fill in the missing data point. 1 mark

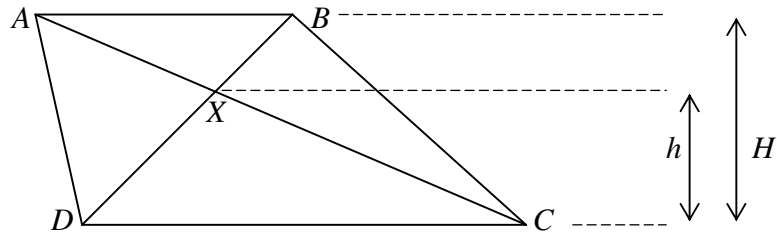


b. The above data can be described approximately by  $P = kt^2 + c$ . Find the values of  $k$  and  $c$ , correct to the nearest ten. 2 marks

c. Predict the Australian population in 2021, correct to the nearest million. 1 mark

**Module 2: Geometry and trigonometry**

**Question 1**  $ABCD$  is a trapezium as shown in the diagram below. Side  $AB$  is parallel to side  $DC$  and  $\frac{AB}{DC} = \frac{3}{5}$ .  $AC$  intersects  $BD$  at  $X$ .

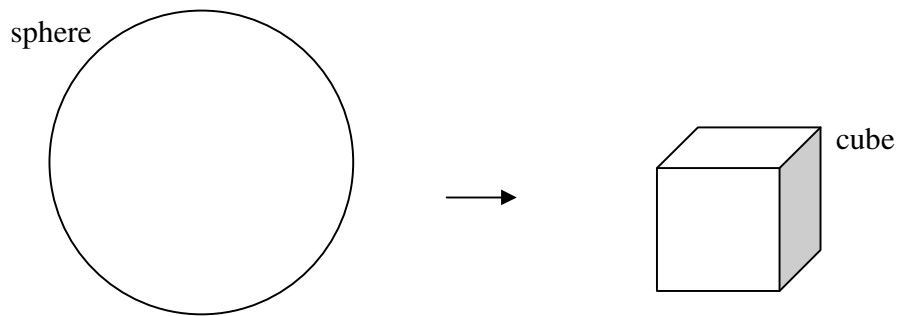


a. Find the value of the ratio  $\frac{h}{H}$ . 1 mark

b. Find the value of  $\frac{\text{area.of } \triangle ABX}{\text{area.of } \triangle CDX}$ . 1 mark

c. Find the value of  $\frac{\text{area.of } \triangle BCX}{\text{area.of } \triangle CDX}$ . 1 mark

**Question 2** A solid cube is to be cut out from a solid sphere which has a radius of 0.50 m.

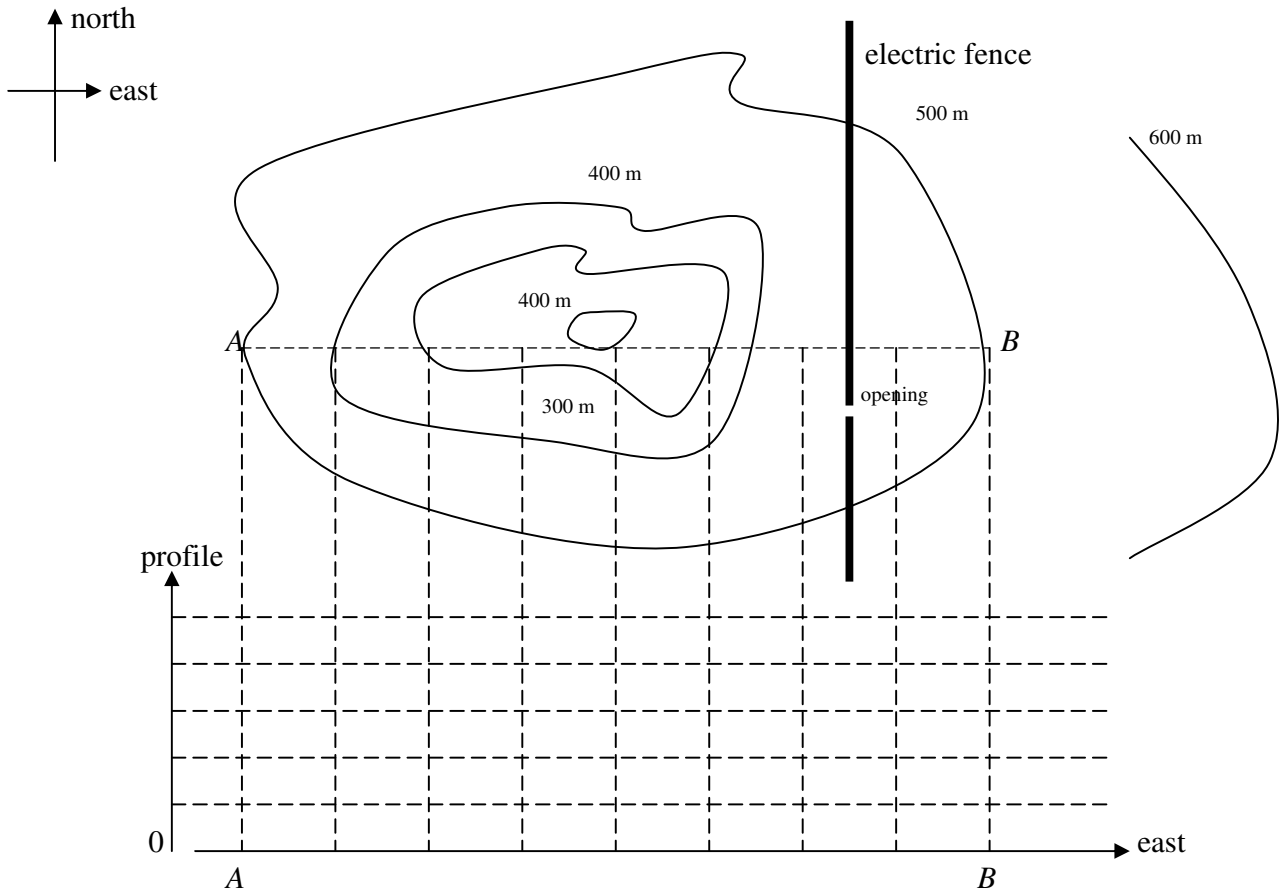


a. Find the volume (in  $\text{m}^3$ , correct to 2 decimal places) of the largest cube possible. 2 marks

b. Find the **exact** value of the ratio, *surface area of the sphere* : *surface area of the largest cube*. 2 marks

**Question 3** Consider the following contour map of a terrain. There is a tall electric fence (with an opening) across the terrain. Distance  $AB = 2$  km.

a. Sketch the profile of the terrain between point  $A$  and point  $B$ . 1 mark



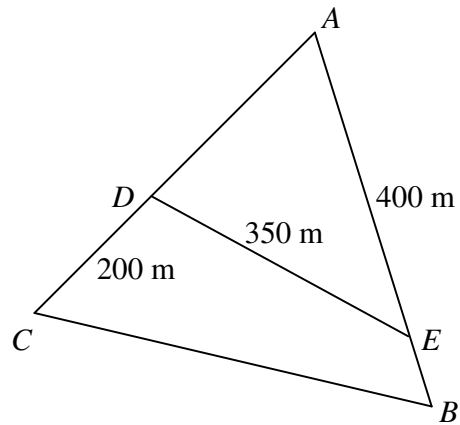
b. Mark a cross on the dotted line  $AB$  on the contour map and label it as  $P$ , where the slope is the steepest. 1 mark

c. Estimate the gradient of the slope at cross  $P$ . 1 mark

d. The land shown in the map is cleared for grazing. Draw on the contour map a short and easy route to travel from  $A$  to  $B$  through the opening of the electric fence. 1 mark



**Question 4** A flat parcel of land is in the shape of an *equilateral* triangle. The measurements are shown in the following diagram.



a. Find the obtuse  $\angle CDE$ , correct to the nearest degree. 1 mark

b. Let  $BC = x$  metres. Show that  $x^2 - 800x + 157500 = 0$ . 2 marks

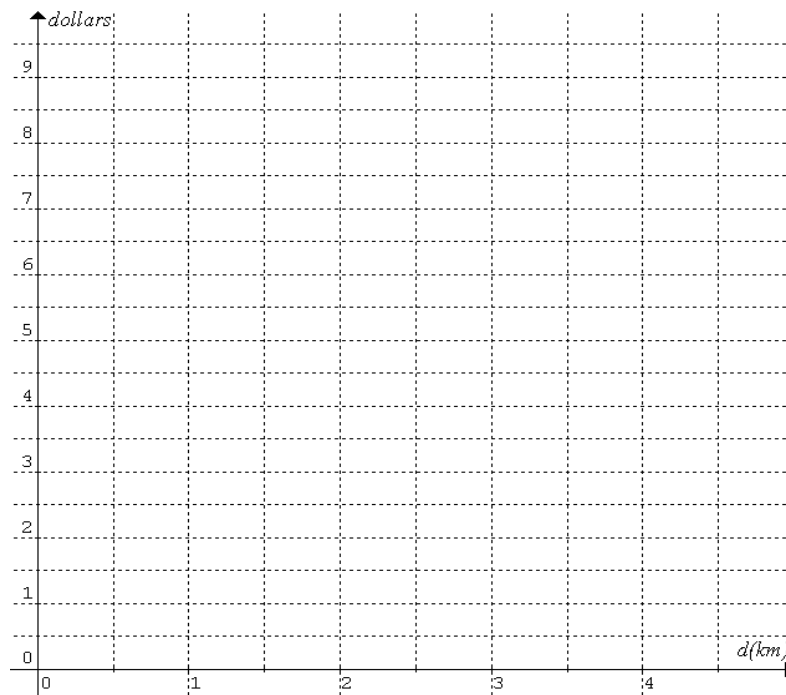
c. Given that  $BC = 450\text{ m}$ , calculate the area of the parcel of land in  $\text{m}^2$ . 1 mark

### Module 3: Graphs and relations

**Question 1** There is a minimum charge (called flag fall charge) of \$3.20 to travel by Taxi in Melbourne. For each kilometre distance of travel, \$1.617 is added to the minimum charge. Let  $C$  be the total taxi fare in dollars and  $d$  be the distance travelled in km.

a. Write an equation for the taxi fare that a passenger pays to travel  $d$  km. 1 mark

b. Draw accurately the graph of Melbourne taxi fare for travelling  $d$  km in the axes below. 1 mark



c. Calculate the distance travelled in Melbourne if the taxi fare is \$51.70. 1 mark

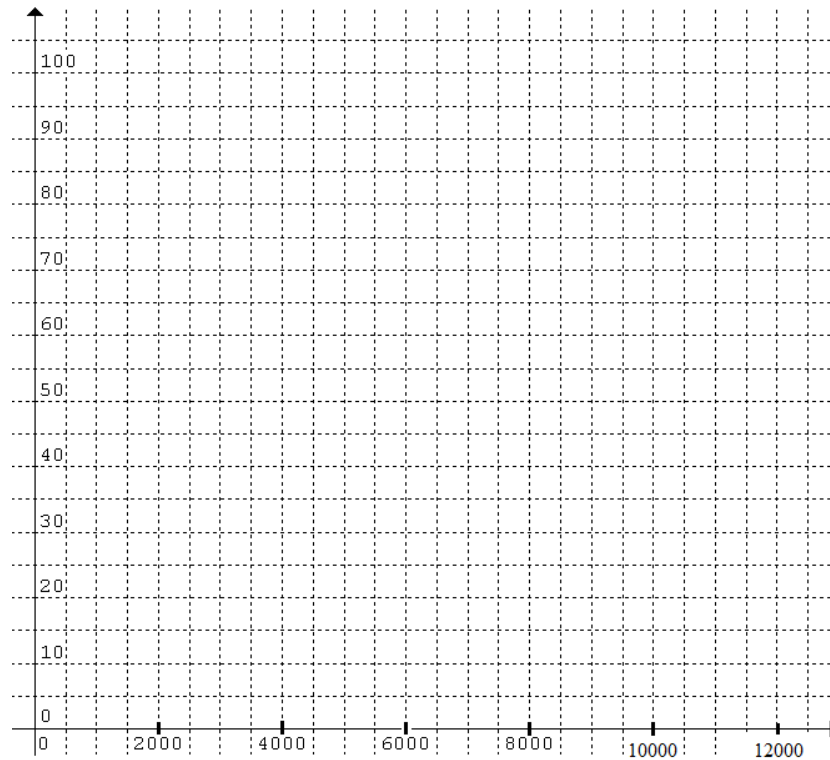
d. Taxi fare in Perth is \$1.59 per km and \$3.90 flag fall charge. Find the distance travelled, correct to the nearest km, for the meters in a Melbourne taxi and a Perth taxi to show the same fare. 1 mark

**Question 2** The braking distance  $d$  metres of a car at speed  $v$  km per hour is measured and the data is shown in the following table.

$v$ (km/h)	30	40	60	70	80	90	100	110
$d$ (m)	8	14	31	42	55	70	85	104

a. Plot  $d$  against  $v^2$  in the following axes. Label the axes.

1 mark

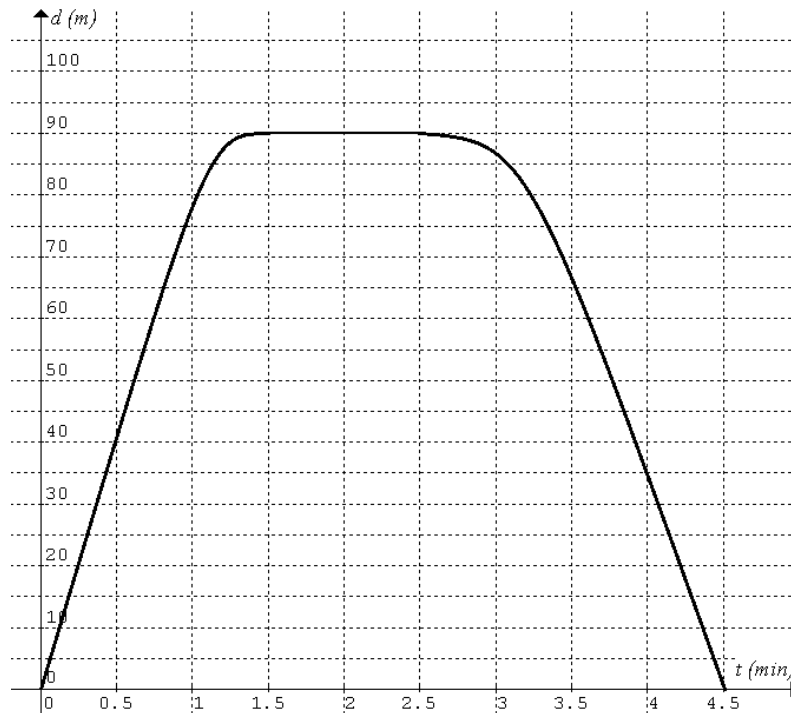


b. Your plot is expected to show that  $d$  is proportional to  $v^2$ . Find the constant of proportionality and its unit.  
2 marks

c. Interpolate the braking distance, correct to the nearest metre, when the speed is 55 km/h.

1 mark

**Question 3** A student walks along a straight road to a train station to meet a friend and then return home. The graph of the distance (metres) from home against time (minutes) is shown below.



a. How far is the train station from the student's home? 1 mark

b. Find the average speed (metres per minute) of the student in the first 4 minutes. 1 mark

c. Find the average rate of change of the student's *distance from home* (metres per minute) in the first 4 minutes. 1 mark

**Question 4** A manufacturer of certain product has two factories A and B at separate locations. Factory A can produce 75 items and Factory B can produce 85 items each week. The items are transported to two retail shops X and Y located in different shopping centres. Each week Shop X requires 75 items, and Shop Y requires 45 items. The following table shows the transportation costs in dollars per item.

	Factory A	Factory B
Shop X	21	16
Shop Y	24	19

Let  $x$  be the number of items transported from Factory A to shop X, and  $y$  the number of items transported from Factory A to shop Y.

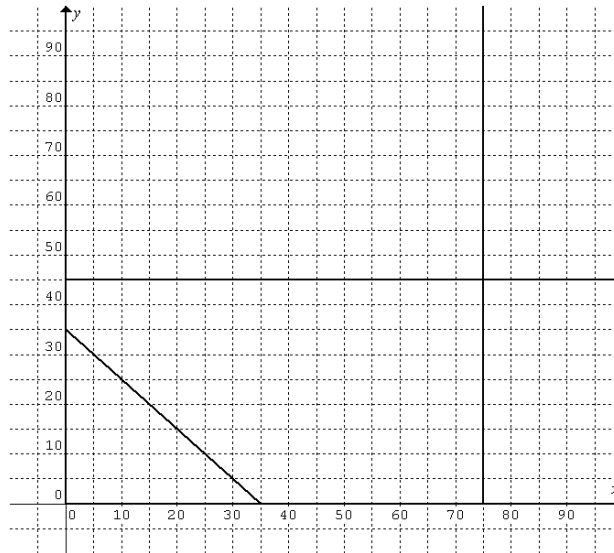
a. Complete the following statements in terms of  $x$  and  $y$ . 1 mark

The number of items transported from Factory B to Shop X =

The number of items transported from Factory B to Shop Y =

b. The constraints are: (1)  $x \geq 0$  (2)  $y \geq 0$  (3)  $x \leq 75$  (4)  $y \leq 45$  (5)  $x + y \leq 75$  (6)  $x + y \geq 35$

5 of the 6 constraints (equalities only) are shown in the following graph. Sketch the equality of the remaining constraint on the same graph. 1 mark



c. Write an equation for the total cost,  $C$  dollars, to transport the items required by Shop X and Shop Y each week, in terms of the variables  $x$  and  $y$ .

1 mark

c. Determine the lowest total cost to transport the items required by Shop X and Shop Y each week. 1 mark

#### Module 4: Business-related mathematics

**Question 1** Carbon pricing causes electricity to increase in price by 9.0% according to the Australian government. A latest electricity bill shows the amount \$375.00 per quarter before GST.

a. Calculate the increase in price, correct to the nearest cent, due to carbon pricing. 2 marks

b. Calculate the amount to be paid, correct to the nearest cent, if there were no increase due to carbon pricing. 1 mark

**Question 2** \$650000 is invested in an ordinary perpetuity at an interest rate of 4.80% per annum to provide a series of regular payments to the investor.

a. Find the monthly payment provided by the perpetuity. 1 mark

b. After three years of monthly payments, how much money remains invested in the perpetuity? 1 mark

c. Determine the effective interest rate on the \$650000 invested in the perpetuity. 2 marks

**Question 3** A \$550 000 house was purchased with a housing loan of \$450 000 from a bank at 6.7% p.a. interest compounded monthly. The loan is to be paid off over 30 years by monthly payments.

a. Calculate the monthly repayment.

1 mark

Interest rate was reduced by 0.18% after 8 repayments. The borrower decided to continue the same repayment.

b. Calculate the number of months (correct to the nearest month) that the term of the loan will be shortened, assuming there is no further change in interest rate.

2 marks

c. How much interest (correct to the nearest \$100) will he save over the term of the loan?

1 mark

**Question 4** The following table shows the Australian quarterly CPI from 2010 September quarter to 2012 June quarter.

Quarter	CPI
Sep-2010	173.3
Dec-2010	174.0
Mar-2011	176.7
Jun-2011	178.3
Sep-2011	179.4
Dec-2011	179.4
Mar-2012	179.5
Jun-2012	180.4

a. Calculate the annual percentage rise (correct to 1 decimal place) in the cost of living in the financial year 2011-12.

1 mark

b. If you earn simple interest on your deposit, find the minimum interest rate (correct to 1 decimal place) needed to maintain the worth of your money in the financial year 2011-12.

1 mark

From the September quarter 2012, all index numbers will be calculated on a new index reference period of 2011-12. This will result in the index numbers being reset to 100.0 for the financial year 2011-12. (Source: ABS)

c. If the reset to 100.0 is done for the financial year 2010-11, calculate and fill in the missing entry (correct to 1 decimal place) in the table showing the Australian quarterly CPI from 2011 September quarter to 2012 June quarter.

2 marks

Quarter	CPI
Sep-2011	100.6
Dec-2011	100.6
Mar-2012	100.7
Jun-2012	

**End of Exam 2**