

2011 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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FURTHER MATHEMATICS

Written examination 2

Reading Time: 15 minutes

Writing Time: 1 Hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
2	2	15
Module		
<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question and answer book of 29 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This paper consists of a core and six modules. Students are to answer all questions in the core and then select **three** modules and answer all questions within those modules. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve e.g. π , surds, fractions. Diagrams are not drawn to scale unless specified otherwise.

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Core	4
 Module	
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TURN OVER

Core**Question 1**

A cricket eleven training schedule showed the number of hours players trained and the number of runs as a result of their training that they scored the following week in a 20-20 match.

Runs	Training time (Hours)
0	0
20	0
10	1
20	1
15	2
33	2
49	2
28	3
52	3
41	4
60	4

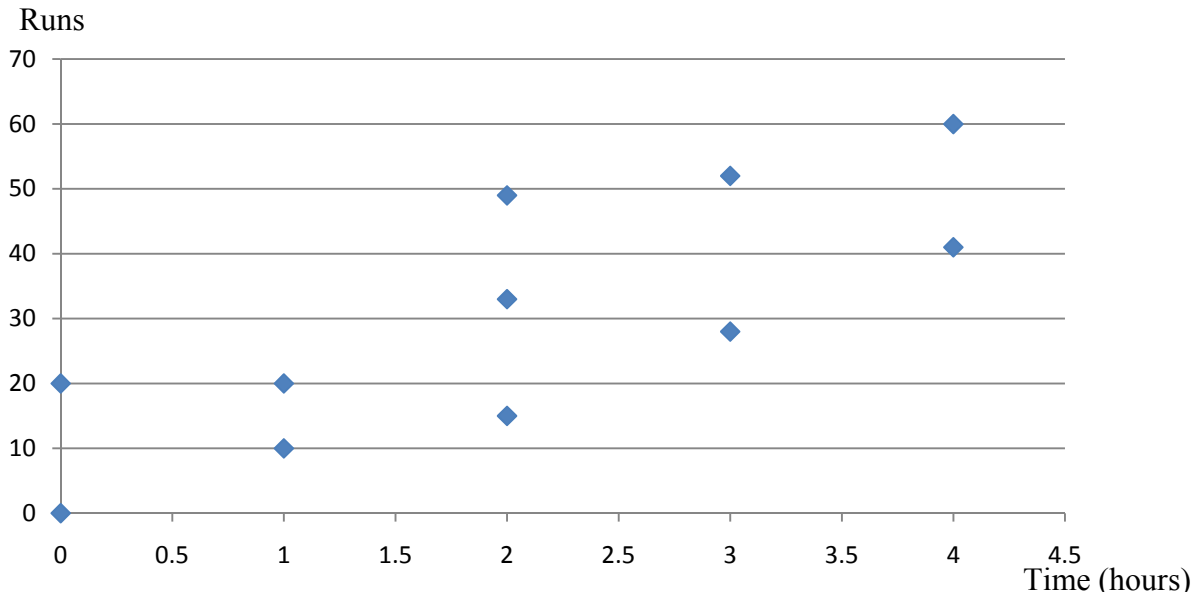
- a. The independent variable is

1 mark

- b. Calculate the r-correlation to 4 decimal places

1 mark

Core - Question 1- continued



c. Circle the data point (4,60) on the graph

1 mark

d. In terms of form, strength and direction, describe the scatterplot.

3 marks

e. Complete the paragraph below.

The coefficient of determination is . This indicates that % of the variation in the number of runs can be explained by the variation in

3 marks

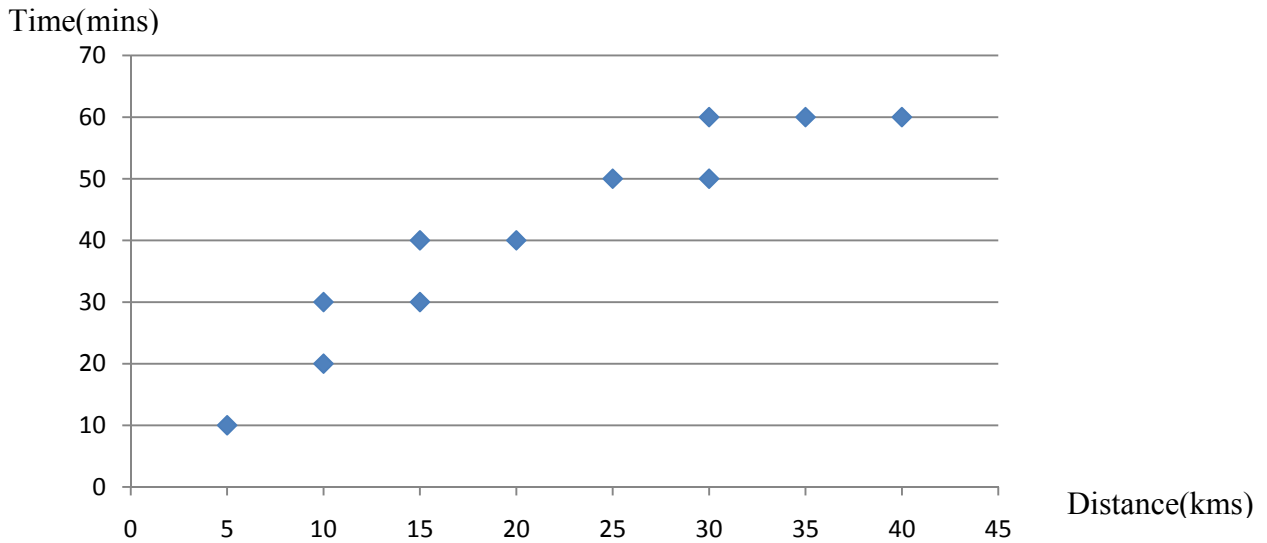
**Core - Question 1- continued
TURN OVER**

- f. Find the least squares regression line in terms of the variables runs scored and training hours, giving coefficients to 3 decimal places.

2 marks

Question 2

Players live varying distances (in kms) from the playing ground. The times (in minutes) to get to the ground are graphed below. The scatterplot is clearly non-linear.



- a. Using a **log** distance transformation, find the equation of the least squares regression that can be used to predict the time it takes to get to the playing ground in terms of **log** distance, giving coefficients to 3 decimal places.

2 marks

Core - Question 2- continued

- b.** Use the equation of your regression line found in **a.** to predict the time taken to get to the playing ground of a cricketer who lives 50 kms from the ground. Give your answer to the nearest minute.

1 mark

- c.** Comment on the reliability of your answer in **b.**

1 mark
Total 15 marks

**END OF CORE
TURN OVER**

Module 1: Number patterns

Question 1

Kolora-Noorat Football Netball Club has kept statistics of its AUSKICK program since the year 2006, when 30 players signed up. The yearly figures are:

30 37 44 51 58

- a. What type of sequence do the above statistics represent? Give reasons for your choice.

2 marks

- b. If growth continues as shown by the statistics above, how many participants will there be in 2015 ?

2 marks

- c. How many participants would have done AUSKICK from 2006 to 2015 inclusive?

2 marks

Question 2

Nearby Terang-Mortlake Football Club have also has kept statistics of its AUSKICK program since the year 2006. These statistics can be represented by a difference equation by letting A_n be the number of AUSKICK participants at the start of its n^{th} year of operation. The difference equation to model the number of participants is

$$A_{n+1} = 1.1 \times A_n + 5 \quad A_0 = 20$$

- a. Write down the first 3 terms of this sequence.

1 mark

- b. What year will the number of participants at Terang-Mortlake overtake the number of participants at Kolora-Noorat?

2 marks

The Kolora-Noorat Netball Club also set out on a publicity campaign to attract more Netball participants. In 2006 they had 50 participants and as a result of the campaign expected growth increased by 10% each year from 2006.

Question 3

The treasurer of the club recognises this will result in a geometric sequence.

- a. What is the value of the second term?

1 mark

- b. What is the common ratio?

1 mark

Module 1: Number patterns – Question 3 – continued

TURN OVER

- c. If this growth pattern continues how many participants are expected to play Netball from 2006 to 2012 inclusive? Give your answer as a whole number.

2 marks

- d. Represent this geometric sequence as a difference equation, where N_n is the number of Netball participants in the n^{th} year if initially there were 50 participants in 2006

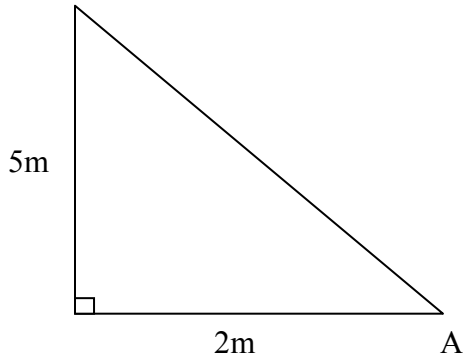
2 marks
Total 15 marks

Module 2: Geometry and trigonometry

Question 1

The local newsagent in Terang is very patriotic and puts a flag pole, with the Australian flag, at the front of his house. The flag pole is 5m high.

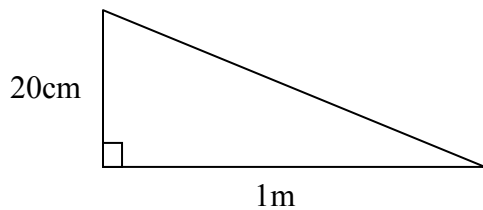
Flag Pole



- a. What is the angle of elevation of the top of the flag pole from point A, 2 m from the flagpole? Give your answer to the nearest degree.

1 mark

The road at the front of the house rises 20cm for every 1m travelled horizontally.



- b. Find the angle of the slope of the road to the nearest degree.

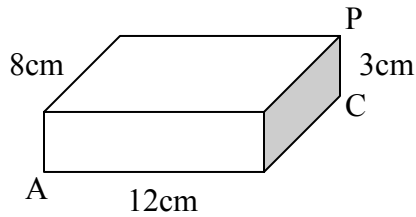
2 marks

Module 2: Geometry and trigonometry – continued

TURN OVER

Question 2

The newsagent's son, Thomas is asked to construct a wooden rectangular pencil box in woodwork class at school, length 12cm, width 8cm and height 3cm.



- a. Determine the length of AC, correct to one decimal place.

1 mark

- b. **Hence**, find the length of the longest pencil, AP, that could fit inside the pencil case, giving your answer to one decimal place.

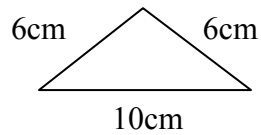
2 marks

- c. Calculate the surface area (in cm^2) and volume (in cm^3) of the pencil box.

2 marks

Module 2: Geometry and trigonometry – Question 2 - continued

- d. Thomas is looking at placing a triangular insignia on the top side of his pencil box.



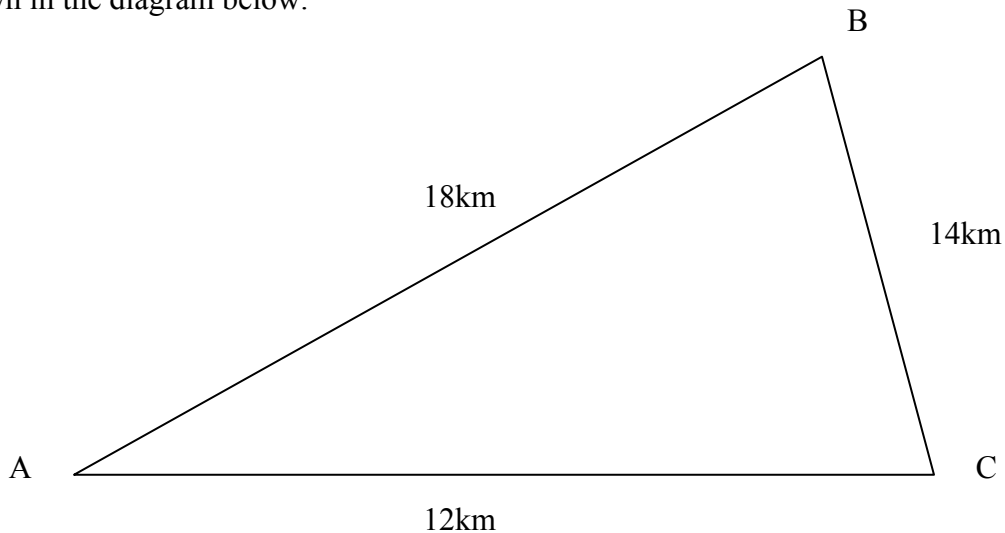
- i. Find using Herons's formula the area of this insignia in cm^2 giving your answer to one decimal place.

- ii. **Hence**, what percentage of the top side is **not** covered by the insignia? Give your answer to the nearest percentage.

Total 3 marks

Question 3

From his house Tom can see 3 oil rigs. These, along with the distances between each oil rig are shown in the diagram below.



Module 2: Geometry and trigonometry – Question 3 – continued
TURN OVER

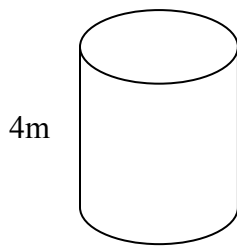
- a. Calculate angle BCA, to one decimal place.

2 marks

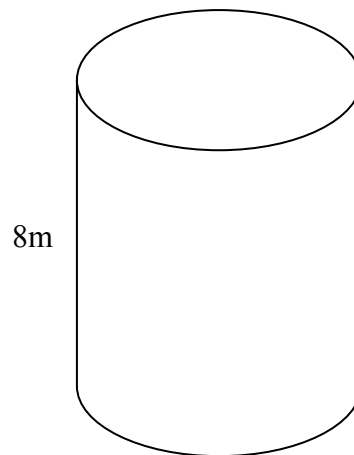
- b. **Hence**, determine the bearing of oil rig C from oil rig B, to one decimal place.

1 mark

Oil rigs B and C have **similar** water tanks, as shown below.



Tank B



Tank C

- c. Tank B the smaller tank of height 4m can hold 6000 litres. How many litres of water can tank C hold if the height of tank C is 8m?

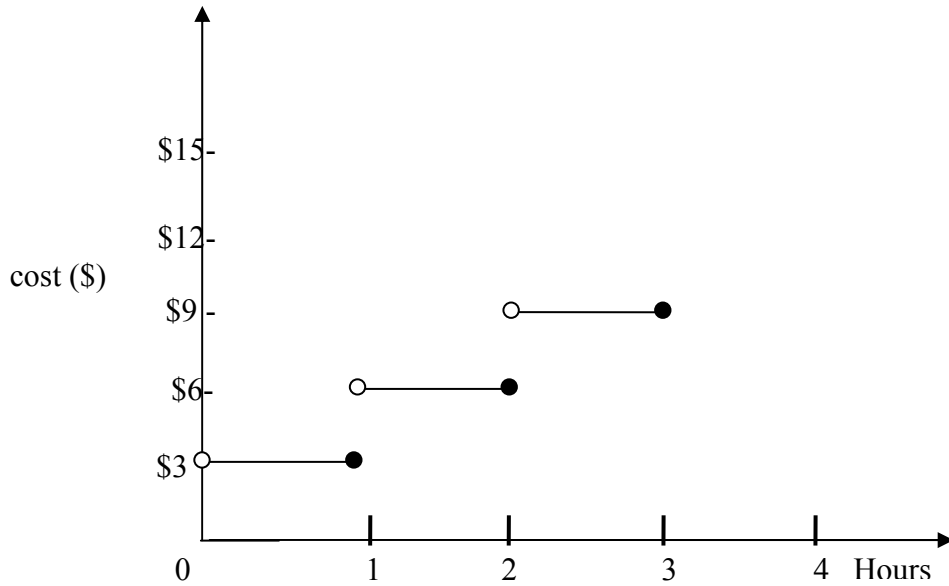
4 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

Customers can park their car near the Terang Newsagency. The cost schedule is shown in the graph below. All day parking costs a flat \$10.



a. How much does it cost to park for 2.5 hours?

_____ 1 mark

The newsagent finds the time t (in hours) it takes to travel at a speed s (in km /hour) to the newsagency from his home and is determined by the formula $t = \frac{80}{s}$

b. If he travels at a speed of 100 km/h, how long would it take him to get to the newsagency? Give your answer in minutes.

1 mark

Module 3: Graphs and relations – Question 1 – continued

TURN OVER

- c. If it took him 50 minutes to get to the newsagency, what speed did he drive at?

2 marks

Question 2

The Terang newsagency has a Sub-agency X which has a receptacle to fit newspapers in when the sub-agency is closed. The receptacle can fit only a total of 100 Ages and Herald-Sun newspapers. The newsagent knows he needs at least 40 Ages and no more than 50 Herald-Suns each day.

If x = the number of Ages and
 y = the number of Herald-Suns

Three of the 5 constraints are given below:

1. $x \geq 0$
2. $y \geq 0$
3. $3 \times x + y \leq 100$

4.

5.

- a. Complete the remaining 2 constraints above in the boxes provided

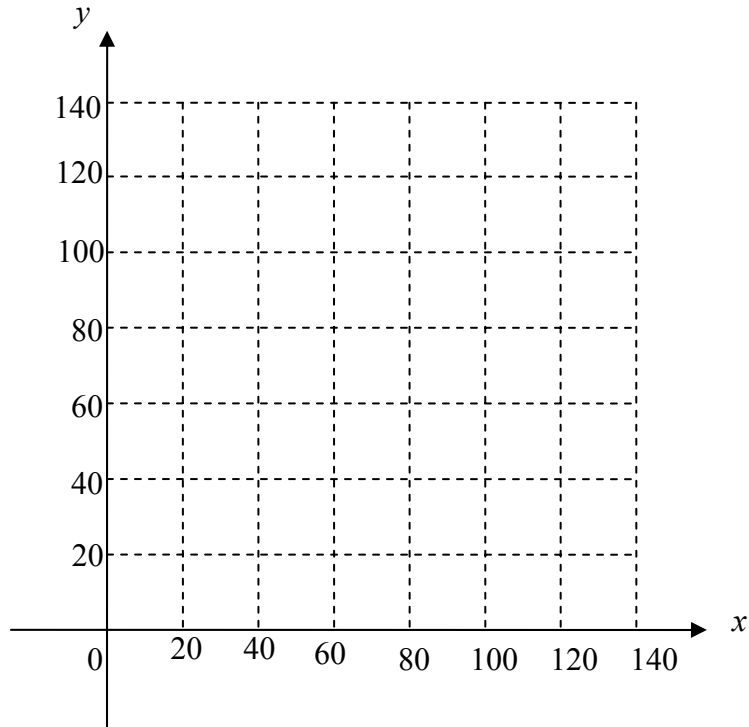
2 marks

If the profit on a Herald-Sun is 30 cents and the profit on an Age is 20 cents

- b. Write down the Objective function, the maximum profit, P, in dollars, in terms of x and y

1 mark

Module 3: Graphs and relations – Question 1 – continued



- c. On the graph above, draw in the 5 constraints from part **b** and **shade** in the feasible region, **labelling** the axial intercepts.

4 marks

- d. **Hence**, calculate the maximum profit made by the newsagent.

2 marks

- e. How many Ages and Herald-Suns need to be sold to achieve this maximum profit?

2 marks

Total 15 marks

Module 4: Business-related mathematics

Question 1

The Terang Newsagency purchased a sub-agency at Argyle St in 2000 for \$75000 and hopes to sell it this year for \$320000.

- a. What percentage increase does this represent, correct to one decimal place.

1 mark

- b. If the newsagent wishes to make a percentage increase of 500%, what will he need to sell the sub-agency at Argyle St for?

1 mark

- c. The newsagent sells a geometric set for \$13.20 including a GST (Goods and Services Tax) of 10%. What is the cost of the geometric set excluding the GST?

1 mark

Question 2

The newsagent sells notebook computers for \$650. Customers can purchase the notebook computers on a hire purchase plan arrangement putting down a deposit of \$150 plus \$50 per month for one year.

- a. What is the overall cost of the notebook computer using this hire purchase plan?

2 marks

Module 4: Business-related mathematics – Question 2 – continued
TURN OVER

- b. What flat rate of interest has been charged?

2 marks

- c. What effective rate of interest does this represent? Give your answer to one decimal place.

2 marks

- d. If the computer depreciates in value by 15% each year, what will the value of the computer be worth in three years time?

2 marks

- e. If the computer is worth \$200 in 5 years time, what annual depreciation rate does this represent, to one decimal place.

1 mark

Question 3

The newsagent decides to invest in the purchase of another sub-agency. He takes out a reducing balance loan of \$500000 at 8.57% per annum, compounded monthly over 20 years.

- a. What are his monthly repayments to the nearest cent?

1 mark

- b. If he can afford \$5000 per month, how long will the loan take to be paid off? Give your answer to the nearest month.

1 mark

- c. If he had taken out an **interest only** loan, what would his repayment be per month, to the nearest cent?

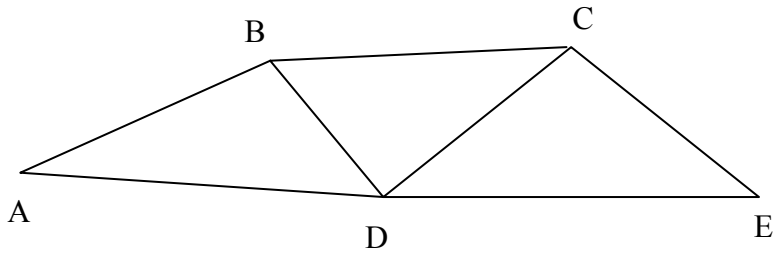
1 mark

Total 15 marks

TURN OVER

Module 5: Networks and decision mathematics

Question 1



- a. Write down the degree of all the vertices of the above graph.

Deg A = ___ Deg B = ___ Deg C = ___ Deg D = ___ Deg E = ___

1 mark

- b. Give reasons why Euler paths exist for the graph above.

1 mark

- c. Add another edge to the graph above to make an Euler Circuit

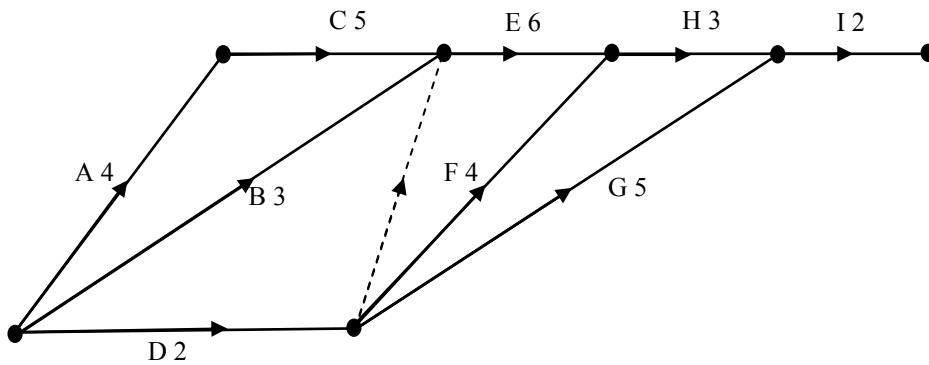
1 mark

- d. Hence write down an Euler Circuit below.

1 mark

Question 2

The following network shows the activities that are needed to complete a project and their completion times (in hours).



- a. Determine the minimum time, in hours, to complete the project.

1 mark

- b. Write down the critical path

1 mark

- c. Determine the float time, in hours, for activity D.

1 mark

In order to speed up the project, which will incur a cost, some activities can be reduced in time. The activities that can be reduced in time are activities A, D, E, F, and G.

- d. Which of these activities, if reduced in time individually, would **not** result in an earlier completion time of the project?

2 marks

The cost of reducing an activity is \$60 per hour for each activity. The maximum reduction in time for each of the five activities A, D, E, F and G, is 2 hours.

- e. What now is the minimum time in hours for the project to be completed given that certain activities can be reduced in time?

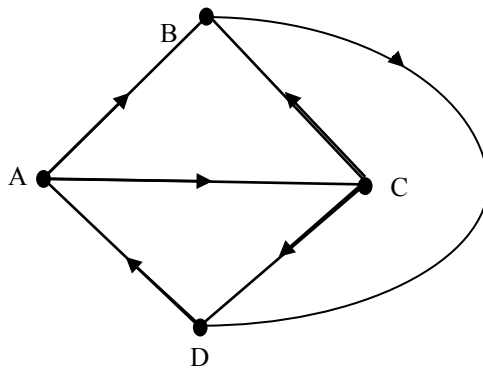
1 mark

- f. What extra cost is incurred in completing the project now?

1 mark

Question 3

Four players compete in a round robin chess championship. The players are labelled A, B, C, and D. An arrow from A to B indicates player A defeated player B.



- a. Name two players who have equal totals of one-step dominances.

1 mark

- b. Over which player does player C have both a one-step and two-step dominance?

1 mark

Module 5: Networks and decision mathematics – Question 3 – continued

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In determining the final order of dominance, the number of one-step and two-step dominances are added together.

- c. List the players in order from first to fourth.

2 marks

Total 15 marks

TURN OVER

Module 6: Matrices**Question 1**

The Terang newsagency has altogether, 3 sub-agencies X,Y,Z. The number of Ages(A) and Herald-Suns(H) delivered to these agencies each day are recorded in Matrix N.

$$N = \begin{array}{cc} & \begin{array}{cc} \text{A} & \text{H} \end{array} \\ \begin{bmatrix} 80 & 110 \\ 70 & 120 \\ 50 & 90 \end{bmatrix} & \begin{array}{c} \text{X} \\ \text{Y} \\ \text{Z} \end{array} \end{array}$$

- a. What is the order of matrix N

1 mark

- b. A price matrix P, shows the price in dollars for an Age and Herald-Sun respectively

$$P = \begin{array}{cc} \begin{bmatrix} 1.90 \\ 1.80 \end{bmatrix} & \begin{array}{c} \text{A} \\ \text{H} \end{array} \end{array}$$

- i. Find the matrix product NP

Module 6: Matrices – Question 1 – continued

ii. What does this product represent?

1+1 = 2 marks

Question 2

The newsagencies records statistics about three different magazines:

x = X-Factor, y = Y-Factor and z = Z-Factor

sold in a particular week and the revenue(in \$'s) from the sales of the magazines

$$4x + 3y + 7z = 57$$

$$9x + 5y + 2z = 64.50$$

$$7y + 3z = 47$$

a. Write these equations in matrix form below

1 mark

b. Is there a **unique** solution to this matrix equation? Justify your answer.

2 marks

c. Using your graphics calculator, write down the **inverse** matrix that may be used to solve this matrix equation. Express your answer in fraction form.

1 mark

Module 6: Matrices – Question 2 – continued

TURN OVER

d. Hence calculate the price of the X-Factor, Y-Factor and Z-Factor magazines.

2 marks

Question 3

The newsagent tracks the patronage of his 3 sub-agencies X, Y and Z and finds

- 80% of customers who go to sub-agency X today will return there tomorrow
- 5% of customers who go to sub-agency X today will go to sub-agency Y there tomorrow
- 15% of customers who go to sub-agency X today will go to sub-agency Z there tomorrow
- 90% of customers who go to sub-agency Y today will return there tomorrow
- 8% of customers who go to sub-agency Y today will go to sub-agency X there tomorrow
- 2% of customers who go to sub-agency Y today will go to sub-agency Z there tomorrow
- 92% of customers who go to sub-agency Z today will return there tomorrow
- 7.5% of customers who go to sub-agency Z today will go to sub-agency X there tomorrow
- 0.5% of customers who go to sub-agency Z today will go to sub-agency Y there tomorrow

a. Enter this information as Transition matrix T

$$T = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

2 marks

Yesterday the newsagent found 200 entered sub-agency X, 250 went to sub-agency Y and 350 to sub-agency Z.

b. Write this information in the matrix $S_0 =$ $\begin{bmatrix} & \\ & \end{bmatrix}$

1 mark

Module 6: Matrices – Question 3 – continued

- c. How many customers are expected to enter sub-agencies X, Y and Z respectively tomorrow?
Write your answers to the nearest whole number.

1 mark

- d. How many customers are expected to enter the sub-agencies X, Y and Z in 4 days time.
Write your answers to the nearest whole number.

1 mark

- e. How many customers are expected to enter the sub-agencies X, Y and Z each day in the long term?
Justify your answer.

1 mark
Total 15 marks

END OF QUESTION AND ANSWER BOOK