

**The Mathematical Association of Victoria**

**FURTHER MATHEMATICS 2011**

**Trial Written Examination 1--SOLUTIONS**

**ANSWERS**

**SECTION A: Core--Data analysis**

- |       |       |       |      |       |
|-------|-------|-------|------|-------|
| 1. B  | 2. B  | 3. D  | 4. E | 5. B  |
| 6. C  | 7. E  | 8. A  | 9. A | 10. A |
| 11. D | 12. D | 13. C |      |       |

**SECTION B: MODULES**

**Module 1: Number Patterns**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. C | 2. A | 3. B | 4. A | 5. D |
| 6. D | 7. D | 8. E | 9. C |      |

**Module 2: Geometry and trigonometry**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. B | 2. E | 3. D | 4. A | 5. A |
| 6. E | 7. B | 8. C | 9. D |      |

**Module 3: Graphs and relations**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. D | 2. E | 3. C | 4. E | 5. D |
| 6. C | 7. B | 8. D | 9. B |      |

**Module 4: Business-related mathematics**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. B | 2. A | 3. D | 4. B | 5. C |
| 6. A | 7. D | 8. D | 9. E |      |

**Module 5: Networks and decision mathematics**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. E | 2. A | 3. B | 4. C | 5. A |
| 6. C | 7. E | 8. B | 9. B |      |

**Module 6: Matrices**

- |      |      |      |      |      |
|------|------|------|------|------|
| 1. B | 2. A | 3. C | 4. C | 5. E |
| 6. D | 7. B | 8. D | 9. E |      |

**Worked solutions--Core: Data analysis****Question 1**

The mode is the most frequently occurring data value, which in this case is 2.

*Answer B*

**Question 2**

The median is the middle value when the data is ordered from smallest to largest. There are 25 data points, so the median will be the  $\frac{25+1}{2} = 13^{\text{th}}$  score, which in this case is 2.

*Answer B*

**Question 3**

This data is positively skewed.

*Answer D*

**Question 4**

75% of 150 = 112.5

$\approx 112$

*Answer E*

**Question 5**

z-score given by  $z = \frac{x - \bar{x}}{s_x}$        $z = \frac{2.5 - 3.25}{0.5} = -1.5$

*Answer B*

**Question 6**

95% of scores lie in the interval between  $\bar{x} - 2s$  and  $\bar{x} + 2s$ , which in this case between 2.25 and 4.25. Hence 47.5% of scores, or half of 95%, lie between the mean and  $\bar{x} + 2s$  which in this case is between 3.25 and 4.25. Approximately 68% of scores lie in the interval between  $\bar{x} - s$  and  $\bar{x} + s$  which in this case is between 2.75 and 3.25. Hence 34% of scores lie between the mean and  $\bar{x} + s$  which in this case between 2.75 and 3.25. Therefore  $34 + 47.5 = 81.5\%$  of scores will lie between 2.75 and 4.25



*Answer C*

**Question 7**

The mean of grouped data is given by  $\bar{x} = \frac{\Sigma xf}{\Sigma f}$

$$\frac{4 + 12 + 24 + 4a}{4 + 6 + 8 + a} = 3$$

$$\frac{40 + 4a}{18 + a} = 3$$

Using the solve function on a calculator gives  $a = 14$

Algebraically:  $40 + 4a = 3(18 + a)$

$$40 + 4a = 54 + 3a$$

$$40 + a = 54$$

$$a = 14$$

*Answer E*

**Question 8**

Plotting  $y$  against  $x^2$  will not produce a straight line. The other options obtain correlation coefficients of  $r$  over 0.95, whereas for  $y$  against  $x^2$ ,  $r = -0.64$

*Answer A*

**Question 9**

The least squares regression equation will be in the form  $y = a + bx$  where  $a = \bar{y} - b\bar{x}$  and  $b = r \frac{s_y}{s_x}$

$$b = 0.909 \times \frac{87.19}{4.90} = 16.2$$

$$a = \bar{y} - b\bar{x}$$

$$a = 152.38 - 16.17 \times 9 = 6.85$$

$$y = 6.85 + 16.2x$$

The least squares regression line is closest to  $y = 7 + 16x$

*Answer A*

**Question 10**

The points are randomly distributed above and below the linear model, hence the residual plot would show a random distribution of points.

*Answer A*

**Question 11**

$$\text{Year 1: } \bar{x} = \frac{7234}{4} = 1808.5$$

$$\text{Year 2: } \bar{x} = \frac{7195}{4} = 1798.75$$

$$\text{Seasonal index for the second quarter: } \frac{\frac{407}{1808.5} + \frac{425}{1798.75}}{2} = 0.2307$$

Answer D

**Question 12**

The sum of the seasonal indices is equal to the number of seasons.  $0.62 + 0.84 + 1.26 + 1.30 + x = 5$

Hence the seasonal index for Friday is 0.98.

$$\begin{aligned} \text{Deseasonalised figure} &= \frac{\text{actual figure}}{\text{seasonal index}} \\ &= \frac{62}{0.98} \\ &= 63.27 \\ &\approx 63 \end{aligned}$$

Answer D

**Question 13**

Week	Sales		
1	17		
2	21		
		$\frac{17 + 21 + 19 + 23}{4} = 20$	
3	19		
		$\frac{21 + 19 + 23 + 18}{4} = 20.25$	
4	23		
		$\frac{19 + 23 + 18 + 16}{4} = 19$	
5	18		$\frac{19 + 19.25}{2} = 19.125$
		$\frac{23 + 18 + 16 + 20}{4} = 19.25$	
6	16		
7	20		

Answer C

**Module 1: Number patterns****Question 1**

$$r = \frac{t_2}{t_1} = \frac{10}{5} = 2, \quad r = \frac{t_3}{t_2} = \frac{20}{10} = 2, \quad r = \frac{t_4}{t_3} = \frac{40}{20} = 2$$

*Answer C***Question 2**

$$t_3 = 412 \times r^2 = 3708$$

$$r^2 = 9$$

$$r = \pm 3$$

$$t_4 = 412 \times 3^3 = 11124 \quad \text{or} \quad t_4 = 412 \times (-3)^3 = -11124$$

*Answer A***Question 3**

$$f_n - f_{n-1} = 7, \quad f_0 = -2$$

$$f_n = f_{n-1} + 7, \quad f_0 = -2$$

$$f_1 = -2 + 7 = 5 \quad f_2 = 5 + 7 = 12$$

Sequence:  $-2, 5, 12$ *Answer B***Question 4**

$$a_2 + 3 = -5$$

$$a_2 = -8$$

*Answer A***Question 5**

Geometric sequence with  $a = 1000$ ,  $r = 1.05$ .  $t_6$  represents the value at the beginning of the 6<sup>th</sup> year, so the end of the 6<sup>th</sup> year will be  $t_7$ .

*Answer D***Question 6**

$$t_1 = 4 \Rightarrow a = 4$$

$$t_4 = 4 + 3d = 13$$

$$d = 3$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[8 + (6)3] = 91$$

*Answer D*

**Question 7**

$$a = 6, d = -7, t_n = a + (n-1)d$$

$$6 + (n-1)(-7) = -281$$

$$6 - 7n + 7 = -281 \text{ or use "solve" function on calculator}$$

$$13 = -281 + 7n$$

$$7n = 294$$

$$n = 42$$

Answer D

**Question 8**

There are two infinite series:

$$1. \text{ Falling: } a = 3, r = \frac{2}{3} \quad S_{\infty} = \frac{3}{1 - \frac{2}{3}} = 9$$

$$2. \text{ Rebounding: } a = 2, r = \frac{2}{3} \quad S_{\infty} = \frac{2}{1 - \frac{2}{3}} = 6$$

Hence total distance travelled is  $9 + 6 = 15$

Answer E

**Question 9**

$$12000 \times 1.1^7 = 23385$$

Answer C

**Module 2: Geometry and trigonometry****Question 1**

Using Heron's formula

$$\text{Area} = \sqrt{s(s-9)(s-11)(s-15)} \quad \text{where } s = \frac{9+11+15}{2} = 17.5$$

$$\text{Area} = \sqrt{17.5 \times (17.5 - 9)(17.5 - 11)(17.5 - 15)}$$

$$\text{Area} = \sqrt{2417.1875}$$

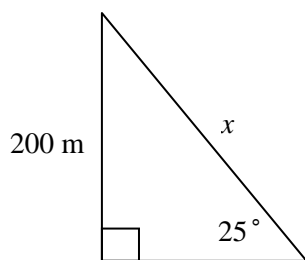
$$\text{Area} = 49.2 \text{ m}^2$$

*Answer B***Question 2**Vertical height of the ski run is  $1850 - 1650 = 200$  metres

$$\sin(25^\circ) = \frac{200}{x}$$

$$x = \frac{200}{\sin(25^\circ)}$$

$$x = 473.2$$

*Answer E***Question 3**

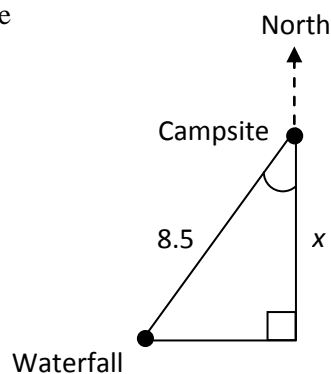
$$\text{Angle } AOB = \frac{360^\circ}{5} = 72^\circ$$

*Answer D***Question 4**Let  $x$  be the distance of the waterfall south of the campsite

$$\cos(52^\circ) = \frac{x}{8.5}$$

$$x = 8.5 \cos(52^\circ)$$

$$x = 5.2$$

*Answer A*

**Question 5**

$$4 \text{ km} = 4 \times 1000 \times 100 = 400,000 \text{ cm}$$

$$1 : 500,000 = x : 400,000$$

$$\frac{1}{500,000} = \frac{x}{400,000}$$

$$x = 0.8 \text{ cm}$$

*Answer A*

**Question 6**

$$\text{Angle } Z = 180^\circ - 30^\circ - 40^\circ = 110^\circ$$

Using the sine rule:

$$\frac{XY}{\sin 110^\circ} = \frac{14}{\sin 30^\circ}$$

$$XY = \frac{14 \sin 110^\circ}{\sin 30^\circ}$$

*Answer E*

**Question 7**

TSA = 2 × triangles + base + 2 × sloping sides

$$\text{TSA} = 2 \times \left( \frac{1}{2} \times 12 \times \sqrt{10^2 - 6^2} \right) + 12 \times 20 + 2 \times (10 \times 20)$$

$$\text{TSA} = 2 \times 48 + 240 + 2 \times 200$$

$$\text{TSA} = 736 \text{ cm}^2$$

*Answer B*



**Question 8**

$$BD = \sqrt{BC^2 + CD^2} = \sqrt{4^2 + 10^2} = \sqrt{116}$$

$$\tan \theta = \frac{AB}{BD}$$

$$\theta = \tan^{-1}\left(\frac{5}{\sqrt{116}}\right)$$

$$\theta = 24.9$$

Answer C

**Question 9**

Triangles  $JGH$  and  $LGK$  are similar so the corresponding sides are in the same ratio.

$$\frac{LK}{JH} = \frac{GK}{GH}$$

$$\frac{LK}{5} = \frac{(6+4)}{4}$$

$$LK = 5 \times \frac{10}{4}$$

$$LK = 12.5 \text{ metres}$$

Answer D

**Module 3: Graphs and relations****Question 1**

Gradient  $m = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$ ; y-intercept:  $c = 3$

Straight line equation:  $y = \frac{3}{2}x + 3$

Multiplying by 2:  $2y = 3x + 6$

Transposing:  $3x - 2y = -6$

Answer D

**Question 2**

$y = mx - 2$  ( $x = 3, y = 4$ )

$4 = 3m - 2$

$6 = 3m$

$m = 2$

Hence equation:  $y = 2x - 2$ , which transposes to  $2x - y = 2$

Answer E

**Question 3**

$151 = 0.1d + 120$

$31 = 0.1d$  or solve using calculator

$d = 310$

Answer C

**Question 4**

The point (1,3) makes both inequalities true.

Answer E

**Question 5**

Let  $a$  = cost of adult ticket;  $c$  = cost of children's ticket

$$2a + 3c = 425 \quad \text{I}$$

$$1a + 4c = 380 \quad \text{II}$$

$$2 \times \text{II} \quad 2a + 8c = 760 \quad \text{III} \quad \text{or solve on calculator}$$

$$\text{III} - \text{I} \quad 5c = 335$$

$c = 67$  substitute into II to find  $a$ .

$$a + 4(67) = 380 \quad a = 112$$

For two adult and two children's tickets Vivienne will pay  $2(112) + 2(67) = \$358$

Answer D

**Question 6**

Substitute  $d = 2$  into each equation to find the equation that produces a corresponding  $h$ -value of  $h = 1$ .

The correct equation is  $h = \frac{4}{d^2}$ .

*Answer C*

**Question 7**

Between Sunbury and Kyneton, Felicity travelled at an average speed of 100 km/h for half an hour, therefore travelling a distance of 50 km. Hence, find the gradient between  $((2.0, 90)$  and  $(2.75, 160)$ .

$m = \frac{160 - 90}{2.75 - 2.0}$  which is closest to 93.

*Answer B*

**Question 8**

The equation will be of the form  $y = kx^3$ , where  $k$  is the gradient of the straight line.

Hence  $y = 4x^3$

*Answer D*

**Question 9**

Test each of the corner points within the feasible region, to determine the point which maximises the objective function  $P = 3x + 2y$ .

$$(0, 10) \quad P = 0 + 20 = 20$$

$$(2, 8) \quad P = 6 + 16 = 22$$

$$(4, 4) \quad P = 12 + 8 = 20$$

$$\left(5\frac{1}{3}, 0\right) \quad P = 16 + 0 = 16$$

Note  $(3, 7)$  is not within the feasible region.

The point  $(2, 8)$  achieves the maximum value for  $P$ .

*Answer B*

**Module 4: Business-related mathematics****Question 1**

Price after discount = 60% of \$300 = \$180

*Answer B*

**Question 2**

$$A = PR^n \quad R = \left(1 + \frac{3}{100}\right) = 1.03 \quad n = 4 \times 4 = 16$$

$$A = 50,000 \times 1.03^{16}$$

$$I = 50,000 \times 1.03^{16} - 50,000$$

*Answer A*

**Question 3**

Amount paid annually = \$1000  $\times$  26 = \$26,000

$$\begin{aligned} \text{Annual interest rate} &= \frac{26,000}{500,000} \times 100\% \\ &= 5.2\% \end{aligned}$$

*Answer D*

**Question 4**

Let  $x$  be the cost for staying one night in a motel before 10% GST is added

$$110\% \text{ of } x = \$171.60$$

$$1.1x = 171.60$$

$$x = \frac{171.60}{1.1} = \$156$$

*Answer B*

**Question 5**

Let  $P$  be the amount invested

$$I = \frac{PRT}{100}$$

$$1000 = \frac{P \times 4 \times 5}{100}$$

$$P = 5000$$

*Answer C*

**Question 6**

$$\text{Depreciation} = \$30,000 - \$7000 = \$23,000$$

$$\$23,000 = 0.20 \times \text{kilometres travelled}$$

$$\text{Kilometres travelled} = \frac{23,000}{0.20} = 115,000$$

Answer A

**Question 7**

Using the TVM solver on a calculator produces an answer closest to \$387,700

Answer D

**Question 8**

$$\text{Value of oven after five years} = 8000 \times 0.85^5 = \$3549.64$$

$$\text{Depreciation} = \$8000 - \$3549.64 = \$4450.36$$

Answer D

**Question 9**

$$\text{Amount borrowed: } P = \$500 - \$100 = \$400$$

$$\text{Total amount paid} = \$100 + \$15 \times 12 \times 3 = \$640$$

$$\text{Interest} = \text{amount paid} - \text{cost price} = \$640 - \$500 = \$140$$

Let  $R$  be the flat interest rate per annum

$T$  = time in years

$I$  = interest charged

$$I = \frac{PRT}{100}$$

$$140 = \frac{400 \times R \times 3}{100}$$

$$R = 11.\dot{6} \% \text{ per annum}$$

The flat interest rate is  $11.\dot{6} \% \text{ per annum}$

Effective interest rate

$$\approx \frac{2n}{n+1} \times \text{Flat interest rate}$$

$$\approx \frac{2 \times 36}{36+1} \times 11.\dot{6} \%$$

$$\approx 22.7 \%$$

Answer E

**Module 5: Networks and decision mathematics****Question 1**

Vertex	Degree
<i>A</i>	5
<i>B</i>	2
<i>C</i>	1
Total	8

Note: A loop has a degree of 2.

Answer *E*

**Question 2**

For any connected planar graph  $v + f = e + 2$ . For 5 faces, only option A with 6 vertices and 9 edges satisfies this relationship.

Answer *A*

**Question 3**

An Eulerian path uses every edge exactly once, but doesn't finish back at the start. Discard options D and E since they finish where they started. Option C does not use edges *CD*, *EF*, *FD* or *BE*. Option A does not use *AE*, *ED*, *CF* or *CD*. Only option B uses every edge.

Answer *B*

**Question 4**

Construct adjacency matrix, *M*. Enter into calculator to find  $M^2$ ,  $M^3$  and  $M^4$ .

$$M = \begin{matrix} & & & & & & E \\ A & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$M^2 = \begin{matrix} & & & & & & E \\ A & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

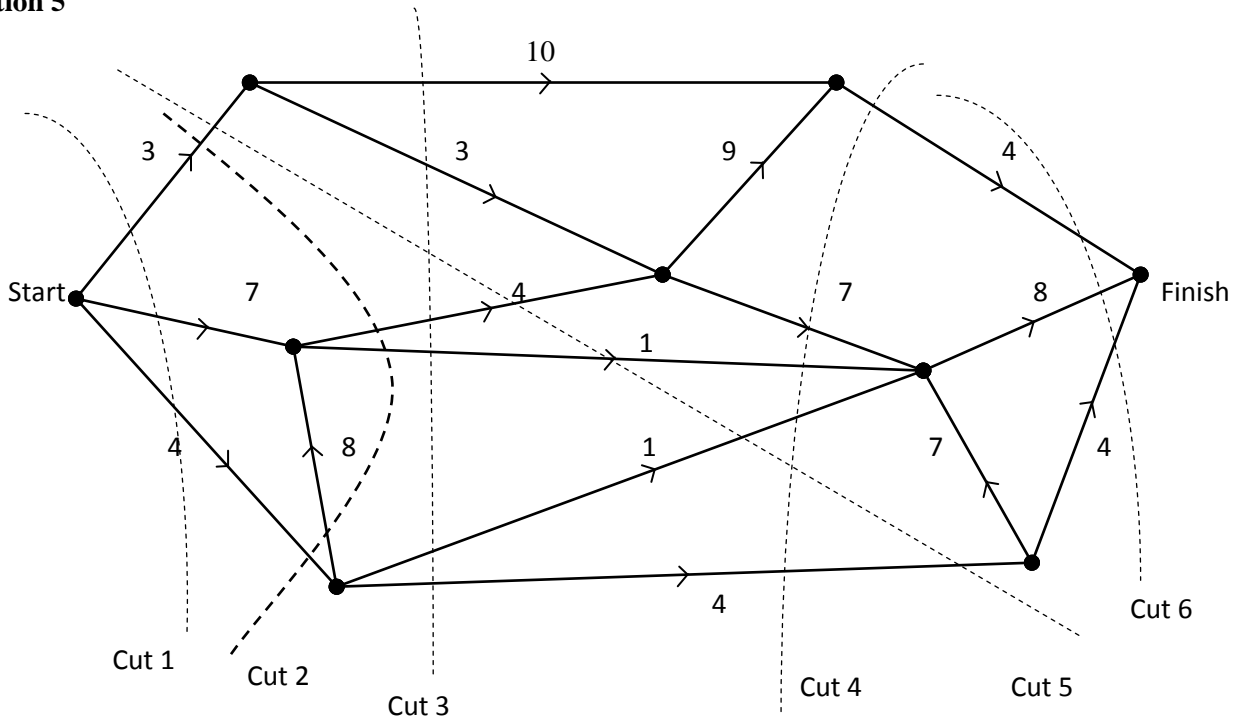
$$M^3 = \begin{matrix} & & & & & & E \\ A & \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$M^4 = \begin{matrix} & & & & & & E \\ A & \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$M^3$  and  $M^4$  both have a 1 in the position indicating a path between A and E.

Answer *C*

**Question 5**



Cut 1: capacity =  $3 + 7 + 4 = 14$

Cut 2: capacity =  $3 + 4 + 1 + 0$  (the 8 is against the flow) +  $4 = 12$

Cut 3: capacity =  $10 + 3 + 4 + 1 + 1 + 4 = 23$

Cut 4: capacity =  $4 + 7 + 1 + 1 + 4 = 17$

Cut 5: capacity =  $3 + 4 + 1 + 1 + 4 = 13$

Cut 6: capacity =  $4 + 8 + 4 = 16$

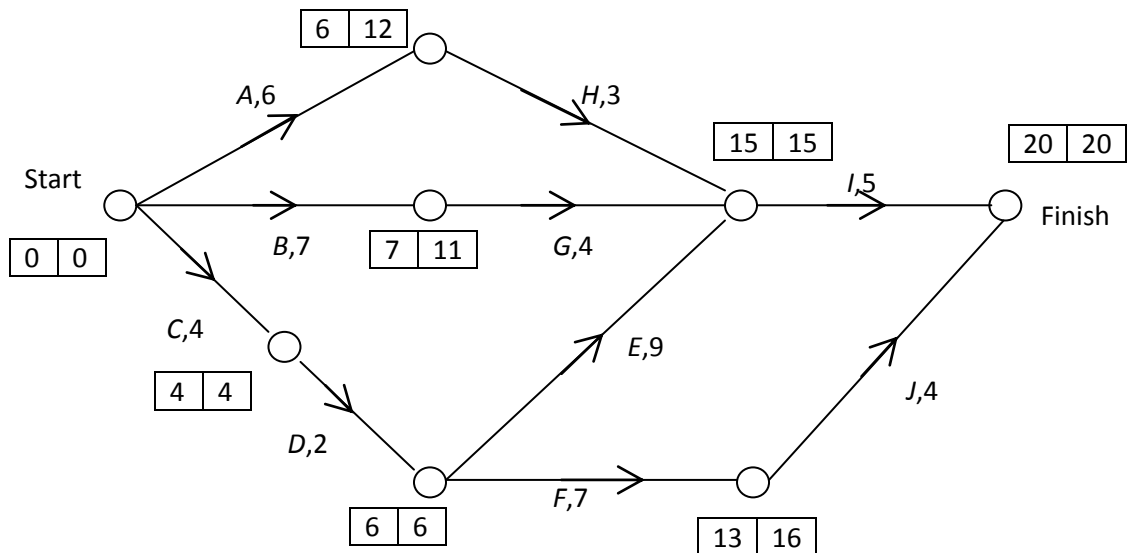
Answer A

**Question 6**

The immediate predecessors of activity F are C, D and E

Answer C

**Question 7**



Activity	Duration	Earliest Start time	Latest Start time	Slack
<i>A</i>	6	0	6	$6 - 0 = 6$
<i>B</i>	7	0	4	$4 - 0 = 4$
<i>C</i>	4	0	0	$0 - 0 = 0$
<i>D</i>	2	4	4	$4 - 4 = 0$
<i>E</i>	9	6	6	$6 - 6 = 0$
<i>F</i>	7	6	9	$9 - 6 = 3$
<i>G</i>	4	7	11	$11 - 7 = 4$
<i>H</i>	3	6	12	$12 - 6 = 6$
<i>I</i>	5	15	15	$15 - 15 = 0$
<i>J</i>	4	13	16	$16 - 13 = 3$

The critical path will be *CDEI* with a length of  $4 + 2 + 9 + 5 = 20$

*Answer E*

### Question 8

Reducing *D* by 1 allows *B* to be reduced by 2, so that *D* takes 5 days and *B* and *E* take 5 days in total.

Savings occur by reducing *G* by 2 days.

Reduce *B* by 2 to 3 days; reduce *D* by 1 to 5 days; reduce *G* by 2 to 3 days.

The project completion time is now  $4 + 3 + 2 + 3 + 6 + 3 = 21$  days

*Answer B*

### Question 9

Applying the Hungarian algorithm:

$$\begin{array}{c}
 D \quad E \quad F \\
 A \begin{bmatrix} 4 & 6 & 6 \end{bmatrix} \\
 B \begin{bmatrix} 3 & 5 & 7 \end{bmatrix} \\
 C \begin{bmatrix} 5 & 2 & 3 \end{bmatrix}
 \end{array}$$

Subtract the smallest number from each row.

$$\begin{array}{c}
 D \quad E \quad F \\
 A \begin{bmatrix} 0 & 2 & 2 \end{bmatrix} \\
 B \begin{bmatrix} 0 & 2 & 4 \end{bmatrix} \\
 C \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}
 \end{array}$$



Subtract the smallest number from each column.

$$\begin{array}{r} D \quad E \quad F \\ A \begin{bmatrix} 0 & 2 & 1 \end{bmatrix} \\ B \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \\ C \begin{bmatrix} 3 & 0 & 1 \end{bmatrix} \end{array}$$

Draw the minimum number of lines to cover all zeros.

$$\begin{array}{r} D \quad E \quad F \\ A \begin{bmatrix} 0 & 2 & 1 \end{bmatrix} \\ B \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \\ C \begin{bmatrix} 3 & 0 & 0 \end{bmatrix} \end{array}$$

Subtract the smallest uncovered number from all the uncovered numbers.

Add the smallest uncovered number (1) to any value where the lines intersect

$$\begin{array}{r} D \quad E \quad F \\ A \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ B \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \\ C \begin{bmatrix} 4 & 0 & 0 \end{bmatrix} \end{array}$$

Belinda does task  $D$ ; Abigail does task  $F$ ; Clare does task  $E$


$$3 + 6 + 2 = 11$$

*Answer B*

**Module 6: Matrices****Question 1**

$NM$  does not exist since the number of columns of  $N \neq$  the number of rows of  $M$

$$(2 \times 2) \times (1 \times 2)$$

  
 not equal

*Answer B*

**Question 2**

Determinant of  $N$

$$= ad - bc$$

$$= 2 \times 5 - 3 \times 4$$

$$= 10 - 12$$

$$= -2$$

*Answer A*

**Question 3**

Express equations in appropriate form

$$y = 2x - 1 \quad \Rightarrow \quad 2x - y = 1$$

$$6y + 3x = 3 \quad \Rightarrow \quad 3x + 6y = 3$$

Write in matrix form  $\begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

*Answer C*

**Question 4**

The bushwalkers visit the locations in the following order

$W, Y, Z, V, X$

The hike ends at location  $X$ .

*Answer C*

**Question 5**

$$\text{Cost (\$)} = \begin{matrix} & \text{petrol} & \text{oil} & & \text{car} & \text{van} \\ \text{station A} & \begin{bmatrix} 1.60 & 5.00 \end{bmatrix} & & \times & \begin{bmatrix} 40 & 70 \end{bmatrix} & \text{petrol} \\ \text{station B} & \begin{bmatrix} 1.50 & 6.00 \end{bmatrix} & & & \begin{bmatrix} 5 & 8 \end{bmatrix} & \text{oil} \end{matrix}$$

$$\text{Cost (\$)} = \begin{matrix} & \text{car} & \text{van} \\ \text{station A} & \begin{bmatrix} 1.60 \times 40 + 5 \times 5 & 1.60 \times 70 + 5 \times 8 \end{bmatrix} \\ \text{station B} & \begin{bmatrix} 1.50 \times 40 + 6 \times 5 & 1.50 \times 70 + 6 \times 8 \end{bmatrix} \end{matrix}$$

$$\text{Cost (\$)} = \begin{matrix} & \text{car} & \text{van} \\ \text{station A} & \begin{bmatrix} 89 & 152 \end{bmatrix} \\ \text{station B} & \begin{bmatrix} 90 & 153 \end{bmatrix} \end{matrix}$$

Answer E

**Question 6**

$$\begin{bmatrix} 0 & 0 & 1.2 \\ 0 & 0.7 & 0 \\ 1.1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \text{publicity} \\ \text{sales} \\ \text{technology} \end{bmatrix} = \begin{bmatrix} 1.2 \times \text{technology} \\ 0.7 \times \text{sales} \\ 1.1 \times \text{publicity} \end{bmatrix}$$

Technology staff has increased by 20%

Sales staff has decreased by 30%

Publicity staff has increased by 10%

Answer D

**Question 7**

$$S_5 = T S_4$$

$$\text{then } S_4 = T^{-1} S_5$$

$$\text{and } S_3 = T^{-1} S_4 = T^{-2} S_5$$

$$S_3 = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}^{-2} \begin{bmatrix} 1150 \\ 850 \end{bmatrix} = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

Answer B

**Question 8**

The percentage of members who stay at location  $C$  and those who leave it must total 100%.

Percentage of members who leave  $C$  and go to  $B$  is  $100\% - (40\% + 20\%) = 40\%$

*Answer D*

**Question 9**

If Jerry buys a Hawaiian pizza this week there is a 20% chance he will buy an Italian pizza next week. Option A is false.

There is no evidence to support the suggestion that Jerry prefers to buy Gourmet pizzas over Hawaiian pizzas. The rows and columns of both types of pizzas total 1. Discard option B

Jerry's favorite type of pizza is irrelevant to the question. Discard option C

Jerry has a 70% chance of buying a Gourmet pizza this week and a Gourmet pizza next week. Option D is false.

Option E is true. The following calculations show that in the long run Jerry will buy an equal proportion of each type of pizza.

$$\begin{bmatrix} 0.7 & 0.1 & 0.1 & 0.1 \\ 0.05 & 0.6 & 0.2 & 0.15 \\ 0.15 & 0.2 & 0.6 & 0.05 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}^{100} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0.7 & 0.1 & 0.1 & 0.1 \\ 0.05 & 0.6 & 0.2 & 0.15 \\ 0.15 & 0.2 & 0.6 & 0.05 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}^{101} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

*Answer E*

**END OF SOLUTIONS**