INSIGHT

YEAR 12 *Trial Exam Paper*

2011
FURTHER MATHEMATICS
UNIT 3
Written examination 1

Solutions book

This book presents:

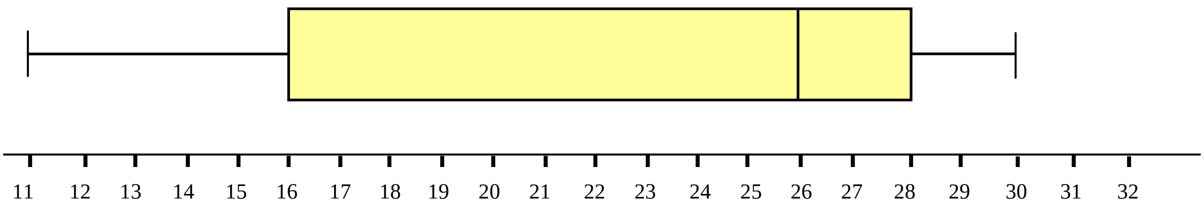
- worked solutions, giving you a series of points to show you how to work through the questions
- tips and guidelines.

SECTION A

Core: Data analysis

Question 1

Which of the following sets of data does the box plot below represent?



A. 11, 11, 16, 26, 26, 28, 30
B. 11, 12, 12, 13, 15, 16, 16, 17, 18, 20, 22, 26, 26, 27, 27, 28, 28, 28, 29, 29, 30, 30, 30
C. 11, 11, 12, 12, 12, 16, 17, 18, 25, 26, 27, 28, 28, 29, 29, 30, 30, 31, 32
D. 11, 11, 11, 16, 16, 26, 26, 28, 28, 30
E. 11, 11, 11, 12, 13, 14, 16, 16, 18, 20, 25, 26, 27, 28, 28, 29, 29, 30, 30, 30, 30

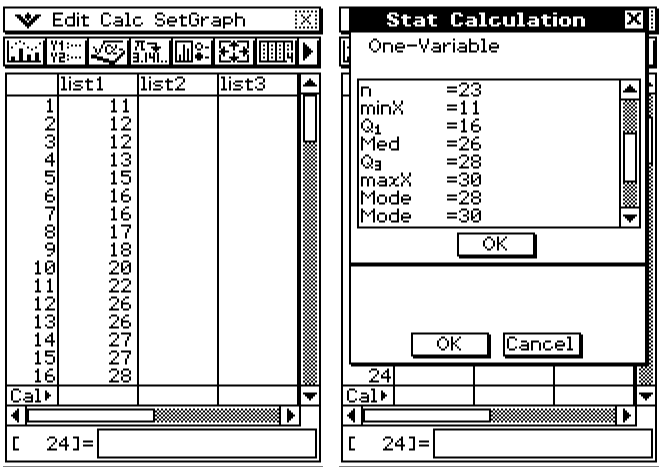
Answer is B

Worked Solution

Calculate the five-figure summary for each data set.

Option A: Minimum value = 11, Q_1 = 11, Median = 26, Q_3 = 28, Maximum value = 30

Option B: Minimum value = 11, Q_1 = 16, Median = 26, Q_3 = 28, Maximum value = 30
You can use a calculator:



Option C: Minimum value = 11, Q_1 = 12, Median = 26, Q_3 = 29, Maximum value = 32

Option D: Minimum value = 11, Q_1 = 11, Median = 21, Q_3 = 28, Maximum value = 30

Option E: Minimum value = 11, Q_1 = 13.5, Median = 25, Q_3 = 29, Maximum value = 30

SECTION A – continued

The following information refers to Questions 2 and 3.

A survey is conducted to investigate the amount of time per day each student in a randomly chosen group of 50 high school students spends on social networking sites. These times are approximately normally distributed with a mean of 4.25 hours and a standard deviation of 1.25 hours.

Question 2

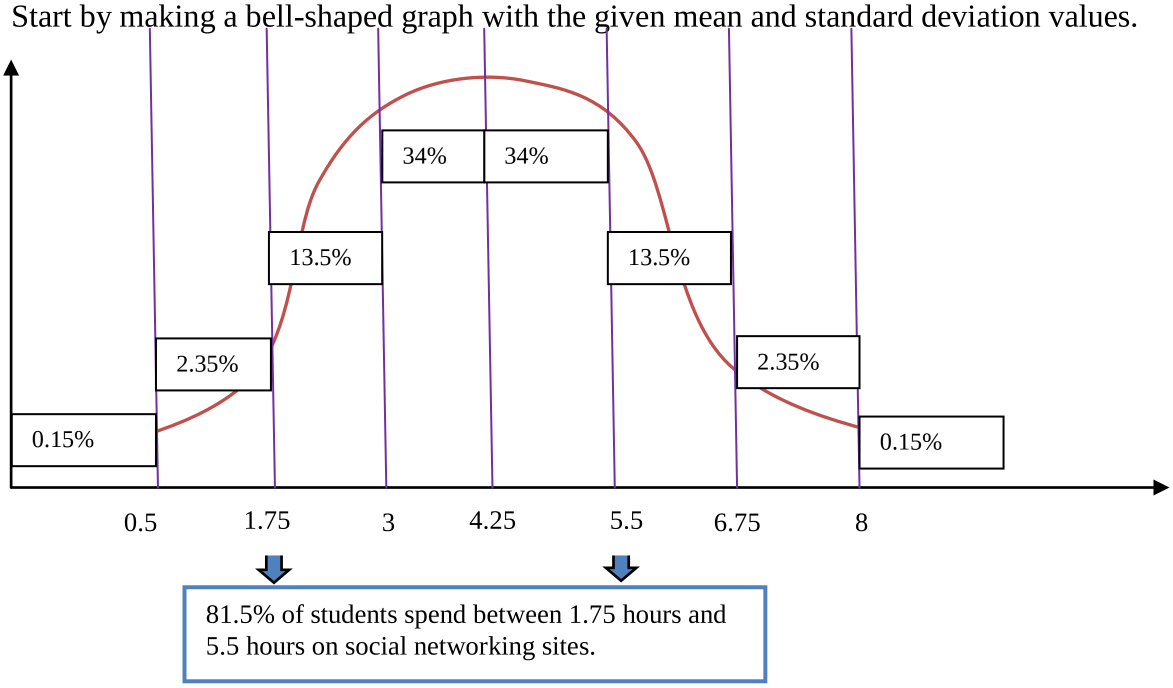
The number of students who do not spend between 1.75 hours and 5.5 hours on social networking sites is closest to

A. 68
B. 18
C. 82
D. 41
E. **9**

Answer is E

Worked Solution

Start by making a bell-shaped graph with the given mean and standard deviation values.



So 100% – 81.5% = 18.5% of students do not spend between 17.5 hours and 5.5 hours on social networking sites.

$50 \times \frac{18.5}{100} = 9.25$

$\cong 9$

So 9 students do not spend between 17.5 hours and 5.5 hours on social networking sites.

Note: If you obtained answer B, the percentage of students was found, not the number of students.

Question 3

Linda is a high school student who participated in this survey. She spends 3.75 hours a day on social networking sites. Which of the following is closest to the standardised time Linda spent on social networking sites (her standard *z* score)?

A. 0.59 hours
B. 0.8 hours
C. 0.4 hours
D. -0.8 hours
E. **-0.4 hours**

Answer is E

Worked Solution

$$z = \frac{x - \bar{x}}{s_x}$$
$$= \frac{3.75 - 4.25}{1.25}$$
$$= -0.4 \text{ hours}$$

The following information refers to Questions 4 and 5.

The following table shows the recycling attitudes of adults with different education levels in a local community group.

	Education level		
Recycling attitudes	**Year 10 or less**	**Year 11 or 12**	**University**
Always recycles	3	9	13
Sometimes recycles	8	7	9
Never recycles	8	5	2

Question 4

We can conclude from the table that the type of investigation is

A. time series analysis of the recycling attitudes of adults in a local community group.
B. **bivariate analysis of the recycling attitudes of adults in a local community group versus their education level.**
C. bivariate analysis of the education level of adults in a local community group versus their recycling attitudes.
D. univariate analysis of the recycling attitudes of adults in a local community group.
E. univariate analysis of the education level of adults in a local community group.

Answer is B

Worked Solution

'Education level' is presented as an independent variable and 'recycling attitudes of adults in a local community group' is presented as a dependent variable. So this is the bivariate analysis of the recycling attitudes of adults in a local community group versus their education level.

Note: If you obtained answer C, *x* versus *y* was written, not *y* versus *x*. Bivariate analysis should be written as *y* versus *x*.

Question 5

We can conclude from the table that

A. 29.2% of adults with Year 11 or 12 education level sometimes recycle.
B. 33.3% of adults with Year 11 or 12 education level never recycle.
C. 15.8% of adults with Year 10 or less education level sometimes recycle.
D. **54.2% of adults with university education level always recycle.**
E. 37.5% of adults with university education level never recycle.

Answer is D

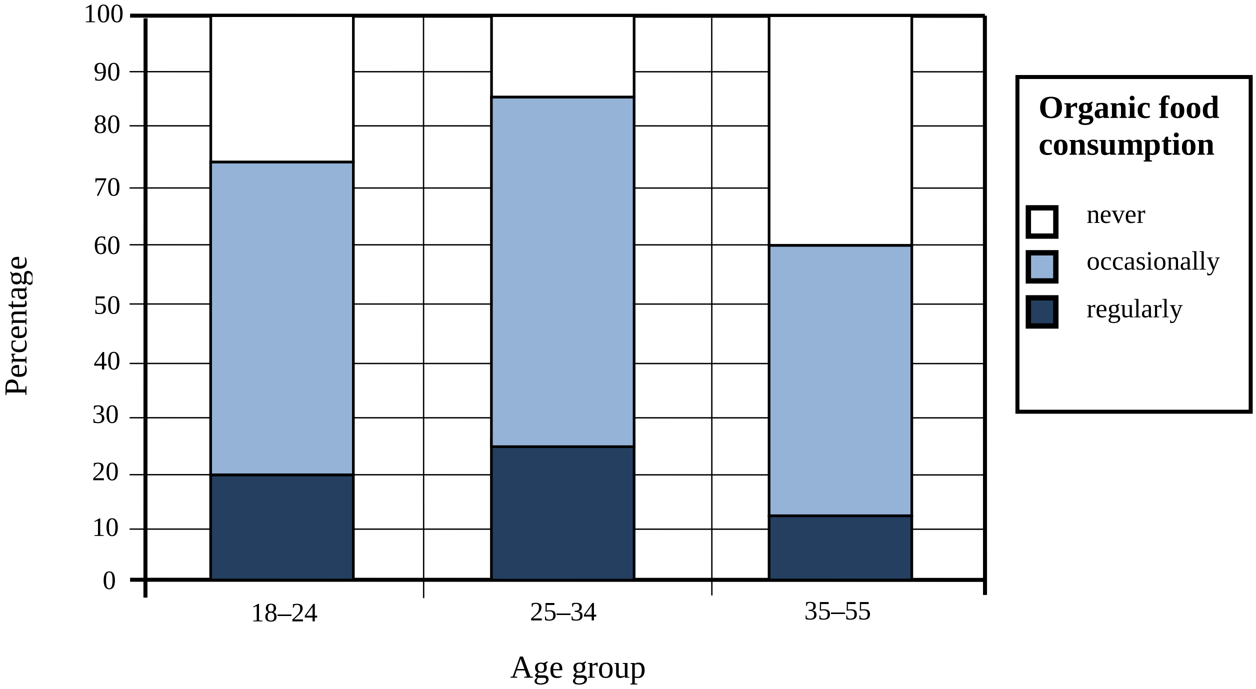
Worked Solution

Find the percentages for all five options.

Option A: 7 adults with Year 11 or 12 education level out of 21 adults with Year 11 or 12 education level sometimes recycle. Expressed as a percentage, this is $\frac{7}{21} \times 100\% = 33.3\%$.
Option B: 5 adults with Year 11 or 12 education level out of 21 adults with Year 11 or 12 education level never recycle. Expressed as a percentage, this is $\frac{5}{21} \times 100\% = 23.8\%$.
Option C: 8 adults with Year 10 or less education level out of 19 adults with Year 10 or less education level sometimes recycle. Expressed as a percentage this is $\frac{8}{19} \times 100\% = 42.1\%$.
Option D: 13 adults with university education level out of 24 adults with university education level always recycle. Expressed as a percentage this is $\frac{13}{24} \times 100\% = 54.2\%$**.**
Option E: 2 adults with university education level out of 24 adults with university education level never recycle. Expressed as a percentage this is $\frac{2}{24} \times 100\% = 8.3\%$.

Question 6

A study investigating the relationship between a person's organic food consumption (never, occasionally or regularly) and age group (18–24, 25–34, 35–55) was conducted. The results are summarised in the percentage segmented bar chart below.

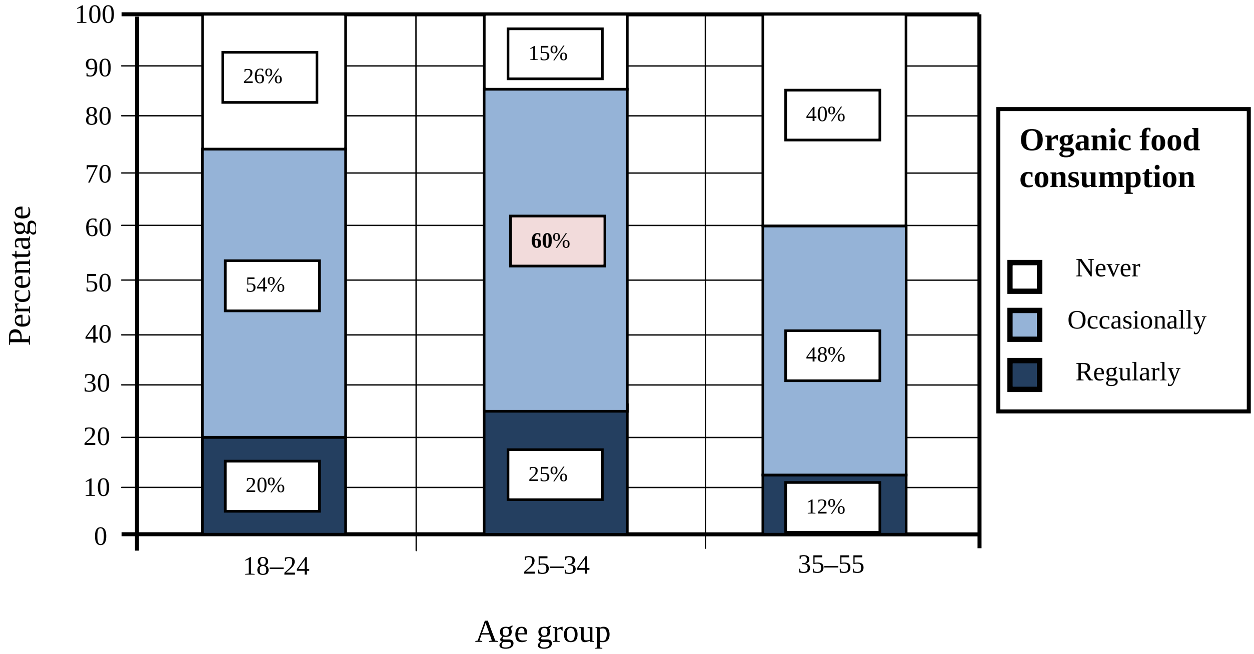


The percentage of people 25–34 years old who consumed organic food occasionally is closest to

A. 20
B. 25
C. 54
D. **60**
E. 84

Answer is D

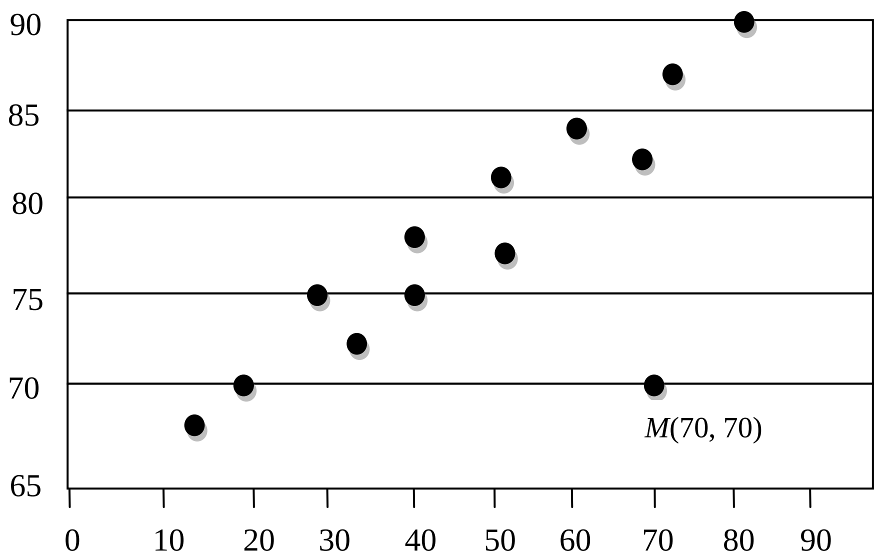
Worked Solution



The percentage of people 25–34 years old who consumed organic food occasionally is 60%.

Question 7

Consider the following scatterplot and point *M* with coordinates (70, 70).



Removal of point *M* would cause

A. **an increase in the gradient of the least squares regression line equation.**
B. a decrease in the gradient of the least squares regression line equation.
C. an increase in the range of *x* values.
D. a decrease in the range of *y* values.
E. a decrease in the Pearson's correlation coefficient.

Answer is A

Worked Solution

When we remove the outlier which is under the least squares regression line, the gradient of the line increases.

Note: Since the x-coordinate and y-coordinate of point M are not the minimum or the maximum values of the ranges of x or y values, removing point M would not change the range of the x or y values. So, neither option C nor option D can be the right answer.
Removing an outlier would cause an increase in the Pearson's correlation coefficient. So, option E cannot be the right answer.

The following information refers to Questions 8, 9 and 10.

A study is conducted to investigate the association between babies' first crawling age and the average temperature during the month they first try to crawl. Researchers want to learn whether babies take longer to learn to crawl in cold months than they do in warmer months (perhaps due to them wearing warmer outfits that restrict their movement in the colder months).

The table below lists the *crawling ages* (in weeks) of 15 babies and the *average monthly temperature* (in degrees Celsius).

Baby's name	**Crawling age (weeks)**	**Average monthly temperature when baby crawled (°Celsius)**
Lily	33	15
Cooper	30	20
Jack	29	25
Rose	34	10
Ruby	26	35
Oliver	28	31
Charlotte	29	29
Hannah	33	12
Georgia	30	19
Thomas	27	38
Noah	28	39
Lucas	31	25
Emma	32	26
Matilda	28	40
William	34	14

Question 8

Using average monthly temperature as the independent variable, a least squares regression line is fitted to the data.

The equation of the least squares regression line is closest to which of the following?

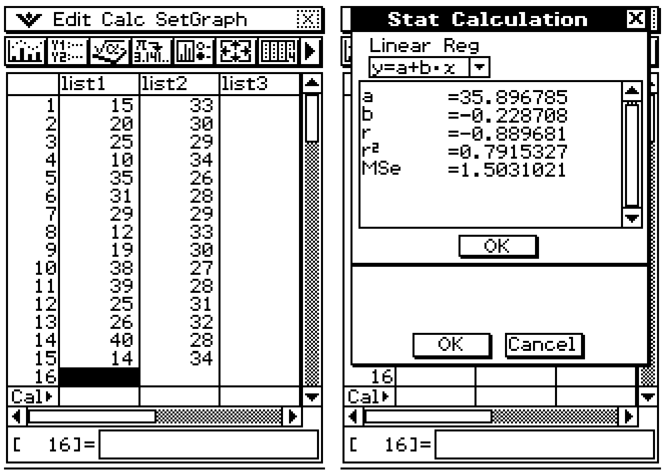
A. average monthly temperature = 129.49 – 3.46 × crawling age
B. average monthly temperature = 35.90 – 0.23 × crawling age
C. average monthly temperature = -3.46 + 129.49 × crawling age
D. crawling age = -0.23 + 35.90 × average monthly temperature
E. **crawling age = 35.90 – 0.23 × average monthly temperature**

Answer is E

Worked Solution

This is a calculator exercise (Stat→Calc→Linear Regression). It must be noted that *average monthly temperature* is the independent variable (the variable x) and *crawling age* is the dependent variable (the variable y).

crawling age = 35.90 – 0.23 × average monthly temperature



Note: If you obtained an answer of D, the gradient and the y-intercept of the equation were swapped.
If you obtained answer A, the dependent and independent variables were swapped while doing the calculation.

Question 9

The value of Pearson's product-moment correlation coefficient (PMCC) and the coefficient of determination for *crawling age* and *average monthly temperature* are respectively closest to

A. 0.79, -0.89
B. **-0.89, 0.79**
C. 0.95, 0.90
D. -0.95, 0.90
E. -0.35, 0.13

Answer is B

Worked Solution

$r = -0.89$ and $r^2 = 0.79$

Note: If you obtained answer A, the positions of r and r^2were swapped. Since the question asks for them to be written 'respectively', the order is important.

Question 10

The least squares regression line equation from Question 8 is then used to predict the crawling age of Matilda.

The residual value (in weeks) for this prediction will be closest to

A. 7.4
B. -7.4
C. **1.3**
D. -1.3
E. 3.5

Answer is C

Worked Solution

Matilda started crawling at 28 weeks when the average monthly temperature was 40°C.
Let's substitute 40 for x into the equation to find the predicted y value:

crawling age = 35.90 – 0.23 × average monthly temperature
$y_{\text{predicted}} = 35.90 - 0.23 \times 40$
$= 26.7$
residual = $y_{\text{actual}} - y_{\text{predicted}}$
$= 28 - 26.7$
$= 1.3$ months

Note: If you obtained answer A, dependent and independent variables were swapped during the calculation.

The following information refers to Questions 11 and 12.

A trend line that can be used to forecast **deseasonalised** quarterly profit for a flower shop is given by:

deseasonalised profit = 12 850 + 109 × quarter number

where Quarter 1 is the summer of 2011, Quarter 2 is the autumn of 2011, and so on.

The seasonal indices for summer, autumn, winter and spring are shown in the table below. The seasonal index for autumn and spring are missing but are known to be equal to each other.

Seasonal index			
Summer	**Autumn**	**Winter**	**Spring**
1.26		0.42	

Question 11

The profit that the flower shop obtains in Spring is typically

A. 32% above the quarterly average.
B. 58% below the quarterly average.
C. 34% below the quarterly average.
D. **16% above the quarterly average.**
E. 18% above the quarterly average.

Answer is D

Worked Solution

We first need to find the seasonal index for spring and autumn.

$$1.26 + 0.42 + x + x = 4$$
$$2x = 2.32$$
$$x = 1.16$$

So, the profit that the flower shop obtains in spring is typically 16% above the quarterly average.

Question 12

The **actual** profit for autumn 2013 is predicted to be closest to

A. \$32 340
B. \$13 940
C. **\$16 170**
D. \$15 918
E. \$13 831

Answer is C

Worked Solution

Autumn 2013 is Quarter number 10. Therefore, deseasonalised profit for autumn 2013 can be calculated in the following way:

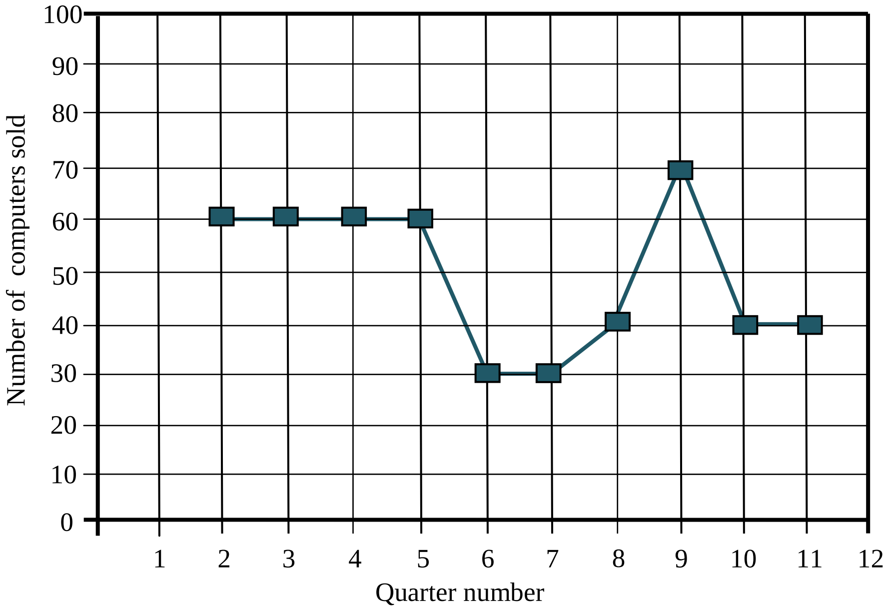
deseasonalised profit = 12 850 + 109 × 10
= 13 940

The deseasonalised profit is $13 940.

actual profit = deseasonalised profit × seasonal index
= 13 940 × 1.16
= 16 170.4
≅ $16 170

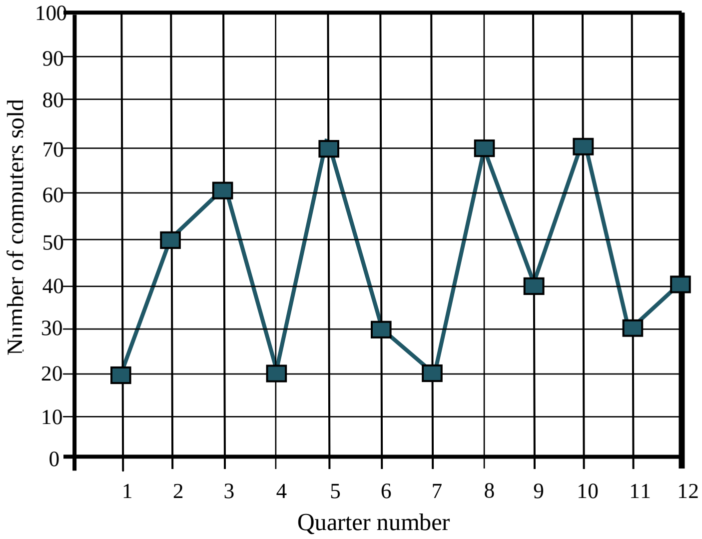
Note: A common mistake students make is to choose option B. If you obtained answer B, you calculated the deseasonalised profit, not the actual profit.

Question 13

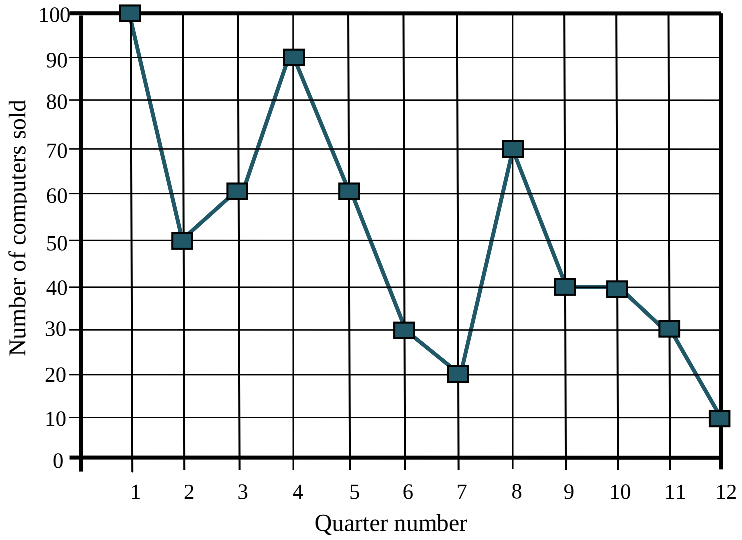


The data on the previous page has been smoothed using three median smoothing. Which one of the following graphs shows the data before it was smoothed?

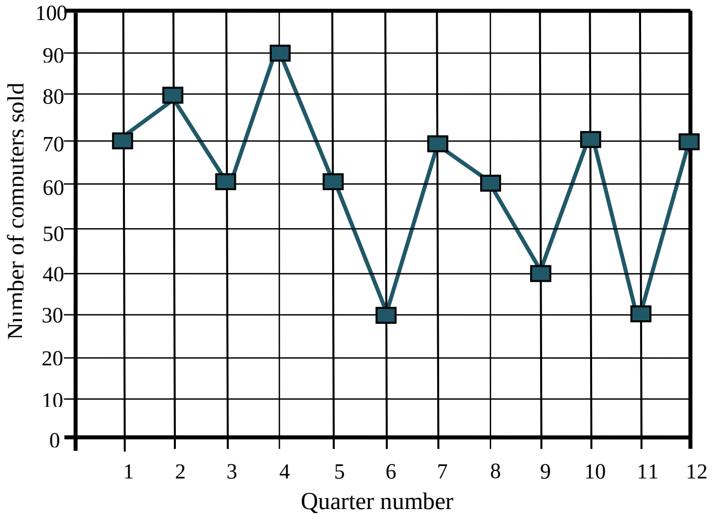
A.



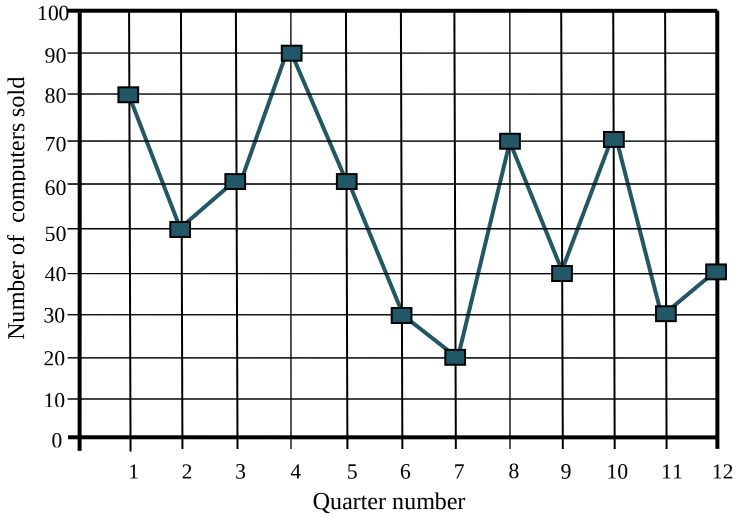
B.



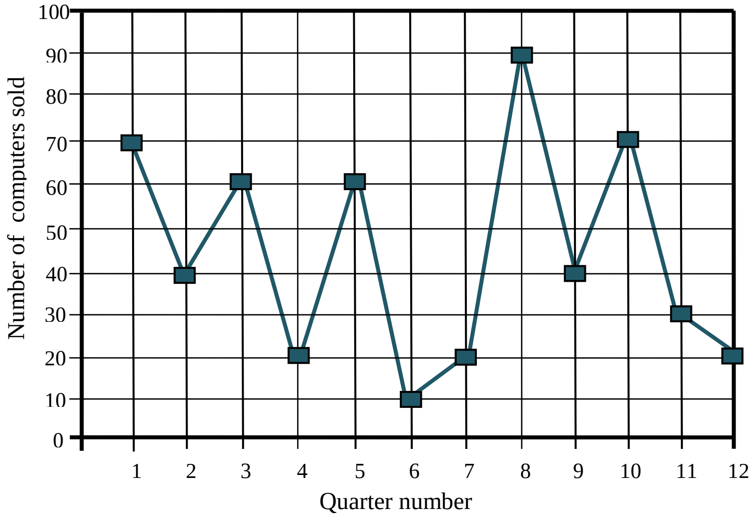
C.



D.



E.



Answer is D

Worked Solution

Smooth the data from all five options.

Option A:

***x* values**	1	2	3	4	5	6	7	8	9	10	11	12
***y* values**	20	50	60	20	70	30	20	70	40	70	30	40
Smoothed *x* values		50	50	60	30	30	30	40	70	40	40	

Option B:

***x* values**	1	2	3	4	5	6	7	8	9	10	11	12
***y* values**	100	50	60	90	60	30	20	70	40	40	30	10
Smoothed *x* values		60	60	60	60	30	30	40	40	40	30	

Option C:

***x* values**	1	2	3	4	5	6	7	8	9	10	11	12
***y* values**	70	80	60	90	60	30	70	60	40	70	30	70
Smoothed *x* values		70	80	60	60	60	60	60	60	40	70	

Option D:

***x* values**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
***y* values**	**80**	**50**	**60**	**90**	**60**	**30**	**20**	**70**	**40**	**70**	**30**	**40**
Smoothed *x* values		**60**	**60**	**60**	**60**	**30**	**30**	**40**	**70**	**40**	**40**	

Option E:

***x* values**	1	2	3	4	5	6	7	8	9	10	11	12
***y* values**	70	40	60	20	60	10	20	90	40	70	30	20
Smoothed *x* values		60	40	60	20	20	20	40	70	40	30	

END OF SECTION A

Module 1: Number patterns

Question 1

For the arithmetic sequence -15, $-11\frac{1}{4}$, $-7\frac{1}{2}$, $-3\frac{3}{4}$, … the values of a (the first term) and d (the common difference) are

A. $a = -15$, $d = 12.25$
B. $\boldsymbol{a = -15, d = 3.75}$
C. $a = -15$, $d = -3.75$
D. $a = 15$, $d = -3.75$
E. $a = 15$, $d = 12.25$

Answer is B

Worked Solution

$a = -15$
$d = t_2 - t_1$
$= -11\frac{1}{4} - -15$
$= 3.75$

Question 2

For the sequence -56, 28, -14, 7, … the sum of the first eight terms is closest to

A. -37.5
B. **-37.2**
C. 37.5
D. 37.2
E. -434

Answer is B

Worked Solution

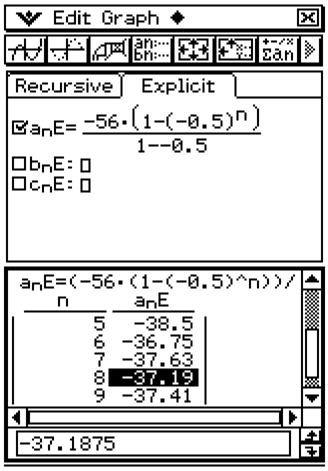
$\frac{t_2}{t_1} = \frac{28}{-56} = -0.5$ and $\frac{t_3}{t_2} = \frac{-14}{28} = -0.5$

So, the sequence is geometric.
$S_n = \frac{a\,(1 - r^n)}{1 - r}$, where $a = -56$, $r = -0.5$ and $n = 8$.

$S_8 = \frac{-56(1 - [-0.5]^8)}{1 - (-0.5)}$
$\cong -37.2$

SECTION B – continued

Alternatively, we can use a calculator to generate the sum of the geometric sequence.



Note: If you obtained answer E, you calculated the sum for an arithmetic sequence.

Question 3

Initially there were 500 trees at a tree farm, and every month 30% of these trees are cut down to produce paper. If t_1 is the initial number of trees and t_2 gives the number of trees remaining in the farm one month later, then which of the following is true for t_n?

A. $t_n = 500(0.3^{n-1})$
B. $t_n = 350(0.7^{n-1})$
C. $\boldsymbol{t_n = 500(0.7^{n-1})}$
D. $t_n = 350(0.3^{n-1})$
E. $t_n = 500(0.7^{n})$

Answer is C

Worked Solution

The first term is 500. So $a = 500$.
If 30% of trees are cut down in a month, 70% remain. So $r = 0.7$.
$t_n = a(r^{n-1})$
So: $t_n = 500(0.7^{n-1})$

Question 4

Clair, the florist, prepared 6150 wedding bouquets for the summer wedding season.
On the first day, she sold 300 bouquets.
Every day after that, she sold 70 more bouquets than the previous day.
On the fifth day, she prepared 3210 more bouquets and she continued selling them together with the rest of the bouquets in the same pattern.

The total number of days that she could sell her bouquets is

A. **13**
B. 10
C. 8
D. 100
E. 85

Answer is A

Worked Solution

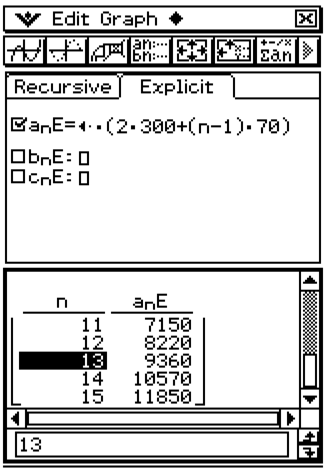
6150 + 3210 = 9360 bouquets in total (everything is correct)
We need to use the arithmetic sum formula:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$9360 = \frac{n}{2}[2 \times 300 + (n-1)70]$$

$$n = 13$$

Alternatively, we can use the calculator to generate the sum of the arithmetic sequence and find the week number which has a total of 9360 bouquets.



Question 5

The trunk diameter of a tree was 36.6 cm at the start of the first year. Each year the diameter increased by 4%. The trunk diameter of the tree after 9 years of growth is closest to

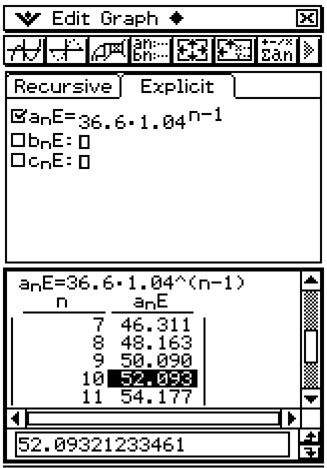
A. 48
B. 50
C. **52**
D. 54
E. 56

Answer is C

Worked Solution

We will use the geometric sequence rule: $t_n = a(r^{n-1})$
$t_{10} = 36.6(1.04^{10-1})$
$= 52$

Alternatively, we can use the calculator to generate the sequence:

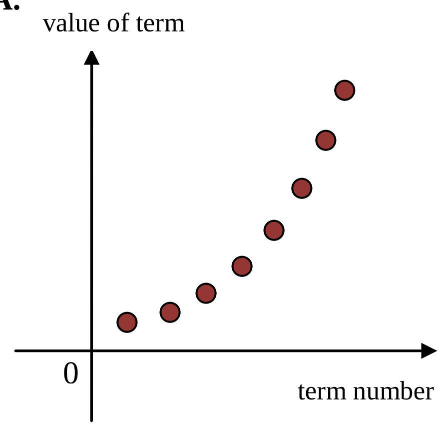


Note: If you obtained answer B, you calculated the trunk diameter of the tree at the start of the ninth year, not after 9 years.

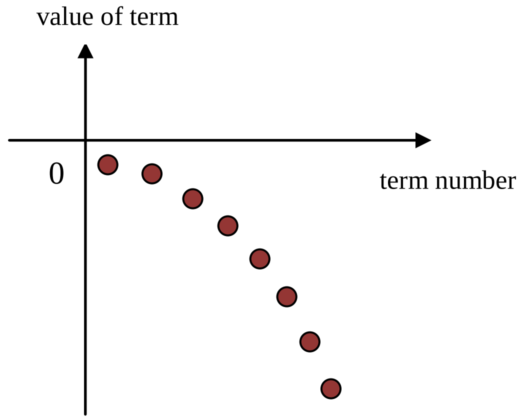
Question 6

For a particular geometric sequence, the common ratio is positive and less than 1. The graph that most closely matches this sequence is

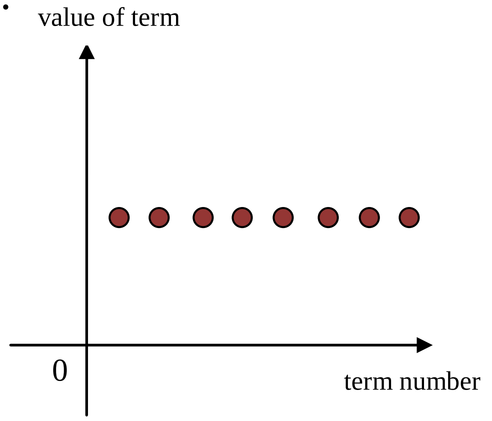
A.



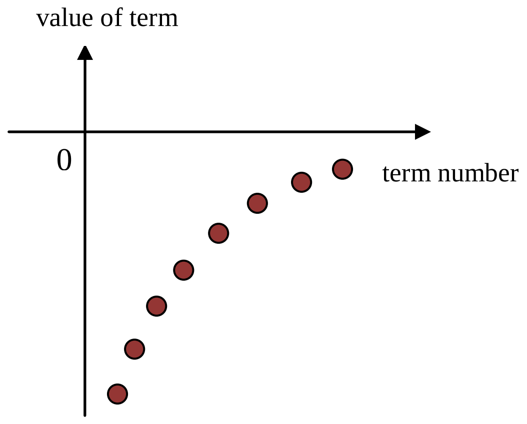
B.



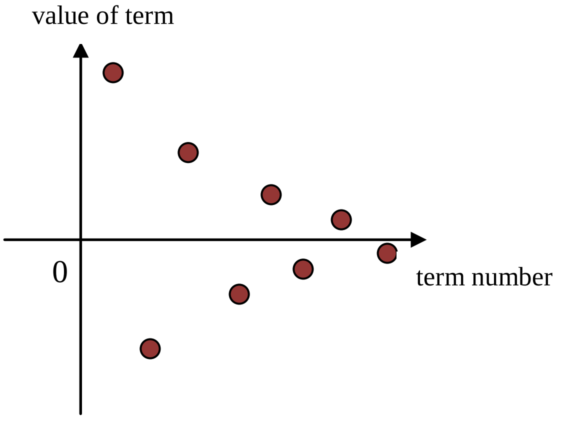
C.



D.



E.



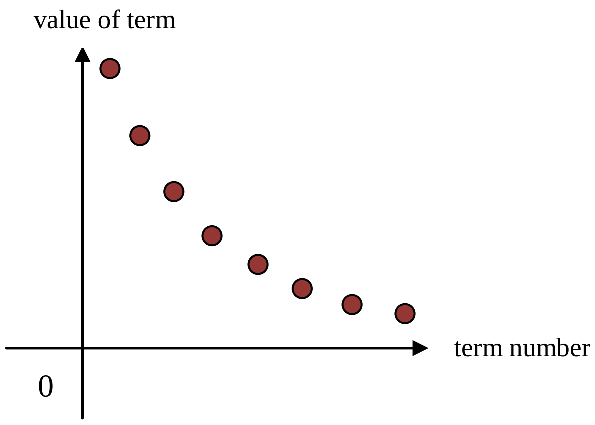
Answer is D

SECTION B – continued

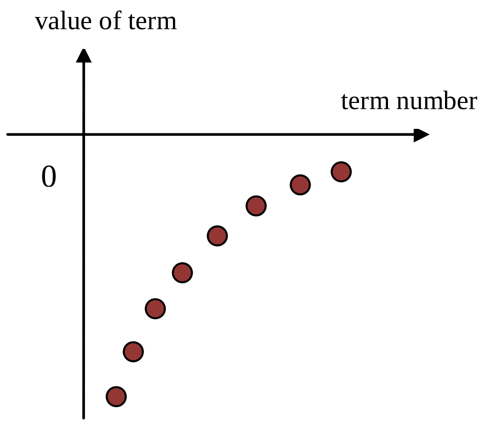
Worked Solution

There are only two possible shapes of graphs for a geometric sequence with a positive common ratio which is less than 1.

i. When $a > 0$, the following graph would match the required sequence:



ii. When $a < 0$, the following graph would match the required sequence:



Question 7

A second order difference equation is defined by: $t_n = 5t_{n-1} - 3t_{n-2}$

Given that $t_7 = 7$ and $t_9 = 24$, the eighth term of this difference equation is

A. 7
B. **9**
C. 11
D. 13
E. 15

Answer is B

Worked Solution

Substitute $n = 9$ into the second order difference equation.
$t_n = 5t_{n-1} - 3t_{n-2}$
$t_9 = 5t_8 - 3t_7$
$24 = 5 \times t_8 - 3 \times 7$
$t_8 = 9$

Question 8

The first four terms of a sequence are: 5, 7, 17, 31.

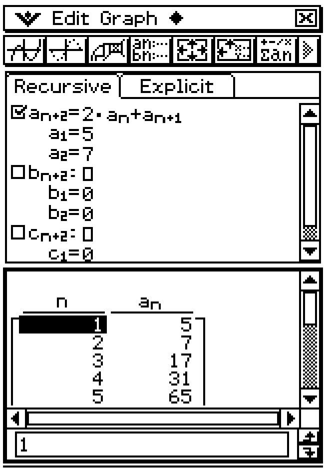
The difference equation for the sequence could be

A. $t_{n+1} = 2t_n - 3, \quad t_1 = 5$
B. $t_{n+1} = t_n + 2, \quad t_1 = 5$
C. $t_{n+2} = t_n + 2t_{n+1} - 2, \quad t_1 = 5$ and $t_2 = 7$
D. $\boldsymbol{t_{n+2} = 2t_n + t_{n+1}, \quad t_1 = 5}$ **and** $\boldsymbol{t_2 = 7}$
E. $t_{n+2} = t_n + t_{n+1} + 5, \quad t_1 = 5$ and $t_2 = 7$

Answer is D

Worked Solution

Generate the sequences for all five options.
Option A: 5, 7, 11, 19
Option B: 5, 7, 9, 11
Option C: 5, 7, 17, 39
Option D: 5, 7, 17, 31



Option E: 5, 7, 17, 29

Question 9

Alex bought a brand new car for \$54 000. The value of the car changes according to the rule:

$C_{n+1} = 0.76C_n + 2000, \qquad C_0 = 54\,000$

where C_n is the value of the car n years after Alex bought it.

Knowing that Alex does yearly renovations to the car to increase its value, from this difference equation, it can be concluded that

A. the value of the car depreciates by 76% every year and increases by \$2000 due to the renovations.
B. the value of the car depreciates by 76% every year and decreases to \$43 040 after the first year's renovations.
C. the value of the car depreciates by 24% every year and increases by \$54 000 due to the renovations.
D. the value of the car depreciates by \$2000 every year and increases by \$0.76 due to the renovations.
E. **the value of the car depreciates by 24% every year and decreases to \$43 040 after the first year's renovations.**

Answer is E

Worked Solution

$C_{n+1} = 0.76C_n + 2000, \qquad C_0 = 54\,000$

i. First let's find the amount of the yearly depreciation in the car's value.

$r = 0.76$

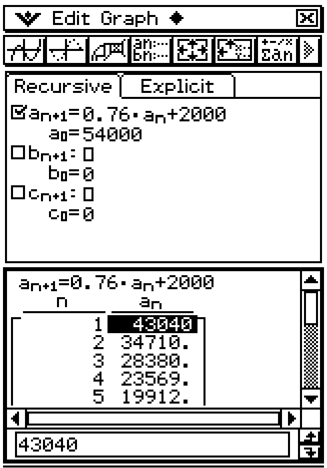
$1 - \frac{x}{100} = 0.76$ So: $x = 24\%$ depreciation

ii. The car's value increases by \$2000 a year due to the renovations.

iii. Let's calculate the car's value after one year:

$C_1 = 0.76 \times 54\,000 + 2000$
$\quad = 43\,040$

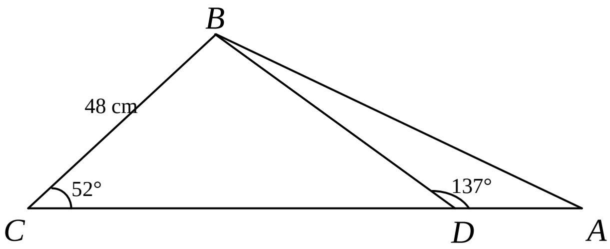
Alternatively, we can use a calculator to generate the sequence.



So, from this difference equation, it can be concluded that the value of the car depreciates by 24% every year and decreases to \$43 040 after the first year's renovations.

Module 2: Geometry and trigonometry

The following information refers to Questions 1 and 2.



Question 1

The size of the angle CBD is

A. 43°
B. **85°**
C. 37°
D. 45°
E. 115°

Answer is B

Worked Solution

Use the exterior angle property.

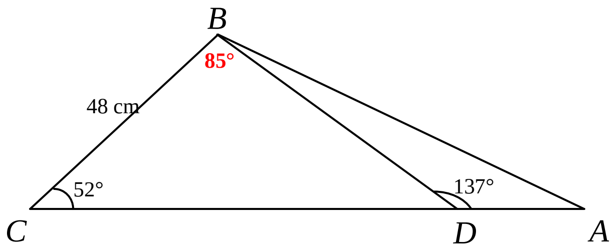
$\angle BCD + \angle CBD = \angle BDA$

$\angle CBD = \angle BDA - \angle BCD$

$= 137° - 52°$

$= 85°$

So $\angle CBD = 85°$



Question 2

Given that the length of *BC* is 48 cm, the length of CD, in cm, is

A. $\dfrac{48\sin 43^\circ}{\sin 85^\circ}$ **B.** $\dfrac{48\sin 52^\circ}{\sin 137^\circ}$

C. $\mathbf{\dfrac{48\sin 85^\circ}{\sin 43^\circ}}$ **D.** $\dfrac{\sin 52^\circ}{48\sin 43^\circ}$

E. $\dfrac{\sin 43^\circ}{48\sin 52^\circ}$

Answer is C

Worked Solution

Using the sine rule:

$$\frac{CD}{\sin 85^\circ} = \frac{48}{\sin 43^\circ}$$

$$CD = \frac{48\sin 85^\circ}{\sin 43^\circ}$$

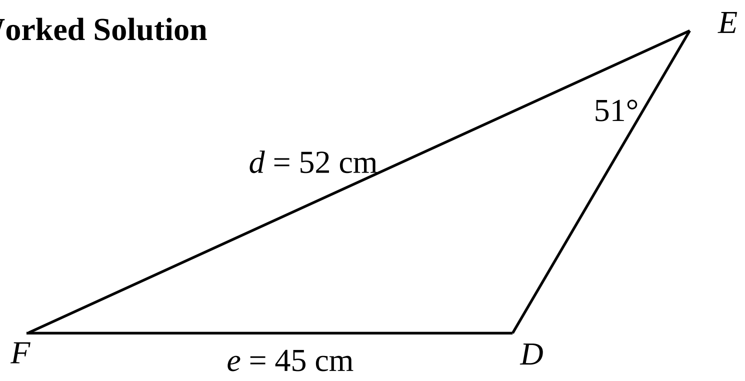
Question 3

A triangle, *DEF*,has side lengths $d = 52$ cm and $e = 45$ cm, and the size of angle *DEF* is 51°. Find the size of angle *EFD*, to the nearest degree, given that angle *EDF* is an obtuse angle.

A. 116°
B. 64°
C. 42°
D. 65°
E. **13°**

Answer is E

Worked Solution



We will use the sine rule to evaluate the size of angle *EDF*.

$$\frac{e}{\sin\ 51°} = \frac{d}{\sin\ (EDF)}$$

$$\frac{45}{\sin\ 51°} = \frac{52}{\sin\ (EDF)}$$

$$\sin\ (EDF) = \frac{52 \times \sin 51°}{45}$$

sin (*EDF)* = 63.9° or 116.1°

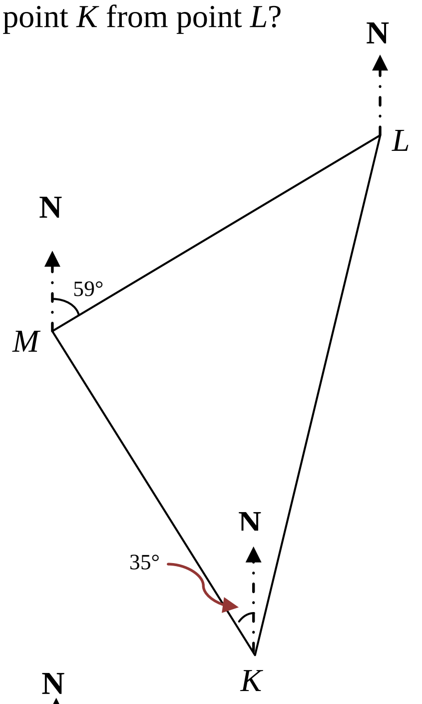
Since angle *EDF* is an obtuse angle: $\angle EDF \cong 116°$
So: $\angle EFD = 180° - 116° - 51°$
$= 13°$

Note: If you did not pay attention to the information that angle *EDF* is an obtuse angle, you would have obtained answer D.

Question 4

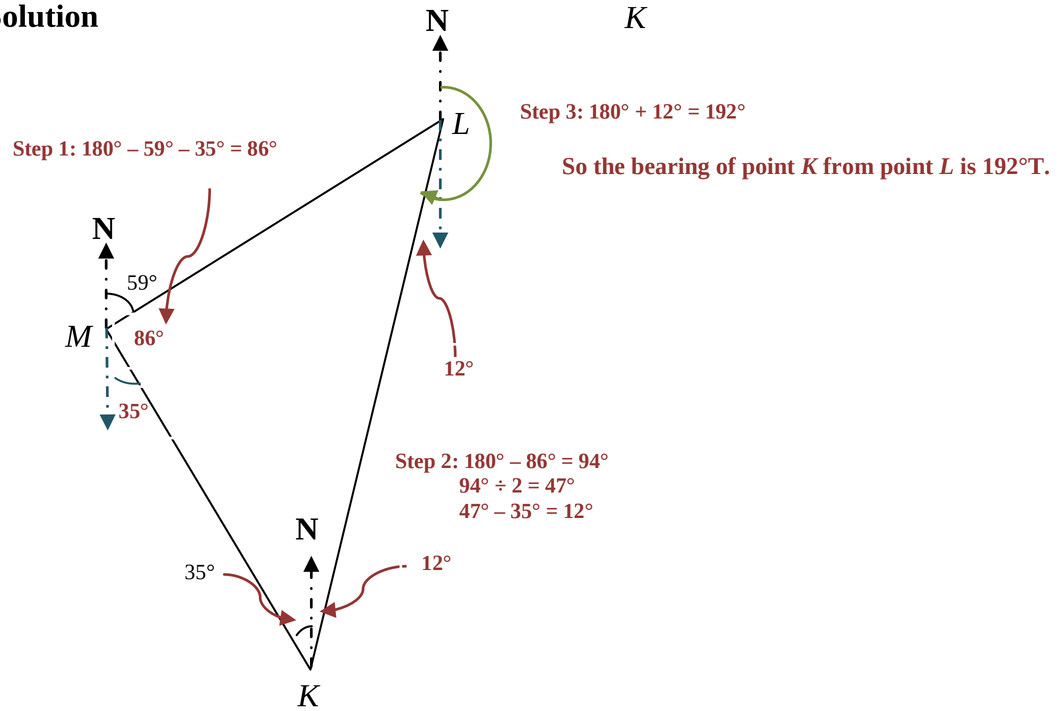
On the diagram below, the length of *KM* and the length of *LM* are equal. The bearing of point *L* from point *M* is 59° and the left part of angle *MKL* is 35°, as indicated.
Which of the following is the bearing of point *K* from point *L*?

A. 135°T
B. 059°T
C. 186°T
D. 012°T
E. **192°T**



Answer is E

Worked Solution



Step 1: Use equal alternate angles between parallel lines to find the 35° angle, and then find what needs to be added to 59° and 35° to give180° (the straight angle). This is the size of ∠*LMK*.

Step 2: Subtract the size of *LMK* (86°) from 180° (the sum of the interior angles of a triangle). Triangle *KLM* is isosceles because *KM* = *LM*. This means ∠*LKM* = ∠*KLM*, so the remaining number of degrees is divided by two to find the size of each angle (47°). Then 35° (given) is subtracted to leave 12°.

Step 3: Equal alternate angles between parallel lines gives us 12° as the amount required to be added to 180° to find the bearing of point *K* from point *L*.

Question 5

The five side lengths of a pentagon are 12 cm, 28 cm, 28 cm, 31 cm and 33 cm. Another pentagon has a side length of 16.5 cm on its longest side, and is similar to the larger pentagon. The perimeter of this smaller pentagon, in cm, is

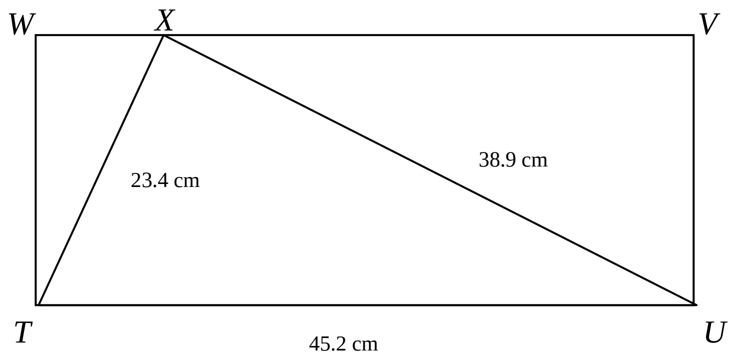
A. 132
B. 115.5
C. **66**
D. 85
E. 18

Answer is C

Worked Solution

Since the two pentagons are similar and the ratio of the longest sides is 33 : 16.5, or 2 : 1, the sides of the smaller pentagon must measure half the sides of the bigger pentagon. The perimeter is therefore 6 + 14 + 14 + 15.5 + 16.5 = 66 cm.

Question 6



The diagram above shows a triangle inside a rectangle. The size of angle *XTU* is closest to

A. 89°
B. 76°
C. **59°**
D. 31°
E. 51°

Answer is C

Worked Solution

We will use the cosine rule to calculate the size of angle *XTU*.

$$\cos(XTU) = \frac{23.4^2 + 45.2^2 - 38.9^2}{2 \times 23.4 \times 45.2}$$

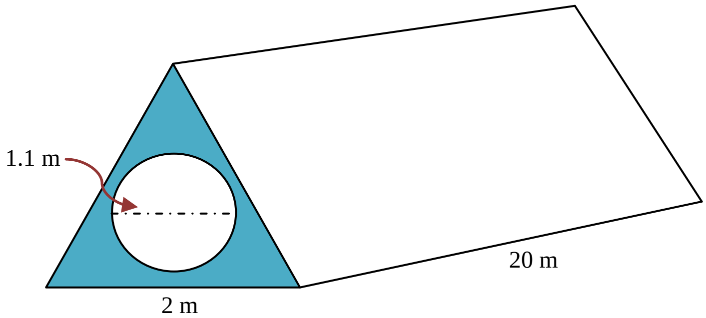
$$= 0.5093175$$

$\cos^{-1} 0.5093175 = 59.38° \cong 59°$

Note: If you obtained answer A, you calculated angle *TXU*.
If you obtained an answer of D, you calculated angle *TUX*.

Question 7

A concrete pipe is in the shape of a triangular prism of length 20 m. The cross-section of the pipe is an equilateral triangle with a side length of 2 m and it has a circular hole of diameter 1.1 m through its centre.



The volume of concrete in the pipe is closest to

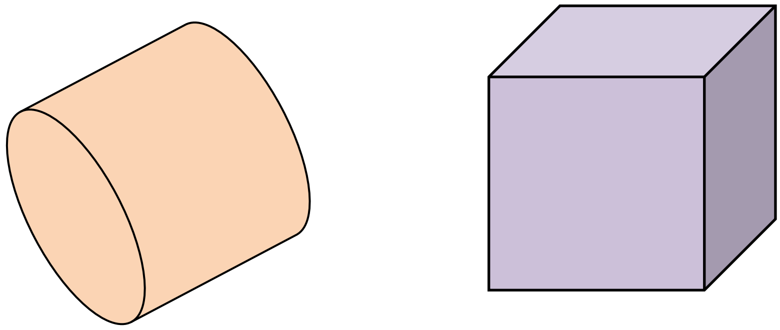
A. **15.6 m^3**
B. 44.0 m^3
C. 34.6 m^3
D. 48.2 m^3
E. 50 m^3

Answer is A

Worked Solution
radius of the circle = 1.1 ÷ 2 = 0.55
volume of the prism = $(\frac{1}{2} \times$ base of triangle × height of triangle) × height of prism
volume of the cylinder = π × radius^2 × height of prism
volume of concrete in pipe = volume of prism – volume of cylinder

$$= (\frac{1}{2} \times 2 \times \sqrt{3}) \times 20 - \pi \times 0.55^2 \times 20$$
$$= 15.6 \text{ m}^3$$

Question 8



The cylinder shown above has a surface area of 75.4 cm². The height of the cylinder and its diameter are equal.

The cube shown above has the same side length as the height of the cylinder. The surface area of the cube is closest to

A. 108 cm^2
B. **96 cm^2**
C. 64 cm^2
D. 4 cm^2
E. 2 cm^2

Answer is B

Worked Solution

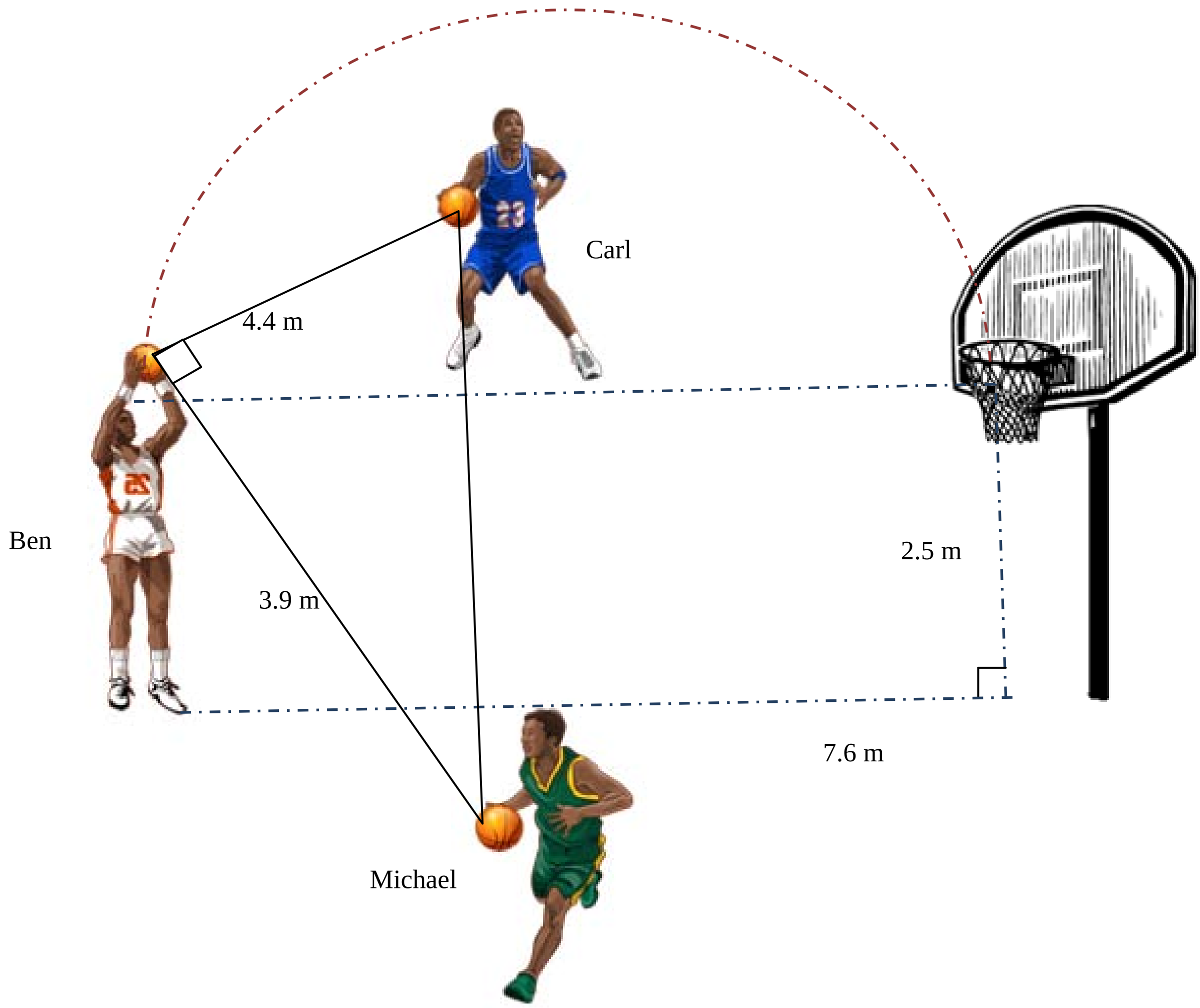
total surface area of cylinder = $2\pi r^2 + 2\pi rh$
Since the cylinder's height is equal to its diameter, $h = 2r$.
$75.4 = 2\pi r^2 + 2\pi r \times 2r$
$r = 2$ cm
$h = 2 \times 2 = 4$ cm
Since the cube has the same side length as the height of the cylinder, the side length of the cube is: 4 cm.

Now let's calculate the surface area of the cube:
total surface area of cube = $6 \times a^2$ where a is the side length
$= 6 \times 4^2$
$= 96$ cm^2

Note: If you obtained answer D, you calculated the side length of the cube. If you obtained answer E, you calculated the radius of the cylinder.

Question 9

Carl, Michael and Ben play basketball together. The diagram below shows their positions on the court. These positions form a right-angled triangle.



At the start of the game, Carl passes the ball to Michael and then Michael passes it to Ben. The distance between Michael and Ben is 3.9 m and the distance between Ben and Carl is 4.4 m.

Ben shoots the ball into the basket, with the ball following a semicircular pattern. The straight line distance between Ben and the basketball hoop is 7.6 m.

Given that the basketball hoop is 2.5 m from ground level, the total distance that the ball travelled during this part of the game, until it reached the ground, is closest to

A. 10.8 m
B. **24.2 m**
C. 21.7 m
D. 30.2 m
E. 52.4 m

Answer is B

Worked Solution

We first need to find the distance between Carl and Michael by using Pythagoras' theorem.
$x^2 = 4.4^2 + 3.9^2$
$x = 5.88$ m

Now we will evaluate the circumference of the semi-circle:

$$\begin{aligned}\text{circumference of the semi-circle} &= \frac{1}{2} \times 2\pi r \\ &= \frac{1}{2} \times 2\pi \times 3.8 \\ &= 11.94 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{total distance that the ball travelled} &= 5.88 + 3.9 + 11.94 + 2.5 \\ &= 24.22 \text{ m} \\ &\cong 24.2 \text{ m}\end{aligned}$$

Note: If you obtained answer C, you did not add the height of the basketball hoop. Since the ball will fall to the ground after the shot scores, the height of the hoop should be added to the total distance travelled.

Module 3: Graphs and relations

Question 1

The lines with equations $2x - 3y = 5$ and $x - 2y = 3$ both pass through the point

A. (9, 1)
B. (4, 1)
C. **(1, -1)**
D. (-2, -3)
E. (2.5, 0)

Answer is C

Worked Solution

Let's solve the equations simultaneously. We will make x the subject in the second equation and substitute it into the first equation.
Substitute $x = 2y + 3$ into $2x - 3y = 5$.

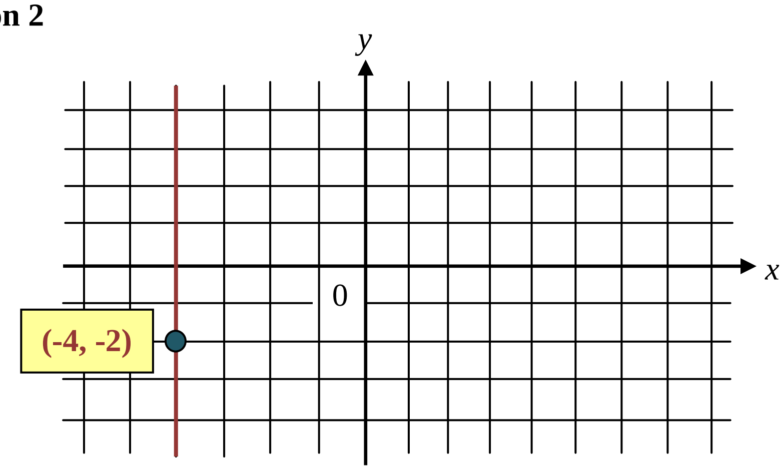
$$2(2y + 3) - 3y = 5$$
$$4y + 6 - 3y = 5$$
$$y = -1$$

Now let's substitute $y = -1$ into $x = 2y + 3$

$$x = 2 \times -1 + 3$$
$$= 1$$

So both lines pass through the point (1, -1).

Question 2



Which of the following is the equation of the line passing through the point (-4, -2) on the graph above?

A. $\mathbf{x = -4}$
B. $y = -4$
C. $x = -2$
D. $y = -2$
E. $y = -2x - 4$

Answer is A

Worked Solution

The line passing through (-4, -2) on the graph is parallel to the *y*-axis. The equation of the line is $x = -4$.

Question 3

A straight line has a *y*-intercept of -4 and passes through the point (-1, 3).
Which of the following is the *x*-intercept of this line?

A. 1
B. $\frac{2}{3}$
C. -4
D. -7
E. $-\frac{4}{7}$

Answer is E

Worked Solution

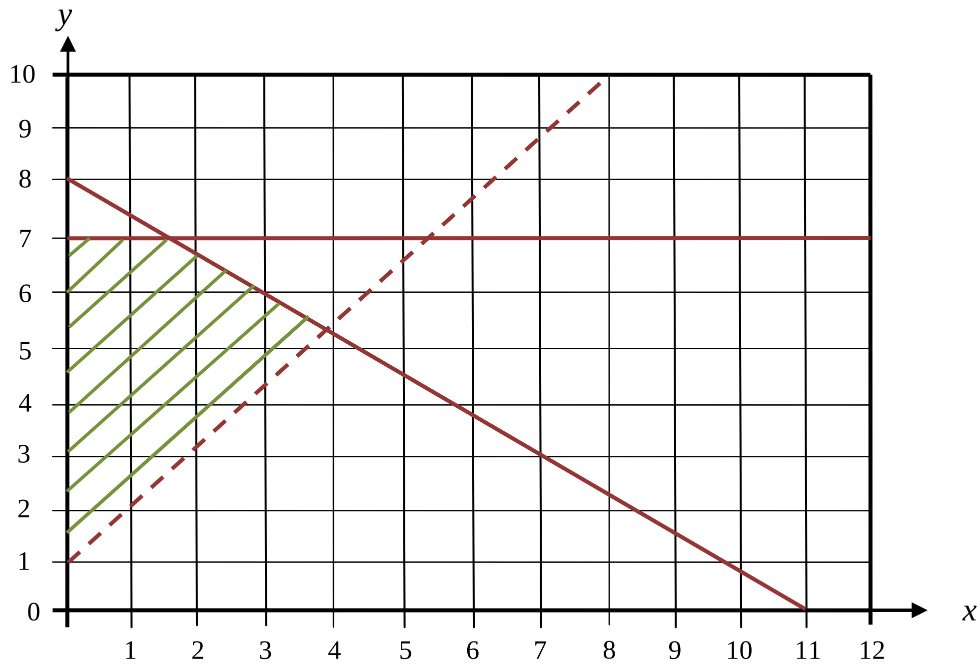
Let's substitute the *y*-intercept and the point (-1, 3) into the straight line equation $y = mx + c$.

$3 = m \times -1 - 4$
$m = -7$

So the equation of the line is: $y = -7x - 4$

In order to determine the *x*-intercept, we need to substitute $y = 0$ into the equation of the line.
$y = -7x - 4$
$0 = -7x - 4$
x-intercept = $-\frac{4}{7}$

Question 4



The shaded region above has been formed by three separate constraints. Which of the following alternatives lists two of these constraints?

A. $8y - 9x \geq 8$
$8x + 11y < 88$

B. $y \leq 7$
$9x + 8y > 8$

C. $\mathbf{8x + 11y \leq 88}$
$\mathbf{8y - 9x > 8}$

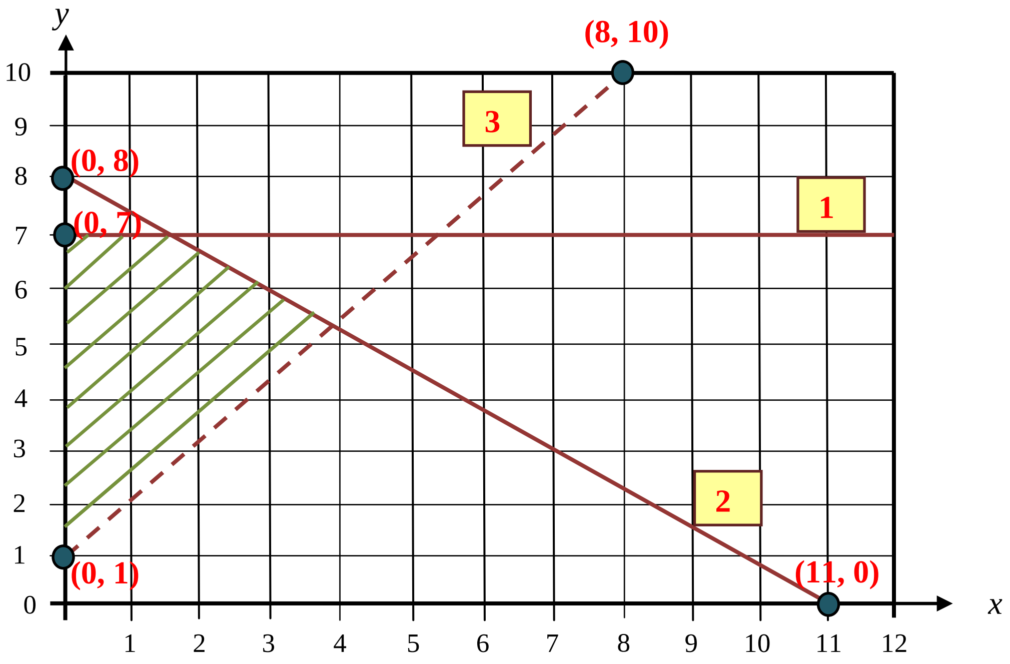
D. $x \leq 7$
$9x - 8y < 8$

E. $y < 7$
$8x + 11y < 88$

Answer is C

Worked Solution

Let's find all three constraints that form the shaded region.



1) y-intercept = 7 and gradient = 0. So the first inequation is $y \le 7$

2) y-intercept = 8 and gradient = $-\frac{8}{11}$.

The equation of the line is: $y = mx + c$

$$y = -\frac{8}{11}x + 8$$

$$11y = -8x + 88$$

$8x + 11y = 88$ The second inequation is $8x + 11y \le 88$.

3) y-intercept = 1 and gradient = $\frac{9}{8}$

The equation of the line is: $y = mx + c$

$$y = \frac{9}{8}x + 1$$

$$8y = 9x + 8$$

$8y - 9x = 8$ The third inequation is $8y - 9x > 8$.

Tips

- *Since the third line is shown with a dotted line, the line itself is not included in the inequation. That is the reason we write the equation with a > sign instead of a ≥ sign.*
- *We can use a very simple method to determine which inequality sign to use. First we make sure that y is on the left-hand side of the equation and that its sign is positive. If this is the case, > means the required region is above the line and < means the required region is below the line. Please bear in mind that this method only works if y is on the left-hand side of the equation and its sign is positive. If not, we should rearrange the equation so that is the case.*

Question 5

Which of the following pairs of simultaneous equations has a unique solution?

A. $3x + 8y = 12$
$1.5x + 4y = 6$

B. $17x + 11y = 10$
$85x + 55y = 50$

C. $\mathbf{3}\boldsymbol{x} + \mathbf{2}\boldsymbol{y} = \mathbf{6}$
$\mathbf{6}\boldsymbol{x} - \mathbf{4}\boldsymbol{y} = \mathbf{12}$

D. $x + 2y = 5$
$-7x - 14y = 7$

E. $4x - 3y = 3$
$20x - 15y = 1$

Answer is C

Worked Solution

For a pair of simultaneous equations to have a unique solution, the ratio of the coefficients of x variables must not be the same as the ratio of the coefficients of y variables. Let's examine the options.

Option A: $\frac{3}{1.5} = \frac{8}{4} = \frac{2}{1}$

So, these simultaneous equations do not have a unique solution.

Option B: $\frac{17}{85} = \frac{11}{55} = \frac{1}{5}$

So, these simultaneous equations do not have a unique solution.

Option C: $\mathbf{\frac{3}{6} \neq \frac{2}{-4}}$

So, these simultaneous equations have a unique solution.

Option D: $\frac{1}{-7} = \frac{2}{-14} = -\frac{1}{7}$

So, these simultaneous equations do not have a unique solution.

Option E: $\frac{4}{20} = \frac{-3}{-15} = \frac{1}{5}$

So, these simultaneous equations do not have a unique solution.

Tips

- *If the ratio of the coefficients of x variables, the ratio of the coefficients of y variables and the ratio of the constants are all equal, then the simultaneous equations have infinitely many solutions. So, they do not have a unique solution.*
- *If the ratio of the coefficients of x variables and the ratio of the coefficients of y variables are equal, but the ratio of the constants is different, then the simultaneous equations have no solution.*

Question 6

Mia can choose between three different tollways to get to different parts of the city: Powerlink, Quicklink and Speedyway.

Powerlink charges fees according to the formula:

fee = \$3.20 × number of sections used

The fee structure for Quicklink is as follows:

$$\text{fee} = \begin{cases} \$6.50, & 1 \le \text{the number of sections used per day} < 3 \\ \$9.40, & 3 \le \text{the number of sections used per day} < 5 \\ \$11.40, & 5 \le \text{the number of sections used per day} \end{cases}$$ (it’s correct)

Speedyway has a fixed daily charge of \$10.10 regardless of the number of sections used.

Mia wants to use four sections of the tollways on Saturday, eight sections on Sunday and two sections on Monday. Which of the following is the minimum total fee that she can pay for using the tollways for the three days?

A. \$44.80
B. \$30.30
C. \$27.30
D. **\$25.90**
E. \$19.50

Answer is D

Worked Solution
Step 1: Let's start by calculating the fees for using four sections on Saturday on all three tollways.

Powerlink: fee = \$3.20 × 4 = \$12.80
Quicklink: fee = \$9.40
Speedyway: Fixed daily fee of \$10.10

Quicklink, with a fee of \$9.40, is the cheapest tollway for using four sections on Saturday.

Step 2: Now let's calculate the fees of all three tollways for using eight sections on Sunday.
Powerlink: fee = \$3.20 × 8 = \$25.60
Quicklink: fee = \$11.40
Speedyway: Fixed daily fee of \$10.10

Speedyway, with a fixed fee of \$10.10, is the cheapest tollway for using eight sections on Sunday.

Step 3: Now let's calculate the fees of all three tollways for using two sections on Monday.
Powerlink: fee = \$3.20 × 2 = \$6.40
Quicklink: fee = \$6.50
Speedyway: Fixed daily fee of \$10.10

Powerlink, with a fee of \$6.40 is the cheapest tollway for using two sections on Monday.

Step 4: Now let's add the three daily fees together.

Total minimum cost for three days = \$9.40 + \$10.10 + \$6.40
= \$25.90

Tip

- *Please take into account that Mia did not need to use the same tollway on each occasion, which is clearly stated in the question. If you obtained answer* A, B *or* C, *you assumed that Mia used the same tollway each day.*

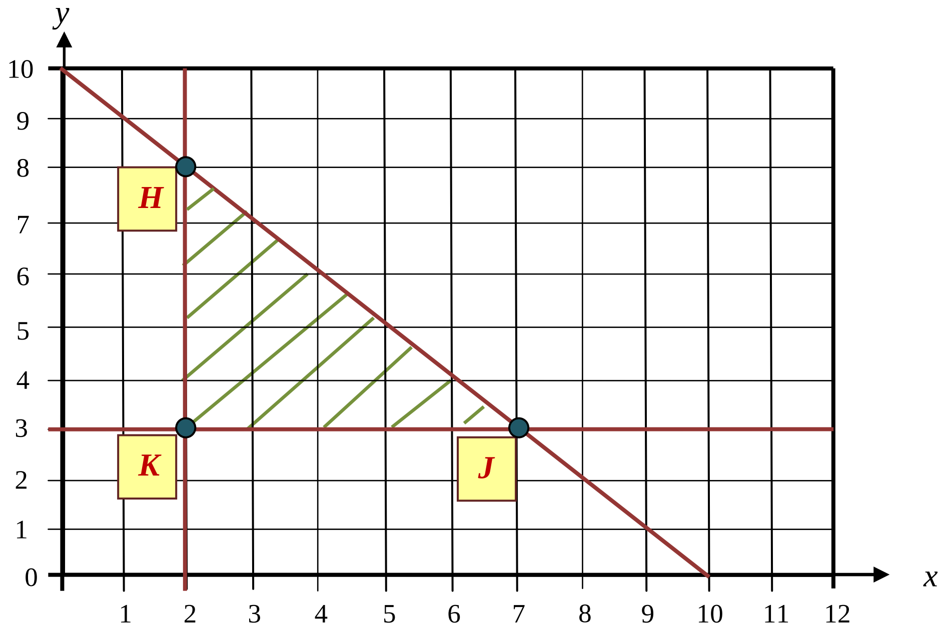
Question 7

A florist sells fresh roses and lilies every day. She prepares a maximum of 10 bouquets of roses and lilies to sell each day. She sells at least two bouquets of roses and three bouquets of lilies daily.

Let x be the number of rose bouquets the florist sells in a day.

Let y be the number of lily bouquets the florist sells in a day.

The constraints above define the feasible region shaded in the graph below.



The profit that the florist makes from selling a bouquet of lilies is \$3 and from a bouquet of roses is \$$a$.

If the maximum profit occurs at every discrete point between the points H and J, and the points H and J are inclusive, then the value of a is

A. 1
B. 2
C. **3**
D. 4
E. 5

Answer is C

Worked Solution

Let's substitute $H(2, 8)$ into the profit function, $P = ax + 3y$:

$P = a \times 2 + 3 \times 8$

Let's substitute J(7, 3) into the profit function, $P = ax + 3y$:

$P = a \times 7 + 3 \times 3$

Since the maximum profit occurs at every discrete point between the points *H* and *J*, and the points *H* and *J* are inclusive, these points must be **equal maximum points**.

$a \times 2 + 3 \times 8 = a \times 7 + 3 \times 3$
$a = 3$

Question 8

Paul manufactures pillows in his factory. The cost of manufacturing pillows starts with a fixed cost of \$2400 plus a cost of \$10 per pillow. For Paul to make a profit, he must sell

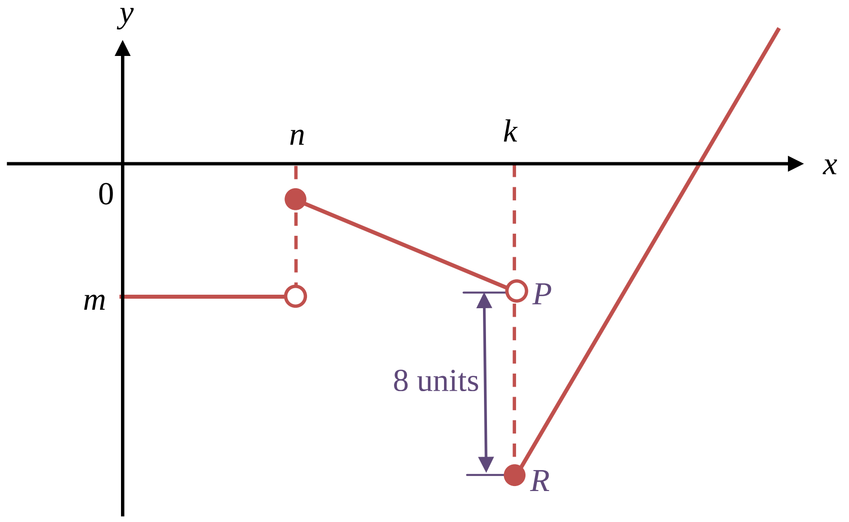
A. 40 pillows at \$70 each
B. 60 pillows at \$50 each
C. **70 pillows at \$45 each**
D. 50 pillows at \$55 each
E. 100 pillows at \$33 each

Answer is C

Worked Solution

Let's calculate the profit or loss situations of all five options.
Option A: $P = 40 \times 70 - (2400 + 10 \times 40) = 0$
This is break-even point, so there is no profit.
Option B: $P = 60 \times 50 - (2400 + 10 \times 60) = 0$
This is break-even point, so there is no profit.
Option C: $P = 70 \times 45 - (2400 + 10 \times 70) = 50$
There is \$50 profit.
Option D: $P = 50 \times 55 - (2400 + 10 \times 50) = -150$
There is a \$150 loss in this situation.
Option E: $P = 100 \times 33 - (2400 + 10 \times 100) = -100$
There is a \$100 loss in this situation.

Question 9



The rule for the graph above is given as:

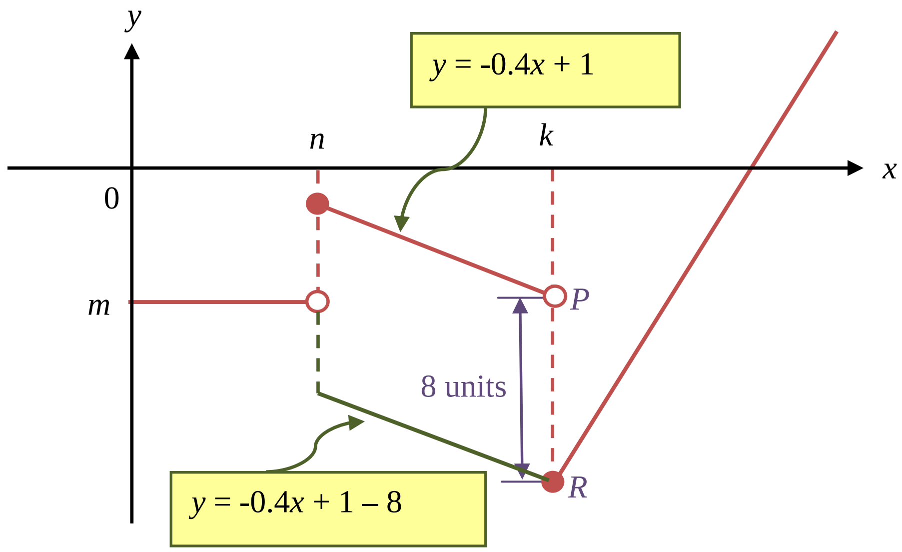
$$y = \begin{cases} m, & 0 < x < n \\ -0.4x + 1, & n \le x < k \\ 1.5x - 45, & k \le x \end{cases}$$

Given that the vertical distance between the points *P* and *R* is 8 units, the value of k is

A. 5
B. 10
C. 15
D. **20**
E. 25

Answer is D

Worked Solution



Let's first find the equation of the line segment which is parallel to the second line segment and is vertically 8 units below it.

This equation would be: $y = -0.4x + 1 - 8$

Since this new line segment and the third line segment intersect at point *R*, we can equate them to each other to find the *x*-coordinate of point *R*. That *x*-coordinate of point *R* is equal to *k*.

$$-0.4x + 1 - 8 = 1.5x - 45$$
$$x = 20$$

So $k = 20$.

Module 4: Business-related mathematics

Question 1

$60 000 is invested at a simple interest rate of 4.5% per annum. Assuming no deposits or withdrawals are made, the amount of money which will be in the bank after 3 years is

A. $62 700
B. **$68 100**
C. $72 600
D. $74 200
E. $78 100

Answer is B

Worked Solution

$I = \frac{Prt}{100}$ (simple interest formula)

$I = \frac{60\,000 \times 4.5 \times 3}{100}$

$P = 8100$

$A = P + I$

$= 60\,000 + 8100 = \$68\,100$

Question 2

Joshua buys a freezer with a purchase price of $3250. He first pays a deposit of $515 and he agrees to make fortnightly payments of $112 for one year. The total interest that he is going to pay in this hire-purchase agreement, and the effective rate of interest, are closest to

A. $112, 6.47%
B. $150, 10.65%
C. $150, 12.46%
D. **$177, 12.46%**
E. $177, 6.47%

Answer is D

Worked Solution

Joshua pays a deposit of \$515.

balance = \$3250 – \$515 = \$2735

He makes payments totalling: 26 × \$112 = \$2912
So: interest = \$2912 – \$2735 = \$177

The annual flat rate of interest = $\frac{177 \times 100}{2735} \cong 6.47\%$

$$\text{effective rate of interest} \cong \frac{2n}{n+1} \times \text{flat rate}$$
$$\cong \frac{2 \times 26}{26+1} \times 6.47$$
$$\cong 12.46\%$$

Question 3

Zoe purchases a brand new car for \$52 000. The car depreciates at a rate of 32 cents per kilometre travelled. Which of the following will be the book value of the car after it has travelled a total distance of 70 000 km?

A. \$46 200
B. \$38 700
C. **\$29 600**
D. \$24 300
E. \$22 400

Answer is C

Worked Solution

amount of depreciation = 0.32 × \$70 000
= \$22 400

book value = original price – amount of depreciation
= \$52 000 – \$22 400
=\$29 600

Note: If you obtained answer E, you found the amount of depreciation.

Question 4

Grace invests \$20 000 at 5.2% per annum with interest compounding monthly for 4 years.

Ethan has the same amount of money and he invests it at a flat rate of interest with the same interest rate and for the same amount of time as Grace.

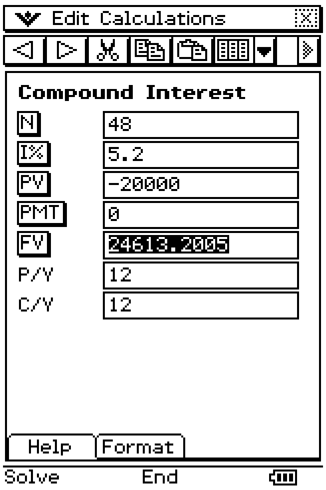
The difference in the interests that they will receive, correct to the nearest cent, is

A. \$613.20
B. \$305.30
C. \$414.00
D. \$524.75
E. **\$453.20**

Answer is E

Worked Solution

First find the amount of money Grace will receive from the bank by using the calculator's TVM solver:



interest amount = \$24 613.20 – \$20 000 = \$4613.20

Now calculate the amount of money Ethan will receive from the bank:

$I = \dfrac{Prt}{100}$ (simple interest formula)

$= \dfrac{20\,000 \times 5.2 \times 4}{100}$

$= \$4160$

The difference between the interest received by Grace and the interest received by Ethan is:

\$4613.20 – \$4160 = \$453.20

Question 5

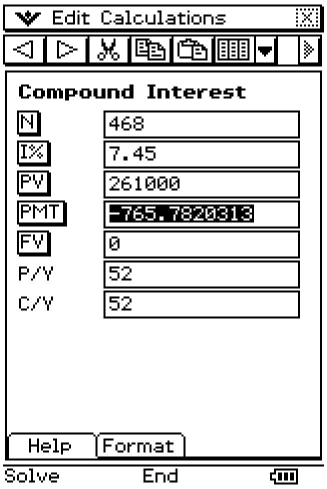
Riley repaid a reducing balance loan of $261 000 in nine years by weekly repayments and with interest charged weekly at 7.45% per annum on the outstanding balance.

The weekly repayment that Riley made to the nearest cent is

A. **$765.78**
B. $785.19
C. $686.77
D. $785.20
E. $765.80

Answer is A

Worked Solution
Find the weekly repayment that Riley made by using the calculator's TVM solver:



So, Riley repaid $765.78 each week.

Question 6

Lucas owns a shoe store. He buys shoes from a manufacturer and sells them in his store at a 90% mark-up on the purchase price. He manages to sell one-quarter of his stock at the marked price. To be able to sell the remainder of his stock, he has to make a 35% discount storewide.

If Lucas then manages to sell all of the shoes in his store, the percentage profit that he makes on the stock is closest to

A. 20%
B. 30%
C. **40%**
D. 50%
E. 60%

Answer is C

Worked Solution

Let's assume that the cost price is $100:

marked-up price = $\$100 + \$100 \times \frac{90}{100} = \$190$

profit from selling one-quarter of the stock at marked price = $(\$190 - \$100) \times \frac{1}{4} = \$22.50$

discounted price = $\$190 - \$190 \times \frac{35}{100} = \$123.50$

profit from selling three-quarters of the stock at discount price = $(\$123.50 - \$100) \times \frac{3}{4}$
$= \$17.625$

total profit = $\$22.50 + \$17.625 = \$40.125$

Since we assumed at the start that the cost price is $100, the $40.125 profit is actually percentage profit. So, the percentage profit is 40.125%. The closest answer is 40%.

Question 7

The Victorian state government stamp duty schedule for the purchase of a home which will be used as a principal place of residence is shown in the table below.

Dutiable value range	Rate
\$0–\$25 000	1.4 per cent of the dutiable value of the property
>\$25 000–\$130 000	\$350 plus 2.4 per cent of the dutiable value in excess of \$25 000
>\$130 000–\$440 000	\$2870 plus 5 per cent of the dutiable value in excess of \$130 000
>\$440 000–\$550 000	\$18 370 plus 6 per cent of the dutiable value in excess of \$440 000
>\$550 000–\$960 000	\$2 870 plus 6 per cent of the dutiable value in excess of \$130 000
More than \$960 000	5.5 per cent of the dutiable value

The amount of stamp duty payable for a home with a value of \$480 000 is

A. \$23 870
B. \$6 720
C. \$11 270
D. **\$20 770**
E. \$20 370

Answer is D

Worked Solution

We can clearly see from the stamp duty schedule table that the stamp duty for a house that costs between \$440 000 and \$550 000is \$18 370 plus 6 per cent of the dutiable value in excess of \$440 000.

\$480 000 – \$440 000 = \$40 000

$\$40\,000 \times \frac{6}{100} = \$2400$

total stamp duty = \$18 370 + \$2400 = \$20 770

Question 8

Connor puts his $32 000 in an account earning 6.1% interest per annum compounding fortnightly. The interest is charged at the end of every fortnight and right after that he regularly adds $140 to his investment.

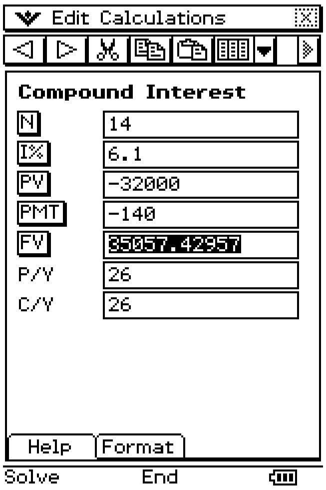
Which of the following is closest to the value of Connor's investment at the end of the fourteenth fortnight before he adds the $140?

A. $35 057
B. **$34 917**
C. $36 814
D. $37 120
E. $37 153

Answer is B

Worked Solution

We'll use the calculator's TVM solver to find the value of Connor's investment:



Since the question is asking for the value of Connor's investment before he pays $140, we need to take away $140 from the amount we found using the calculator.

$35 057.43 – $140 = $34 917.43

Note: If you obtained answer A, you calculated the value of the investment after Connor paid the $140.

Question 9

Liam borrowed \$43 000 to make renovations to his house. He will fully repay the loan in 8 years with equal monthly payments. Interest is charged at the rate of 8.3% per annum, calculated monthly, on the reducing balance.

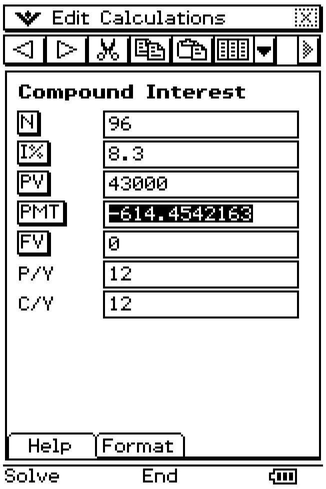
The amount Liam will have paid off the principal immediately following the fifty-sixth repayment is closest to

A. \$21 407
B. \$20 509
C. \$22 349
D. **\$21 593**
E. \$24 254

Answer is D

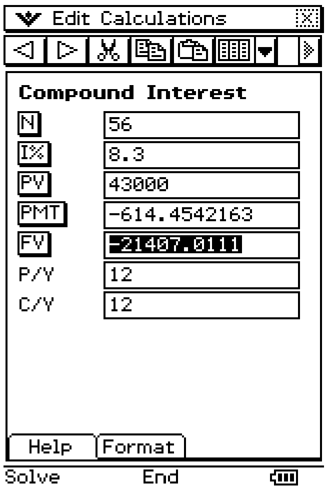
Worked Solution

First we'll use the calculator's TVM solver to calculate the amount of Liam's monthly repayments:



Liam pays \$614.45 per month.

Now we'll use the calculator's TVM solver to calculate the amount Liam still owes after the fifty-sixth repayment.



Liam still owes $21407.0111

Finally we will calculate the amount Liam has paid off the principal immediately following the fifty-sixth repayment.

Amount paid off = $43 000 – $21 407.0111
= $21 592.9889
≅ $21 593

Note: If you obtained answer A, you calculated the amount Liam owes the bank.

Module 5: Networks and decision mathematics

Question 1

A connected, planar graph has 11 vertices and 18 edges. The number of faces it has is

A. 5
B. 7
C. **9**
D. 11
E. 13

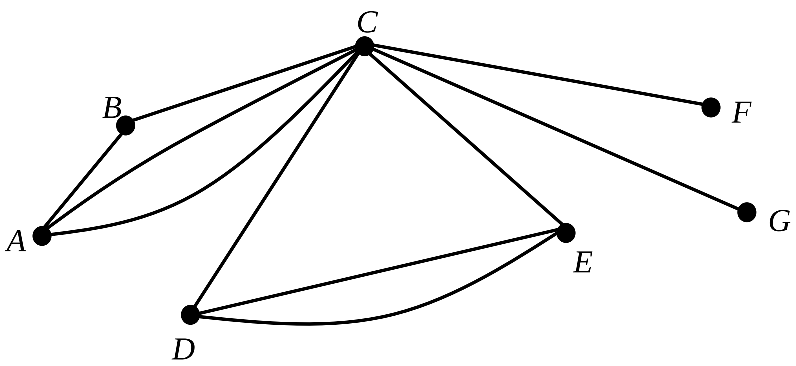
Answer is C

Worked Solution

We will use Euler's formula to determine the number of faces:

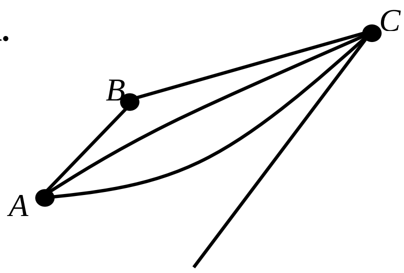
$$v + f = e + 2$$
$$11 + f = 18 + 2$$
$$f = 9$$

The following information refers to Questions 2 and 3.

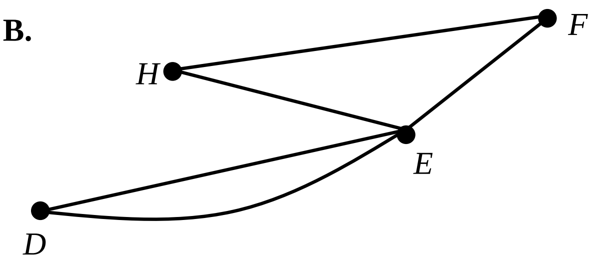


Question 2
Which one of the following is a subgraph of the graph above?

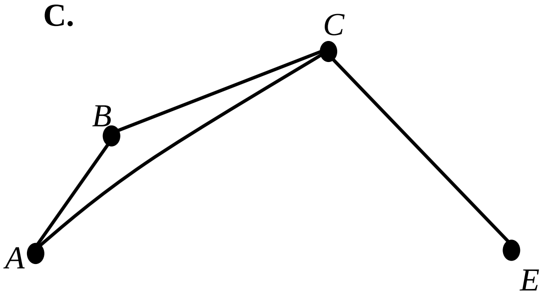
A.



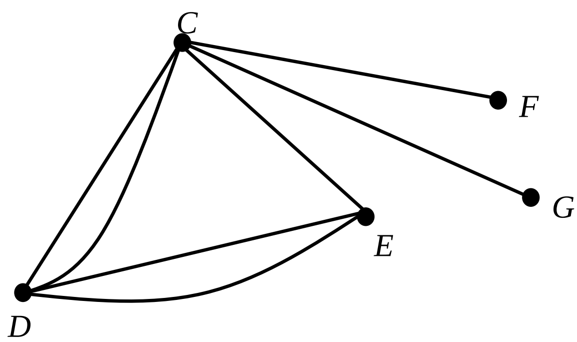
B.



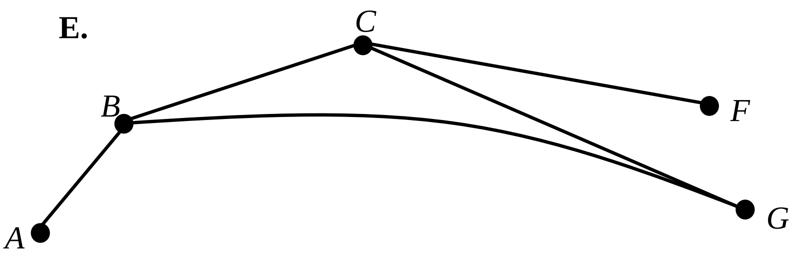
C.



D.



E.



Answer is C

Worked Solution
First, consider the definition of a subgraph: A subgraph is a part of the original graph without the addition of any new vertices or edges. If an edge is included then its connecting vertices must also be included.

Now let's go through all five options.

Option A is not a subgraph because, although an edge is included in it, the connecting vertex *D* is not included.
Option B is not a subgraph because it has an extra vertex, *H*, and two extra edges connecting the vertex *H* to vertices *E* and *F*. It also has an extra edge connecting *E* and *F*.
Option C is a subgraph.
Option D is not a subgraph because it has an extra edge between vertices *C* and *D*.
Option E is not a subgraph because it has an extra edge between vertices *B* and *G*.

Question 3
The adjacency matrix of the graph is shown below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & \textbf{?} & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \textbf{?} & 1 & 0 & \textbf{?} & 1 & 1 & 1 \\ 0 & 0 & \textbf{?} & 0 & \textbf{?} & 0 & 0 \\ 0 & 0 & 1 & \textbf{?} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Six of the entries in the adjacency matrix are missing.
The sum of the six missing entries is

A. 12
B. **10**
C. 8
D. 6
E. 4

Answer is B

Worked Solution

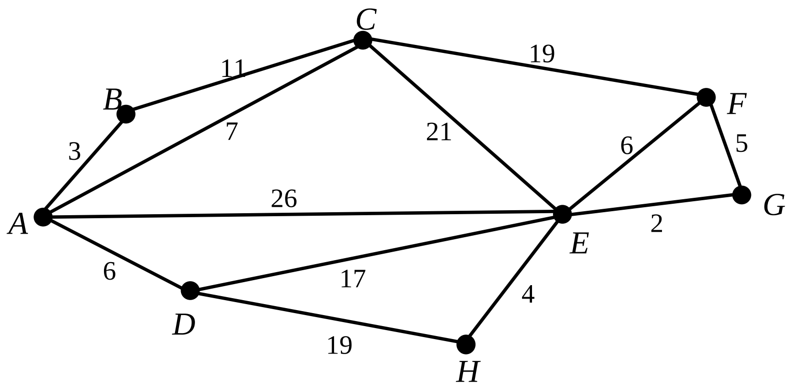
The complete adjacency matrix is as follows:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & \mathbf{2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \mathbf{2} & 1 & 0 & \mathbf{1} & 1 & 1 & 1 \\ 0 & 0 & \mathbf{1} & 0 & \mathbf{2} & 0 & 0 \\ 0 & 0 & 1 & \mathbf{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

There are two paths between *A* and *C*.
There is one path between *C* and *D*.
There are two paths between *D* and *E*.
The sum of the missing entries = 2 + 2 + 1 + 1 + 2 + 2 = 10

SECTION B – continued

Question 4

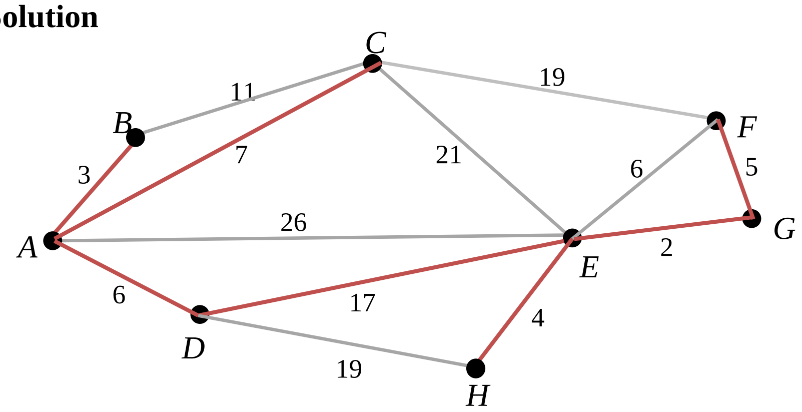


The minimal spanning tree for the network above has a length of

A. 69
B. 56
C. 48
D. **44**
E. 38

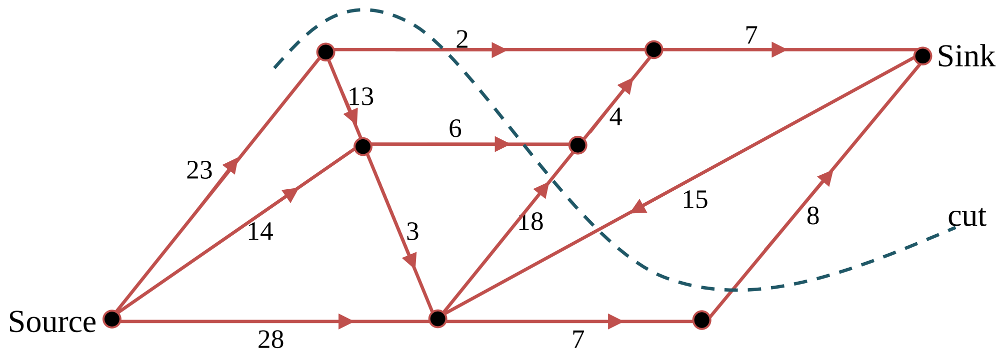
Answer is D

Worked Solution



length of the minimum spanning tree = 3 + 7 + 6 + 17 + 4 + 2 + 5 = 44

The following information refers to Questions 5 and 6.



In the directed graph above the numbers on the edges give the maximum flow possible between each pair of vertices. A cut that separates the source from the sink is also shown.

Question 5

The cut shown in the network has a capacity of

A. **34**
B. 39
C. 49
D. 52
E. 58

Answer is A

Worked Solution

capacity of the cut = 2 + 6 + 18 + 0 + 8 = 34

Tip

- *The side with a flow of 15 is flowing from sink to source across the cut and so is counted as 0.*

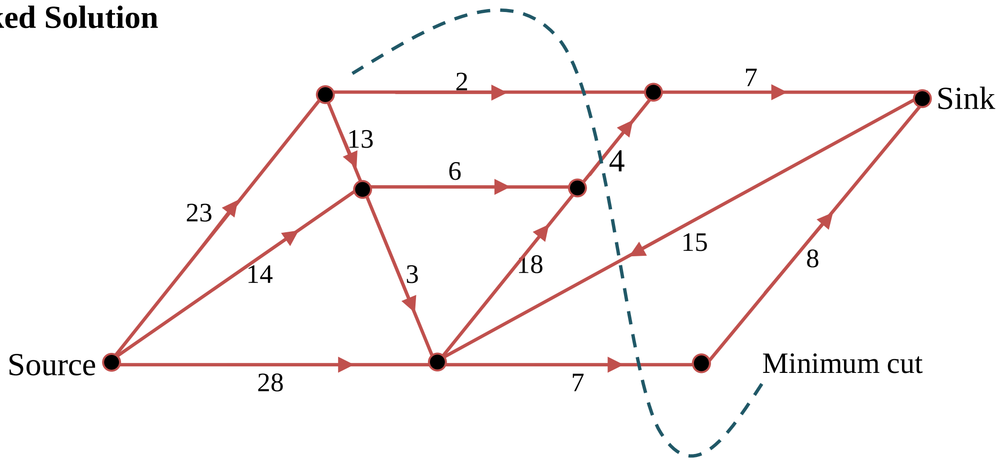
Question 6

Using the 'minimum-cut, maximum-flow' theorem, the maximum flow possible between source and sink through the network is

A. 29
B. 28
C. 15
D. 14
E. **13**

Answer is E

Worked Solution

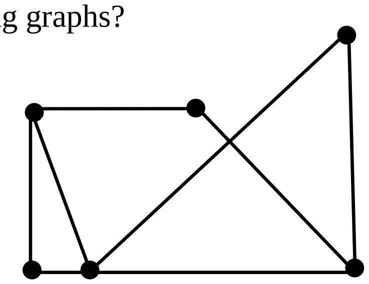


Maximum flow = 2 + 4 + 0 + 7 = 13

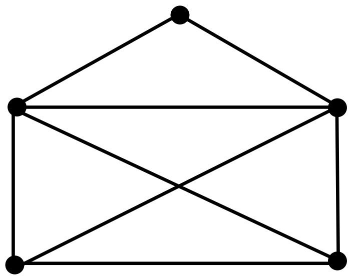
Question 7

Euler's formula, relating vertices, faces and edges, does **not** apply to which one of the following graphs?

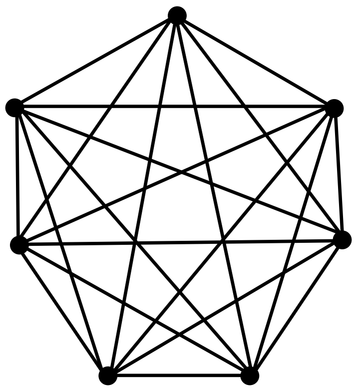
A.



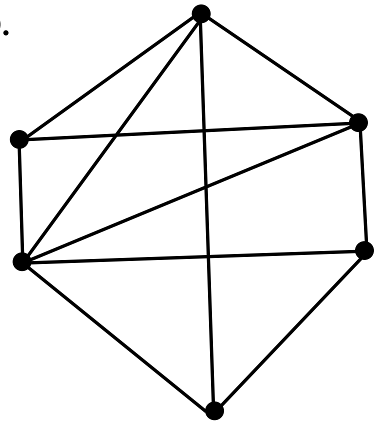
B.



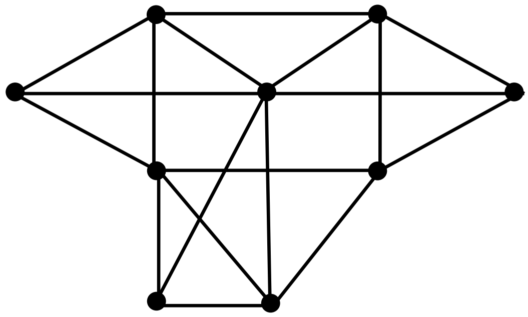
C.



D.



E.

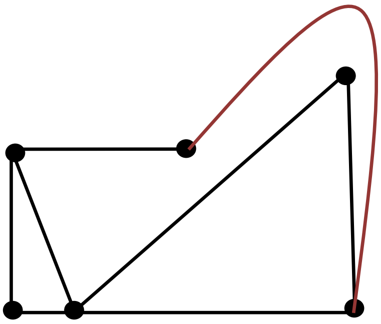


Answer is C

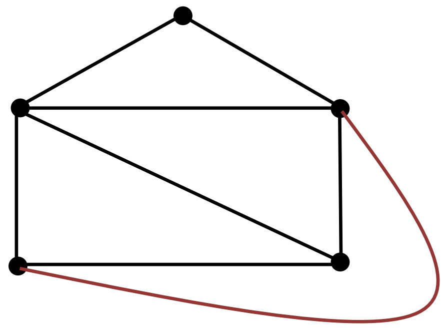
Worked Solution

As we know, only planar graphs satisfy Euler's formula.
Let's see which option cannot be redrawn as a planar graph.

Option A can be redrawn as a planar graph.

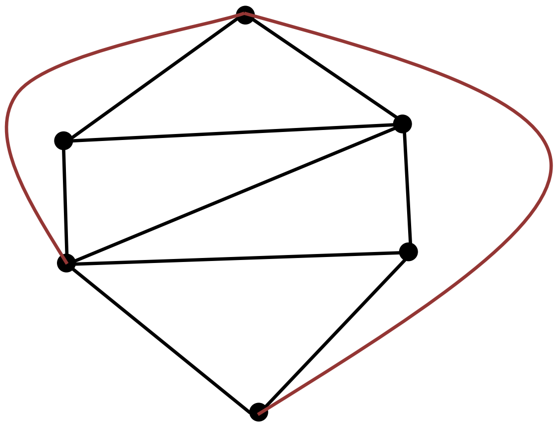


Option B can be redrawn as a planar graph.

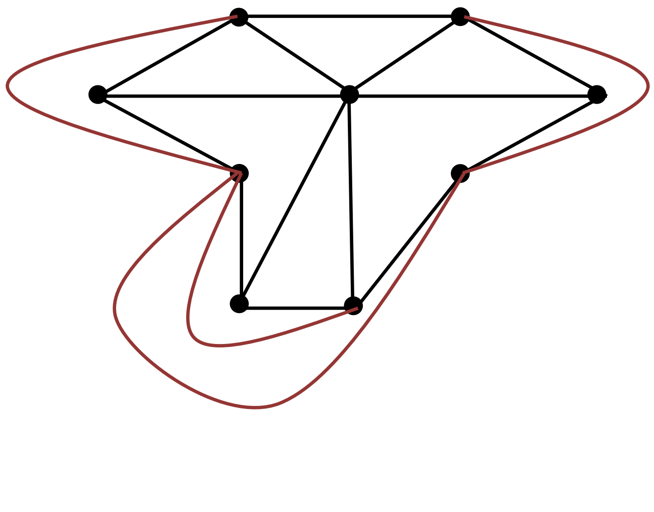


Option C cannot be redrawn as a planar graph.

Option D can be redrawn as a planar graph.



Option E can be redrawn as a planar graph.



Question 8
The table below shows the time (in minutes) that each of five cleaning company employees, Leonie, Luke, Anna, Simon and Felix, would take to complete each of the cleaning tasks: vacuuming, mopping, ironing, cleaning windows and dishwashing.

	Time to complete task (minutes)				
Cleaning company employee	Vacuuming	Mopping	Ironing	Window cleaning	Dishwashing
Leonie	30	15	31	41	15
Luke	15	12	38	32	25
Anna	27	25	42	38	16
Simon	35	22	21	18	19
Felix	28	23	30	25	22

If each person is allocated to one task only, the minimum total time in minutes for this group of people to complete all five tasks is

A. 108
B. 105
C. 96
D. **92**
E. 81

Answer is D

Worked Solution

$$\begin{array}{l} \\ \text{Leonie} \\ \text{Luke} \\ \text{Anna} \\ \text{Simon} \\ \text{Felix} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Vacuuming} & \text{Mopping} & \text{Ironing} & \begin{array}{c}\text{Window}\\ \text{cleaning}\end{array} & \text{Dishwashing} \end{array} \\ \begin{bmatrix} 30 & 15 & 31 & 41 & 15 \\ 15 & 12 & 38 & 32 & 25 \\ 27 & 25 & 42 & 38 & 16 \\ 35 & 22 & 21 & 18 & 19 \\ 28 & 23 & 30 & 25 & 22 \end{bmatrix} \end{array}$$

Step 1: Perform row reduction. Subtract the minimum element of each row from each element of that row. Then cover the zeros with a minimum of lines.

$$\begin{bmatrix} 15 & 0 & 16 & 26 & 0 \\ 3 & 0 & 26 & 20 & 13 \\ 11 & 9 & 26 & 22 & 0 \\ 17 & 4 & 3 & 0 & 1 \\ 6 & 1 & 8 & 3 & 0 \end{bmatrix}$$

Only three lines, so cannot start allocation.

Step 2: Perform column reduction. Subtract the minimum element of each column from each element of that column. Then cover the zeros with a minimum of lines.

$$\begin{bmatrix} 12 & 0 & 13 & 26 & 0 \\ 0 & 0 & 23 & 20 & 13 \\ 8 & 9 & 23 & 22 & 0 \\ 14 & 4 & 0 & 0 & 1 \\ 3 & 1 & 5 & 3 & 0 \end{bmatrix}$$

Only four lines, so cannot start allocation.

SECTION B – continued

Step 3: Perform the Hungarian algorithm. Find the smallest uncovered number from Step 2 (which, in this case is 3). Add this number to all covered numbers. At the intersections of straight lines, add this number twice.

$$\begin{bmatrix} 15 & 3 & 13 & 26 & 3 \\ 3 & 3 & 23 & 20 & 16 \\ 11 & 12 & 23 & 22 & 3 \\ 20 & 10 & 3 & 3 & 7 \\ 6 & 4 & 5 & 3 & 3 \end{bmatrix}$$

Then subtract the overall smallest number from all the numbers in the matrix, and cover the zeros with a minimum of lines.

$$\begin{array}{c} \\ \text{Leonie} \\ \text{Luke} \\ \text{Anna} \\ \text{Simon} \\ \text{Felix} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Vacuuming} & \text{Mopping} & \text{Ironing} & \text{Window cleaning} & \text{Dishwashing} \end{array} \\ \begin{bmatrix} 12 & 0 & 10 & 23 & 0 \\ 0 & 0 & 20 & 17 & 13 \\ 8 & 9 & 20 & 19 & 0 \\ 17 & 7 & 0 & 0 & 4 \\ 3 & 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

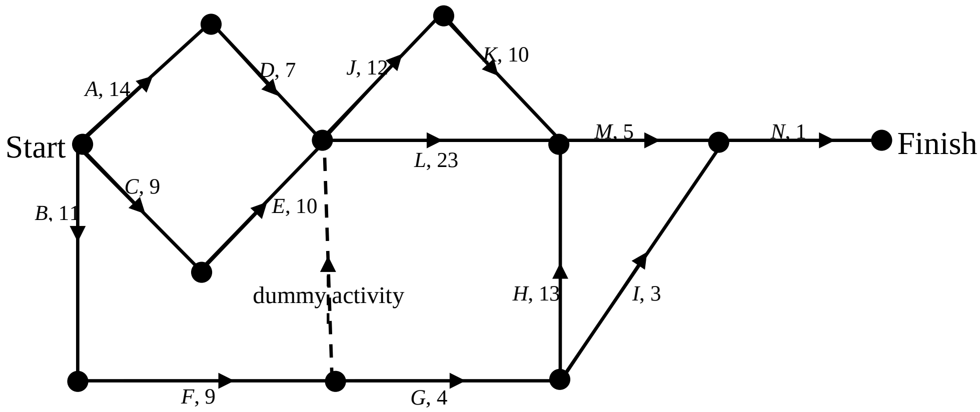
There are now five lines, so we can start our allocations.

- Luke should be allocated vacuuming. (He is the only one for vacuuming.)
- Simon should be allocated ironing (He is the only one for ironing.)
- Leonie and Luke both could be allocated mopping. But, since Luke has already been allocated vacuuming, Leonie must be allocated mopping.
- Simon and Felix both could be allocated window cleaning. But, since Simon has already been allocated ironing, Felix must be allocated window cleaning.
- Leonie, Anna and Felix all can be allocated dishwashing. Since Leonie and Felix have already been allocated other jobs, Anna must be allocated dishwashing.

minimum total time = 15 + 21 + 15 + 25 + 16 = 92 minutes

Question 9

A project has ten activities. The network below shows all these activities together with the time it takes, in days, to complete each activity.

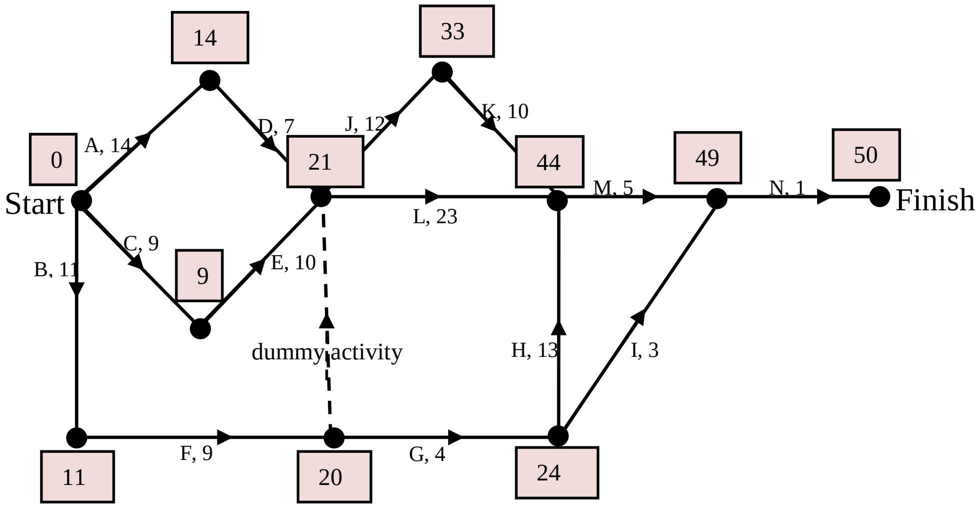


The critical path for this project and its length is

A. *ADJKMN*, 49
B. ***ADLMN*, 50**
C. *CELMN*, 48
D. *BFGIN*, 28
E. *CEJKMN*, 52

Answer is B

Worked Solution



The earliest start times, in days, are on the boxes. The critical path is *ADLMN*.

length of the critical path = 14 + 7 + 23 + 5 + 1 = 50

Module 6: Matrices

Question 1

The matrix $\begin{bmatrix} 30 & 0 \\ 15 & 65 \\ 45 & 5 \\ 15 & 35 \\ 0 & 5 \end{bmatrix}$ can also be written as

A. $\begin{bmatrix} 30 \\ 15 \\ 45 \\ 15 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 65 \\ 5 \\ 35 \\ 5 \end{bmatrix}$

B. $\frac{1}{5} \times \begin{bmatrix} 6 & 0 \\ 3 & 13 \\ 9 & 1 \\ 3 & 7 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 10 & 0 \\ 5 & 13 \\ 15 & 1 \\ 5 & 7 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

D. $\mathbf{5} \times \begin{bmatrix} \mathbf{6} & \mathbf{0} \\ \mathbf{3} & \mathbf{13} \\ \mathbf{9} & \mathbf{1} \\ \mathbf{3} & \mathbf{7} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

E. $\begin{bmatrix} 30 & 0 \end{bmatrix} + \begin{bmatrix} 15 & 65 \end{bmatrix} + \begin{bmatrix} 45 & 5 \end{bmatrix} + \begin{bmatrix} 15 & 35 \end{bmatrix} + \begin{bmatrix} 0 & 5 \end{bmatrix}$

Answer is D

Worked Solution
Let's calculate the matrix operations for all five options.

Option A: $\begin{bmatrix} 30 \\ 15 \\ 45 \\ 15 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 65 \\ 5 \\ 35 \\ 5 \end{bmatrix} = \begin{bmatrix} 30 \\ 80 \\ 50 \\ 50 \\ 5 \end{bmatrix}$

Option B: $\frac{1}{5} \times \begin{bmatrix} 6 & 0 \\ 3 & 13 \\ 9 & 1 \\ 3 & 7 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.2 & 0 \\ 0.6 & 2.6 \\ 1.8 & 0.2 \\ 0.6 & 1.4 \\ 0 & 0.2 \end{bmatrix}$

Option C: $\begin{bmatrix} 10 & 0 \\ 5 & 13 \\ 15 & 1 \\ 5 & 7 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 30 \\ 80 \\ 50 \\ 50 \\ 5 \end{bmatrix}$

Option D: $\mathbf{5} \times \begin{bmatrix} \mathbf{6} & \mathbf{0} \\ \mathbf{3} & \mathbf{13} \\ \mathbf{9} & \mathbf{1} \\ \mathbf{3} & \mathbf{7} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{30} & \mathbf{0} \\ \mathbf{15} & \mathbf{65} \\ \mathbf{45} & \mathbf{5} \\ \mathbf{15} & \mathbf{35} \\ \mathbf{0} & \mathbf{5} \end{bmatrix}$

Option E: $\begin{bmatrix} 30 & 0 \end{bmatrix} + \begin{bmatrix} 15 & 65 \end{bmatrix} + \begin{bmatrix} 45 & 5 \end{bmatrix} + \begin{bmatrix} 15 & 35 \end{bmatrix} + \begin{bmatrix} 0 & 5 \end{bmatrix} = \begin{bmatrix} 105 & 110 \end{bmatrix}$

Question 2

The matrix $12 \times \begin{bmatrix} -2 & 0 & -5 & 8 \\ 3 & 5 & -1 & 0 \\ 1 & 2 & 4 & 6 \end{bmatrix} - 4 \times \begin{bmatrix} 5 & 4 & -8 & 4 \\ 9 & 0 & 6 & 1 \\ 7 & 6 & 9 & 5 \end{bmatrix} + 6 \times \begin{bmatrix} 3 & 0 & 6 & 2 \\ 2 & 1 & 9 & 5 \\ 0 & 2 & -7 & 1 \end{bmatrix}$ equals

A. $\mathbf{0.5} \times \begin{bmatrix} -26 & -16 & 8 & 92 \\ 12 & 66 & 18 & 26 \\ -16 & 12 & -30 & 58 \end{bmatrix}$

B. $\begin{bmatrix} -2 & -2 & 4.5 & 3 \\ -2 & 3 & 1 & 2 \\ -3 & -1 & -6 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 24 & 0 & -60 & 96 \\ 36 & 60 & -12 & 0 \\ 12 & 24 & 48 & 72 \end{bmatrix}$

D. $\mathbf{2} \times \begin{bmatrix} \mathbf{-13} & \mathbf{-8} & \mathbf{4} & \mathbf{46} \\ \mathbf{6} & \mathbf{33} & \mathbf{9} & \mathbf{13} \\ \mathbf{-8} & \mathbf{6} & \mathbf{-15} & \mathbf{29} \end{bmatrix}$

E. $\mathbf{3} \times \begin{bmatrix} -2 & 0 & -5 & 8 \\ 3 & 5 & -1 & 0 \\ 1 & 2 & 4 & 6 \end{bmatrix}$

Answer is D

Worked Solution

$$12 \times \begin{bmatrix} -2 & 0 & -5 & 8 \\ 3 & 5 & -1 & 0 \\ 1 & 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 24 & 0 & -60 & 96 \\ 36 & 60 & -12 & 0 \\ 12 & 24 & 48 & 72 \end{bmatrix}$$

$$-4 \times \begin{bmatrix} 5 & 4 & -8 & 4 \\ 9 & 0 & 6 & 1 \\ 7 & 6 & 9 & 5 \end{bmatrix} = -\begin{bmatrix} 20 & 16 & -32 & 16 \\ 36 & 0 & 24 & 4 \\ 28 & 24 & 36 & 20 \end{bmatrix}$$

$$6 \times \begin{bmatrix} 3 & 0 & 6 & 2 \\ 2 & 1 & 9 & 5 \\ 0 & 2 & -7 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 0 & 36 & 12 \\ 12 & 6 & 54 & 30 \\ 0 & 12 & -42 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 24 & 0 & -60 & 96 \\ 36 & 60 & -12 & 0 \\ 12 & 24 & 48 & 72 \end{bmatrix} - \begin{bmatrix} 20 & 16 & -32 & 16 \\ 36 & 0 & 24 & 4 \\ 28 & 24 & 36 & 20 \end{bmatrix} + \begin{bmatrix} 18 & 0 & 36 & 12 \\ 12 & 6 & 54 & 30 \\ 0 & 12 & -42 & 6 \end{bmatrix} = \begin{bmatrix} -26 & -16 & 8 & 92 \\ 12 & 66 & 18 & 26 \\ -16 & 12 & -30 & 58 \end{bmatrix}$$

$$= \mathbf{2} \times \begin{bmatrix} \mathbf{-13} & \mathbf{-8} & \mathbf{4} & \mathbf{46} \\ \mathbf{6} & \mathbf{33} & \mathbf{9} & \mathbf{13} \\ \mathbf{-8} & \mathbf{6} & \mathbf{-15} & \mathbf{29} \end{bmatrix}$$

Question 3

Matrix **A** is of order $d \times e$.
Matrix **C** and matrix **X** are of order $e \times f$.
d, e and *f* are positive integers.

The set of linear equations defined by the matrix equation **AX** = **C** has a unique solution for **X** when

A. *e* is equal to *f* and matrix **C** is not singular.
B. *e* is greater than *f* and matrix **C** is singular.
C. *e* is less than *f* and matrix **C** is not singular.
D. *d* is greater than *e* and matrix **A** is singular.
E. ***d* is equal to *e* and matrix A is not singular.**

Answer is E

Worked Solution

The set of linear equations defined by the matrix equation **AX** = **C** has a unique solution for **X** when the determinant of matrix **A** is not equal to 0, i.e. matrix **A** is not singular.

The solution of this matrix equation is $\mathbf{X} = \mathbf{A}^{-1} \times \mathbf{C}$..

Matrix **A** has to be a square matrix, so *d* must be equal to *e*.

Question 4

Ruby bought nine lollies and five chewing gums and paid \$19. Ruby's friend Charlie bought eleven lollies and six chewing gums and paid \$23.
Let x be the cost of a lolly and y be the cost of a chewing gum.
The solution matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 9 & 5 \\ 11 & 6 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$

B. $\begin{bmatrix} 9 & 11 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$

C. $\begin{bmatrix} \mathbf{-6} & \mathbf{5} \\ \mathbf{11} & \mathbf{-9} \end{bmatrix}\begin{bmatrix} \mathbf{19} \\ \mathbf{23} \end{bmatrix}$

D. $\begin{bmatrix} -6 & 11 \\ 5 & -9 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$

E. $\begin{bmatrix} 9 & 5 \\ 11 & 6 \end{bmatrix}\begin{bmatrix} 23 \\ 19 \end{bmatrix}$

Answer is C

Worked Solution

$$\begin{bmatrix} 9 & 5 \\ 11 & 6 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \left(\begin{bmatrix} 9 & 5 \\ 11 & 6 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 19 \\ 23 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 & 5 \\ 11 & -9 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

Note: If you obtained an answer of A, you did not take the inverse of the square matrix. Please be careful with the order of the items. If you obtained answer D, the columns and rows of the matrix were swapped.

Question 5

The Diamond Milk factory makes skim milk (S), low-fat milk (L) and full-cream milk (F). Each type of milk comes in either 2-litre bottles or 3-litre bottles.

The price of each type of milk, in dollars, is listed in a price matrix, **A**:

$$\mathbf{A} = \begin{array}{c} \begin{array}{ccc} S & L & F \end{array} \\ \begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{2-litre bottles} \\ \text{3-litre bottles} \end{array}$$

Diamond Milk wants to increase the price of a 2-litre bottle of milk by 25% and decrease the price of a 3-litre bottle of milk by 10%.

The matrix equation used to calculate the new prices is best represented by

A. $\begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix} \times \begin{bmatrix} 1.25 \\ 0.9 \end{bmatrix}$

B. $\begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix} \times \begin{bmatrix} 0.9 \\ 1.25 \end{bmatrix}$

C. $\begin{bmatrix} 1.25 & 0.9 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix}$

D. $\begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix} \times \begin{bmatrix} 1.25 & 0 \\ 0 & 0.9 \end{bmatrix}$

E. $\begin{bmatrix} \mathbf{1.25} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.9} \end{bmatrix} \times \begin{bmatrix} \mathbf{3.50} & \mathbf{3.20} & \mathbf{2.80} \\ \mathbf{4.80} & \mathbf{4.50} & \mathbf{3.90} \end{bmatrix}$

Answer is E

Worked Solution

Increasing the price of a 2-litre bottle of milk by 25% means multiplying the first row of the matrix by 1.25, and decreasing the price of a 3-litre bottle milk by 10% means multiplying the second row of the matrix by 0.9.

So, the new price matrix is $\begin{bmatrix} 1.25 & 0 \\ 0 & 0.9 \end{bmatrix} \times \begin{bmatrix} 3.50 & 3.20 & 2.80 \\ 4.80 & 4.50 & 3.90 \end{bmatrix}$.

Note: Although option D is very similar to the correct answer, it cannot be the right answer because the order of multiplication is reversed. The multiplication in option D is undefined. (The first matrix is of order 2×3 and the second matrix is of order 2×2.)

Question 6

$$\mathbf{T} = \begin{bmatrix} 0.4 & 0.5 & 0.3 \\ 0.5 & 0.2 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{bmatrix}$$ **T** is a transition matrix.

$$\mathbf{S}_{15} = \begin{bmatrix} 400 \\ 400 \\ 200 \end{bmatrix}$$ $\mathbf{S}_{15}$ is a state matrix.

If $S_{15} = TS_{14}$, then S_{14} equals

A. $\begin{bmatrix} 400 \\ 400 \\ 200 \end{bmatrix}$

B. $\begin{bmatrix} 300 \\ 300 \\ 400 \end{bmatrix}$

C. $\begin{bmatrix} 900 \\ 100 \\ 200 \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{600} \\ \mathbf{200} \\ \mathbf{200} \end{bmatrix}$

E. $\begin{bmatrix} 500 \\ 400 \\ 100 \end{bmatrix}$

Answer is D

Worked Solution

If $\mathbf{S}_{15} = \mathbf{T}\mathbf{S}_{14}$, then $\mathbf{S}_{14} = \mathbf{T}^{-1} \times \mathbf{S}_{15}$.

$$\mathbf{S}_{14} = \left(\begin{bmatrix} 0.4 & 0.5 & 0.3 \\ 0.5 & 0.2 & 0.3 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 400 \\ 400 \\ 200 \end{bmatrix} = \begin{bmatrix} 600 \\ 200 \\ 200 \end{bmatrix}$$

Question 7

Emily sat for a multiple-choice test consisting of 40 questions. Each question had five alternative answers: A, B, C, D or E.

Emily randomly guessed the answer to the first question. She then determined her answers to the remaining 39 questions by following this transition matrix:

$$\mathbf{T} = \begin{array}{c} \text{This question} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \;\text{Next question} \\ D \\ E \end{array}$$

Which of the following statements is **not true**?

A. Emily would always give the same answer to the odd numbered questions.
B. Emily would always give the same answer to the sixth and eighth questions.
C. **It's impossible for Emily to give the same answer to all forty questions.**
D. It's possible that Emily gave the same answer to exactly twenty of forty questions.
E. Emily would always give the same answer to the even numbered questions.

Answer is C

Worked Solution

Let's think about all the alternative answers for the first twenty questions based on this transition matrix:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B	E	B

Now let's interpret the table and discuss all five options.
Option A: Whatever answer Emily gave to the first question, all odd numbered questions had the same answer as each other.

Option B: Emily would always give the same answer to the sixth and eighth questions.

Option C: It's possible for Emily to give the same answer to all forty questions.

Option D: We can see from the table that, when Emily started her test with an answer of A, she answered exactly half of the test as A and the rest as C. So it's possible that Emily gave the same answer to exactly twenty of forty questions.

Option E: Whatever answer Emily gave to the second question, all even numbered questions had the same answer as each other.

The following information refers to Questions 8 and 9.

The Diamond Milk factory delivers milk to three different supermarkets: A, B and C.

In April, equal numbers of customers buy milk from supermarkets B and C and the number of customers who buy milk from supermarket A is twice as many as that who buy from supermarket B.

In each month after April, a percentage of customers changed the supermarket at which they bought milk. This movement of customers is described by this transition matrix:

$$\begin{array}{c} \text{This month} \\ \begin{array}{ccc} A & B & C \end{array} \end{array}$$

$$\mathbf{T} = \begin{bmatrix} 0.5 & 0.7 & 0.5 \\ 0.3 & 0.3 & 0.5 \\ 0.2 & 0 & 0 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array} \text{ Next month}$$

Question 8

Assume the pattern of movement described by the transition matrix continues. Which of the following statements is **not true**?

A. In the long term, more customers will buy milk from supermarket A than from supermarket C.
B. 30% of customers who buy milk from supermarket A in a given month will buy milk from supermarket B the next month.
C. There will be 14% more customers buying milk from supermarket A in June than there was in April.
D. **In the long term, no customers will be buying milk from supermarket C.**
E. There will be 40% more customers buying milk from supermarket B in May than there were in April.

Answer is D

Worked Solution

i) We can clearly see from the transition matrix that 30% of customers who buy milk from supermarket A in a given month will buy milk from supermarket B the next month so option B is true.

ii) Let the state matrix in April be $\mathbf{S_0} = \begin{bmatrix}200\\100\\100\end{bmatrix}$.

So the state matrix in May would be $\mathbf{S_1} = \mathbf{T} \times \mathbf{S_0} = \begin{bmatrix}0.5 & 0.7 & 0.5\\0.3 & 0.3 & 0.5\\0.2 & 0 & 0\end{bmatrix} \times \begin{bmatrix}200\\100\\100\end{bmatrix} = \begin{bmatrix}220\\140\\40\end{bmatrix}$.

We can see from this state matrix that there will be 40% more customers buying milk from supermarket B in May than there were in April, so option E is true.

iii) The state matrix in June would be:

$$\mathbf{S_2} = \mathbf{T}^2 \times \mathbf{S_0} = \left(\begin{bmatrix}0.5 & 0.7 & 0.5\\0.3 & 0.3 & 0.5\\0.2 & 0 & 0\end{bmatrix}\right)^2 \times \begin{bmatrix}200\\100\\100\end{bmatrix} = \begin{bmatrix}228\\128\\44\end{bmatrix}.$$

We can see from this state matrix that there will be 14% (that is $\frac{28}{200} \times 100\% = 14\%$) more customers buying milk from supermarket A in June than there were in April, so option C is true.

iv) The steady state matrix would be:

$$\mathbf{S_{100}} = \mathbf{T}^{100} \times \mathbf{S_0} = \left(\begin{bmatrix}0.5 & 0.7 & 0.5\\0.3 & 0.3 & 0.5\\0.2 & 0 & 0\end{bmatrix}\right)^{100} \times \begin{bmatrix}200\\100\\100\end{bmatrix} = \begin{bmatrix}225.81\\129.03\\45.16\end{bmatrix}.$$

We can see from the steady state matrix that, in the long term, more customers will buy milk from supermarket A than supermarket C, so option A is true.

In the long term, there will always be customers buying milk from supermarket C, so option D is false.

SECTION B – continued

Question 9

The number of customers buying milk from supermarket *C* in the long term will be

A. **approximately 55% less than the initial number in April.**
B. 0.
C. equal to the initial number in April.
D. approximately 29% more than the initial number in April.
E. more than the number of people buying milk from supermarket B.

Answer is A

Worked Solution

Let the state matrix in April be $\mathbf{S_0} = \begin{bmatrix} 200 \\ 100 \\ 100 \end{bmatrix}$.

Then the steady state matrix would be:

$$\mathbf{S_{100}} = \mathbf{T^{100}} \times \mathbf{S_0} = \left(\begin{bmatrix} 0.5 & 0.7 & 0.5 \\ 0.3 & 0.3 & 0.5 \\ 0.2 & 0 & 0 \end{bmatrix} \right)^{100} \times \begin{bmatrix} 200 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 225.81 \\ 129.03 \\ 45.16 \end{bmatrix}.$$

The number of customers buying milk from supermarket C in the long term will eventually drop from 100 to 45, which is a 55% decrease from the number in April.

END OF SOLUTIONS BOOK