Neap:

Trial Examination 2010

VCE Further Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

Core

Question 1

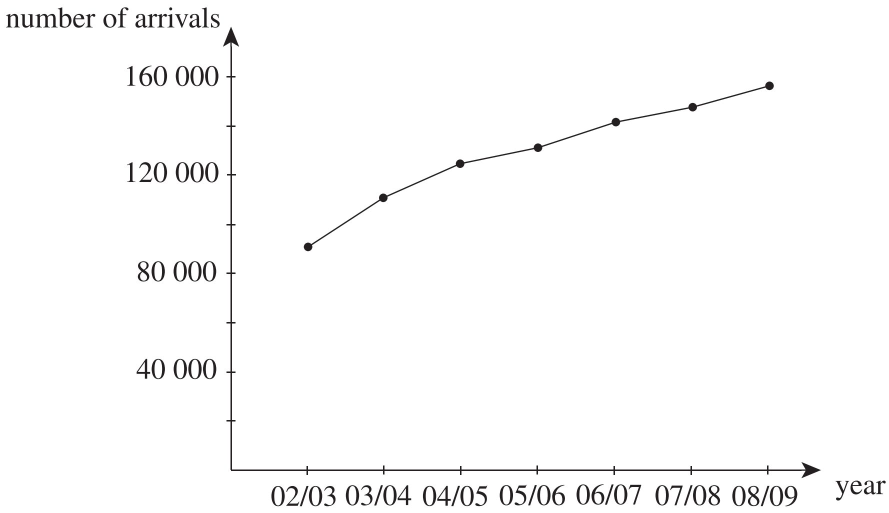
a. Drug B has a median of 28, Drug A has a median of 25, so Drug B appears to be more effective. A1

This was calculated by finding the 6th value out of 11 for Drug A, *and the 8th of 15 for Drug* B.

b. 11 + 15 = 26 patients A1

Question 2

a.



correct time series A1 A1

b.

111 600
123 400
131 600
140 100
149 400

$$\frac{111\,600 + 123\,400 + 131\,600 + 140\,100}{4} = 126\,675$$

$$\frac{123\,400 + 131\,600 + 140\,100 + 149\,400}{4} = 136\,125$$

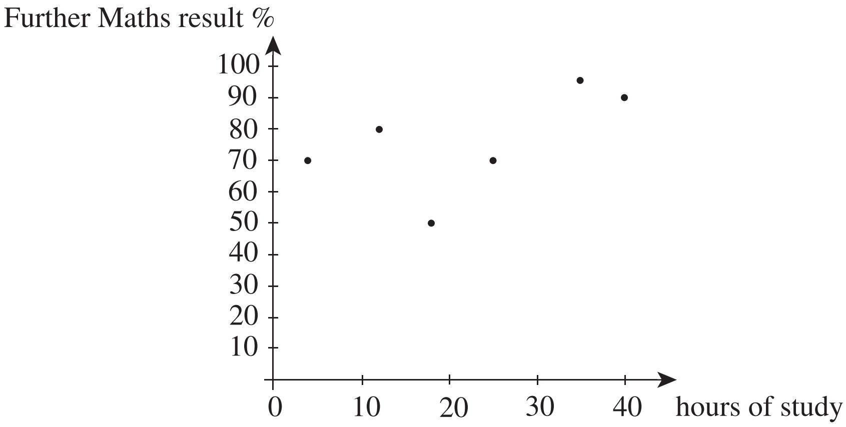
$\longrightarrow$ 131 400 M1 A1

Question 3

a. Mean = adding the hours of study and dividing by 6 = 22.3

Standard deviation = using the *Sx* function on the calculator gives *Sx* = 13.7

b.



hours of study as the horizontal axis A1
accuracy A1

c. Using the regression function on the calculator gives $r = 0.59$. A1

d. **35**% of the variation in the further maths results is due to a variation in **hours of study**. A1

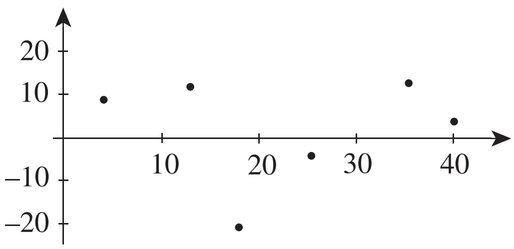
$r^2 = 0.348 = 35\%$

e. **i.**

Hours of study	25	18	4	12	35	40
Actual Maths %	70	50	70	80	95	90
Predicted Maths %	75.5	70.6	60.8	66.4	82.5	86
Residual	–5.5	–20.6	9.2	13.6	12.5	4

calculation of Predicted Maths % A1
calculation of residuals A1

ii.



A1

Module 1: Number patterns

Question 1

a. This can clearly be seen to be an arithmetic sequence with first term 146 and common difference 18. G_n is just the sequence itself. A1

Thus

$$G_n = a + (n-1)d$$

$$= 146 + 18(n-1)$$

$$= 128 + 18n$$ A1

b. This is month 15.

$G_{15} = 128 + 18(15) = 398$ A1

c. $500 = 128 + 18n$ M1

$372 = 18n$

$n = 20.67$

The 20th month will still be insufficient. Month 21 is thus the month required.
This is February 2012. A1

Question 2

a. This is clearly a geometric sequence with first term 27 and common ratio $\frac{2}{3}$. A1

Thus

$$t_n = a.r^{n-1}$$

$$= 27\left(\frac{2}{3}\right)^{n-1}$$ A1

$$= 40.5\left(\frac{2}{3}\right)^n$$

Either of last two lines

b. This is term 5.

$$t_5 = 27\left(\frac{2}{3}\right)^4 = \frac{16}{3}\%$$

$$= 5\frac{1}{3}\%$$ A1

Either of last two lines

c. Students have a choice of trial and error or calculation using logs.

$$3 = 27\left(\frac{2}{3}\right)^{n-1}$$

$$\frac{1}{9} = \left(\frac{2}{3}\right)^{n-1}$$

$$\log\left(\frac{1}{9}\right) = (n-1)\log\left(\frac{2}{3}\right)$$

$$n - 1 = 5.419$$

$$n = 6.419$$

Thus the 7th year is 2016. A1

d. The quantity sought in this question is the sum of 12 terms. M1

$$S_{12} = \frac{a(1-r^{12})}{1-r}$$

$$= \frac{27\left(1-\left(\frac{2}{3}\right)^{12}\right)}{1-\frac{2}{3}}$$

$$= 81\left(1-\left(\frac{2}{3}\right)^{12}\right)$$

Thus the total of all discounts is 80.4%. A1

e. The quantity sought now is the infinite term sum

$$S_{12} = \frac{a}{1-r}$$

$$= \frac{27}{1-\frac{2}{3}} = 81$$

Thus this cumulative discount cannot exceed 81%. A1

Question 3

a. $P_{n+2} = 0.5(P_n + P_{n+1}) + 40$ A1

b. We require term 5. M1

$P_1 = 1230$

$P_2 = 1320$

$P_3 = 1275 + 40 = 1315$

$P_4 = 1317.5 + 40 = 1357.50$

$P_5 = 1336.25 + 40$

$= \$1376.25$ A1

Module 2: Geometry and trigonometry

Question 1

a. $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\tan\theta = \frac{80}{425}$

$\theta = 10.6603\ldots°$

$\theta \approx 11°$ A1

b. 11°

The angle of depression of the post office from the factory is the same size as the angle of elevation of the factory from the post office. A1

Question 2

a. $C = \pi \times d$

$C = \pi \times 84$

$C = 263.894$

$C \approx 264$ mm A1

b. $C = 2\pi r$

$r = \frac{C}{2\pi}$

$r = 14.0056$

$r \approx 14$ mm A1

c. length = 8×14

length = 112 mm A1

d. volume = length × width × height

volume = $112 \times 28 \times 28$

volume = 87808 mm^3 M1

volume = 87.808 cm^3

volume ≈ 88 cm^3 A1

e. Volume of one golf ball $= \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 14^3$

$= 11\ 494 \text{ mm}^3$

Volume of four golf balls $= 4 \times 11\ 494$

$= 45\ 976.2 \text{ mm}^3$

M1

Volume of air

$= 87\ 808 - 45\ 976.2$

$= 41\ 831.8 \text{ mm}^3$

$\approx 42 \text{ cm}^3$ A1

Question 3

a. Map scale 1 : 3000

Actual distance is 420 metres, so map distance is $\frac{420}{3000} = 0.14$ m or 14 cm. A1

b. $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 420^2 + 443^2 - 2 \times 420 \times 443 \times \cos 96°$

$a^2 = 411546.13\ldots$

$a = 641.519\ldots$

$a \approx 642$ m A1

c. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

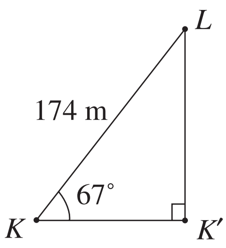
$\cos A = \frac{420^2 + 642^2 - 443^2}{2 \times 420 \times 642}$

$\cos A = 0.727479$

$A = 43.3245\ldots°$

$A \approx 43°$ A1

Question 4



KK'

$$\frac{\text{adj}}{\text{hyp}} = \cos 67°$$

$$\frac{\text{adj}}{174} = \cos 67°$$

$$\text{adj} = 67.9872\ldots$$

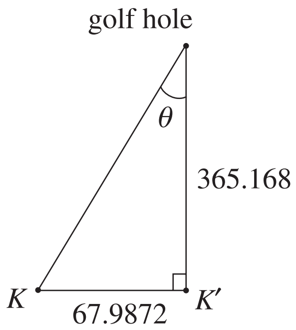
LK'

$$\frac{\text{opp}}{\text{hyp}} = \sin 67°$$

$$\frac{\text{opp}}{174} = \sin 67°$$

$$\text{opp} = 160.168\ldots$$

M1



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \frac{67.9872}{365.168}$$

$$\theta = 10.5466°$$

M1

Bearing from hole to point K:

Bearing = 180 + 10.5°

Bearing = 190.5° A1

Module 3: Graphs and relations

Question 1

a. $12x + 16y \geq 384$

$x + y \leq 30$

$x \geq y + 1$

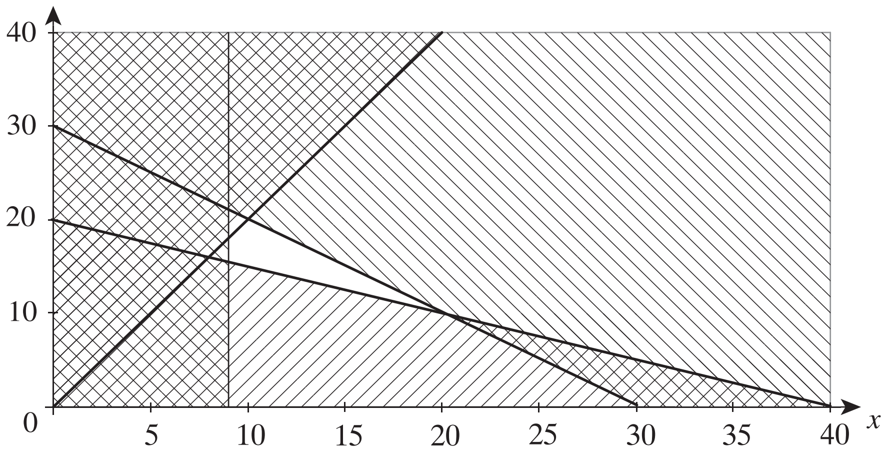
$0 \leq x \leq 20$

$0 \leq y \leq 20$

A2

1 mark lost for each incorrect or missing constraint

b.



Required region unshaded

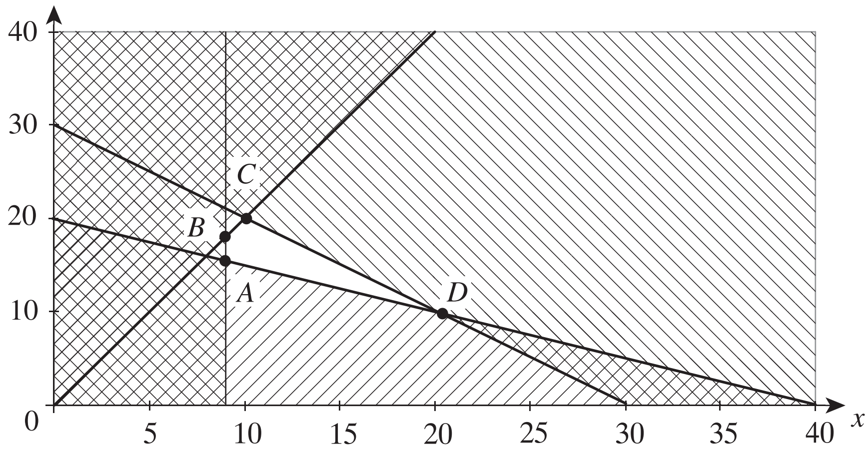
Graph showing shading not just lines M1

Correct lines/boundaries A1

Correct shading A1

c. $C = 20x + 24y$ A1

d.



Required region unshaded

A	$x = 9$ $y = 20 - \frac{9}{2} = 15.5$ (round to 16) $A(9, 15.5)$: $C = 180 + 384 = 564$
B	$x = 9$ $y = 18$ $B(9, 18)$: $C = 180 + 432 = 612$
C	$2x = 30 - x$ $x = 10$ $y = 30 - 10 = 20$ $C(10, 20)$: $C = 200 + 480 = 680$
D	$30 - x = 20 - \frac{x}{2}$ $10 = \frac{x}{2}$ $x = 20$ $y = 30 - 20 = 10$ $D(20, 10)$: $C = 400 + 240 = 640$

Coordinates of all points A1
Calculation of all objective function values A1

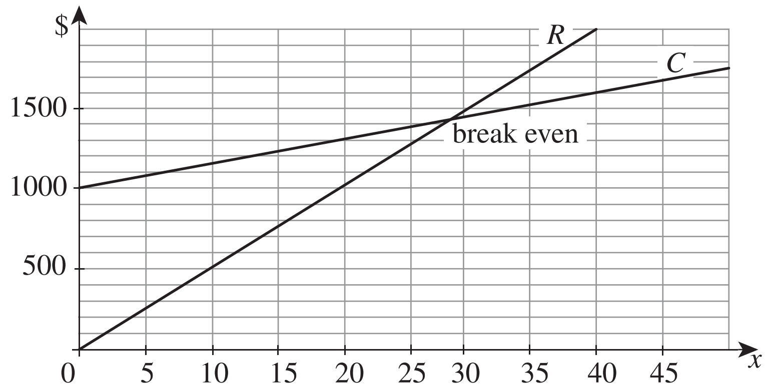
e. Point A on the graph has the least cost. Thus this option (9 classes for Trevor and 16 for Jane) is optimal. The weekly cost is \$584. A1

Question 2

a. $C = 1000 + 15n$ A1

b. $R = 50n$ A1

c.



Both lines correct A1
Break even labelled correctly A1

d.

$$C = R$$
$$1000 + 15n = 50n$$
$$1000 = 35n$$
$$n = 28.57$$ M1

Thus require 29 hours per week. A1

Module 4: Business-related mathematics

Question 1

a. Costs are $5 + 2.5 = \$7.50$

Profit is $13.90 - 7.50 = \$6.40$

$\%\text{ Profit} = \dfrac{6.40}{7.50} \times 100 = 85.3\,\%$ A1

b. Using TVM solver or equivalent,

$N = 10 \times 4 = 400$, PV is –170 000, FV is 0, PMT is the question, C/Y is 4, P/Y is 4 M1

$= \$6546.05$ A1

c. **i.** Scrap value $= 180\ 000 \times 0.88^{10} = \$50\ 130.18$ M1 A1

ii. 8% for ten years $= \dfrac{180\ 000 \times 8 \times 10}{100} = 144\ 000$

Scrap value $= 180\ 000 - 144\ 000 = \$36\ 000$ M1 A1

1 mark for method

Question 2

a. Using TVM solver or equivalent,

$N = 10 \times 12$, PV is –210 000, FV is the question, % is 8.4, PMT is 1900, C/Y is 12, P/Y is 12

Amount owing = \$129 553.24 A1

Note: as some students will realise, there are not exactly 26 fortnights per year. They may use 26.07 or 26.08 for a leap year. Accept answers within \$500 of given solution.

b. As in part **a** using TMV solver or equivalent, but change $N = 260$, P/Y to 26 and PMT to \$950.

Amount owing = \$99 206.87 A1

c. With fortnightly repayments, an extra payment is made each year.

and/or

Making earlier repayments reduces the balance more quickly and less interest will be charged.

A1

1 mark for either or both answers

Question 3

a. Allowing for inflation, the price would be $= 32000 \times 1.023^3 = \$34\ 259.17$. M1

Therefore the profit is $41\ 500 - 34\ 259.17 = \$7240.83$. A1

b. **i.** Outstanding money is $0.8 \times 32\ 000 = \$25\ 600$.

Repayments total $60 \times 600 = \$36\ 000$.

Interest is \$10 400 on a loan of \$25 600.

1 mark for calculation of interest M1

Using Simple interest formula gives:

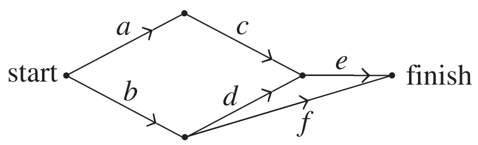
$r = \dfrac{100 \times I}{P \times T} = \dfrac{100 \times 10\ 400}{25\ 600 \times 5} = 8.1\%$ per annum. A1

ii. $R_e = \dfrac{2n \times r_f}{n + 1} = \dfrac{2 \times 60 \times 8.1}{60 + 1} = 15.9\%$ A1

Module 5: Networks and decision mathematics

Question 1

a.



A1

b. $a \to c \to e = 30$ minutes A1

c. b, d or f A1

Question 2

a. 14 kilometres

Tamsville Market $\to$ ABC café $\to$ Flower cafeé $\to$ Sun café $\to$ Cuppa café

7 + 2 + 3 + 2 = 14 A1

b. **i.** Hamiltonian path

ii. Crisp cafe A1

Visiting Flower café or ABC café first will make it impossible to create a Hamiltonian path. A1

Question 3

Amanda must be allocated to Task IV

Carly must be allocated to Task II

A1

Ben must be allocated to Task I. David must be allocated to Task III. This leaves tasks II and IV for Amanda and Carly. Amanda cannot be allocated to Task II.

Question 4

a. 50 minutes A1

b. 45 minutes A1

c. a, e, h A1

d. 55 minutes A1

e. Activity a

Activities e and h also lie on the critical path but they are limited in relation to their capacity to reduce the project completion time. A1

f. $220 A1

Activity a should be reduced by 20 minutes. Doing so will result in another critical path:

$a \to e \to h = 35$ minutes

$c \to g \to i = 35$ minutes

Reducing activity a by more than 20 minutes will not reduce the project completion time below 35 minutes. This is because path cgi becomes a new critical path of 35 minutes duration. M1

Module 6: Matrices

Question 1

a. The matrix required is $\begin{bmatrix} 1650 & 1230 & 1710 & 1710 \\ 2170 & 1610 & 2430 & 2160 \\ 2980 & 2130 & 2750 & 3040 \end{bmatrix}$. A1

b. **i.** $$AB = \begin{bmatrix} 1650 & 1230 & 1710 & 1710 \\ 2170 & 1610 & 2430 & 2160 \\ 2980 & 2130 & 2750 & 3040 \end{bmatrix} \begin{bmatrix} 0.92 & 1.00 & 0.91 \\ 0.90 & 0.98 & 0.96 \\ 0.95 & 0.79 & 0.92 \\ 0.92 & 0.96 & 0.98 \end{bmatrix}$$

$$= \begin{bmatrix} 5822.7 & 5847.9 & 5931.3 \\ 7741.1 & 7741.1 & 7872.7 \\ 10067.9 & 10158.3 & 10265.8 \end{bmatrix}$$ A1

ii. The matrix product gives the cost of each of the nine different scenarios. The rows represent economy, business and first class respectively. Column 1, 2 and 3 represent packages *Q*, *R* and *S* respectively. A1

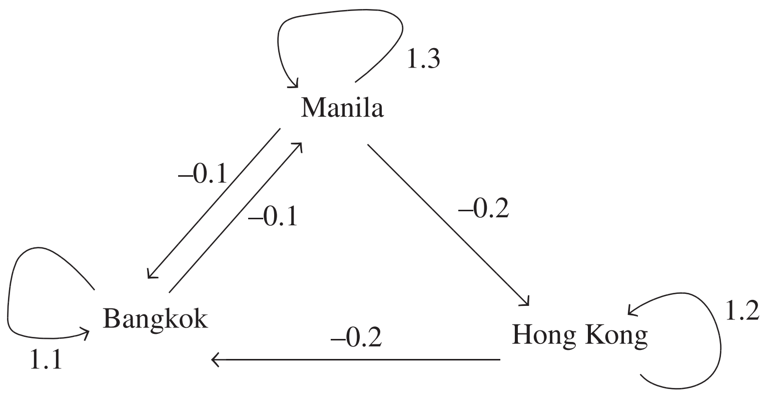
For example, element 10158.3 in row 3, column 2, represents the cost of first class travel to all 4 destinations under package *R*. M1

iii. Firstly, we should consider the issue of whether *BA* is defined. For this purpose, it is necessary that the numbers of columns of *B* match the number of rows of *A*. Since this true, *BA* is indeed defined, and is in fact, a 4×4 matrix. A1

Secondly, we need to discern meaning from this product. There simply is none. Each column of *B* represents a different plan while each row of *A* represents a different flight class. Multiplying these values is entirely pointless and meaningless. A1

Question 2

a.



M1

Thus the transition matrix is $\begin{bmatrix} 1.3 & 0 & -0.1 \\ -0.1 & 1.2 & 0 \\ -0.2 & -0.2 & 1.1 \end{bmatrix}$ A1

b. $AB = \begin{bmatrix} 1.2 & 0 & -0.1 \\ -0.1 & 1.2 & 0 \\ -0.1 & -0.2 & 1.1 \end{bmatrix} \begin{bmatrix} 0.91 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.92 \end{bmatrix}$

$= \begin{bmatrix} 1.092 & 0 & -0.092 \\ -0.091 & 1.14 & 0 \\ -0.091 & -0.19 & 1.012 \end{bmatrix}$ A1

c. The matrix that we require here is the inverse matrix, A^{-1}.

$A^{-1} = \begin{bmatrix} 0.8408 & 0.0127 & 0.0764 \\ 0.0701 & 0.8344 & 0.0064 \\ 0.0892 & 0.1529 & 0.9172 \end{bmatrix}$ M1

$A^{-1}S_0 = \begin{bmatrix} 0.3599 \\ 0.3217 \\ 0.3185 \end{bmatrix}$ A1

d.

$S_3 = A^3 S_0$

$= \begin{bmatrix} 1.761 & 0.070 & -0.398 \\ -0.433 & 1.726 & 0.035 \\ -0.4503 & -1.2278 & 1.5321 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.35 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.6294 \\ 0.4397 \\ -0.0691 \end{bmatrix}$ M1

Students may choose a later state

Calculation of A^2 and A^3 thus reveals problems. $S_3 = A_3 S_0$ contains negative values for Bangkok travel so this is clearly unrealistic. It also becomes obvious that no steady state exists. A1

e. It is necessary firstly to produce a matrix from the flight price data. Three elements exist. The choice that exists is whether a row or column matrix should be used and how the multiplication should be formed. The proportions are already in the form of a column and two column matrices cannot be multiplied, so prices must form a row, order 1×3. The second choice is whether prices should be premultiplied by proportions or post-multiplied. The former results in a 3×3 matrix, the latter in a single element. Given that a single element is all that we require, it is clearly the latter that we require. M1

$\begin{bmatrix} 1650 & 1230 & 1710 \end{bmatrix} \begin{bmatrix} 0.455 \\ 0.38 \\ 0.165 \end{bmatrix} = \begin{bmatrix} 1500.3 \end{bmatrix}$ A1