Mathematical Association of Victoria
Trial Examination 2010

STUDENT NAME ____

FURTHER MATHEMATICS

Written Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core			
	Number of questions	*Number of questions to be answered*	*Number of marks*
	5	5	15
Module			
	Number of modules	*Number of modules to be answered*	*Number of marks*
	6	3	45 Total 60

• Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
• Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

• Question and answer book of 31 pages, with a detachable sheet of miscellaneous formulas at the back.
• Working space is provided throughout the book.

Instructions

• Detach the formula sheet from the centre of this book during reading time.
• Write your **student name** in the space provided above on this page.
• All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

TURN OVER

Core

Question 1

In a small survey, twenty-five students were asked how they travelled to school (walk, train, car, bus, tram) on a particular school day.

Their responses are recorded below.

car	train	bus	walk	walk
bus	car	train	bus	tram
car	bus	walk	car	car
train	car	walk	walk	tram
bus	walk	car	bus	train

Use the data to

a. complete the following frequency table

Travel to school	**Frequency**
Car	
Public Transport (tram, train or bus)	
walk	
Total	25

1 mark

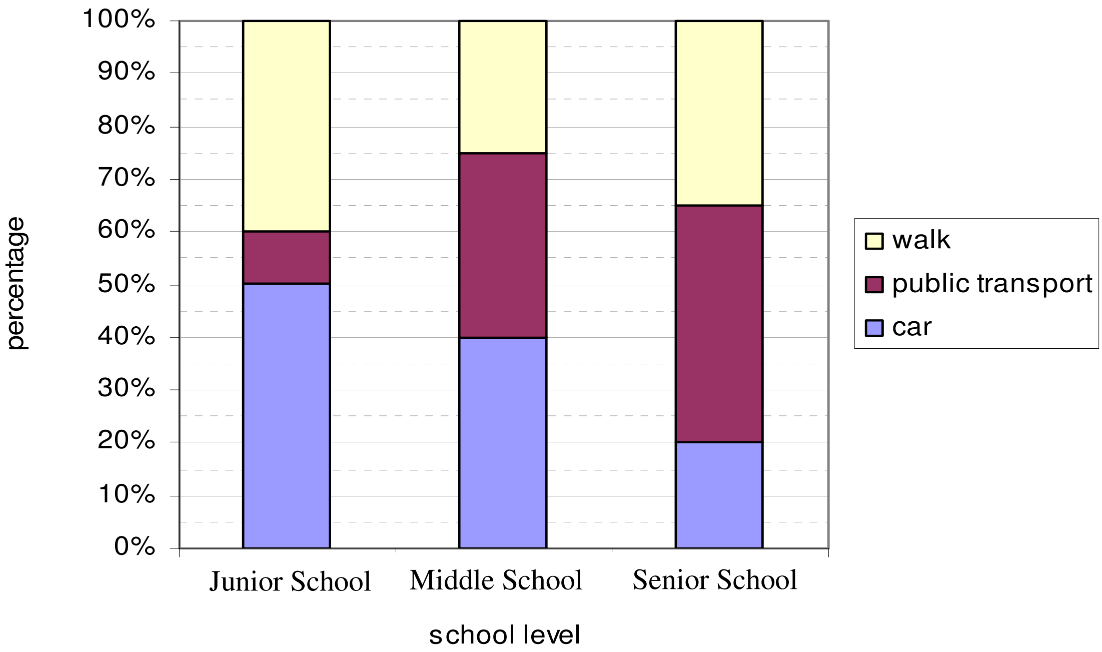
b. determine the percentage of students who travelled by car.

1 mark

Core - continued

Question 2

In a larger survey, students from the junior school, middle school and senior school were asked how they travelled to school (car, public transport or walk) on a particular school day. The results are displayed in the percentaged segmented bar chart below.



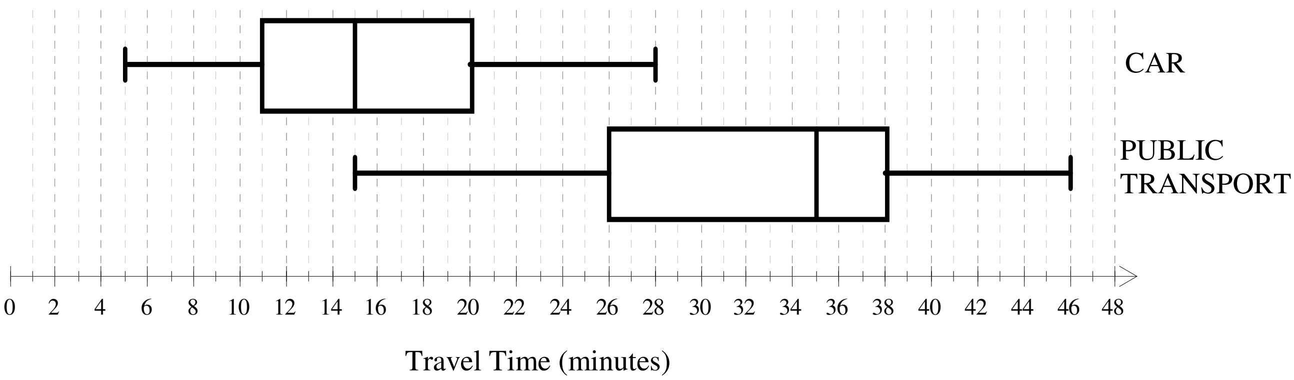
Does the percentage segmented bar chart support the opinion that, for this school, the type of travel (car, public transport or walk) undertaken is associated with their school level?
Justify your answer by quoting appropriate statistics.

2 marks

Core - continued
TURN OVER

Question 3

The travel time (in minutes) was also recorded in the larger survey. The results for students who travelled by public transport and those who travelled by car are summarised in the boxplots below.



a. Complete the following sentence.

75% of students who travel by public transport take at least ________ minutes to get to school

1 mark

b. Describe the shape of the distribution of travel times for those students who travelled to school by public transport.

____

____

1 mark

c. 80 students travelled by car and 120 travelled by public transport. How many of these 200 students took between 15 and 35 minutes to travel to school?

____

____

____

____

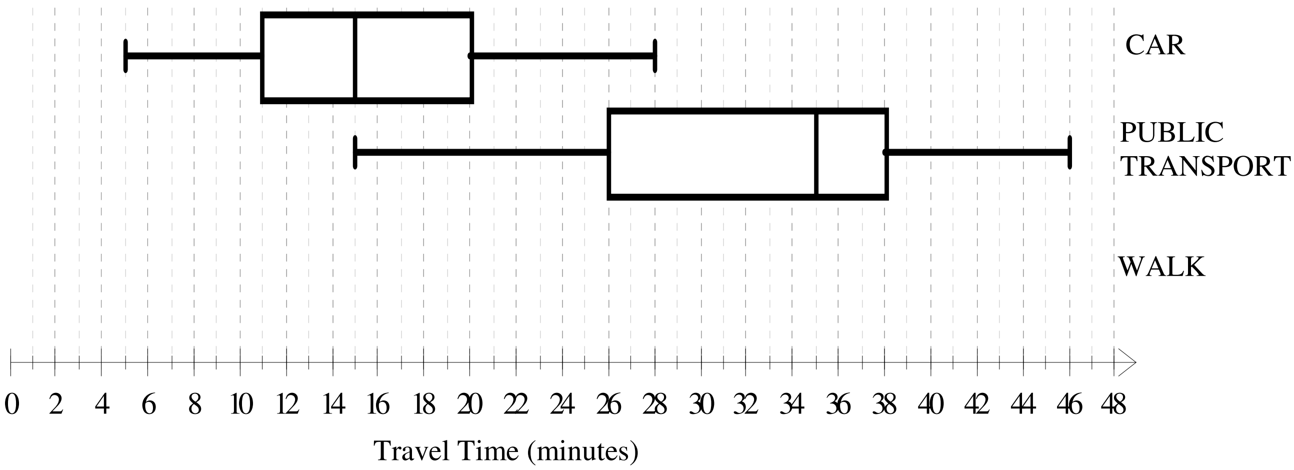
1 mark

Core - continued

The time taken for those who walk to school are shown in the table below.

walk (minutes)	14	25	20	16	44	34	24	18	16	25	6	14	25	22	10

d. Complete the boxplot display below by constructing and drawing a boxplot that shows the travel times for students who walk to school.



2 marks

e. The student who took 44 minutes to walk to school on that particular day had forgotten his lunch money and walked back home some of the way to collect it before walking to school again. If it normally takes this student 35 minutes to walk to school, determine whether this time of 35 minutes would be regarded as an outlier. Give reasons for your answer.

___

___

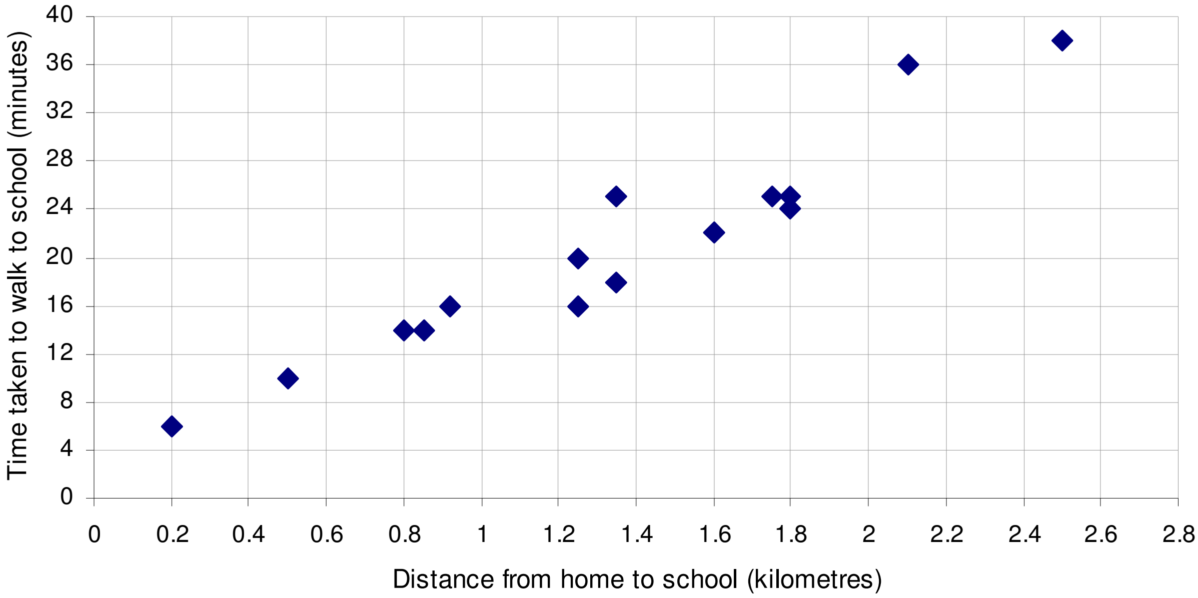
___

___

1 mark

Core - continued
TURN OVER

Question 4
The scatterplot below shows the time taken to walk to school and the distance from home to school.



An equation of the least squares regression line for this data set is

$$\text{Time taken to walk} = 2.5 + 13.5 \times \text{distance to school}$$

a. Draw this line on the scatterplot.

1 mark

b. Interpret the slope

____

____

1 mark

One of the students on the scatterplot lives 2.1 km from school. The equation of the regression line is used to predict the time it takes for this student to walk to school.

c. Find the residual value for this prediction in minutes correct to one decimal place.

____

____

1 mark

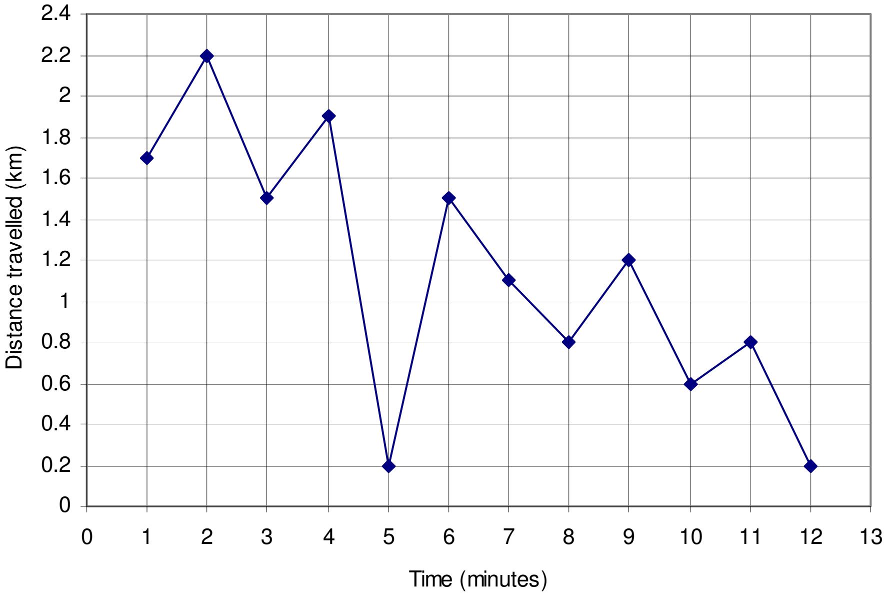
Core - continued

Question 5

The distance travelled to school in peak hour traffic is recorded every minute.

The results are shown in the table below together with a time series plot.

Minute	Distance
1	1.7
2	2.2
3	1.5
4	1.9
5	0.2
6	1.5
7	1.1
8	0.8
9	1.2
10	0.6
11	0.8
12	0.2



a. Use the three-median smoothing method to smooth the time series. Plot the smoothed time series on the plot above. Mark each smoothed data point with a cross (×).

1 mark

b. Describe the general pattern in distance travelled each minute that is revealed by the time series plot.

____

____

1 mark

Total 15 marks

END OF CORE
TURN OVER

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Module 1: Number Patterns

Question 1

Jane is a keen jogger. Preparing for an upcoming fun run, she runs laps of the primary school oval. On the first day Jane runs 5 laps, on the second day she runs 8 laps and on the third day she runs 11 laps.

a. If this sequence continues, find how many laps Jane will run on the 8^{th} day?

1 mark

b. Using the formula, $t_n = a + (n-1)d$ for the above sequence find a rule in terms of n for the number of laps Jane completes on any given day.

1 mark

c. Find how many laps Jane will run in **total** between the 7^{th} and 13^{th} day, inclusive.

1 mark

Module 1: Number Patterns - continued
TURN OVER

Anton is training for a marathon run with the number of laps, l, that he runs each day in terms of n following the sequence $l_n = -3n + 44$. He began to wind down his training program on the same day as Jane.

d. By completing the table of values for both t_n (Jane) and l_n (Anton), find **the date** on which Jane will first run more laps than Anton if they both started the programme on the 3rd November.

Day	Jane, t_n	Anton, l_n
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

2 marks

Module 1: Number Patterns - continued

Question 2

A sport clinic recommends a training program where the number of laps completed each day follows the following sequence.

Day 1	Day 2	Day 3	Day 4	Day 5
3	1	4	5	9

a. Write an appropriate difference equation to describe the above **fibonacci** sequence.

1 mark

b. Find the number of laps to be completed on the 8^{th} day for the above sequence.

1 mark

Question 3

In a vegetable patch, Hypo the pet snail is moving towards a green lettuce. In the first hour the snail moves 100 cm closer to the lettuce, in the second hour it moves 40cm closer, on the third hour it moves 16 cm and so on.

a. Show that the move each hour by the snail follows a geometric sequence.

1 mark

b. If the green lettuce was originally 200 cm away from the snail, will the snail ever reach the lettuce if it continued the geometric pattern? Show with calculations.

2 marks

Module 1: Number Patterns - continued

TURN OVER

c. If the green lettuce was instead only 165 cm away from snail originally, in which hour will the snail reach the green lettuce?

1 mark

Question 4

The number of carpentry jobs completed by Rob the Builder's business in 2009 was 160. Over the years in business the number of jobs completed has been decreasing by 15%. Each year, a steady 20 new jobs were signed up at the annual home shows. Let the number of jobs completed for the n^{th} year be J_n.

a. Write an appropriate difference equation for the number of jobs.

2 marks

b. Determine the number of jobs expected in 2011.

1 mark

c. Instead of the 15% decrease, find what percentage decrease would result in the number of jobs remaining stable, that is, so that there are 160 jobs every year.

1 mark

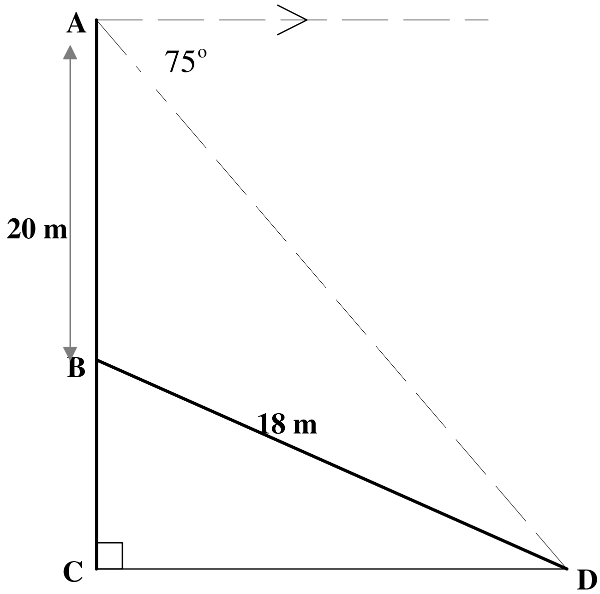
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

The angle of depression from the top of a vertical pole, A, to the bottom of a supporting beam, D, is 75°. The length of the supporting beam, BD, is 18 m and the length from A to B is 20 m.



a. Calculate the angle ABD correct to one decimal place.

2 marks

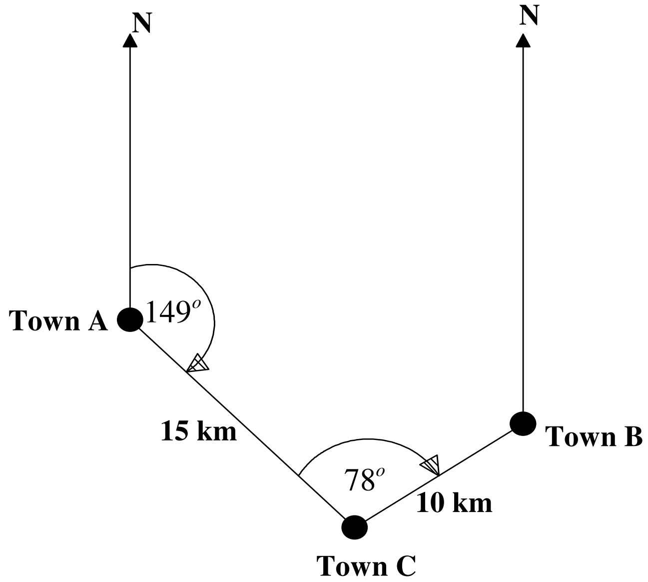
b. Hence calculate the angle of elevation of the supporting beam BD correct to one decimal place.

1 mark

Module 2: Geometry and trigonometry – continued
TURN OVER

Question 2

The locations of town A, B and C are shown in the diagram below.



Not to scale

Sam's delivery service travels from Town A to Town B and then to Town C.

a. Calculate the distance from Town A to Town B.
Give your answer in kilometres correct to one decimal place.

2 marks

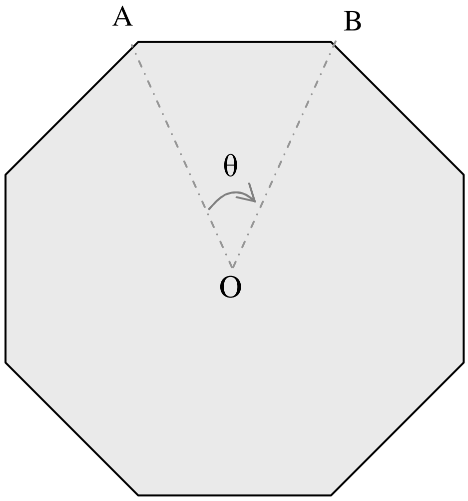
b. Calculate the bearing from Town B to Town C.

2 marks

Module 2: Geometry and trigonometry - continued

Question 3

Sam delivers cement pavers in the shape of regular octagons as shown in the diagram below.



a. Show that angle θ is 45°

1 mark

b. The length OA=OB = 15 cm.

i. Calculate the area of OAB. Give your answer in square centimetres correct to two decimal places.

ii. Hence, calculate the area of the octagonal paver, in square centimetres, correct to one decimal place.

1+1=2 marks

c. Each concrete paver has a depth of 2.5 cm. Determine the volume of 100 pavers. Give your answer in cubic metres correct to two decimal places.

2 marks

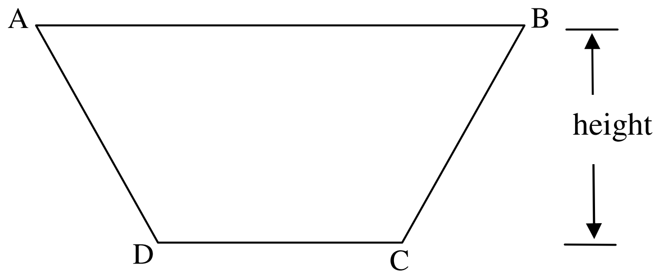
Module 2: Geometry and trigonometry – continued
TURN OVER

Question 4

Sam also delivers ceramic pots. The cross section of a pot is in the shape of a trapezium as shown in the diagram below.

Angle ADC is 115^o
CD is 50 cm in length and is on the floor line.
AB is 70 cm in length and is along the top of the pot.

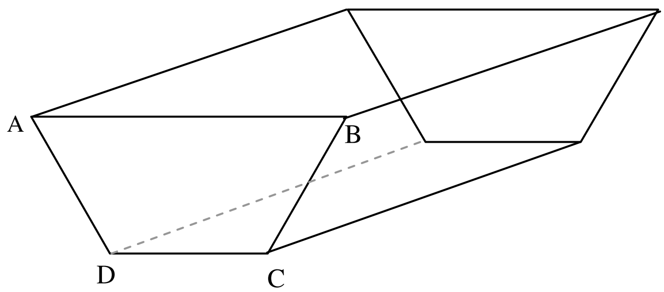
AB and CD are parallel
length AD = length BC



a. Calculate the vertical height of the ceramic pot.
Give your answer in centimetres correct to one decimal place.

1 mark

b. The ceramic pot is in the shape of a trapezoidal prism. The total volume inside the pot is given by V cm^3. Soil is put into the pot until it is half the vertical height of the pot. What fraction of V is full of soil?



2 marks
Total 15 marks

END OF MODULE 2

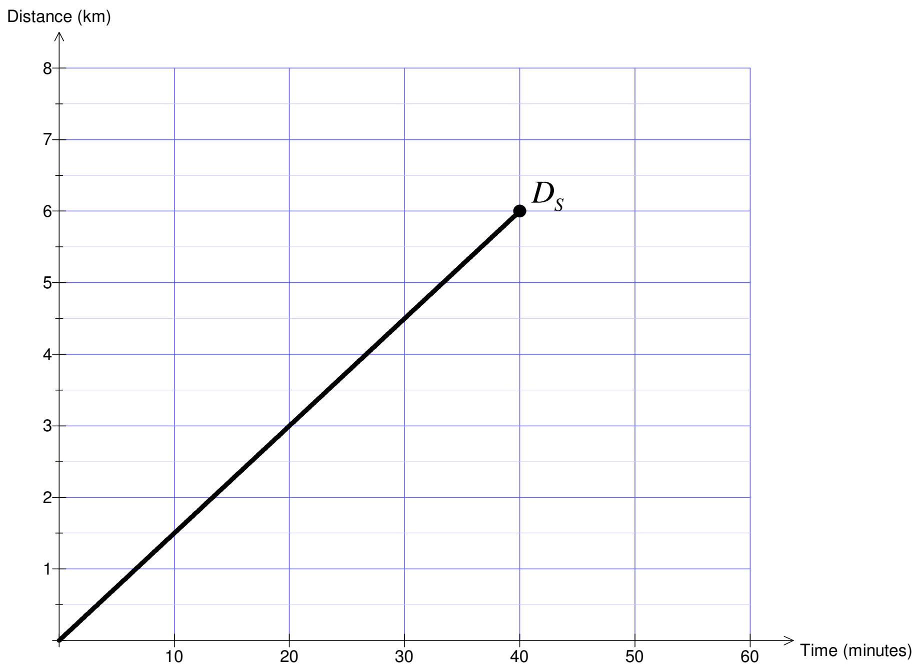
Module 3: Graphs and relations

Question 1

Sam and Niko are two brothers who ride to school on their bikes. The distance from their house to school is 6 km. They start at the same time and follow the same route.

Sam rides at a constant speed of 9 km per hour for the entire journey and takes no rest periods.

A graph showing the distance travelled by Sam, in kilometres, against time, in minutes, is shown below.



a. How many minutes did it take Sam to ride to school?

1 mark

Niko sets off to school at a speed of 18 km/h. However, after riding at this speed for 15 minutes, he develops a cramp in his leg and rather than stopping he slows down to 3 km/hr for the remainder of the trip.

b. How far did Niko travel in 15 minutes? Give your answer in kilometres.

1 mark

c. Draw the graph of Niko's journey on the axes above and label it as D_n

2 marks

Module 3: Graphs and Relations- continued
TURN OVER

d. The equation below gives the distance, D_n , in kilometres, travelled by Niko at any time *t* minutes

$$D_n = \begin{cases} 0.3t, & 0 \le t \le 15 \\ bt + c, & 15 \le t \le d \end{cases}$$

Find the values of *b*, *c* and *d*

3 marks

e. Sam eventually catches up to Niko. How many minutes after they start riding does this happen?

1 mark

Module 3: Graphs and Relations- continued

Question 2

A company produces two types of bicycles: mountain bikes and racers. The metal finishing machine can process at most 20 bicycles. It takes 2 hours to produce a mountain bike and 3 hours to produce a racer. There are 48 hours available each week for the production of bikes.

Let x represent the number of mountain bikes produced each week
y represent the number of racers produced each week

Inequalities 1 to 4 represent the production constraints.

Inequality 1: $x \geq 4$

Inequality 2: $y \geq 0$

Inequality 3: $x + y \leq 20$

Inequality 4: $2x + 3y \leq 48$

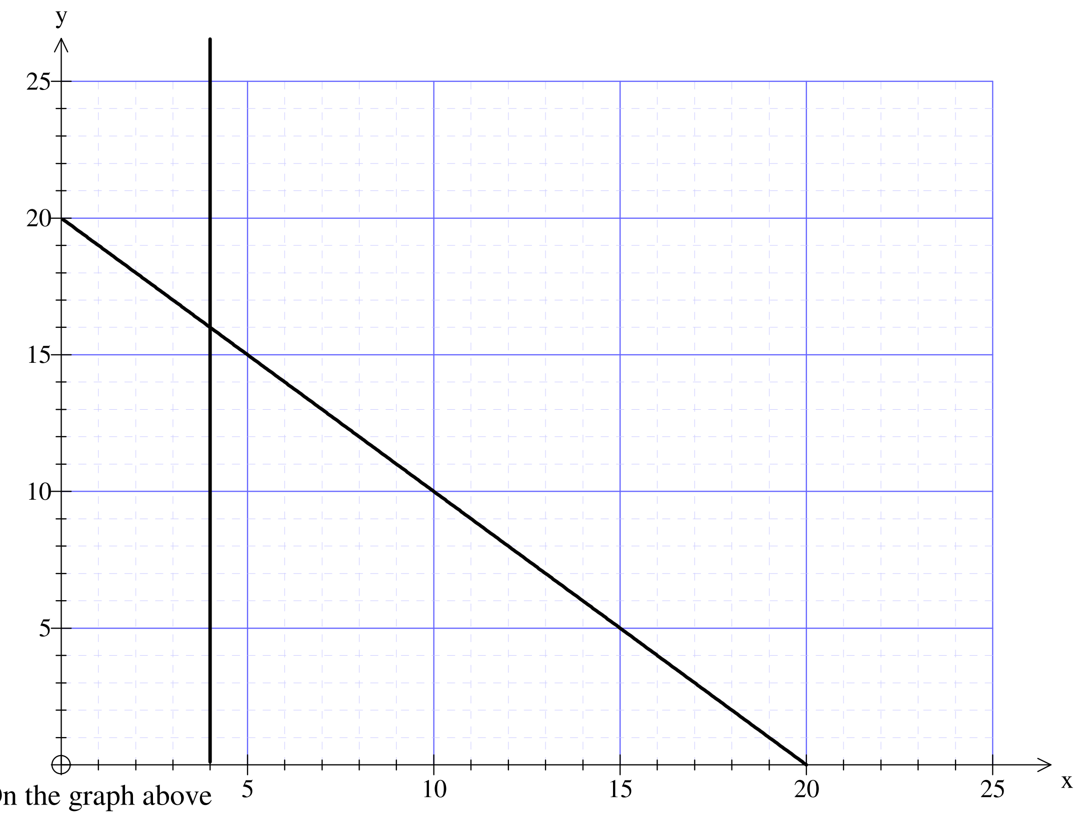
a. Explain the meaning of Inequality 1 in terms of the context of this problem.

____

____

1 mark

The lines $x = 4$ and $x + y = 20$ are drawn on the graph below



b. On the graph above

i. **draw** a line that represents $2x + 3y = 48$

ii. clearly **indicate** the feasible region represented by Inequalities 1 to 4

1+1 = 2 marks

Module 3: Graphs and Relations- continued

TURN OVER

The company makes \$30 profit on each mountain bike and \$45 on each racer sold.
Let P be the weekly profit obtained from selling the bicycles.

c. Write an equation for the Profit in terms of x and y.

1 mark

d. **i.** Find all solutions for the number of completed mountain bikes and the number of racers that should be produced each week in order to maximise the profit.

ii. What is the maximum total profit that can be obtained each week?

2+1=3 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

A new car is purchased for a drive away price of $42 000 (which includes GST).

a. Find the amount of GST tax to be received by the federal government from the $42 000 sale (to the nearest dollar).

1 mark

New cars generally depreciate at the rate of 22½% p.a. reducing balance.

b. Calculate the bookvalue after two years (to the nearest hundred dollars).

1 mark

Question 2

The $42 000 car is to be purchased on a hire purchase agreement with $2000 deposit at 7.2% p.a. flat rate with monthly repayments over a 2 year period.

a. Find the extra amount paid to finance the car compared to the cash price.

1 mark

b. Calculate the monthly repayment.

1 mark

Module 4: Business-related mathematics - continued
TURN OVER

Question 3

Josephine has just turned 50 and is planning to retire at 55. Her annual salary is $60 000 and her employer superannuation contributions are 9% of her gross monthly income. Josephine also contributes a **further $400 a month** as a salary sacrifice (that is, she pays $400 from her salary into the superannuation fund).

a. Calculate her employer's monthly contribution to the superannuation fund.

1 mark

b. Calculate the total monthly contribution to Josephine's superannuation fund.

1 mark

The superannuation fund has been returning an interest rate of 9.0% p.a. compounded monthly and her current balance in the fund is $120 000.

Using the formula

$$A = PR^n + \frac{Q(R^n - 1)}{R - 1}$$

c. State the values of the following to be used in the above given annuities formula for superannuation funds.

n =	P =	R =

2 marks

d. Calculate the lump sum that she can receive for her planned retirement at age 55 (to the nearest thousand dollars)?

1 mark

Module 4: Business-related mathematics - continued

Question 4

Using the sum of money in her funds as calculated in **Question 3**, Josephine has two options for setting up an annuity to provide a regular income after she retires at 55.

Option 1. A perpetuity that offers monthly payments at 8% p.a. compounded monthly.

Option 2. A reducing balance annuity, also paid monthly at 8% p.a., compounded monthly.

a. Calculate the monthly annuity using **option 1**.

1 mark

b. Using the answer from **part a.**, express the annual salary from this option as a percentage of her current salary of $60 000. Give answer to one decimal place.

1 mark

c. Calculate the monthly annuity using **option 2** if the fund needs to **last for 25 years.**

2 marks

d. Should Josephine die after **only 15 years**, how much inheritance can her estate expect (to the nearest thousand dollars).

i. For the perpetuity.

ii. For the reducing balance annuity.

1 +1 = 2 marks

Total 15 marks

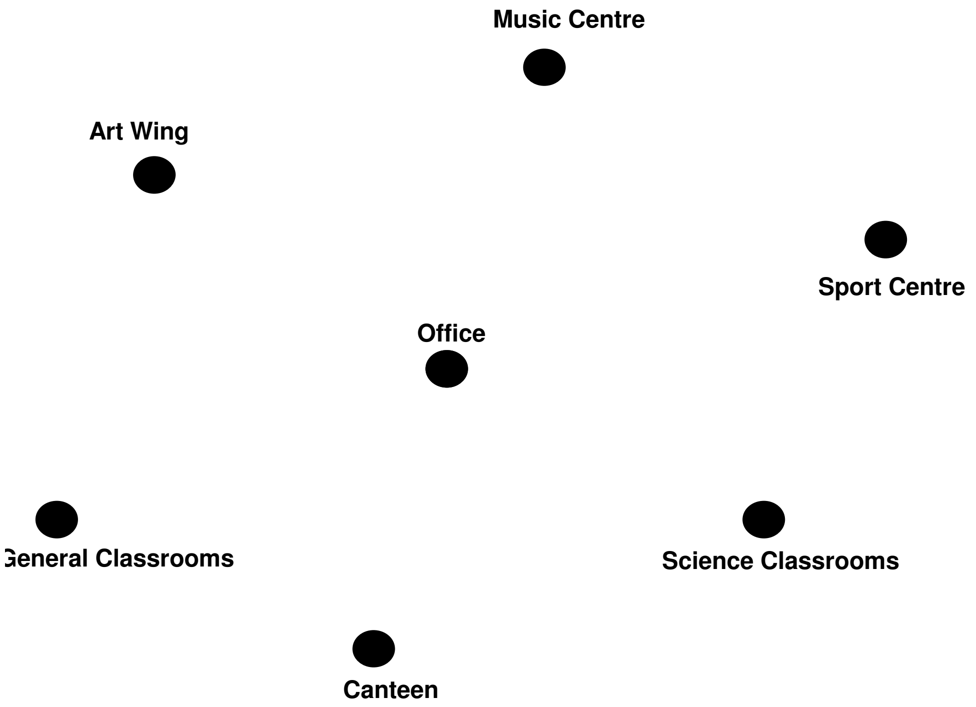
END OF MODULE 4

TURN OVER

Module 5: Networks and decision mathematics

Question 1

Seven major buildings in a school are shown in the diagram below.



Peter, the school captain, is called to the office and asked to show a new student around the school.

a. What is the minimum number of edges that Peter can take in order to show the new student each building once and then return to the office?

___

1 mark

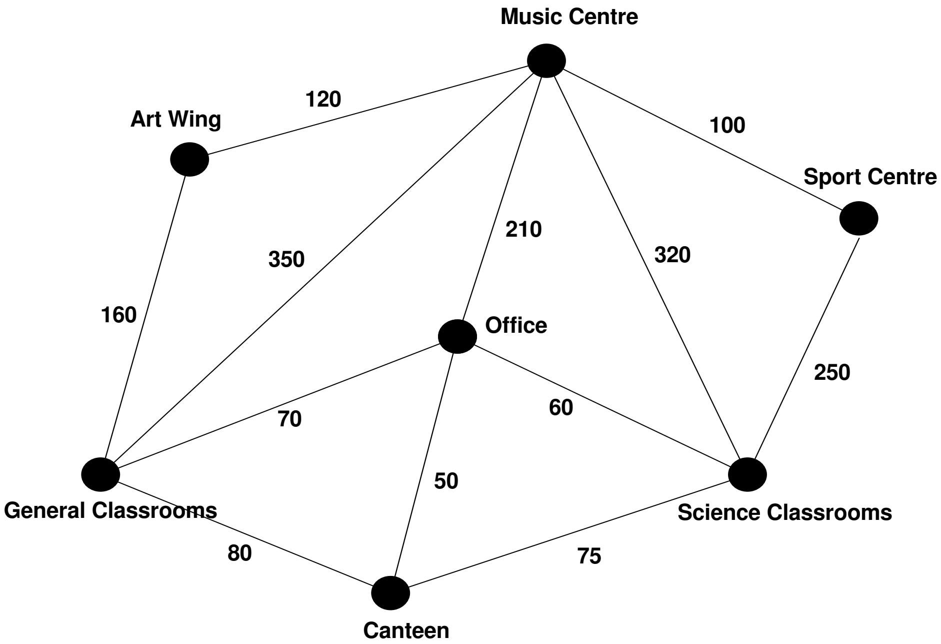
b. On the diagram above, draw a connected graph with this number of edges.

1 mark

Module 5: Networks and decision mathematics - continued

Question 2

A diagram of the undercover walk ways connecting the buildings in the school are shown as edges in the network below. The numbers on the edges represent the length of each walkway in metres.



a. What fraction of the vertices in the network have an odd degree?

____

1 mark

b. Maureen needs to deliver the daily bulletin to the teaching staff. She wishes to follow a route described as the **shortest** Hamiltonian circuit. She is currently in the Office. Write down the route that Maureen can take.

____

____

1 mark

c. Every morning, John the maintenance man, inspects the condition of each of the walkways.

i. State a vertex at which John could begin his inspection so that he only walks through each walkway once.

____

ii. Regardless of which route John decides to take, how many buildings (including those at the start and finish) will he pass on exactly two occasions?

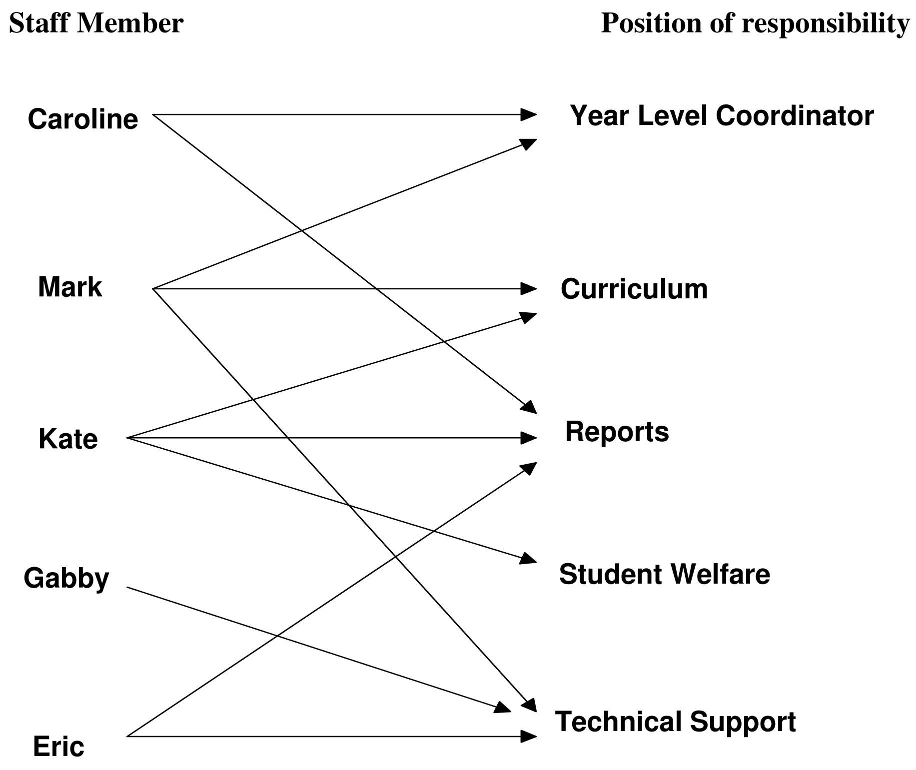
____

1+1=2 marks

Module 5: Networks and decision mathematics – continued
TURN OVER

Question 3

Eric, Mark, Caroline, Kate and Gabby are a group of staff members at the school.
Each staff member needs to fill one position of responsibility only.
The following bipartite graph illustrates the positions of responsibility that each is able to fill.



a. Which staff member must be in charge of Reports?

1 mark

b. Complete the table showing the positions that the following staff must full.

Staff Member	Position
Gabby	
Mark	
Kate	

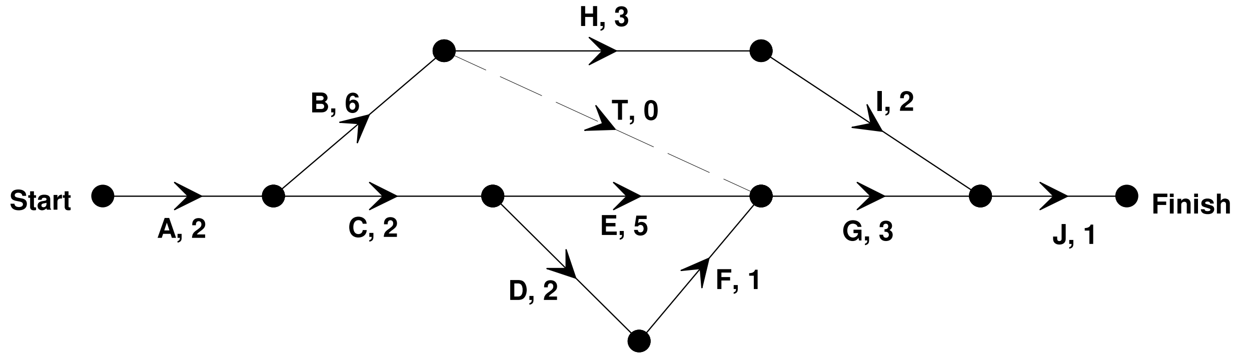
2 marks

Module 5: Networks and decision mathematics - continued

Question 4

Storm damage to a school building needs to be repaired.
Ten activities have been identified for this project.

The directed network below shows the activities and their completion times in hours.



The incomplete table below shows this same information and includes predecessor activities and the earliest starting times (EST).

Activity	Immediate Predecessor (s)	Duration of activity (hours)	Earliest starting time EST (hours)
A	–	2	0
B	A	6	2
C	A	2	2
D	C	2	4
E	C	5	4
F	D	1	6
G		3	
H	B	3	8
I	H	2	11
J	G, I	1	13

a. Complete the two shaded cells for activity G in the table above.

2 marks

Module 5: Networks and decision mathematics – continued
TURN OVER

b. What is the minimum time, in hours, needed to complete this project from the beginning?

____

1 mark

c. Explain why the completion of the project will be delayed if the duration of activity H is increased.

____

____

1 mark

d. Several activities in this project can be delayed without increasing the minimum completion time. Which of these activities can be delayed for the longest time?

____

1 mark

e. Activity G is delayed by 3 hours. How will this affect the minimum completion time of the project?

____

____

1 mark
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

An insurance company needs to measure its risk if they are to underwrite a policy for the Australian Grand Prix planned in 2010. They used the following information about Melbourne's weather.
The weather for the next day in Melbourne, from long-run data, suggests that there is a 15% chance that if today is dry, then the next day is wet. Also, if today is wet, there is a 52% chance that the next day will also be wet.

a. Complete the transition matrix that represents the probabilities quoted above.

	Today is dry	Today is wet
Next day is dry		0.48
Next day is wet		

1 mark

b. Find the long-term probabilities if initially the day is wet. Show your workings and complete the following sentences to the nearest percentage.

If initially the day is wet then there is a _____ % probability it will be wet the next day.
If initially the day is wet then there is a _____ % probability it will be dry the next day.

2 marks

c. If **Thursday**, the first day of the Grand Prix is wet, find the probability that it will be **dry** on the **Sunday** when the main race is run. Give your answer to two decimal places.

1 mark

Module 6: Matrices - continued
TURN OVER

Question 2

For the 2010 Australian Grand Prix, organizers have decided to analyse the ticket sales from the previous year. The set prices from 2009 are summarized in **Table 1** below

Table 1 – 2009 Ticket Prices

	General Admission	Major Stands	Corporate/VIP Boxes
Single Day	$40	$160	$800
Weekend	$72	$320	$1600
Four Day package	$120	$440	$2400

The number of tickets sold in 2009 is summarized in **Table 2** below.

Table 2 – 2009 Ticket Sales

	Single Day	Weekend	Four Day package
General Admission	80 000	120 000	20 000
Major Stands	1000	3000	4000
VIP Boxes	1000	1600	400

a. To investigate general admission tickets **only**, represent the **general admission ticket prices,** as a suitable matrix labelled T_{2009} that will be used with 2009 ticket sales matrix, labelled N_{2009}. Hence, using matrices, determine the **revenue** ($) from **general admission ticket only** sales in 2009.

2 marks

Module 6: Matrices - continued

The total number of ticket sales as 1, 2 and 4 day packages can be summarized as follows.

	Total Number of Ticket Sales
Single Day	82 000
Weekend	124 600
Four Day package	24 400

or as a matrix $A_{2009} = \begin{bmatrix} 82000 \\ 124600 \\ 24400 \end{bmatrix}$

b. Given that there were 1, 2 and 4 day passes, show this as a suitable matrix that can be **pre- multiplied** with the above matrix A_{2009}.

1 mark

c. Using matrices determine the **total number** of people that passed through the gates for **all four days** of the 2009 event**.**

1 mark

Question 3

For the 2010 Australian Grand Prix, organizers have decided to increase the prices of the tickets. The 2010 AGP tickets are to be increased by 12½% on the prices of the 2009 AGP tickets, T_{2009}.

a. Show the 2010 ticket prices, T_{2010} as a suitable **scalar** matrix equation.

$T_{2010} =$

1 mark

b. State **type of ticket** and its **new 2010 price** of that found in element $t_{1,3}$.

---

2 marks

Module 6: Matrices – continued
TURN OVER

Question 4

The mobile call monthly charges ($\$C$) for two mobile phone carriers is related to the number of minutes of talk time (m) and the fixed monthly fee as given below.

Red Phone $C = 20 + 0.60m$ which represents a set fee of \$20 and \$0.60 per minute.
Blue Phone $C = 30 + 0.30m$ which represents a set fee of \$30 and \$0.30 per minute.

a. Represent the two simultaneous equations using matrices.

1 mark

b. For the above simultaneous equation matrix representation, the determinant is +0.3
Show how this was arrived at and explain what a determinant not equal to zero means in the context of the two phone plans.

2 marks

c. Using matrix methods, calculate the number of minutes of talk time that represents the same total cost for both phone companies.

1 mark

Total 15 marks

END OF MODULE 6

END OF QUESTION AND ANSWER BOOK

Exam 1 & 2 Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET