2010

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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Core: Data analysis

Question 1 C

The mean will change because the sum of the values will be less. This means that the variance and standard deviation will also change.
The range will change because the maximum value will now be 17 instead of 19
The interquartile range will remain $15 - 10 = 5$

Question 2 B

The highest reading at both sites is 78.
The lowest reading at site A is 34
The median at site A is 51.5
The median at site B is 53
The interquartile range for site B is
$62.5 - 44 = 18.5$

Question 3 A

The whiskers each contain 25% of the scores.
Only one of Anna's whiskers is longer than the largest of Belinda's scores, so 75% of Anna's scores are less than Belinda's highest score.
The box for Anna is longer than the box for Belinda so Anna's IQR is larger than Belinda's IQR.
Belinda's scores are not skewed.
Anna's scores are positively skewed.

Question 4 B

Number female labour voters = 27
Total number of labour voters = 52

$$\% \text{ labour voters} = \frac{52}{55+62} \times 100 = 44\%$$

Question 5 D

$$Z_{Eng} = \frac{80-70}{10} = 1$$

$$Z_{Biol} = \frac{85-70}{15} = 1$$

$$Z_{Phys} = \frac{95-67}{15} = 1.9$$

$$Z_{Chem} = \frac{75-55}{10} = 2$$

$$Z_{Math} = \frac{80-60}{12} = 1.7$$

Chemistry has the highest Z score so it is the subject she did best in relative to the others in her class.

Question 6 D

$$Z = \frac{19-25}{3} = -2$$

95% lie within 2 standard deviations of the mean, so 5% lie outside. 2.5% are less than -2 so 2.5% are less than 19.

Hence $100 - 2.5 = 97.5\%$ are greater than 19

Core: Data analysis

Question 7 E
Gender and employment classification are both categorical data. The back to back stem and leaf and the parallel box plots are used for comparing the numerical values of one set of categorical data, e.g. gender. A scatter plot is used for two numerical variables. A histogram is used for a frequency distribution of numerical data.

Question 8 B
The time series shows a seasonal pattern only. Since the pattern is roughly the same for each year, there is no linear trend. A cyclical pattern is a pattern over a longer period of time than a quarter or season.

Question 9 E
For a TI-83 calculator go to stat edit. Enter the Age values in L_1 and the blood pressures in L_2 Go to stat calc linear regression ($ax + b$). Press L_1, $L_2$and read the value $a = 1$, $b = 84.98$

Question 10 A
Using the same calculator screen as for question 9,
$r^2 = 0.75$

Question 11 C
The median of the lowest three points is (25,110) and the median of the upper three points is (73,164)

$$m = \frac{164 - 110}{73 - 25} = 1.125$$

Question 12 A
Residual Value = Actual value – Predicted Value. Hence, points above the line will be positive and points below the line will be negative.
The distance above or below the line in the question is the distance above or below the zero line in the answer.

Question 13 B

$$\text{SI for third quarter} = 4 - (0.59 + 1.24 + 0.81) = 1.36$$

$$\text{SI} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

$$\text{Deseasonalised figure} = \frac{\text{actual figure}}{\text{SI}} = \frac{32465}{1.36} = \$23,871$$

Module 1 Number patterns

Question 1 B

The pattern of this arithmetic sequence is subtracting 5, so a is 5 more than -2, i.e. $a = 3$

Question 2 D

$a + d = 9$

$a + 4d = 12$

$a + d + 3d = 12$

$9 + 3d = 12$

$3d = 3$

$d = 1$

$a = 9 - 1 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_6 = \frac{6}{2}\left[16 + 5 \times 1\right] = 63$$

Question 3 C

This is a geometric sequence with $a = 2$ and $r = 4$

$t_n = ar^{n-1} = 2 \times 4^{n-1} = 32768$

$4^{n-1} = 16384$

$4^{n-1} = 4^7$

$n - 1 = 7$

$n = 8$

You can also use your calculator in sequence mode to solve this question.

Press

$y =$

$nMin = 1$

$u(n) = 4 \wedge (n-1)$

$u(nMin) = 0$

Press 2nd table to get when $n = 8$, $y = 16384$

Here $4 \wedge (8-1) = 16384$

so $4^7 = 16384$

$n - 1 = 7$

$n = 8$

Module 1 Number patterns

Question 4 D

$r = \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

$S_\infty = \frac{a}{1-r} = 6$

$S_4 = \frac{a(1-r^4)}{1-r} = \frac{a}{1-r} \times (1-r^4) = 6 \times (1-\left(\frac{1}{2}\right)^4)$

$S_4 = 5\frac{5}{8}$

Question 5 A

$t_{n+1} = t_n + 3n$

$t_2 = t_1 + 3 \times 1 == 4 + 3 = 7$

$t_3 = t_2 + 3 \times 2 == 7 + 6 = 13$

$t_4 = t_3 + 3 \times 3 == 13 + 9 = 22$

$t_5 = t_4 + 3 \times 4 == 22 + 12 = 34$

4,7,13,22,34

Question 6 E

$t_3 = 2t_2 + 3t_1 = 2 + 3 = 5$

$t_4 = 2t_3 + 3t_2 = 2 \times 5 + 3 \times 1 = 13$

Question 7 C

This is an arithmetic sequence with $a = 64$ and $d = -4$

Each term is found by subtracting 4 from the previous term.

$t_{n+1} = t_n - 4$

Question 8 C

Put calculator in sequence mode.

Press

$y =$

$nMin = 1$

$u(n) = 1.0045u(n-1) + 2600$

$u(nMin) = 20{,}000$

Press 2nd table to get when $n = 11$, $u(n) = 47451$

Question 9 D

10, 8, 6.4

This is a geometric sequence with $r = 0.8$

$S_\infty = \frac{a}{1-r} = \frac{10}{1-0.8} = 50$ cm.

Module 2 Geometry and trigonometry

Question 1 B

$\sin\theta = \frac{12}{28}$

$\theta = \sin^{-1}\left(\frac{12}{28}\right) = 25^0$

Question 2 E

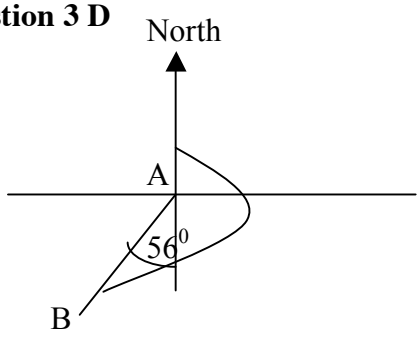
$\angle XYZ = 180 - (30 + 55) = 95$

Using the sine rule

$\frac{x}{\sin 30^0} = \frac{y}{\sin 95^0}$

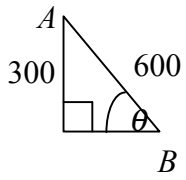
$x = \frac{y \sin 30^0}{\sin 95^0}$

Question 3 D



Bearing of B from A is measured from North
= 180 + 56 = 236^0 T

Question 4 D



$\sin\theta = \frac{300}{600}$

$\theta = \sin^{-1}\left(\frac{300}{600}\right) = 30^0$

Gradient = $\tan\theta = \tan 30^0 = 0.58$

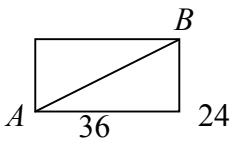
This is closest to 0.6

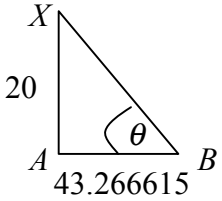
Question 5 D

Triangles *ABE* and *EDC* are similar (SAS)
Hence, the sides are in the same ratio.
Each side in triangle *ABE* is 4 times its corresponding side in triangle *EDC*.
AB = 4 × 4.5 = 18 m.

Module 2 Geometry and trigonometry

Question 6 A





$AB = \sqrt{36^2 + 24^2} = 43.266615$

$\tan\theta = \dfrac{20}{43.266615}$

$\theta = \tan^{-1}\left(\dfrac{20}{43.266615}\right) = 24.8^0$

This is closest to 25^0

Question 7 D

Volume of cube = l^3

Volume of sphere = $\dfrac{4}{3}\pi \times \left(\dfrac{l}{2}\right)^3$

Fraction occupied by ball = $\dfrac{4}{3}\pi \times \dfrac{l^3}{8} \times \dfrac{1}{l^3} = \dfrac{\pi}{6}$

% occupied by ball = 52.4%

% not occupied by ball = 100 – 52.4 = 47.6%

This is closest to 48%

Question 8 A

Area of right angled triangle = $\dfrac{1}{2}PQ \times QR$

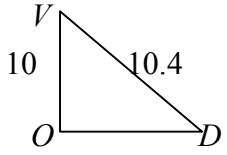
$PQ = a\cos\theta$

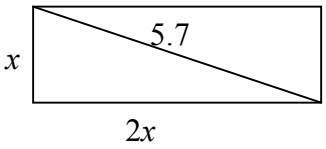
$QR = a\sin\theta$

Area of right angled triangle = $\dfrac{1}{2}a\cos\theta a\sin\theta$

Area of right angled triangle = $\dfrac{1}{2}a^2\sin\theta\cos\theta$

Question 9 C





$OD = \sqrt{10.4^2 - 10^2} = 2.8566$

$BD = 2 \times OD = 5.71314$

$(2x)^2 + x^2 = 5x^2 = 32.64$

$x^2 = 6.528$

$x = 2.55$

$2x = 5.1$ m

Module 3 Graphs and relations

Question 1 A
From April to May the rainfall decreases from 40 to 20 mm. This is the biggest decrease between any two consecutive months.

Question 2 D
At least 20 means 20 **or** more than 20. This happens in January, February, March, April, **for most of September**, October, November, December. i.e. approximately 8 months.

Question 3 C
The line $2x + y = 18$ is the line with the points (0,18) and (9,0). y is greater than or equal to this line so the regions are limited to R and T. The other line, containing the points (0,-2) and (4,0) is the line $x - 2y = 4$. $-y$ is less than or equal to this line so y must be greater than or equal to this line. This gives the regions P, Q and R . the only region common to both is R.

Question 4 B
$$\text{Toll} = 12 + x(p-1) = 12 + xp - x$$
$$\text{Toll} = \$(xp - x + 12)$$

Question 5 C
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{212 - 50}{100 - 10} = \frac{162}{90} = 1.8$$

Question 6 C
$$F = 1.8C + a$$
$$50 = 1.8 \times 10 + a$$
$$a = 50 - 18 = 32$$
$$F = 1.8c + 32$$
$$F = 1.8 \times 30 + 32 = 86^0$$

Module 3 Graphs and relations

Question 7 D

Let basic wage be x and wage after 11.00 pm be
$19x + 6y = 784 \quad (1)$
$13x + 8y = 700 \quad (2)$
$(1) \times 4 \rightarrow 76x + 24y = 3136 \quad (1a)$
$(2) \times 3 \rightarrow 39x + 24y = 2100 \quad (2a)$
$(1a) - (2a) \rightarrow 37x = 1036$
$x = \$28$
Substitute this value for x in (1)
$19 \times 28 + 6y = 784$
$6y = 252$
$y = \$42$

Question 8 D

$y = -px - r$
$-p$ is negative since the slope of the line is negative
so p is positive.
-r is positive as can be seen from the y intercept
so r is negative

Question 9 B

The vase would fill rapidly at first, since the base of the vase is narrow, then more slowly as the vase broadens. Hence, the graph is not linear at any stage so not C, D, or E. A shows a vase that is filling more quickly with time. The answer is B.

Module 4 Business-related mathematics

Question 1 C

$$I = \frac{PRT}{100}$$

$$I = \frac{45{,}000 \times R \times 6}{100} = 25{,}110$$

$$R = \frac{25{,}110 \times 100}{45{,}000 \times 6} = 9.3\%$$

Question 2 B

Let amount of depreciation per year = $\$x$

$7500 - 10x = 1400$

$6100 = 10x$

$x = \$610$

Question 3 E

Amount owing = $6600 - 1000 = 5600$

Amount paid over 36 months = $180 \times 36 = 6480$

Interest = $6480 - 5600 = \$880$

Question 4 C

$100\% - 7\% = 93\%$

Value = $3500(0.93)^9 = \$1821$

Question 5 A

The scrap value is \$2000

The office equipment is written off after 6 years.

The rate of depreciation is the gradient of the line

$$\frac{2-8}{6-0} == -1$$

The negative indicates depreciation. Hence depreciation is \$1,000 per year.

Question 6 D

Tax payable on $\$78{,}000 = 4{,}350 +$

$$(78{,}000 - 35{,}000) \times \frac{30}{100}$$

Tax payable on $\$78{,}000 = \$17{,}250$

Tax payable on $\$85{,}000 = 17{,}850 +$

$$(85{,}000 - 80{,}000) \times \frac{38}{100}$$

Tax payable on $\$85{,}000 = \$19{,}750$

Difference = $\$19{,}750 - \$17{,}250 = \$2{,}500$

Module 4 Business-related mathematics

Question 7 E

Use TVM solver

$N = 5 \times 4 = 20$

$I = 8$

$PV = -16000$

$PMT = 0$

$FV = ?$

$P / Y = 4$

$C / Y = 4$

End

Alpha solve gives $FV = 23775.16$

Interest = $23775.16 - 16000 = 7775.16$

This is closest to \$7775

Question 8 B

Use TVM solver

$N = ?$

$I = 6$

$PV = 82000$

$PMT = -2100$

$FV = 0$

$P / Y = 12$

$C / Y = 12$

End

Alpha solve gives $N = 43.55$ months = 3.6 years

This is closest to 4 years

Question 9 B

Use TVM solver

$N = 20$

$I = 7.5$

$PV = 36000$

$PMT = -880$

$FV = ?$

$P / Y = 4$

$C / Y = 4$

End

Alpha solve gives $FV = -31080.56825$

$N = ?$

$I = 7.5$

$PV = 31080.56825$

$PMT = -1400$

$FV = 0$

$P / Y = 4$

$C / Y = 4$

End

Alpha solve gives $N = 28.977$ quarters = 7.2 years

Total = 7.2 + 5 = 12.2 years.

Module 5 Networks and decision mathematics

Question 1 D

Number of vertices = 6

Number of edges = $\frac{6\times(6-1)}{2}=15$

Question 2 B

A simple graph has no multiple edges or loops.

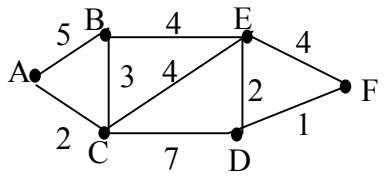
Question 3 D

For an Euler path to exist, there must be just two odd vertices.
A, B and E have more than 2 odd vertices.
C has 1 odd vertex

Question 4 A

For a tree, $v=e+1$
The number of edges is one less than the number of vertices.

Question 5 B

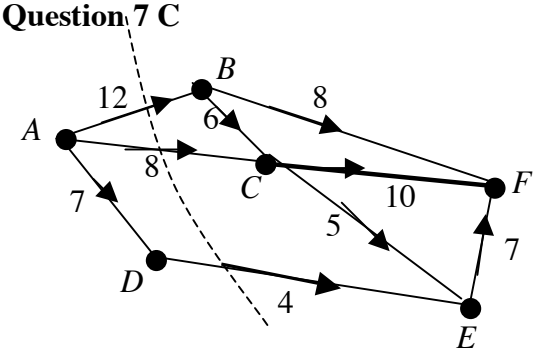


Shortest route is ACEDF = 2 + 4 + 2 + 1 = 9 km.

Question 6 C

Euler's formula only applies to planar graphs.
C is not planar.

Question 7 C



Maximum flow = Minimum cut
= 12 + 8 + 4 = 24

Module 5 Networks and decision mathematics

Question 8 E

To minimize the problem take each value in the table away from the maximum value in the table, 20

	Tennis	Netball	Squash	Darts
Andy	**14**	**10**	**10**	**13**
Ben	**13**	**16**	**16**	**12**
Cathy	**11**	**11**	**10**	**12**
Demi	**6**	**12**	**11**	**0**

Reduce the rows by subtracting the smallest number in each row from each element in the row.

	Tennis	Netball	Squash	Darts
Andy	**4**	**0**	**0**	**3**
Ben	**1**	**4**	**4**	**0**
Cathy	**1**	**1**	**0**	**2**
Demi	**6**	**12**	**11**	**0**

Reduce the columns by subtracting the smallest number in each column from each element in the column.

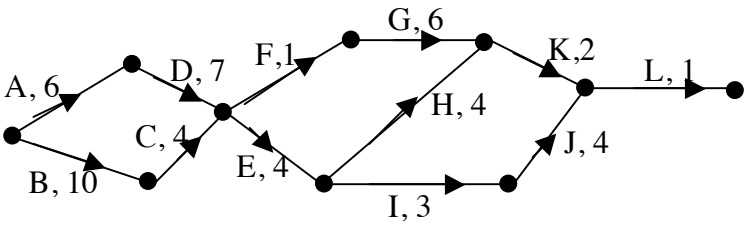
	Tennis	Netball	Squash	Darts
Andy	**3**	**0**	**0**	**3**
Ben	**0**	**4**	**4**	**0**
Cathy	**0**	**1**	**0**	**2**
Demi	**5**	**12**	**11**	**0**

Allocate sport. Darts – Demi; Netball – Andy; Tennis – Ben; Squash – Cathy.

Module 5 Networks and decision mathematics

Question 9 B

Critical Path is **B-C-F-G-K-L**
Total time before crashing = 10 + 4 + 5 + 6 + 2 + 1 = 28 days.
After crashing, project becomes



Now critical path is **B-C-E-I-J-L**
Total time now is 10 + 4 + 4 + 3 + 4 + 1 = 26 days.
Time is reduced by 2 days.

Module 6 Matrices

Question 1 E

$$\text{kA} = -3\begin{bmatrix} -1 & 2 \\ 6 & -4 \\ 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} -1\times -3 & 2\times -3 \\ 6\times -3 & -4\times -3 \\ 0\times -3 & 3\times -3 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -18 & 12 \\ 0 & -9 \end{bmatrix}$$

Question 2 C

$$P+Q = \begin{bmatrix} 2 & -1 & 4 \\ -3 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 0 & -3 \\ 1 & 8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & 1 \\ -2 & 11 & 13 \end{bmatrix}$$

$$R = \begin{bmatrix} 8 & -1 & -2 \\ 0 & 16 & 20 \end{bmatrix} - \begin{bmatrix} 4 & -1 & 1 \\ -2 & 11 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & -3 \\ 2 & 5 & 7 \end{bmatrix}$$

Question 3 A

$$A^{-1} = \frac{1}{12-2}\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$$

$s = 0.4$

Question 4 C

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 6.3 & -2.3 \\ 0.5 & -4.4 \end{bmatrix} = \begin{bmatrix} -1.3 & 2.3 \\ -0.5 & 9.4 \end{bmatrix}$$

Question 5 D

The equations can be moved into the order

$1a + 0b + 0c + 1d = 5$

$0a + 4b + 7c + 0d = 8$

$3a + 2b + 0c + 0d = 6$

$0a + 0b + 5c + 2d = 9$

This gives the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 4 & 7 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 5 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 6 \\ 9 \end{bmatrix}$$

Question 6 B

A singular matrix occurs when $\Delta = 0$

$\Delta = a^2 - 2ab = 0$

$a(a-2b) = 0$

$a = 0$ or $a = 2b$

Module 6 Matrices

Question 7 D

To multiply matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. Only **D** satisfies this requirement.

Question 8 E

0.6 LB this year are SB next year so 0.6 goes in the first column second row. 0.4 LB this year are LB next year so this goes in the first column first row. 0.3 SB this year are LB next year, so this goes in the second column, second row. The remaining spot is for the 0.7 SB that remains.

Question 9 B

$$2A^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2A^2 = A^{-1} = 2\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 2p^2 + 2qr & 2pq + 2qs \\ 2pr + 2rs & 2qr + 2s^2 \end{bmatrix}$$

End of Suggested Solutions 2010 Further Mathematics VCE Trial Examination 1

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