INSIGHT

Trial Exam Paper

2010

FURTHER MATHEMATICS

Written examination 2

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 35 pages.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** question within the modules selected. You do not need to give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

SECTION A

Core: Data Analysis

Question 1

2006 FIFA World Cup performances of 18 of the 32 qualified countries are given in the table below. The table shows the total goals scored, the total goals conceded and the total games won by these 18 countries.

Countries	Total goals scored	Total goals conceded	Total games won
Germany	18	8	6
England	7	5	3
Paraguay	2	2	1
Argentina	13	7	3
Netherlands	3	2	2
Cote d'Ivoire	5	6	1
Portugal	9	3	5
Mexico	5	5	1
Italy	17	5	6
Ghana	4	6	2
United States	2	6	0
Brazil	10	2	4
Australia	5	6	1
Japan	2	7	0
Switzerland	4	3	2
France	12	8	4
Korea Republic	3	4	1
Spain	9	4	3

Table 1

a. List the names of two countries whose standardised scores of total goals conceded are closest to 1.

Country 1: ☐

Country 2: ☐

1 mark

b. Germany, the host of 2006 FIFA World Cup, scored a total of 18 goals. Find the standardised score for the total goals scored by Germany and interpret its meaning.

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1 mark

c. Germany and Italy both won 6 games. Are these values outliers? Justify your answer by showing appropriate calculations.

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1 mark

d. For all 32 countries qualified to play in 2006 World Cup, the mean number of total goals conceded was 5.125. For the sample of 18 countries in table 1, determine the percentage of countries that conceded more goals than the World Cup mean of 5.125.Write your answer correct to two decimal places.

Percentage: [ ]

1 mark

e. Use the data in table 1 to determine the equation of the least squares regression line that will enable the total number of goals scored to be predicted from the total number of games won. Write the coefficients correct to three decimal places.

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2 marks

f. Complete the following sentences by filling in the spaces.

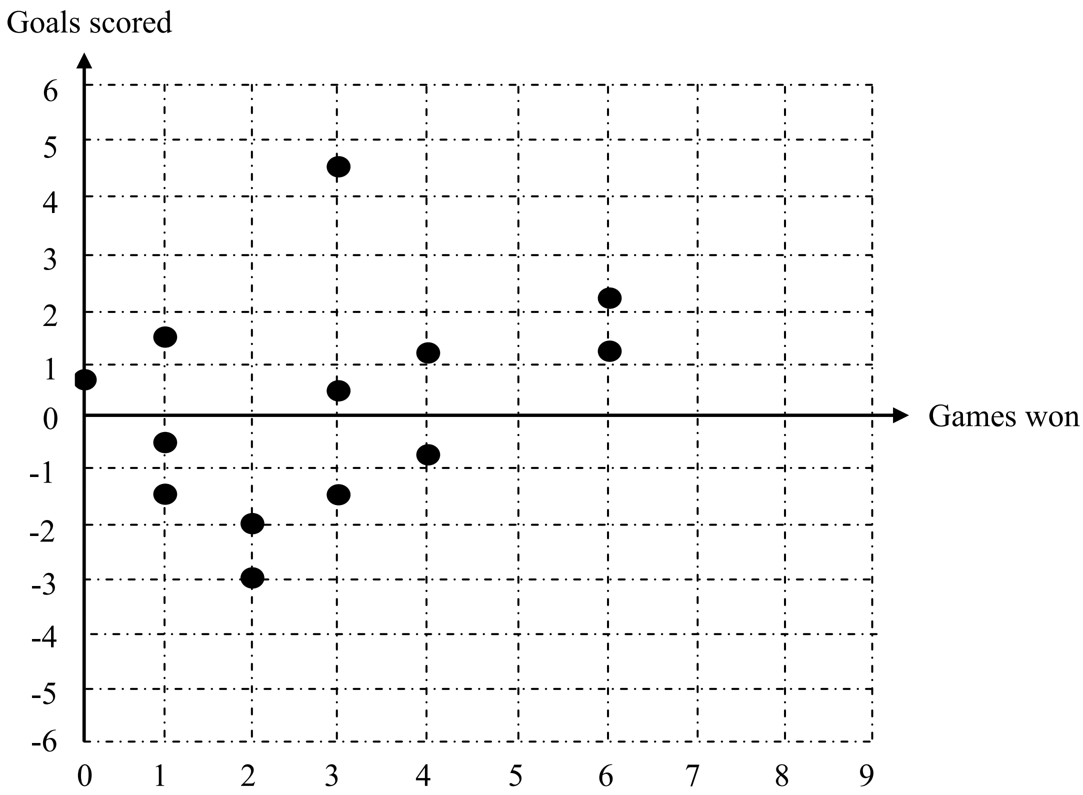
i. [ ] % of the variation in the total number of goals scored can be explained by the variation in the total number of games won.

ii. On average, a country's total scored goals increase by [ ] goals per game won.

1+1=2 marks

Question 2

To investigate the form of the relationship between the total number of games won and the total number of goals scored, the following residual plot is constructed. It is incomplete.



a. Complete the residual plot above by marking in the missing residual value that belongs to the country that won 5 games.

1 mark

b. When complete does the residual plot suggest that the original data probably has a linear relationship? Justify your response.

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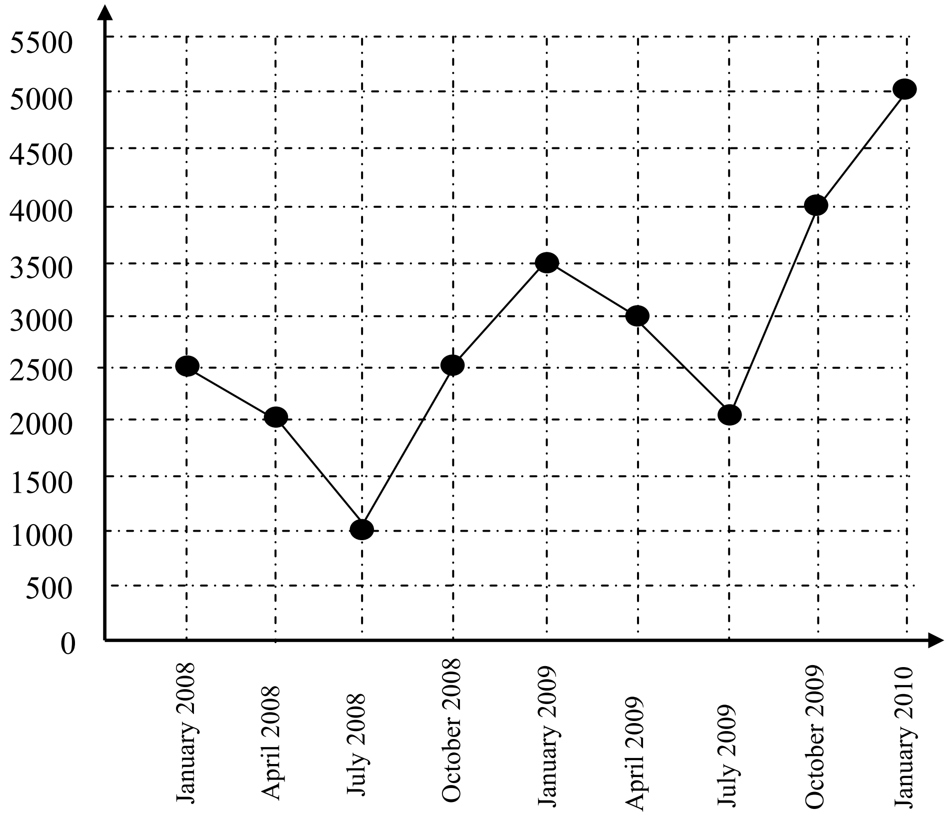
1 mark

Working space

Question 3

Charlie has been a casual employee in a roof insulation company since 2008. His quarterly net pay is shown on the time series plot below.



a. Use four-median smoothing with centring to smooth the time series plot. Plot the smoothed series on the same graph above. Mark each smoothed data point with a cross (×).

2 marks

b. Describe the general pattern in the quarterly pay that is revealed by the smoothed time series plot.

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1 mark

c. Find the equation of the regression line that is fitted to the smoothed time series graph above.

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1 mark

d. Use the regression equation to predict Charlie's quarterly net pay for October quarter in 2010. Give your answer correct to the nearest thousand dollars.

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1 mark

Total 15 Marks

END OF CORE

TURN OVER

SECTION B

Module 1: Number Patterns

Question 1

A group of students from Bright Starts Kindergarten is taken for monthly excursions to Melbourne Luna Park. Melbourne Luna Park has got some height regulations for most of the rides. The children need to be at least 110 cm tall to ride twin dragon and at least 120 cm tall to ride sky rider which both happen to be the favourite rides of the students. The teachers measure the height of the children and find out that 35% of the group are eligible to ride twin dragon. Tony, one of the teachers in the group, believes that each month a further 6% of the children will exceed the 110 cm height limit.

a. If this occurs, what type of sequence will be formed by the monthly percentage of students who are eligible to ride twin dragon?

1 mark

b. If in their *n*th visit, the percentage of students who are eligible to ride twin dragon is given by t_n, write down a formula for t_n in terms of *n*.

1 mark

c. If their first visit to Luna Park is in February 2010, determine the percentage of students who will be eligible to ride twin dragon in July 2010.

1 mark

d. Under this scenario, in which month will all the students in this group be eligible to ride twin dragon?

1 mark

Question 2

Another group of students from the same kindergarten also visits Luna Park once a month, starting from June 2010. Jessie, a student in this group, is 105cm tall initially. She grows 6 cm in the first month and each consecutive month she grows 45% less than the previous month.

a. How tall is Jessie going to be in her fifth visit to Luna Park? Give your answer correct to two decimal places.

1 mark

b. Write an expression that will determine the total growth in Jessie's height in her n^{th} visit to the Luna Park?

1 mark

c. Totally how many centimetres will she grow between her eight and eleventh visits to the Luna Park? (The eighth and eleventh visits are inclusive.) Write your answer correct to three decimal places.

1 mark

d. If she continues growing with this pattern, in which month will she be eligible to go on sky rider? Explain your answer with doing appropriate calculations.

2 marks

Question 3

The kindergarten started operating at the start of 2006 with only 50 children. The number of students enrolled at the kindergarten at the start of its n^{th} year of operation is given by a difference equation

$A_{n+1} = 0.8 \times A_n + 80 \quad , where\ A_1 = 50$

a. How many students are enrolled at this kindergarten at the start of 2008?

1 mark

b. Show that the number of students enrolled at the kindergarten at the start of its n^{th} year of operation does not follow an arithmetic or a geometric sequence.

1 mark

c. Show that the solution to the difference equation $A_{n+1} = 0.8 \times A_n + 80 \quad , where\ A_1 = 50$ is given by $A_n = -350 \times 0.8^{n-1} + 400$.

1 mark

d. Explain why the total number of students in this kindergarten can never exceed 400?

1 mark

Question 4

The yearly fees paid by the kindergarten students at the start of its n^{th} year of operation is specified with the second order difference equation

$$F_{n+2} = 0.6F_n + 0.75F_{n+1}\ ,\ where\ F_1 = 2\,500,\quad and\quad F_2 = 3\,100$$

a. What is the yearly fee of the kindergarten at the start of its 3rd year of operation?

1 mark

b. Find the total amount of fees collected by the kindergarten owners in its first three years of operation. Give your answer correct to the nearest thousand dollars.

1 mark

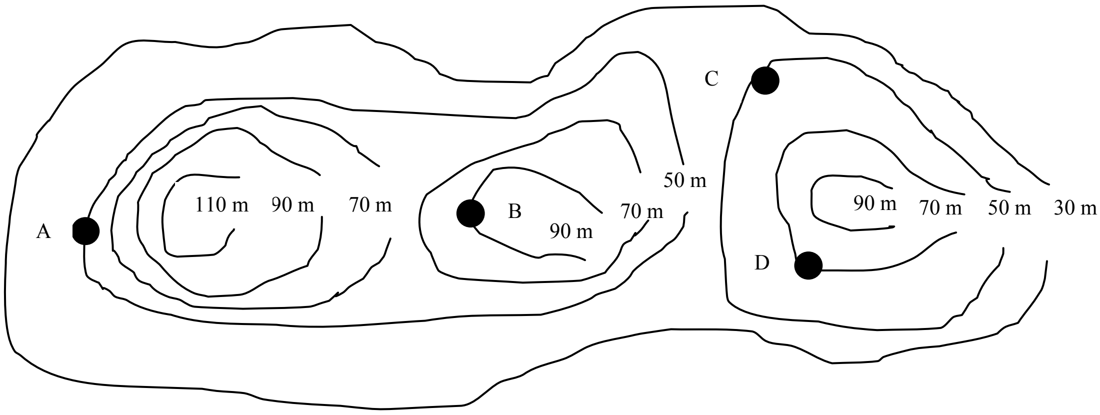
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

A contour map of the amusement park is shown below. It has contours drawn at intervals of 20 metres. The map shows four observation decks in four different spots.



Scale 1:2500

a. The horizontal distance between observation deck A and observation deck B on the contour map is 36 mm.

i. Determine the horizontal distance between observation deck A and observation deck B to the nearest metre.

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ii. Hence determine the angle of depression of a person looking at observation deck A from observation deck B. Give your answer correct to the nearest degree.

1+1=2 marks

b. What is the difference in altitudes of observation deck C and observation deck A?

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1 mark

c. The horizontal distance between observation deck C and observation deck D is 44 metres. The owners of the botanical garden want to make a top tree walk between these observation decks. What is the shortest possible length of this top tree walk? Write your answer correct to two decimal places.

1 mark

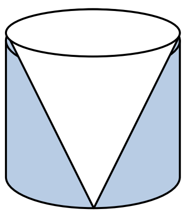
d. Find the slope (gradient) of the top tree walk. Write your answer correct to three decimal places.

1 mark

Question 2

Martha uses two measuring cups to cook her mother's favourite cupcakes from her secret recipe. One of the measuring cups is in cylindrical shape and the other one is a cone. They both have the same radii and equal heights.

Her mother told her to use a secret ingredient in a specific amount to give the cupcakes their unique taste. She measured it in a very strange way. She first put some secret ingredient into the cylindrical cup and then she put the cone in the cylinder. The part between the cylinder and the cone is meant to be full of secret ingredient as shown in the figure below.

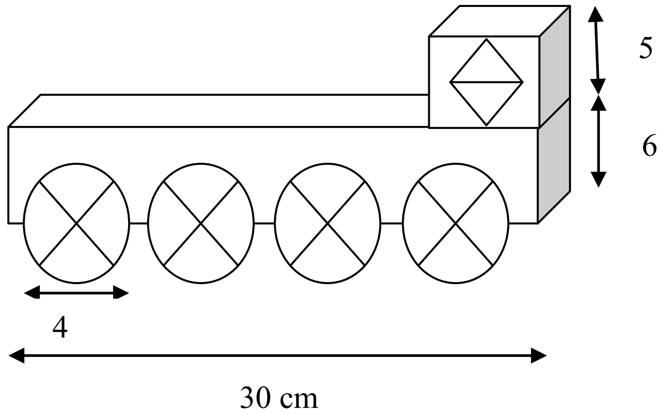


If the volume of the cylinder is V cm^3 what fraction of V is the volume of the secret ingredient?

2 marks

Question 3

Josh made a little train with his wooden blocks. The train consists of a rectangular prism shaped body, a cube shaped conductor compartment, eight circular wheels (four on each side), two triangular windows. The wheels are stuck to the train from their centres and half of each wheel is below the train. He wants to paint the train including the windows but excluding the wheels.



Find the total surface area of the train that needs to be painted. Give your answer correct to three decimal places.

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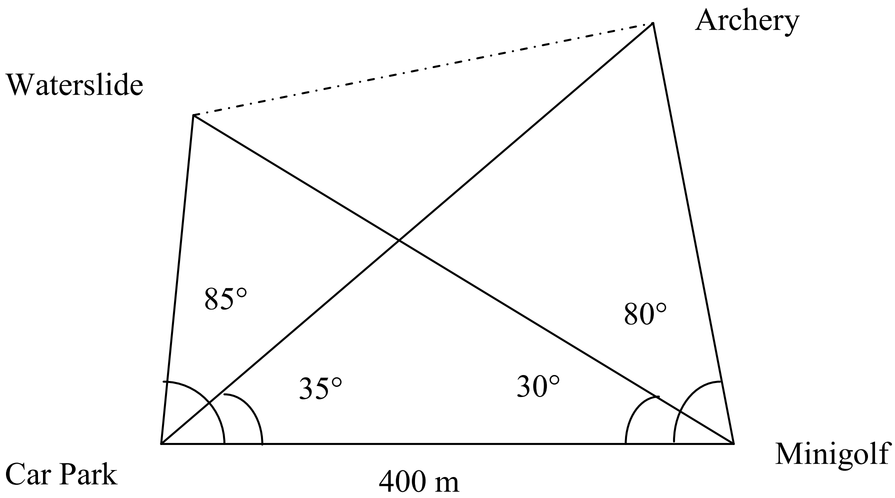
2 marks

Working space

Question 4

An amusement park in Melbourne is visited by so many tourists every year. Jack and Kate came from Mildura to visit the amusement park with their three year old daughter Jenny. They all started to walk from the car park. Kate and Jenny first went on the waterslides while Jack went to the archery. Then they all joined in the minigolf course. After playing minigolf they went back to the car park together. The diagram below shows the angles and distances between the car park, waterslide, archery and minigolf course.



a. How far did Jack walk on his way from the car park to the archery? Write your answer correct to three decimal places.

1 mark

b. How far did Kate and Jenny walk from the car park to the waterslides? Give your answer correct to the nearest metre.

1 mark

c. Find the shortest distance between waterslide and the archery. Write your answer correct the nearest metre.

1 mark

d. The minigolf course is due east of the car park. By using the answers obtained from questions 4a, 4b and 4c, find the bearing of the archery from the waterslides. Write your answer correct to the nearest degrees and minutes.

2 marks

e. What is the area of the quadrilateral that is formed by the car park, waterslides, archery and minigolf course? Give your answer, in km^2, correct to three decimal places.

1 mark

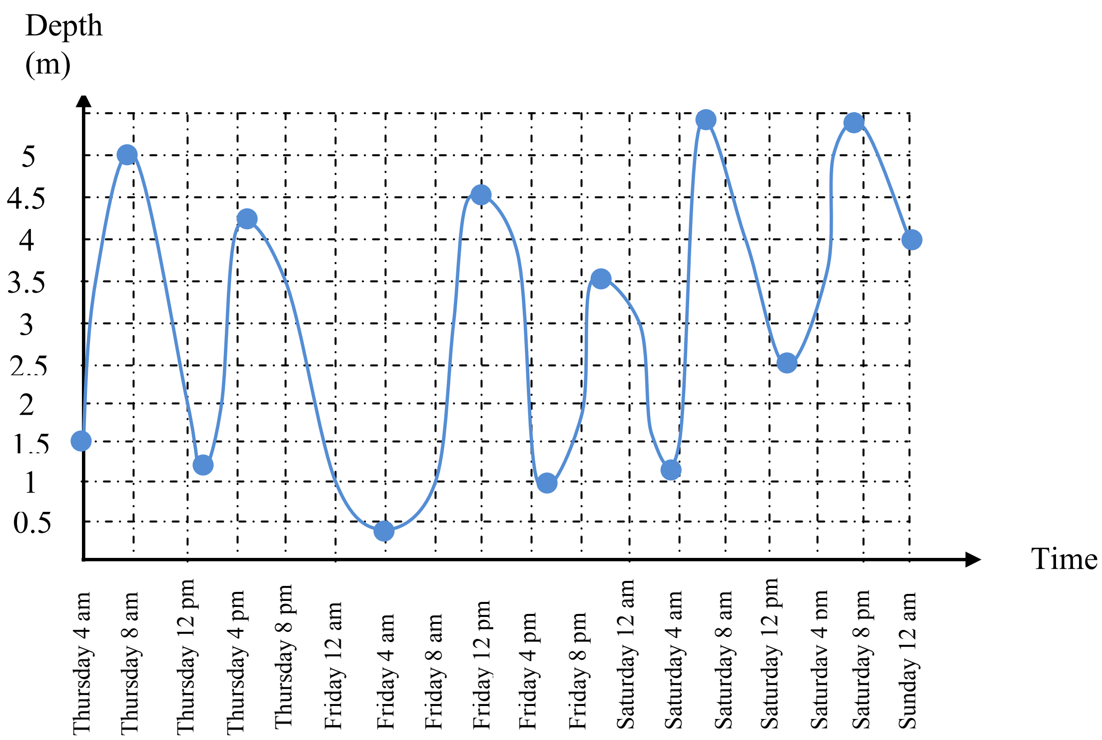
Total 15 marks

END OF MODULE 2

Module 3: Graphs and Relations

Question 1

Jennifer, a professional swimmer does her swimming practice in the natural environment. The depth of water across the sand bar that she prefers swimming in varies depending on the tide as shown in the graph below.



a. When does the third high tide occur?

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1 mark

b. The water levels should be on or above 1metre level for Jennifer to swim comfortably. Approximately between what times and on which day can Jennifer not swim comfortably?

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1 mark

c. At what time on Friday morning will there be the least movement of water in the sand bar?

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1 mark

d. What is the difference in heights of the first high tide and the fourth low tide?

1 mark

e. How many high tides and low tides occur between Thursday 8 pm and Saturday 4 pm?

1 mark

Question 2

Jennifer is a member of a swimming club which has many other male and female members. The club has a swimming coach training each swimmer for two different swimming styles, butterfly and freestyle. On average, it takes 10 minutes for the swimming coach to train each female swimmer and 25 minutes to train each male swimmer for butterfly. He has a total of 250 minutes a day to train for the butterfly. He also spends an average of 45 minutes with each male swimmer and 40 minutes with each female swimmer to train them for freestyle and he has a total of 720 minutes a day for training freestyle. Let x be the number of female swimmers and y be the number of male swimmers in the club in one day.

a. Two of the constraints are $x \geq 0 \; and \; y \geq 0$.

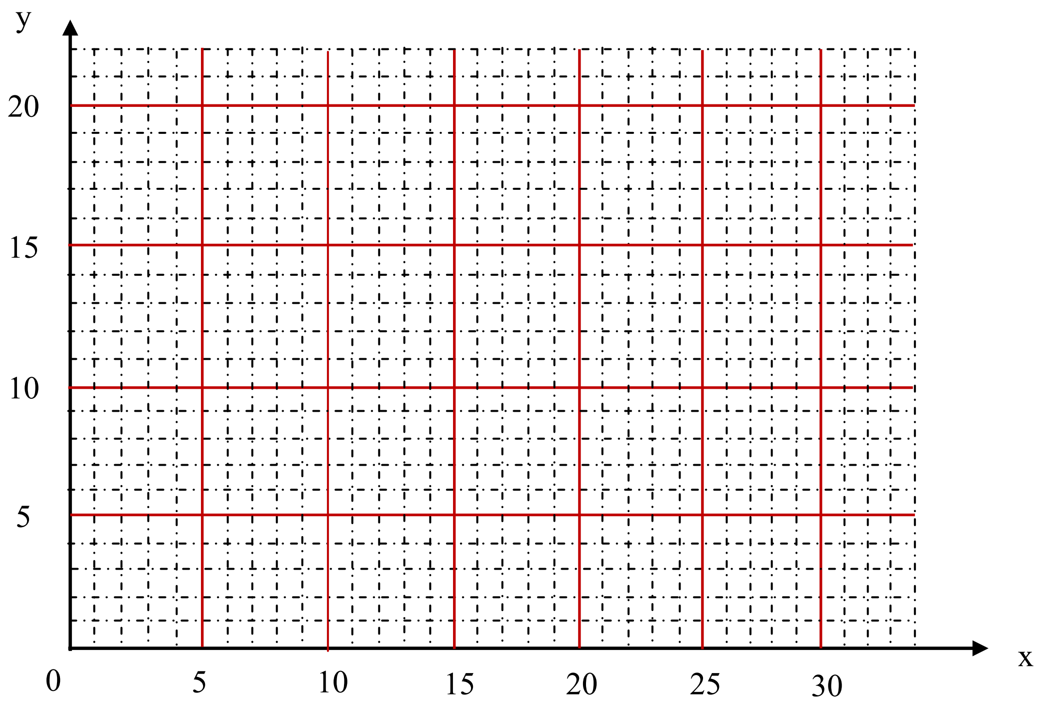
i. Write down the constraint that is formed by the daily available times of the coach to train female and male swimmers for butterfly.

ii. The following inequation represents the constraint of the daily available times of the coach to train female and male swimmers for freestyle. Complete the following constraint by filling in the spaces.

$$\frac{x}{\square} + \frac{y}{\square} \leq 1$$

1+1=2 marks

b. Sketch and shade the feasible region described by the four constraints on the graph below. Label each line and clearly identify the coordinates of all vertices of the feasible region.



3 marks

c. The profit made by the owners of the club on each male swimmer is $50 and on each female swimmer is 60% less than a male swimmer.

i. Write down an equation for the profit P.

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ii. How many male swimmers and female swimmers are needed in a day in order to maximise the profit.

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1+1=2 marks

d. One month before the swimming competition the club decided to hire another swimming coach to train the swimmers for breaststroke. The new coach trains a female swimmer in 20 minutes and a male swimmer in 10 minutes and he is available in the club for a total of 200 minutes a day.

i. 5 male and 10 female swimmers want to train for breaststroke in the club on an average day during the busy competition preparations. Will the club be able to meet the needs of these swimmers?

ii. What is the maximum number of female and male swimmers that the club can accept on a busy day?

1+2=3 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

a. Adam needs $45 000 to renovate his first class restaurant. He wants to borrow money from the Bestrates Bank at $9\frac{4}{9}$% p.a. simple interest for four years. How much interest will Bestrates Bank charge Adam in these four years?

1 mark

b. Adam's friend, Sheila recommended him the Cheaper Building Society. Adam finds that the Cheaper Building Society will lend the $45 000 to him at 0.8% per month simple interest for four years. Find the total interest that Adam will have to pay to the Cheaper Building Society if he decides to borrow the loan from them?

1 mark

c. Hence or otherwise, which institution would you recommend Adam to borrow the loan from?

1 mark

Question 2

Sheila has $63 000 invested in an account which pays interest at the rate of 4.2% per annum compounding monthly.

a. Show that the interest rate per month is 0.35%.

1 mark

b. Determine the value of the $63 000 investment after five years. Write your answer in dollars correct to the nearest cent.

1 mark

c. If Sheila had decided to invest her money into another account find the interest rate per annum that would enable her \$63 000 investment to grow to \$80 000 over five years if interest is compounded quarterly. Write your answer correct to two decimal places.

2 marks

Question 3

A school bought 20 new computers with each of them valuing at \$1 950 and having a scrap value of \$220.

a. If they depreciate by 11.1% per annum by flat rate depreciation method, how long will it be before the scrap value is reached?

2 marks

b. Use the reducing balance depreciation method with a depreciation rate of 30.5% to calculate the total amount of time that they can be used before they will need to be replaced.

1 mark

c. Which method, flat rate depreciation or reducing balance depreciation is better to use for school to utilise the computers for a longer period of time? Write down the difference in the lifetime of the school computers calculated by two different methods.

1 mark

Question 4

a. Joseph's monthly rent payments are $1 300. Assuming an average inflation rate of 5.6% per annum, calculate the amount of monthly rent he will be paying in five years' time. Write your answer in dollars correct to the nearest cent.

1 mark

b. His annual gross salary is $45 000. His employer decided to raise his salary by 3% every year. Calculate his annual salary in five years' time. Write your answer in dollars correct to the nearest cent.

1 mark

c. John is 42 years old and is planning to retire at 61. His employer contributions to his superannuation fund are 8% of his gross monthly income. John also contributes a further $450 a month as a salary sacrifice. The superfund has been returning an interest rate of 4.2% p.a. compounded monthly and his current balance in the superfund is $52 000.

i. Calculate John's total monthly contributions to the superannuation account made by him and his employer.

ii. Calculate the lump sum that he can receive for his planned retirement at age 61 assuming that his salary remains constant every year. Write your answer in dollars correct to the nearest cent.

1+1=2 marks

Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

Santos and Carla are married with four children, Maria, Arlene, Tommy and Peter.
Arlene married Jim and had one child, Jenny.
Tommy married Tina and had three children, Adele, John and Bony.
Peter married Cindy and had two children, Juliet and Sammy.
Jenny married Charlie and had one child, Morgan.
Maria married David and they don't have any children.

a. Construct a network representing the given family tree. Use a single node to represent each married couple and each child.

1 mark

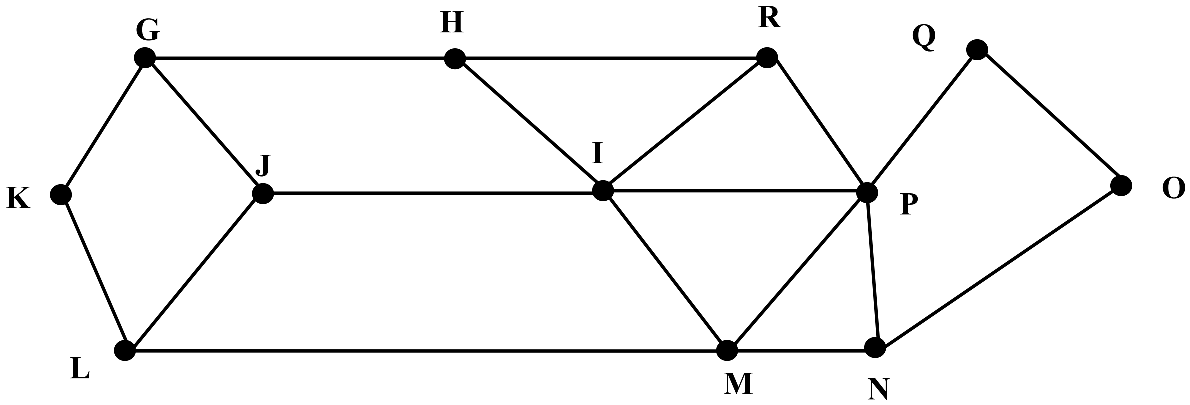
b. How many vertices and edges does the network have?

____

1 mark

Question 2

Santos and Carla visit an art gallery during their trip to Spain. The art gallery contains 12 drawings denoted as vertices G to R on the network diagram below. The edges on this network represent the pathways that link the 12 drawings.



a. Write down the degree of vertex P.

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1 mark

b. Carla wishes to examine each drawing only once. She wants to begin her tour with drawing H and end it at drawing G.

i. Write down the order Carla will examine the drawings.

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ii. How many pathways doesn't she need to use at all during her tour? List their names.

___

iii. Suppose she starts and finishes her tour with drawing L. List two different ways that she can examine the drawings.

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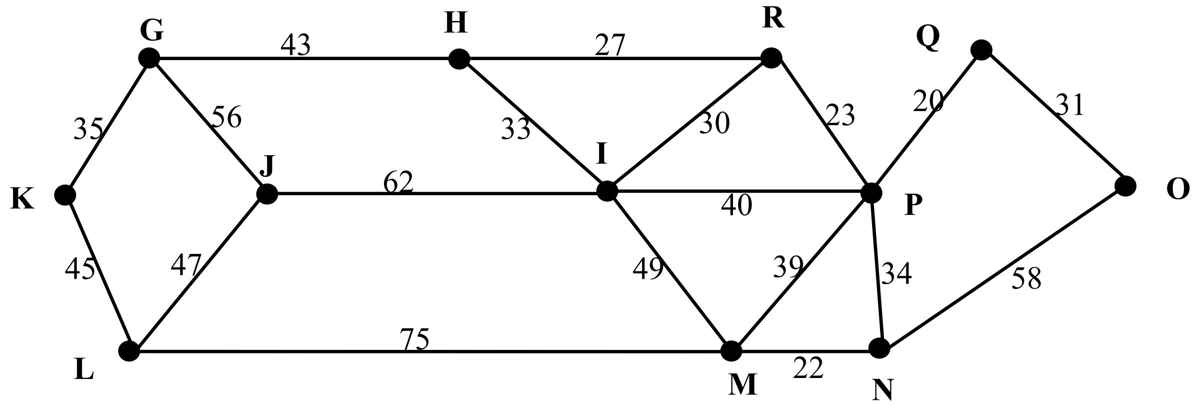
1+1+1=3 marks

Question 3

The following network represents the distances (in metres) of pathways connecting 12 drawings.



a. Explain why an Euler circuit does not exist for this graph.

1 mark

b. **i.** Explain why L-K-G-H-R-P-Q-O-N-P-M-I-J doesn't form a tree.

ii. Draw the minimum spanning tree for this network on the above graph.

iii. Find the total distance of the minimum spanning tree.

1+1+1=3 marks

Question 4

The following table represents the flow capacity of the water pipes between some of the main streets in a town. The letters represent the street names and flow capacities are given in litres per minute.

From	To	Flow capacity
H	I	400
H	L	400
I	J	300
I	M	100
L	M	200
J	K	240
M	K	240
I	K	100

a. Convert the information presented in the following table to a network diagram, clearly indicating the direction and quantity of the flow.

1 mark

b. Show the minimum cut and hence, determine the maximum flow capacity of the network.

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2 marks

Question 5

In the "Brightest Brains" chess tournament, each player competes against each of the other players in a chess game after which one player is judged the winner and the other is the loser. The results of these games between five competitors; Susan, Keith, Cathy, Malcolm and Terry, are shown in the one-step dominance matrix below.

$$A = \begin{matrix} & S & K & C & M & T \\ S & 0 & 1 & 0 & 1 & 1 \\ K & 0 & 0 & 0 & 0 & 1 \\ C & 1 & 1 & 0 & 1 & 0 \\ M & 0 & 1 & 0 & 0 & 0 \\ T & 0 & 0 & 1 & 1 & 0 \end{matrix}$$

a. Find the two step dominance matrix.

1 mark

b. Using the information on one-step and two-step dominances, find the winner of the chess competition.

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

The table below displays the goods to be delivered from a technological equipment company's warehouse to its four different shops.

	To shop A	**To shop B**	**To shop C**	**To shop D**
Mobile phones	24	15	35	22
Televisions	32	23	31	20
Cameras	45	76	63	55
Laptops	12	7	9	11

a. Write down a 2×4 matrix that displays the number of mobile phones, televisions, cameras and laptops to be delivered to shop B and shop D.

$$A = \begin{matrix} & \text{M} & \text{T} & \text{C} & \text{L} \\ \textit{To shop B} & \square & \square & \square & \square \\ \textit{To shop D} & \square & \square & \square & \square \end{matrix}$$

1 mark

b. Let A be the matrix found in the previous question and B be a 4×1 matrix representing the unit cost prices (in dollars) of mobile phones, televisions, cameras and laptops produced by the warehouse. Matrix B is defined as follows.

$$B = \begin{bmatrix} 560 \\ 1250 \\ 360 \\ 1850 \end{bmatrix}$$

i. Evaluate the matrix product AB.

ii. What information does the matrix product AB represent? Explain its meaning.

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iii. Explain why the matrix product BA does not exist.

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iv. If K is a 1×6 matrix, what is the order of the matrix product BK?

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1+1+1+1=4 marks

c. All of the mobile phones, televisions, cameras and laptops in the shops were sold within a month. Selling price of each item was constant in every store. The revenues that shop A, shop B, shop C and shop D received from selling all these technological items are \$119 560, \$106 035, \$129 725 and \$103 105, respectively.

Let the selling price, in dollars; of each of the items be represented by ***m*** for the mobile phones, ***t*** for the televisions, ***c*** for the cameras and ***l*** for the laptops.

This situation is represented in the matrix equation below.

$$\begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \times \begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{t} \\ \boldsymbol{c} \\ \boldsymbol{l} \end{bmatrix} = \begin{bmatrix} 119\ 560 \\ 106\ 035 \\ 129\ 725 \\ 103\ 105 \end{bmatrix}$$

i. On the above equation, find the 4×4 matrix that displays the number of mobile phones, televisions, cameras and laptops that were delivered to shops A, B, C and D.

ii. Solve the matrix equation and hence, find the values of **m**, **t, c** and **l**.

1+2=3marks

Question 2

The technological equipment company did a questionnaire on the customer behaviour in order to attract more customers. The feedback of the questionnaire showed that:

20% of the people who went to shop A this month will go to shop C next month.
15% of the people who went to shop A this month will go to shop B next month.
Only 5% of the people who went to shop A this month will go to shop D next month.

One fifth of the people who went to shop B this month will go to the same shop next month.
None of the people who went to shop B this month will go to shop C next month.
8% of the people who went to shop B this month will go to shop D next month.

16% of the people who went to shop C this month will go to shop B next month.
Three quarters of the people who went to shop C this month will go to shop A next month.
4% of the people who went to shop C this month will go to shop D next month.

Half of the people who went to shop D this month will go to the same shop next month.
Half of the remaining people will go to shop B and the other half will go to shop C next month.

a. Enter this data into the transition matrix T below.

$$\begin{array}{c} \\ \\ T = \end{array} \begin{array}{c} \text{this month} \\ \begin{array}{cccc} A & B & C & D \end{array} \end{array}$$

$$T = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \text{next month}$$

2 marks

b. This questionnaire was conducted in May, 2010. At that time shop A, B, C and D had 315, 426, 790 and 1200 customers respectively. Write down the initial state matrix, S_0, for the number of customers that each shop had when the questionnaire was conducted.

1 mark

c. How many of the people involved in the questionnaire moved from shop C to shop B one month later?

1 mark

d. How many customers will shop C have in September?

1 mark

e. The technological equipment company decided to close down the shops if they have less than 300 customers a month.

i. Write down the number of customers that each shop is going to have in the long run.

ii. Which one of the four shops is going to have the highest number of customers and which one will have to be closed down?

1+1=2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK