2009

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

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Core

Question 1

a. $\frac{9}{13} \times 100 = 69.2\%$

(1 mark)

b.

For T183 calculator enter data in stat edit in column L_1. Go to stat calc, 1-var stats and enter L_1. This gives standard deviation = 26.9

(1 mark)

c.

The Levi stewards have a much larger spread in their number of flying hours than the Hippo stewards.

(1 mark)

d.

Using calculator window from **1 b.** gives $\bar{x} = 106.1, Q_1 = 85, Q_3 = 129$

$\bar{x} \pm 1.5IQR = 106.1 \pm 1.5 \times (129 - 85)$

So values outside 40.1 and 172.1 are outliers.

Hence, there are no outliers.

(1 mark)

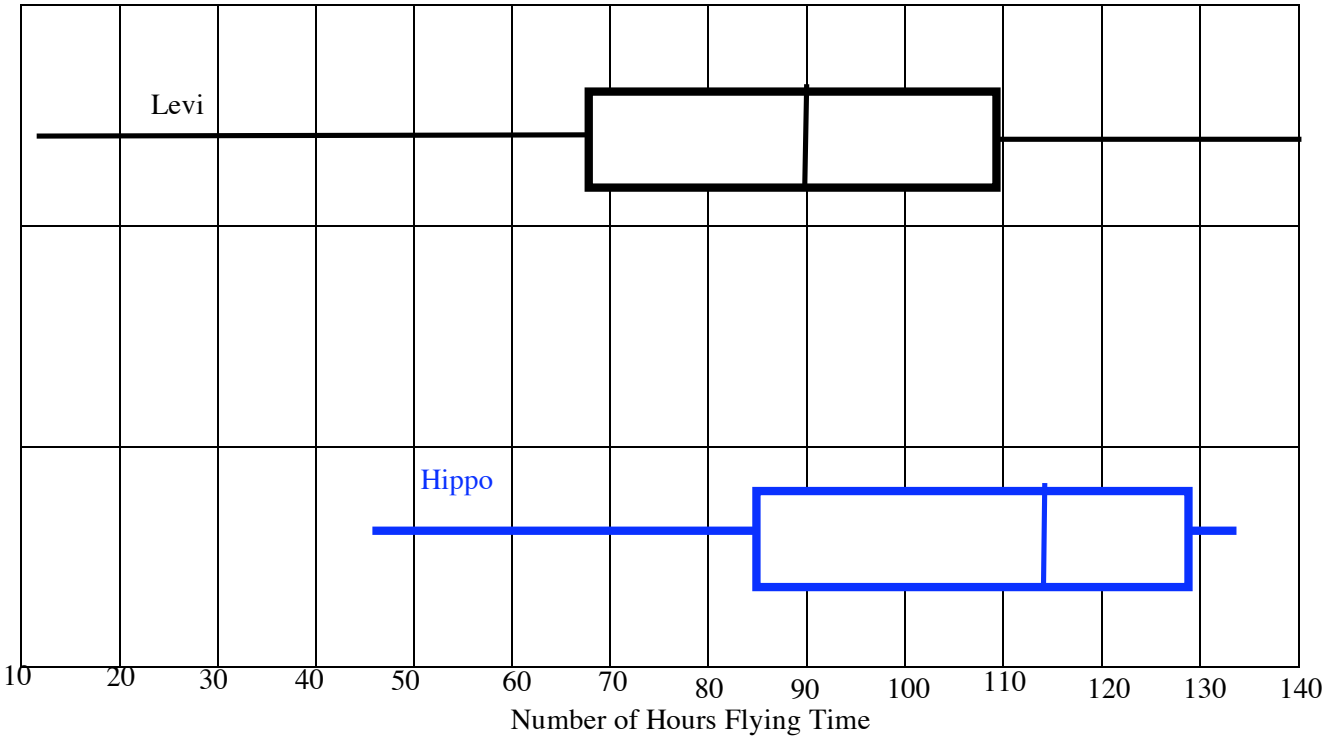
Core

Question 1 (continued)

e.

Using the calculator window from 1(b) gives $Q_1 = 85$, Med = 114, $Q_3 = 129$
Min = 45, Max = 135

(1 mark)



f.

Hours spent flying by Hippo stewards is more negatively skewed than hours spent flying by Levi stewards.

(1 mark)

Core

Question 2

a.

Advertising is the independent variable (1 mark)

b.

The variables are basically linearly related, and there are no outliers. (Only one of these points required for a mark)

(1 mark)

c.

For T183 calculator, enter data in stat edit in column L_1 and L_2 Go to stat calc, LinReg($ax+b$) and enter L_1 , L_2 to get

$Sales = \boxed{8609.0} + Advertising \times \boxed{0.5}$

(1 mark)

d.

Predicted Value = $0.5 \times 6000 + 8609 = 11609$
Residual = $12500 - 11609 = \$891$

(1 mark)

e.

Using same calculator screen as for 2c. gives $r^2 = 0.98$

(1 mark)

f.

98% of the variation in sales income can be explained by the variation in advertising expenditure.

(1 mark)

g.

$20000 = 0.5 \times$ advertising $+ 8609$
$11391 = 0.5 \times$ advertising
Advertising $= \$22782$

(1 mark)

Core

Question 3

a.

Year	**2005**	**2006**	**2007**	**2008**
Average	162.5	163.75	163	164.25
Autumn	$\frac{172}{162.5}$ $=1.06$	$\frac{170}{163.75}$ $=1.04$	$\frac{171}{163}$ $=1.05$	$\frac{174}{164.25}$ $=1.06$

$$SI = \frac{1.06+1.04+1.05+1.06}{4}$$

$$=1.1$$

(1 mark)

b.

In autumn, sales are 10% higher than sales in an average season.

(1 mark)

Module 1 Number patterns and applications

Question 1

a.

$300, 350, 400...$ Arithmetic sequence

$a = 300, d = 50, n = 10$

$t_{10} = 300 + 9 \times 50 = 750$ tonnes

(1 mark)

b.

$t_n = 300 + (n-1)50$

$t_n = 300 + 50n - 50 = 250 + 50n$ tonnes

(1 mark)

c.

This is an arithmetic sequence

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 300 + 19 \times 50]$

$S_{20} = 10 \times 1550 = 15500$ tonnes

(1 mark)

d.

Use T1-83 graphics calculator in sequence mode.

$y =$

enter

$nMin = 1$

$\mu_n = 300 + (n-1) \times 50$

$\mu(nMin) = 300$

Press 2nd table

Scroll down to $n = 20$

This gives the value $\mu_n = 1250$

This is amount extracted in 20th year.

Amount extracted in 21st year = $1250 - 75 = 1175$ tonnes

(1 mark)

Module 1 Number patterns and applications

Question 1 (continued)

e.	**f.**
Use graphics calculator in sequence mode. y = enter $nMin = 1$ $\mu_n = 1175 - (n-1)75$ $\mu(nMin) = 1175$ Press 2nd table Scroll down to find the first μ_n value which is less than 300 This is 275 when $n = 13$ Since $n = 1$ corresponds to year 21 then $n = 13$ corresponds to year 33 (1 mark)	Use same calculator screen as for 1e. If first year is 2000 then 21st. year is 2020. This corresponds to n = 1 on calculator. Cannot mine a negative amount of copper, So mining ceases when n = 16 which is year 2035 (1 mark)

Module 1 Number patterns and applications

Question 2

a.

Geometric sequence

$a = 800, r = 1 - 0.085 = 0.915$

$t_2 = ar = 800 \times 0.915 = 732$ tonnes

(1 mark)

b.

$S_n = \dfrac{a(1-r^n)}{(1-r)}$

$S_{10} = \dfrac{800(1-0.915^{10})}{(1-0.915)} = 5540.24$ tonnes

(1 mark)

c.

$S_\infty = \dfrac{a}{(1-r)}$

$\Rightarrow S_\infty = \dfrac{800}{(1-0.915)}$

$\Rightarrow S_\infty = 9411.76$ tonnes

(1 mark)

d.

Use calculator in sequence mode. Press $y =$

$nMin = 1$

$\mu_n = 800 \times 0.915^{(n-1)}$

$\mu(nMin) = 800$

Press 2nd table

Scroll down to find the first μ_n value

which is less than 100

This is 94.885 when $n = 25$

So mine will close in 25th year of production

(1 mark)

Module 1 Number patterns and applications

Question 2 (continued)

e.	**f.**
Use calculator in sequence mode and keep same input as for d. Then add $v_n = 300 + (n-1)50$ $\mu(nMin) = 300$ Press 2nd table Scroll down to find the first v_n value which is greater than μ_n This is 550 > 513.09 when $n = 6$ So in the 6th year of production (1 mark)	Use same screen as **e.** When $n = 10$ *Acme* is 750 and *Beta* is 359.65 750 – 359.65 = 390.35 tonnes. (1 mark)

Module 1 Number patterns and applications

Question 3

a.

$P_n = 1.15P_{n-1} \qquad P_0 = 250000$

(1 mark)

b.

$P_4 = 1.15^4 \times 250000$

$= \$437251.56$

(1 mark)

c.

$t_4 = 1100 + 700 = 1800$

$t_5 = 1800 + 1100 = 2900$

$t_6 = 2900 + 1800 = 4700$ m.

(1 mark)

Module 2 Geometry and trigonometry

Question 1

a.

$AC^2 = 10^2 + 12^2$

$AC = \sqrt{10^2 + 12^2} = 15.62$ km.

(1 mark)

b.

$\tan \angle ACB = \frac{12}{10}$

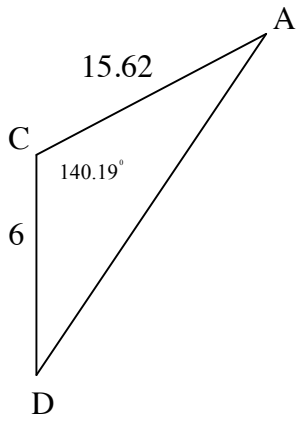
$\angle ACB = \tan^{-1}\left(\frac{12}{10}\right) = 50.19^0$

$\angle ACD = 50.19^0 + 90^0 = 140.19^0$

1 mark for $\angle ACB$,1 mark for $\angle ACD$

(2 marks)

c.



$AD^2 = 6^2 + 15.62^2 - 2 \times 6 \times 15.62 \times \cos 140.19$

$AD = \sqrt{6^2 + 15.62^2 - 2 \times 6 \times 15.62 \times \cos 140.19}$

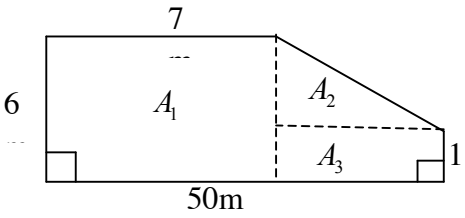
$AD = 20.6$ km.

(1 mark)

Module 2 Geometry and trigonometry

Question 2

a.



$A_1 = 6 \times 7 = 42$

$A_2 = \frac{43 \times 5}{2} = 107.5$

$A_3 = 43 \times 1 = 43$

Total area $= 42 + 107.5 + 43 = 192.5$ m^2

(1 mark)

b.

Volume = Area of base × height

Volume $= 192.5 \times 12 = 2310$ m^3

(1 mark)

c.

Area to be tiled $= 2(12 \times 8) + 2(50 \times 8) + (12 \times 50)$

Area to be tiled $= 1592$ m^2

(1 mark)

d.

Number tiles $= \frac{1592 \times 100 \times 100}{300} = 53066.67$

Need to buy 53100 tiles.

(1 mark)

e.

Number of packs $= \frac{53100}{1000} = 53.1$

Need 54 packs

Cost = 120 × 54 = \$6480

(1 mark)

f.

Area under water

$= 2(192.5) + (6 \times 12) + (1 \times 12) + (50 \times 12)$

Area under water $= 1069$

% under water $= \frac{1069}{1592} \times 100 = 67.1\%$

(1 mark) for Area under water

(1 mark) for percentage.

(2 marks)

Module 2 Geometry and trigonometry

Question 3

a.

From 2b. Vol. of water in pool = 2310

7.5 m^3 in 1 min

2310 m^3 in $\frac{2310}{7.5} = 308$ mins $= \frac{308}{60} = 5.1$ hrs.

(1 mark)

b.

$V = \pi r^2 h$

$50 = \pi r^2 4.9$

$r^2 = \frac{50}{4.9\pi}$

$r = \sqrt{\frac{50}{4.9\pi}} = 1.8$

$d = 2 \times 1.8 = 3.6$ m

(1 mark)

Question 4

a.

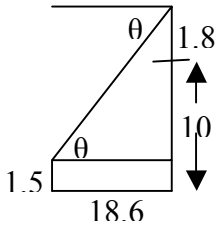
$XY = \frac{1}{2} AC$

$AC = \sqrt{30^2 + 22^2}$

$XY = \frac{1}{2}\sqrt{30^2 + 22^2} = 18.6$ m

(1 mark)

b.



$\tan\theta = \frac{10.3}{18.6}$

$\theta = \tan^{-1}\left(\frac{10.3}{18.6}\right) = 29^0$

(1 mark)

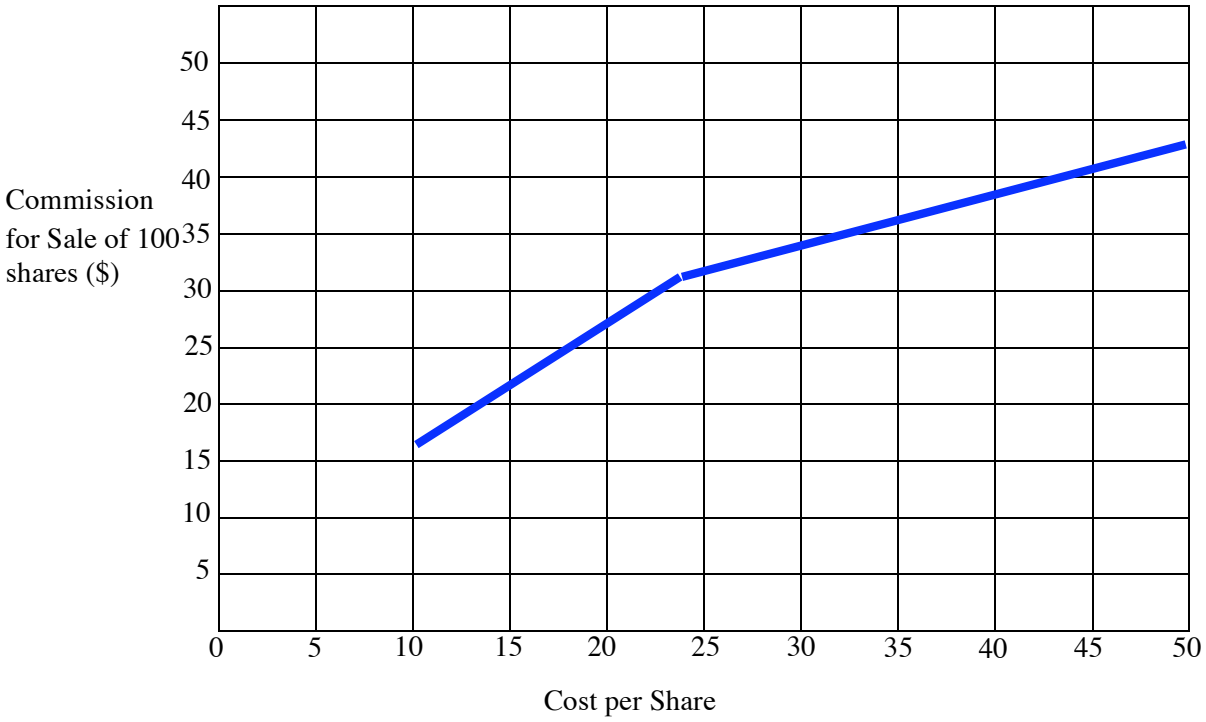
Module 3 Graphs and relations

Question 1

a.	**b.**
$80 \times 100 = \$8000$ (1 mark)	$75 > 50$ $C = 39 + 0.1 \times 75 = \$46.50$ (1 mark)
c. $p = \frac{160}{100} = 1.6$ 1.6 is between 1.5 and 4 $C = 3 + 2 \times 1.6 = \$6.20$ (1 mark)	

Module 3 Graphs and relations

Question 1 (continued)



d.

When $p = 10$, $C = 7 + 10 = 17$
When $p = 23.9$, $C = 7 + 23.9 = 30.9$
When $p = 24$, $C = 19 + 0.5 \times 24 = 31$
When $p = 49.9$, $C = 19 + 0.5 \times 49.9 = 43.95$
When $p = 50$, $C = 39 + 0.1 \times 50 = 44$
Plot points on graph and join with straight lines since all equations are equations of straight lines.

For correct straight line from 10 to 24 (1 mark)

For correct straight line from 24 to 50 (1 mark)

Module 3 Graphs and relations

Question 2

a. $C = 4245 + 5.40x$ (1 mark)	**b.** $S = 62x$ (1 mark)
c. $4245 + 5.4x = 62x$ $56.6x = 4245$ $x = 75$ 75 shirts must be sold to break even. (1 mark)	**d.** $P = SP - CP$ $P = 62x - 4245 - 5.4x$ $P = 56.6x - 4245$ (1 mark)
e. $56.6x - 4245 = 25639.80$ $56.6x = 29884.80$ $x = 528$ (1 mark)	

Module 3 Graphs and relations

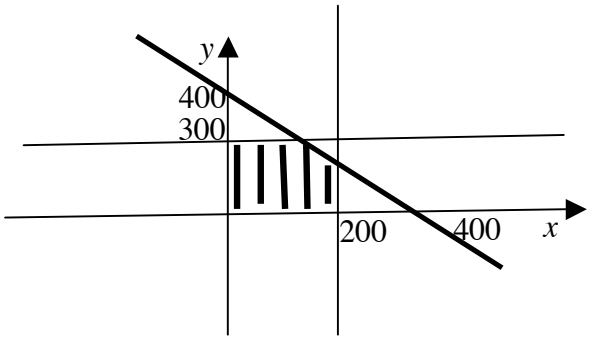
Question 3

a.

$x + y \leq 400$

(1 mark)

b. and c.



For the straight line (1 mark)

For the shaded region (1 mark)

d.

$P = 52x + 48.5y$

(1 mark)

e.

Corner point (0,0) $P = 52 \times 0 + 48.5 \times 0 = 0$

Corner point (0,300) $P = 52 \times 0 + 48.5 \times 300 = 14550$

Corner point (100,300) $P = 52 \times 100 + 48.5 \times 300 = 19750$

Corner point (200,200) $P = 52 \times 200 + 48.5 \times 200 = 20100$

Corner point (200,0) $P = 52 \times 200 + 48.5 \times 0 = 10400$

Maximum profit is \$20100

(1 mark)

Module 4 Business-related mathematics

Question 1

a.

Tax paid = 18000 + 0.40(8000) = $21,200

(1 mark)

b. i.

Last year paid 0.15(33000 – 6000) = 4050
This year pays 4200 + 0.30 × 4000 = 5400
Extra tax paid = 5400 – 4050 = $1350

(1 mark)

b. ii.

Last year take home = 33000 – 4050 = 28950
This year take home = 38000 – 5400 = 32600
Extra take home this year
= 32600 – 28950 = 3650
Gets to actually take home $3650 more.

(1 mark)

Question 2

a.

$$\frac{20}{100} \times 7200 = \$1440$$

(1 mark)

b.

$$\text{Amount for work} = 7200 \times \frac{100}{110} = 6545.45$$

$$\text{GST} = 7200 - 6545.45 = \$654.55$$

(1 mark)

c.

Amount repaid = 475.20 × 20 = $9504

(1 mark)

d.

$$\text{Interest paid} = 9504 - (7200 - 1440) = 3744$$

(1 mark)

$$R = \frac{100I}{PT} = \frac{100 \times 3744}{(7200 - 1440) \times 5} = 13\%$$

(1 mark)

Module 4 Business-related mathematics

Question 3

a.

Use graphics calculator
Press Apps Finance Enter TVM Solver Enter
$N = 120$
$I = 6.5$
$PV = 25000$
$PMT =$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
End
Put cursor on PMT and press alpha solve.
$PMT = \$283.87$

(1 mark)

b.

Total amount repaid = 120 × 283.86994
=34064.39
Interest = 34064.39 – 25000 = $9064.39
=$9064 to nearest dollar.

(1 mark)

c.

Use graphics calculator
Press Apps Finance Enter TVM Solver Enter
$N = 60$
$I = 6.5$
$PV = 25000$
$PMT = -283.86994.....$
$FV =$
$P/Y = 12$
$C/Y = 12$
End
Put cursor on PMT and press alpha solve.
$FV = \$14508.21818$

(1 mark)

Again use graphics calculator
Press Apps Finance Enter TVM Solver Enter
$N =$
$I = 6.5$
$PV = 14508.21818$
$PMT = -400$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
End
Put cursor on PMT and press alpha solve.
$N = 40.4911$ months = 3.4 years

(1 mark)

Module 4 Business-related mathematics

Question 4

a.

$0.15 \times 100000 = \$15000$

(1 mark)

b.

$\text{Depreciation} = 25000 - 5000 = 20000$

$20000 = x \times 0.15$

$x = \dfrac{20000}{0.15} = 133333 \text{ km.}$

(1 mark)

c.

With calculator in function mode enter

$y = 25000(0.92)^x$

Press 2nd table and scroll down to $y = 11804$

This corresponds to $x = 9$

Hence, car will be valued at less than

\$12000 after 9 years.

(1 mark)

Module 5 Networks and decision mathematics

Question 1

a.

Since all the vertices are not of an even degree, then an Euler circuit does not exist.

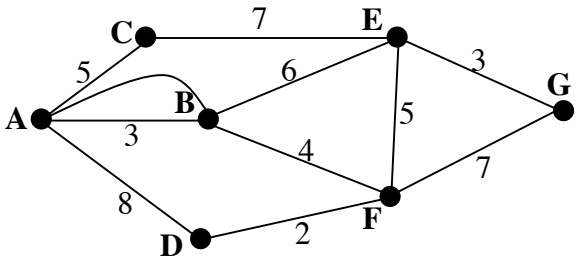
(1 mark)

b.

B as this is the only other vertex of odd degree.

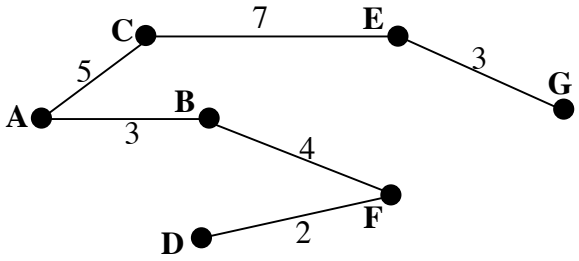
(1 mark)

c.



(1 mark)

d.



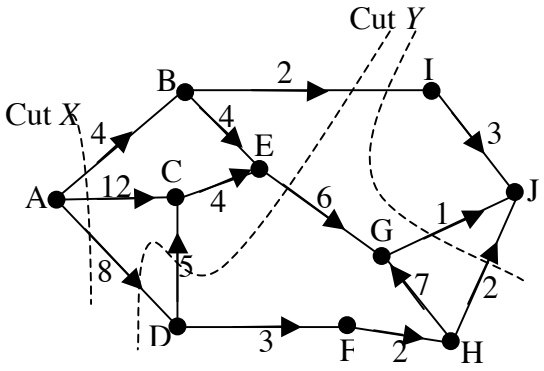
2 + 4 + 3 + 5 + 7 + 3 = 24

There are other routes that will also give 24.

(1 mark)

Module 5 Networks and decision mathematics

Question 2

a. i. 4 + 12 + 8 = 24 (1 mark)	**a. ii.** 2 + 6 + 8 = 16 (1 mark)
b. 2 + 1 + 2 = 5 (1 mark)	**c.**  (1 mark)

Module 5 Networks and decision mathematics

Question 3

a.

Activity	Immediate Predecessor	Earliest Starting Time (days)
A	None	0
B	A	5
C	A , E	**24**
D	B	11
E	A, D	19
F	**C , E , D**	**37**

(3 marks)

(1 mark) for each of the three values

b.

Critical path is the longest path.
A – B – D – E – C – F – G

(1 mark)

c.

There is no slack time for E because E lies on the critical path.

(1 mark)

d. i.

There is now a new critical path:
A – E – C – F – G
This takes 35 days instead of the original 38 days, so 3 days are saved.

(1 mark)

d. ii.

He saves \$3000 by saving 3 days but he has to pay 4 workers \$200 per day for 1 day, so he would save 3000−800 = \$2200. It is worth doing.

(1 mark)

Module 6 Matrices

Question 1

a.

$$R\begin{bmatrix} x \\ y \end{bmatrix}$$

(1 mark)

b.

3×2

(1 mark)

c. i.

$$\begin{bmatrix} 7 & 3 \\ 2 & 5 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 200 \\ 120 \end{bmatrix}$$

(1 mark)

c. ii.

$$\begin{bmatrix} 7 & 3 \\ 2 & 5 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 200 \\ 120 \end{bmatrix} = \begin{bmatrix} 1760 \\ 1000 \\ 920 \end{bmatrix}$$

1760 of a, 1000 of b and 920 of c.

(1 mark)

Module 6 Matrices

Question 2

a.

$$\begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix}$$

(1 mark)

b.

Let $\$x$ be the price per kilogram of raisins

Let $\$y$ be the price per kilogram of sultanas

Let $\$z$ be the price per kilogram of currants

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix}$$

For correct matrices (1 mark)

For correct equation (1 mark)

c.

Use graphics calculator.

Enter

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix} \text{ in } A$$

Enter

$$\begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix} \text{ in } B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=A^{-1}B=\begin{bmatrix} 7.20 \\ 1.20 \\ 5.20 \end{bmatrix}$$

1 kg. raisins cost \$7.20, 1kg. sultanas cost \$1.20, 1 kg currants cost \$5.20

(1 mark)

d.

$X = Q - P$

$$X=\begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix}-\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix}$$

$$=\begin{bmatrix} -0.3 & -0.1 & 0.4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(1 mark)

e.

X says that the mixture in Home brand is changed by increasing the currants by 0.4 kg and decreasing the raisins and sultanas by 0.3 kg and 0.1 kg respectively.

(1 mark)

Module 6 Matrices

Question 3

a.

$$T = \begin{array}{c} \\ \text{Next Week} \end{array} \begin{array}{c} \begin{array}{cc} \text{This Week} & \\ \text{Buy} & \text{Not Buy} \end{array} \\ \begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Buy} \\ \text{Not Buy} \end{array}$$

(1 mark)

b.

$$S_0 = \begin{bmatrix} 500 \\ 200 \end{bmatrix}$$

(1 mark)

c.

$$S_3 = T^3 S_0$$

$$S_{30} = \begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 500 \\ 200 \end{bmatrix} = \begin{bmatrix} 322.4 \\ 377.6 \end{bmatrix}$$

Expect 322 people to buy goats cheese in three weeks time.

(1 mark)

d.

$$S_{100} = T^{100} S_0$$

$$S_{100} = \begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix}^{100} \begin{bmatrix} 500 \\ 200 \end{bmatrix} = \begin{bmatrix} 307 \\ 393 \end{bmatrix}$$

In the long term expect 393 customers to not buy goats cheese.

(1 mark)

e.

Let number of people who bought blue, cheddar, brie and gouda last week be x, $3x$, $2x$ and x, respectively.

$$\begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.9 \end{bmatrix} \begin{bmatrix} x \\ 3x \\ 2x \\ x \end{bmatrix} = 0.1x + 0.6x + 0.4x + 0.9x = 2x = 800$$

$x = 400$

Total number to buy four cheeses $= x + 3x + 2x + x = 7x$

$7x = 7 \times 400 = 2800$

(1 mark)

End of suggested solutions 2009 Further Mathematics VCE Trial Examination 2

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