2009

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: (03) 9817 5374
Fax: (03) 9817 4334
kilbaha@gmail.com
http://kilbaha.googlepages.com

Core: Data analysis

Question 1 E

At least 10 means shoe size of 10, 11, 12, or 13.

No. students with these sizes = 15 + 8 + 6 + 4

= 33

Total frequency = 1 + 1 + 3 + 2 + 5 + 5 + 15 + 8 + 6 + 4 = 50

$$\% = \frac{33}{50} \times 100 = 66\%$$

Question 2 C

On a TI-83 calculator press Stat edit

Enter shoe size in L_1 and frequency in L_2

Press stat calc 1-Var stats 2^{nd} L_1 , 2^{nd} L_2 enter

This gives $\bar{x} = 9.76$

Question 3 B

From the calculator results of question 2

Median = 10,

Lower quartile = 9, Upper quartile = 11.

Hence, the interquartile range = 11 – 9 = 2

Question 4 C

Total in year 11 = 20 + 3 + 4 + 25 = 52

Total altogether = 45 + 52 + 40 = 137

Total who drink milk = 137 – (47 + 26 + 52)

= 12

Yr. 10 who drink water = 47 – (18 + 20) = 9

Yr. 10 who drink milk = 45 – (9 +21 + 10) = 5

Yr. 12 who drink milk = 12 – (5 + 4) = 3

Question 5 C

$99 = 93 + 2 \times 3$

$99 = \bar{x} + 2s$

$2.5\% > 99$

$\therefore 100 - 2.5 = 97.5\% < 99$

Question 6 E

$$b = \frac{rs_y}{s_x}$$

s_y is the standard deviation of the dependent variable, exam mark

$s_y = 13.5$

s_x is the standard deviation of the independent variable, aptitude mark

r is correlation coefficient

$$b = \frac{0.75 \times 13.5}{11.9} = 0.85$$

Core: Data analysis

Question 7 D

$\frac{375}{250000} \times 100 = 0.15\%$

0.15% is 3 standard deviations above the mean

$52 + 3 \times 12 = 88$

Question 8 A

For the first 3 points the median x value of 1, 2, 3 is 2 and the median y value of 3, 6, 7 is 6.

So the first point is (2,6)

For the last 3 points the median x value of 6, 6, 7 is 6 and the median y value of 4, 5, 7 is 5.
So the second point is (6,5)

Question 9 E

There is a seasonal pattern here, the graph following a similar pattern in the 4 quarters of each of 3 years. There is also a positive trend upward for each year.

Question 10 C

The median of the first three points is (2, 30000) so not D or E. The median of points 2, 3 and 4 is (3, 30000) so not A. The median of points 5, 6 and 7 is (6, 40000) so not B.

Question 11 E

$r^2 = 0.85$

$r = \pm\sqrt{0.85} = \pm 0.92$

A strong correlation exists but it depends on the gradient of the least squares regression line as to whether it is positive or negative. We do not know the gradient of the least squares regression line so not A, C or D

Pearson's correlation coefficient could be −0.92 which is not greater than 0.85 so not B

The coefficient of determination gives the % variation in the dependent variable that is explained by the independent variable. Here, homework is the dependent variable and hours spent watching TV is the independent variable.

Question 12 B

Coordinates of two points on the graph are (20,10) and (100,100)

$m = \frac{100 - 10}{100 - 20} = 1.125$

This represents a one mark increse in English will result in 1.25 increase in the French mark

Question 13 D

There is no y intercept on an x, y graph because the log of zero is not defined so not A.

When $x = 3$, $y = 3\log 3 + 5 \neq 5$ so not B

When $\log x = 3$, $y = 3 \times 3 + 5 \neq 0$ so not C

When y is plotted against $\log x$ the graph is a straight line with a gradient of 3 so not E.

Module 1 Number patterns

Question 1 B

Each term in the sequence is found by multiplying the term before it by 2 and not adding anything, so not C, D or E. The first term is 3 so not A

Question 2 C

The amount each year decreases as 0.97<1, so not B or D

The amount in any year equals 97% of the amount the year before so not A

97% represents a decrease of 3% so C

Question 3 A

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{0.3}{1-0.1}$$

$$S_\infty = \frac{0.3}{0.9}$$

$$S_\infty = \frac{3}{9} = \frac{1}{3}$$

Question 4 A

Geometric sequence

$a = 4, r = -2, n = 15$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{15} = \frac{4((-2)^{15} - 1)}{-2 - 1}$$

$$S_{15} = 43692$$

Question 5 C

With calculator in sequence mode press

$y =$

$nMin = 0$

$\mu(n) = 5000(0.94)^n$

$\mu(nMin) = 5000$

2nd table

Scroll down to find

the first value less than 2000 in $\mu(n)$

This corresponds to $n = 15$

After 15 years.

Question 6 D

$$S_{n+1} = a(n+1)^2 - b(n+1)$$

$$S_{n+1} = an^2 + 2an + a - bn - b$$

$$S_{n+1} - S_n = an^2 + 2an + a - bn - b - an^2 + bn$$

$$S_{n+1} - S_n = 2an + a - b$$

Module 1 Number patterns

Question 7 B

Use sequence mode on graphics calculator and press $y =$

$n(min) = 1$

$\mu(n) = 1.15\mu(n-1) - 10$

$\mu(n \text{ min}) = 2000$

Press second table and scroll down to get ≥ 6000

This corresponds first to $n = 10$

which corresponds to the year 2014

Question 8 B

Use sequence mode on graphics calculator and press $y =$

$n(min) = 1$

$\mu(n) = \mu(n-1) + \mu(n-2)$

$\mu(n \text{ min}) = \{1,1\}$

Press second table and scroll down to get 20000

This corresponds to $n = 23,\ \mu(n) = 28657$

which corresponds to

the first term greater than 20,000

Question 9 B

$t_6 = a + 5d$

$t_3 = a + 2d$

$a + 5d = 2(a + 2d)$

$a + 5d = 2a + 4d$

$d = a$

$S_n = \frac{12}{2}(2a + 11d) = 234$

$S_n = 6(2d + 11d) = 234$

$S_n = 6 \times 13d = 234$

$78d = 234$

$d = 3$

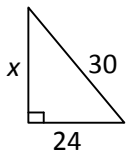
Module 2 Geometry and trigonometry

Question 1 C

In any triangle, any two sides added together must be greater than the third side. Hence, this triangle must have two sides of 10 and one side of 4

Perimeter $= 10 + 10 + 4 = 24$

Question 2 C



$x^2 + 24^2 = 30^2$

$x^2 = 900 - 576$

$x^2 = 324$

$x = \sqrt{324} = 18 \text{ cm}$

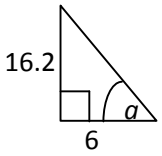
Question 3 A

$\angle A = 180 - (102 + 25) = 53^0$

$$\frac{60}{\sin 25^0} = \frac{BC}{\sin 53^0}$$

$$BC = \frac{60 \sin 53^0}{\sin 25^0}$$

Question 4 E



$$\tan a = \frac{16.2}{6}$$

$$a = \tan^{-1}\left(\frac{16.2}{6}\right) = 69.68^0$$

which is closest to 70°.

Question 5 A

1 on map represents 200000

1 m on map represents 200000 m.

x m on map represents 40000 m.

$$x = \frac{40000}{200000} = \frac{1}{5} \text{ m}$$

$$x = \frac{1}{5} \times 100 = 20 \text{ cm}$$

Question 6 C

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

Base, b ,$= \sqrt{14.6^2 - 8.2^2}$

$\Rightarrow b = 12.0797$

$$\Rightarrow \text{Area} = \frac{1}{2} \times 12.0797 \times 8.2$$

$\Rightarrow \text{Area} = 49.5$

Module 2 Geometry and trigonometry

Question 7 E

Using the Pythagorean triad (3:4:5) on the front right angled triangle, gives a slant height to the triangle of 5 m.

Area of whole front triangle = $\frac{1}{2}\times 8\times 3 = 12$

Area of two triangles = $2\times 12 = 24$

Area of base rectangle = $8\times 12 = 96$

Area of side rectangle = $5\times 12 = 60$

Area of 2 side rectangles = $2\times 60 = 120$

Total area of prism = $120+96+24 = 240$ m^2

Question 8 B

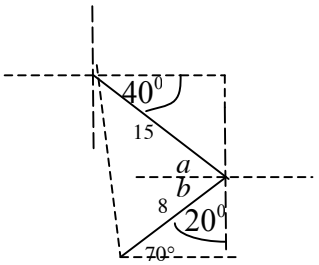
Volume of original cylinder $= \pi\left(\frac{x}{2}\right)^2 y = \frac{\pi x^2 y}{4}$

Volume of second cylinder $= \pi x^2 \frac{y}{2} = \frac{\pi x^2 y}{2}$

Volume of second cylinder

$= 2\times$ Volume of original cylinder

Question 9 E



20 mins = $\frac{1}{3}$ hr.

Distance travelled in $\frac{1}{3}$ hr $= \frac{1}{3}\times 24 = 8$ km

$a = 40°$ (alternate angle)

$b = 70°$ (alternate angle)

$a+b = 110°$

Using the cosine rule

$x^2 = 15^2 + 8^2 - 2\times 15\times 8\cos 110° = 371.08$

$\Rightarrow x = 19.3$

Module 3 Graphs and relations

Question 1 D

Between noon and 2.00pm and between 5.00pm and 9.00pm = 2 + 4 = 6 hours

Question 2 E

14 year old have the same start time as 15 year olds, not 13 year olds.

16 year olds start at 12.30pm.

16, 17 and 18 year olds have the same start time.

All start times cater for two age groups except for the 16 to 18 age group.

10, 11, 12 and 13 year olds swim before 12.00pm. This is 4 age groups so true.

Question 3 C

Total cost = 7 days at \$50 per day + x km at \$30 per km.

$C = 7 \times 50 + x \times 30$

$C = 350 + 30x$

$C = 30x + 350$

Question 4 D

$2x + 3y = 18 \quad (1)$

$5x - 2y = 26 \quad (2)$

$(1) \times 2 \rightarrow 4x + 6y = 36 \quad (1a)$

$(2) \times 3 \rightarrow 15x - 6y = 78 \quad (2a)$

$(2a) + (1a) \rightarrow 19x = 114$

$x = 6$

Substituting $x = 6$ in $(1) \rightarrow 12 + 3y = 18$

$3y = 6$

$y = 2$

Substituting $x = 6$ and $y = 2$ in $3x + 5y$

$\rightarrow 18 + 10 = 28$

Question 5 A

The left segment of the line joins two points (0,6) and (3,4)

Equation of line is $y = mx + c$

c is y intercept = 6

$$m = \frac{4-6}{3-0} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + 6 \quad 0 \le x \le 3$$

Only A has this as the equation of the first segment.

Question 6 A

The line joining the points (0,12) and (15,0)

Equation of line is $y = mx + c$

c is y intercept = 12

$$m = \frac{0-12}{15-0} = -\frac{4}{5}$$

$$y = -\frac{4}{5}x + 12$$

$5y = -4x + 60$

$4x + 5y = 60$

Region required is $4x + 5y \ge 60$

Only A satisfies this condition

Module 3 Graphs and relations

Question 7 E

Anna reaches the 200 metre line in 20 seconds. Ben reaches this line later than 20 seconds, so Anna wins. Hence, not A or C.

Anna runs 100 m in 20 seconds so her speed is 5 m/s. Hence, not B

Ben travels 8 m/s for 20 seconds i.e. 160 m

Ben is at the 160 m mark when Anna is at the 200 m mark, so Anna wins by 40 m.

Question 8 E

Equation of straight line is

$y = mx^3 + c$

$c = 0$ (y intercept)

$m = \frac{18}{3} = 6$

$y = 6x^3$

Question 9 B

R lies on QR and SR

Gradient of $QR = \frac{5-3}{1-0} = 2$

Gradient of $SR = \frac{4-5}{3-1} = -\frac{1}{2}$

$y = -ax + Z$

So gradient of objective function is $-a$

So $-a$ lies between $-\frac{1}{2}$ and 2

So a lies between -2 and $\frac{1}{2}$

Module 4 Business-related mathematics

Question 1 D
May's car is now 85% of value at beginning of year.

Let value at beginning of year = x

Value at end of year = $0.85 \times x = 12{,}360$

$x = 12360 \div 0.85 = \$14541.18$

Question 2 C
8% of 1400 = 112

Discounted price = 1400 – 112 = 1288

G.S.T. = 10% of 1288 = 128.80

Amount paid = 1288 + 128.80 = \$1416.80

Question 3 C
Amount borrowed = 3500 – 500 = 3000

Amount repaid = 160 × 24 = 3840

Interest = 3840 – 3000 = \$840

Question 4 B

$$I = \frac{PRT}{100}$$

$$R = \frac{100I}{PT} = \frac{100 \times 840}{3000 \times 2} = 14\%$$

Question 5 B
Minimum balance in January = 9360

$$\text{Interest} = \frac{5}{1200} \times 9360 = \$39$$

Minimum balance in February = 9360

$$\text{Interest} = \frac{5}{1200} \times 9360 = \$39$$

Minimum balance in March = 9510

$$\text{Interest} = \frac{5}{1200} \times 9510 = \$39.63$$

Minimum balance in January = 9360

Total Interest = $39 + 39 + 39.63 = \$117.63$

Question 6 B
Amount of depreciation each year
= 6.5% of 2500 = 0.065 ×2500 = 162.5
Total depreciation = 2500 – 550 = 1950
Number of years = 1950 ÷ 162.5 = 12 years

Question 7 D
The value of the investment is never zero during this time, so not B or E

The value will increase more rapidly with time because of the compound interest, so the slope of the graph will increase with time. Hence, D.

Module 4 Business-related mathematics

Question 8 B

Use TVM solver

$N = 25 \times 12 = 300$

$I = 5$

$PV = 360000$

$PMT =$

$FV = 0$

$P / Y = 12$

$C / Y = 12$

End

Alpha solve gives $PMT = 2104.5241$

He must repay \$2104.53 per month

Question 9 E

4% per year $= \frac{4}{12}\% = \frac{1}{3}\%$ per month

$R = 1 + \frac{1}{300}$

$n = 3 \times 12 = 36$

$P = 5000$

$\text{Balance} = 5000\left(1 + \frac{1}{300}\right)^{36}$

Module 5 Networks and decision mathematics

Question 1 B

The sum of all the vertex degrees of a network

= 2 × number of edges.

number of edges = 26 ÷ 2 = 13

Question 2 E

A complete graph is a graph with edges connecting all pairs of vertices.

A complete graph with n vertices has

$\frac{n(n-1)}{2}$ edges so not A

A triangular graph is obviously planar and has three vertices, and it is a complete graph, so not B.

A complete graph in the shape of a hexagon has each of its vertices of degree 5. Since there are 6 vertices that are odd, then no Euler circuit or path exists so not C or D

E is true because a Hamiltonian just passes through all the vertices, which are on the outside of the diagram in this case.

Question 3 A

P does not join Q or R so not E.

R does not join P so not B or D.

S joins to T, so not C.

Question 4 A

All the shapes except for A have all vertices of even degree except for two vertices which are odd. This means that all the graphs have an Euler path except for A.

Question 5 C

For a tree, the number of edges = the number of vertices −1, and the graph must be connected. Only C satisfies both these conditions.

Question 6 B

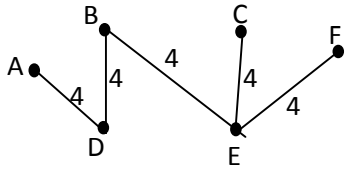
The shortest path is

A − B − E

Distance = 7 + 4 + 4 = 15 km.

Module 5 Networks and decision mathematics

Question 7 D



Weight of spanning tree = 4 + 4 + 4 + 4 + 4 = 20

Question 8 E

The critical path is the one with the longest completion time, i.e.

PQSTWX

Question 9 C

Project takes 30 hours.

Latest starting time for U = 30 – 4 – 4 = 22 hours.

Module 6 Matrices

Question 1 D

$2x-1y+0z=5$

$1x-1y+1z=4$

$1x+0y+3z=7$

This gives the matrix multiplication

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \\ 7 \end{bmatrix}$$

Question 2 E

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ x & -3 \\ 4 & 0 \end{bmatrix}=\begin{bmatrix} 0+3x+8 & -8 \\ 16 & 3 \end{bmatrix}$$

$3x+8=2$

$3x=-6$

$x=-2$

Question 3 A

$$P^2=\begin{bmatrix} 5 & -1 & 2 \\ -1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 5 & -1 & 2 \\ -1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$P^2=\begin{bmatrix} 30 & -6 & 12 \\ -6 & 30 & 12 \\ 12 & 12 & 12 \end{bmatrix}$$

$$=6\begin{bmatrix} 5 & -1 & 2 \\ -1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}=6P$$

Question 4 C

Q and T are not the same order and so cannot be added. Hence, not A.

R + T is a 1×3 matrix but you cannot multiply a 3×2 matrix by a 1×3 matrix .
Hence, not B.

P and Q are3×2 matrices but you cannot multiply a 3×2 matrix by a 3×2 matrix. Hence, not D.

R times P is a 1×2 matrix so cannot be multiplied by Q which is a 3×1 matrix. Hence, not E.

Question 5 D

$$A-kI=\begin{bmatrix} 6-k & 1 \\ 8 & 4-k \end{bmatrix}$$

$\det(A-kI)=(6-k)(4-k)-8=0$

$24-10k+k^2-8=0$

$k^2-10k+16=0$

$(k-8)(k-2)=0$

$k=8,$ or $k=2$

Question 6 E

In E, multiplying the first row of the first matrix by the column matrix gives the cost of the soft centres, and multiplying the second row of the first matrix by the column matrix gives the cost of the hard centres

Module 6 Matrices

Question 7 D

Good this check up and good next check up is 0, so not A.

Fair this check up and good next check up is 0, so not B or C.

Fail this check up and fair next check up is 5% which is 0.05, so not E.

Question 8 E

20% of fair at one inspection fail the next inspection.

$$\frac{20}{100} \times 50 = 10$$

Question 9 C

To be commutative, $XY = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$at + bv = 1 \quad (1)$

$bt + av = 0 \quad (2)$

$(1) \times a \rightarrow a^2t + abv = a \quad (3)$

$(2) \times b \rightarrow b^2t + abv = 0 \quad (4)$

$(3) - (4) \rightarrow a^2t - b^2t = a$

$t(a^2 - b^2) = a$

$$t = \frac{a}{a^2 - b^2}$$

End of suggested solutions 2009 Further Mathematics VCE Trial Examination 1

Kilbaha Multimedia Publishing
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9817 5374
Fax: (03) 9817 4334
kilbaha@gmail.com
http://kilbaha.googlepages.com

