Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 1

Multiple-choice Question Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Marks
A – Core	13	13			13
B – Modules	54	27	6	3	27
					Total 40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 30 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Please ensure that you write your **name** and your **teacher's name** in the space provided on this page and on the answer sheet for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2008 VCE Further Mathematics Units 3 & 4 Written Examination 1.

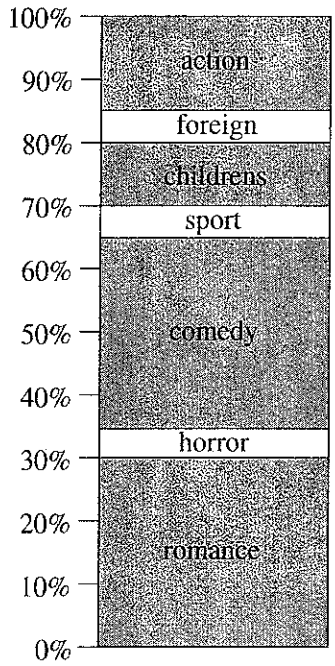
SECTION A – DATA ANALYSIS – CORE MATERIAL

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

In 'Clueless' magazine, teenagers are reported to have the following preferences when attending the movies.

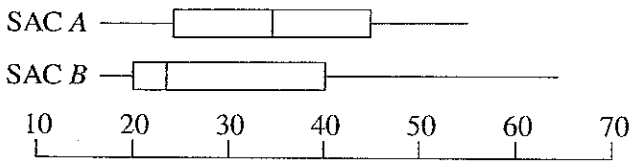


The percentage of teenagers who chose comedy is closest to

A. 30%
B. 35%
C. 40%
D. 50%
E. 60%

Question 2

The results of two different SAC tasks are shown in the box plots below.



Which of the following statements are **true**?

I The interquartile range for both sets of data is the same.

II SAC *B* has a larger range and is negatively skewed.

III The results for SAC *B* contain an outlier.

IV Less than 25% of SAC *A* scores are below the median of SAC *B*.

A. statements I and II only
B. statements I, II and III only
C. statements I and IV only
D. statements I, II, III and IV
E. statements I, II and IV only

Question 3

Shells collected during a beach holiday are found to have a mean weight of 2.3 g with a standard deviation of 0.5 g.

What percentage of shells found were between 2.8 g and 3.3 g in weight?

A. 13.5%
B. 16%
C. 34%
D. 84%
E. 95%

The following information relates to Question 4 and 5.

Wendy recorded the final number on the first 24 number plates she observed. The table below summarises the information.

Number	Frequency
2	3
3	4
4	5
5	2
6	3
7	6
8	1

Question 4

Which one of the following is **true**?

A. The mean is 7 and the range is 6.

B. The mode is 7 and the median is 1.9.

C. The standard deviation is 1.9 median is 7.

D. The interquartile range is 4 and the mode is 7.

E. Both the mean and the median are 4.5.

Question 5

When Wendy checks her original data she finds she has mistakenly recorded two 9s as two 7s.

The effect of correcting her mistake will be to increase

A. both the mean and standard deviation.

B. the range but decrease the median.

C. the mean and median but the mode remains the same.

D. the mean, median and mode.

E. the standard deviation and decrease the mean.

Question 6

At Ingrid's Ice-cream Emporium, bivariate statistics have been kept on ice-cream sales and the temperature of the day.

If we analysed this raw data, we would

A. find the relationship to be strong, positive and non-linear.
B. plot a scatterplot and nominate the temperature to be the dependent variable.
C. probably use a box plot.
D. plot a scatterplot and nominate the sales as the independent variable.
E. find the value of the correlation coefficient to be between 0.5 and 1.

Question 7

For a selected group of teenagers the correlation between height and weight is 0.7.

Which of the following is **true**?

A. 70% of the variation in weight is due to a variation in height.
B. 70% of the variation in height is due to a variation in weight.
C. The relationship is moderately strong, linear and negative.
D. 49% of the variation in height is due to a variation in weight.
E. 49% of the variation in weight is due to a variation in height.

Question 8

Two month period	January	March	May	July	September	November
Seasonal index	0.8	1.2		1.35	0.95	1.2

The missing seasonal index is

A. –0.5
B. 0.5
C. 0.85
D. 1.0
E. 1.25

The following information relates to Question 9, 10 and 11.

Name	Alexis	Belinda	Carrie	Dannah	Ejaz	Freya	Gomez
age	3	5	6	12	14	16	18
shoe size	3	8	9	12	12	14	15

Question 9

When comparing age and shoe size, the value for r is closest to

A. 0.66
B. 0.89
C. 0.90
D. 0.94
E. 3.44

Question 10

After performing a linear regression analysis, the equation of the line of best fit is

A. shoe size $= -3.5 \times$ age $+ 1.35$

B. shoe size $= 1.35 \times$ age $- 3.5$

C. shoe size $= 0.94 \times$ age $+ 0.89$

D. shoe size $= 0.66 \times$ age $+ 3.44$

E. shoe size $= 3.44 \times$ age $+ 0.66$

Question 11

After performing a $\log(x)$ transformation, the equation of the line of best fit is

A. shoe size $= 13.6 \times$ age $- 2.45$

B. shoe size $= 13.5 \times$ age $- 2.45$

C. shoe size $= 13.5 \times \log($age$) + 2.45$

D. shoe size $= 13.6 \times \log($age$) - 2.45$

E. $\log($shoe size$) = 13.5 \times$ age $- 2.45$

Question 12

A set of ten bivariate results has the statistics $s_x = 10$, $s_y = 3.8$, $r = -0.75$, $\bar{x} = 2.3$ and $\bar{y} = 4.8$.

The least squares regression equation is

A. $y = 5.46 + 0.29x$

B. $y = 5.45 + 0.28x$

C. $y = 5.46 - 0.29x$

D. $y = 0.29 - 5.46x$

E. $y = -0.29 + 5.45x$

Question 13

The statistics for the runs scored by members of two local cricket teams over the first two matches are summarised in the following back-to-back stemplot.

Club *A*	Stem	Club *B*
8 5 2	3	1 5 5
7 7 5 4 3	4	2 3 6 7 9
9 9 4 2 1 1 0	5	2 2 3 5 6 8 9 9
7 6 5 5 4 3 2 2 2	6	4 5 5 7
5 4 1 1 0 0 0	7	0 2

Key: 3/1 = 31

Which of the following statements is **completely true**?

A. Club *A*'s statistics have a higher median and are positively skewed.

B. Club *A*'s statistics have a lower median and are negatively skewed.

C. Club *B*'s statistics have a higher median and are symmetrical.

D. Club *B*'s statistics have a lower median and are positively skewed.

E. Club *A*'s statistics have a higher median and are negatively skewed.

END OF SECTION A – DATA ANALYSIS – CORE MATERIAL

SECTION B – MODULES

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Module **Page**

Module 1: Number patterns

Before answering these questions you must **shade** the "Number patterns" box on the answer sheet for multiple-choice questions.

Question 1

The numbers 16, –8, 4, –2 form

A. a geometric sequence with a positive common ratio.
B. a geometric sequence with a negative common ratio.
C. an arithmetic sequence with a positive common difference.
D. an arithmetic sequence with a negative common difference.
E. neither an arithmetic nor a geometric sequence.

Question 2

A certain arithmetic sequence has a first term of 6 and a third term of 24.

The second term could be

A. 2
B. 4
C. 9
D. 12
E. 15

Question 3

The difference equation $t_{n+1} = at_n + b$ generates the sequence 4, 6, 10, 18 ...

The value of b is

A. –6
B. –2
C. 0
D. 2
E. 3

The following information relates to Question 4 and 5.

Sales revenue at a particular show has increased by 5% per annum since 2004. In 2004, sales were $2.4 million.

Question 4

The sales revenue for 2007 was closest to

A. $2.52 million.
B. $2.76 million.
C. $2.78 million.
D. $2.92 million.
E. $3.00 million.

Question 5

The shop increases its revenue by a further $200 000 each year by opening an internet cafe at the start of 2005. Thus its revenue for 2005 is $1.05 \times 2.4 + 0.2 = \$2.72$ million. Let R_n be the revenue in millions of dollars n years after 2003, so $R_1 = 2.4$.

The rule for a difference equation that models the revenue is now

A. $R_{n+1} = 1.05R_n$

B. $R_{n+1} = R_n + 0.05n$

C. $R_{n+1} = 1.05R_n + 0.2$

D. $R_{n+1} = 1.05(R_n + 0.2)$

E. $R_{n+1} = R_n + 0.05n + 0.2$

Question 6

Simon knows that a mikilulu tree grows more slowly as it ages. Growing from a seed, it is 0.5 metres tall after one year. It grows another 0.3 metres in its second year and 0.18 metres in its third year.

The height of the fully-grown tree would be expected to approach but not exceed

A. 0.8 metres.
B. 0.833 metres.
C. 0.9 metres.
D. 1.0 metre.
E. 1.25 metres.

Question 7

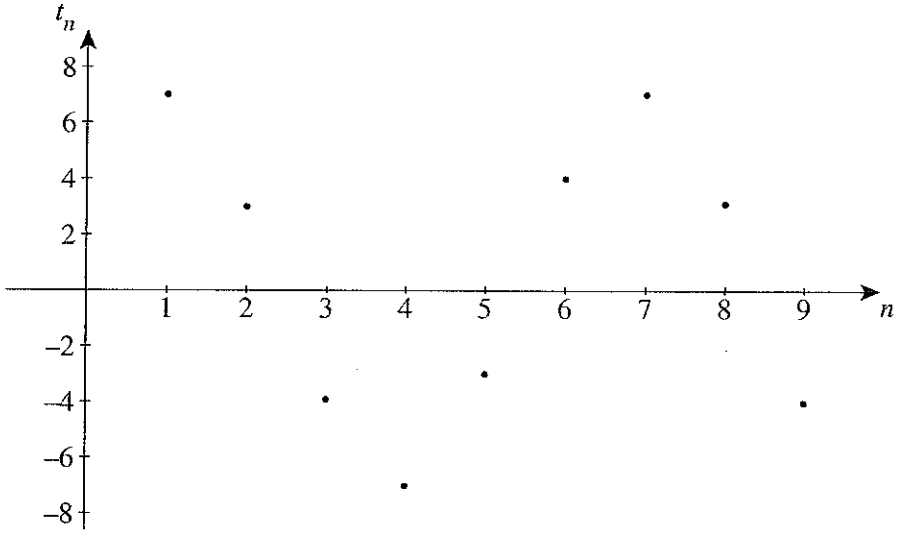
For a certain arithmetic sequence, the sum of the first four terms is 6 more than the sum of the first three.

From this we know that

A. the first term is 2.
B. the first term is 6.
C. the common distance is 6.
D. the fifth term is 6 more than the fourth.
E. the fourth term is 6.

Question 8

The first six terms of a sequence are illustrated in the graph below.



The sequence is **completely** determined by

A. $t_{n+6} = t_n;\ t_1 = 7.$

B. $t_{n+1} = 2t_n;\ t_1 = 7.$

C. $t_{n+1} = 10 - t;\ t_1 = 7.$

D. $t_{n+1} = t_n - t_{n-1};\ t_1 = 7;\ t_2 = 3.$

E. $t_{n+3} = -t_n;\ t_1 = 7;\ t_2 = 3.$

Question 9

The difference equation $t_{n+2} = 2t_{n+1} - t_n$

A. can generate a Fibonacci sequence for suitable values of t_0 and t_1.

B. generates an arithmetic sequence for $t_0 = 1$ and $t_1 = 2$ but not for other values of t_0 and t_1.

C. generates an arithmetic sequence regardless of values of t_0 and t_1.

D. generates a geometric sequence for $t_0 = 1$ and $t_1 = 2$ but not for other values of t_0 and t_1.

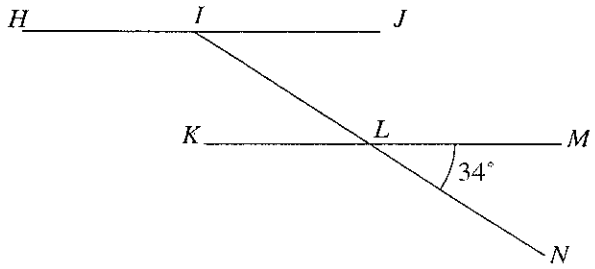
E. generates a geometric sequence for regardless of values of t_0 and t_1.

END OF MODULE 1

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the "Geometry and trigonometry" box on the answer sheet for multiple-choice questions.

The following information relates to Question 1 and 2.



Line segment *HJ* is parallel to line segment *KM*.

Question 1

The size of *NIJ* is

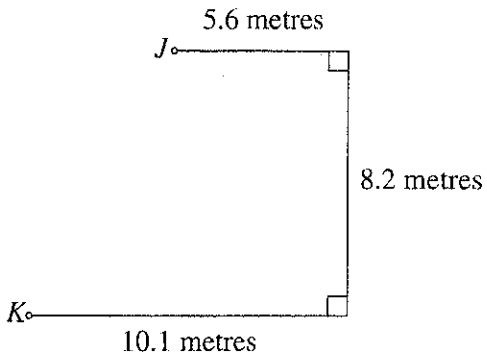
A. 34°
B. 56°
C. 134°
D. 146°
E. not known as not enough information is provided.

Question 2

The size of angle *HIN* is

A. 34°
B. 56°
C. 134°
D. 146°
E. not known as not enough information is provided.

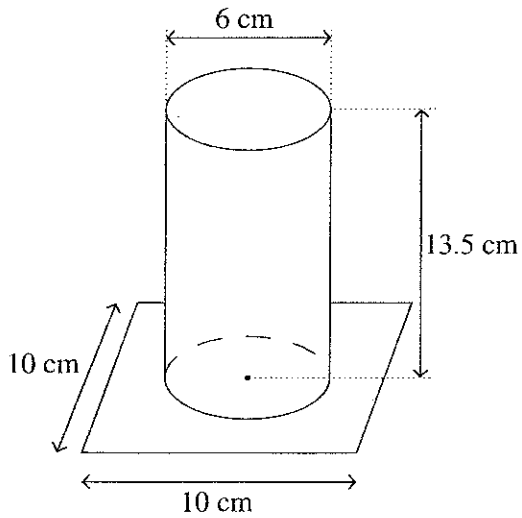
Question 3



The direct distance between point *J* and point *K*, in the diagram above, is closest to

A. 4.5 metres.
B. 8.2 metres.
C. 9.3 metres.
D. 9.4 metres.
E. 10.3 metres.

The following information refers to Question 4 and 5.



A cylinder sits in the centre of a square base such that the centre of the cylinder is directly above the centre of the base. The cylinder is 13.5 cm tall and has a diameter of 6 cm. The length of each side of the square base is 10 cm.

Question 4

The volume of the cylinder, in cubic centimetres, is closest to

A. 81
B. 100
C. 382
D. 675
E. 1060

Question 5

The shortest distance between the base of the cylinder and a corner of the base, in centimetres, is closest to

A. 0.7
B. 1.1
C. 3.8
D. 4.1
E. 7.1

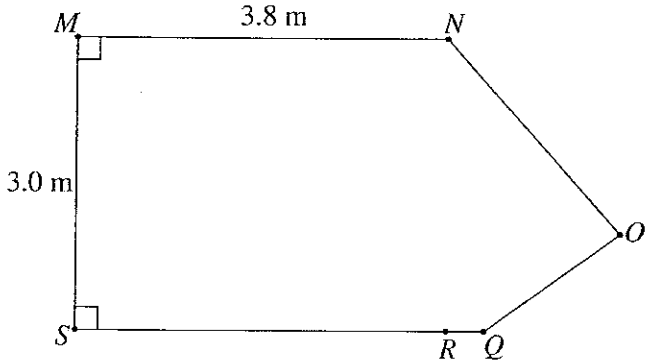
Question 6

From a ship at sea, the True bearing of a warning light on a bridge is 338° T.

The compass bearing of the ship from the warning light is

A. S 22° E.
B. S 22° W.
C. S 68° E.
D. S 68° W.
E. 158° T.

The following information relates to Question 7 and 8.



A five-sided metal frame is constructed for a butcher. The diagram above shows the cross-section of the frame. Length *MN* and length *SQ* are parallel. The length of *MS* is 3.0 m. The length of *MN* is 3.8 m. Point *R* sits directly below point *N*.

Question 7

The size of the angle *NSQ* is closest to

A. 38.3°
B. 48.3°
C. 51.7°
D. 52.1°
E. 54.1°

Question 8

The area of the trapezium bounded by *MNQS* is 11.925 square metres.

The length of *RQ*, in centimetres, is

A. 23
B. 35
C. 53
D. 55
E. 415

Question 9

An advertising board is triangular and has an area of 10 square metres. The base of the advertising board is 3.8 metres.

A larger advertising board is to be built in the same proportion as the original board. The larger board will have an area of 62.5 square metres.

The length of the base of the larger advertising board, in metres, will be

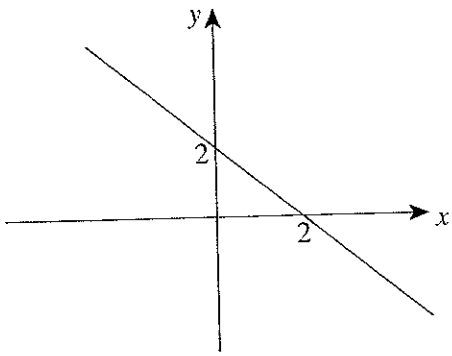
A. 2.5
B. 6.5
C. 9.5
D. 10.5
E. 12.5

END OF MODULE 2

Module 3: Graphs and relations

Before answering these questions you must **shade** the "Graphs and relations" box on the answer sheet for multiple-choice questions.

Question 1



The equation of the line shown above is

A. $y = x + 2$

B. $y = -x + 2$

C. $y = x - 2$

D. $y = 2x + 2$

E. $y = -2x + 2$

The following information relates to Question 2 and 3.

A courier company has the rule $C = 5n + 20$ to determine its charges in transporting items between two offices, where n represents the number of items to be carried.

Question 2

If one additional item is added to the set to be carried, the increase in cost will be

A. \$0

B. \$5

C. \$15

D. \$20

E. \$25

Question 3

On a particular day, one office has \$120 with which to pay for courier charges to the other office. They expect to have ten items to send during the day but they are not all ready at the same time.

The maximum number of courier journeys that may be used is

A. 1

B. 2

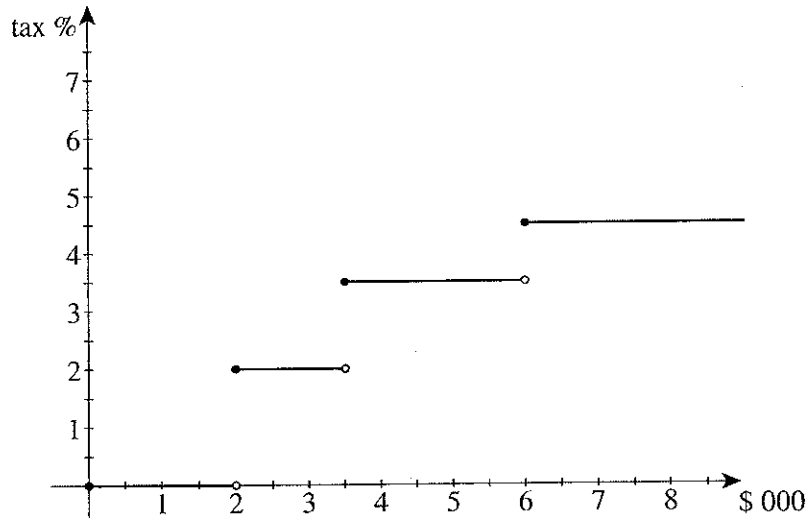
C. 3

D. 4

E. dependent on the number of items in each journey.

Question 4

The marginal tax rates in Yzchekistan are shown in the graph below.



Sam paid 3.5% tax on his earnings in Yzchekistan.

He could have earned

A. \$3.50
B. \$2000
C. \$3200
D. \$3500
E. \$6000

Question 5

Alvin is a bookshop owner. He decides to stock at least 40% more fiction books than non-fiction books this year. Let x represent the number of fiction titles and y the number of non-fiction titles.

An appropriate inequality would be

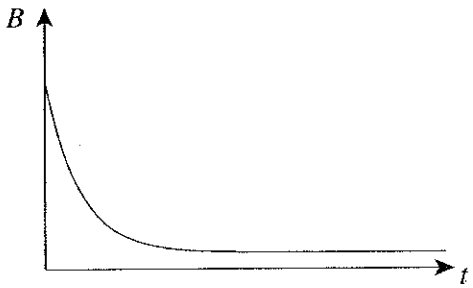
A. $x \geq 40y$
B. $x \geq 0.4y$
C. $y \geq 0.4x$
D. $y \geq 1.4x$
E. $x \geq 1.4y$

Question 6

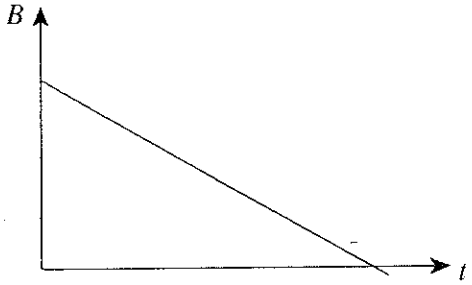
The concentration of a certain type of bacteria (B) after commencement of an antibiotic treatment is given by $B = \frac{10}{t+1}$, $t \geq 0$, where t is the number of hours since the treatment.

Which of the following graphs could represent this rule?

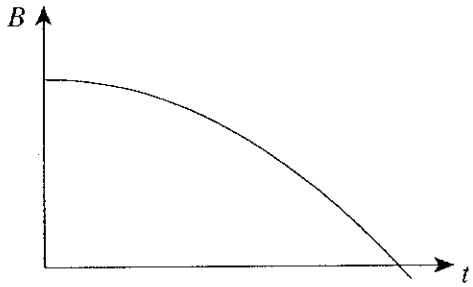
A.



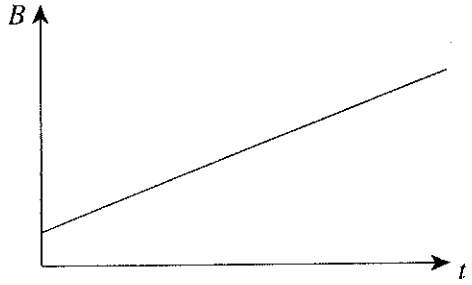
B.



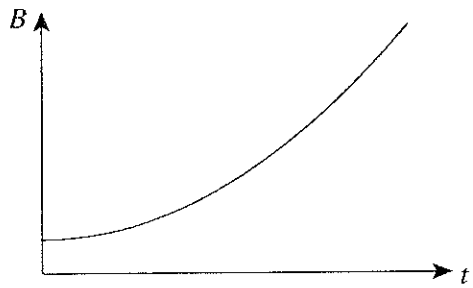
C.



D.



E.



Question 7

Which of the statements about the line $3x - 2y = 5$ is **not** true?

A. It has the same gradient as $6x - 4y = 10$.

B. It intersects with the line $3x + 2y = 5$.

C. It has the same slope as $3x - 2y = 7$.

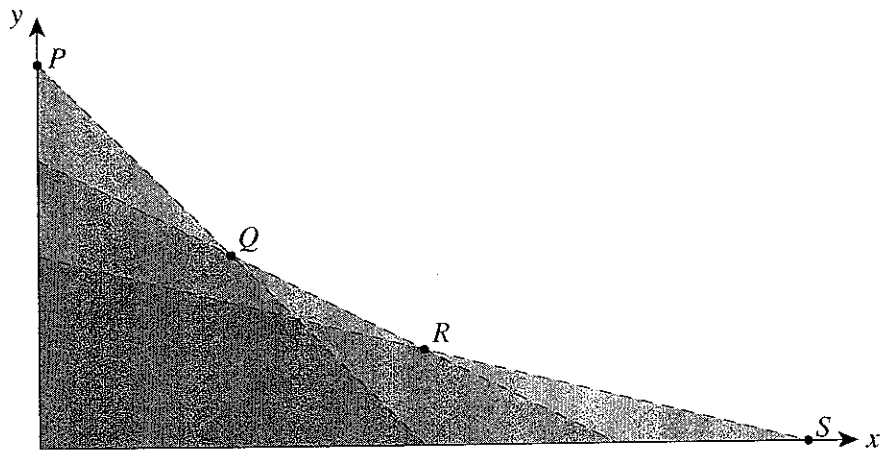
D. It does not pass through the origin.

E. It has a y-intercept at 5.

Question 8

The feasible region in the graph below is that unshaded. The points marked have the following coordinates.

$P(0,12)$ $Q(2,7)$ $R(4,4)$ $S(8,0)$

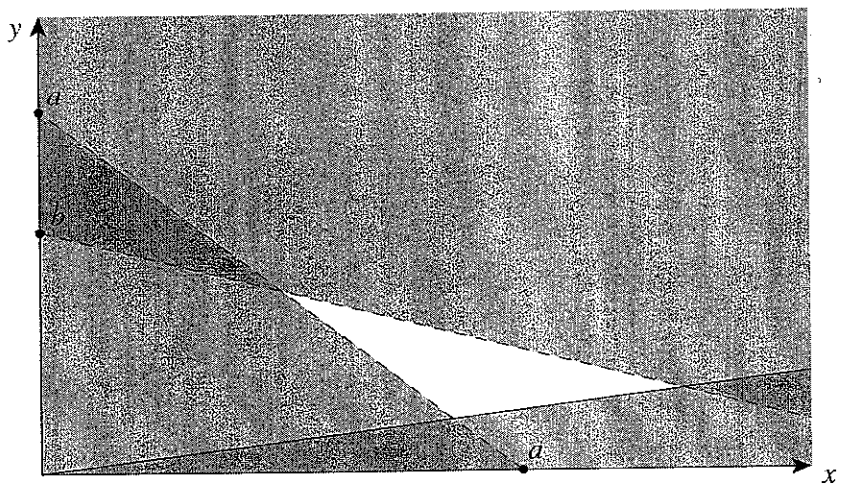


The minimum value of the objective function $C = x + y$ is at

A. P only.
B. Q only.
C. R only.
D. S only.
E. multiple points.

Question 9

The graph below shows a feasible region (unshaded) governed by a series of constraints.



One of the constraints could be

A. $y < a - x$

B. $y < b - x$

C. $y > a - x$

D. $y > b - x$

E. $y > \frac{x}{2}$

END OF MODULE 3

Module 4: Business-related mathematics

Before answering these questions you must **shade** the "Business-related mathematics" box on the answer sheet for multiple-choice questions.

Question 1

George buys Wendy an expensive painting for $83 000. The painting increases in value at an average of 7% per annum over the next ten years.

What is the value of the painting (to the nearest dollar) after six years?

A. $40 170
B. $53 700
C. $88 810
D. $124 561
E. $163 274

Question 2

To buy a new $500 camera under a hire-purchase agreement, Susan agrees to pay a 10% deposit and then pay $23 a month for two years.

The effective interest rate (to one decimal place) is

A. 5.2%
B. 10.4%
C. 11.3%
D. 21.8%
E. 22.6%

Question 3

Cameron buys his first car for $7600 and over the first five years the car depreciates at a reducing balance rate of 15% per annum.

How much is the car worth (to the nearest dollar) at the end of the fourth year?

A. $1140
B. $2866
C. $3372
D. $3967
E. $4000

Question 4

Kate invests $3200 for 300 days at 8.3% per annum simple interest.

The interest earned is

A. $218.30
B. $225.89
C. $265.60
D. $3418.30
E. $3425.89

Question 5

After agreeing on a 15% discount on a new dress for his wife, George pays a price of $68.

How much was the original price?

A. $10.20
B. $12
C. $57.80
D. $78.20
E. $80

Question 6

An investment of $5550 amounted to $7086. The interest rate was 7.2% compounding monthly.

How long (to the nearest month) was the money invested?

A. 4
B. 13
C. 14
D. 40
E. 41

Question 7

Andrew can afford to make monthly repayments of $1900 on a home loan over 20 years at an interest rate of 6.75% per annum compounded monthly.

How much can Andrew afford to borrow (to the nearest dollar)?

A. $20 823
B. $28 148
C. $110 564
D. $249 880
E. $250 000

Question 8

Erin is trying to save $7500 to purchase her first car. She invests an initial $2000 and then adds $400 a month to her investment.

After how many months will she reach her target if the interest rate is 4.8% per annum compounding monthly?

A. 12
B. 13
C. 14
D. 15
E. 16

Question 9

A loan of $240 000 is to be repaid over 15 years.

If the interest is 7.3% calculated and added monthly, which of the following equations should be solved to calculate the monthly repayments?

A. $$Q = \frac{240\,000\left(\frac{7.3}{100}\right)}{\left(\left(1+\frac{7.3}{100}\right)^{15}-1\right)}$$

B. $$Q = \frac{240\,000\left(1+\frac{7.3}{100}\right)^{180}}{\left(\left(1+\frac{7.3}{100}\right)^{180}-1\right)} \times \left(\frac{7.3}{100}\right)$$

C. $$Q = \frac{240\,000\left(1+\frac{7.3/12}{100}\right)^{180}}{\left(\left(1+\frac{7.3/12}{100}\right)^{180}-1\right)} \times \left(\frac{7.3/12}{100}\right)$$

D. $$Q = \frac{240\,000\left(1+\frac{7.3/12}{100}\right)^{15}}{\left(\left(1+\frac{7.3/12}{100}\right)^{15}-1\right)} \times \left(\frac{7.3/12}{100}\right)$$

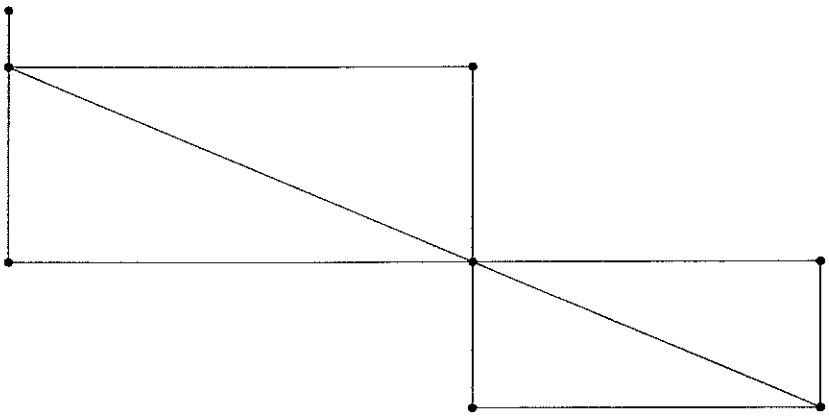
E. $$Q = \frac{240\,000\left(1+\frac{7.3/12}{100}\right)^{15}}{\left(\left(1+\frac{7.3/12}{100}\right)^{15}-1\right)}$$

END OF MODULE 4

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the "Networks and decision mathematics" box on the answer sheet for multiple-choice questions.

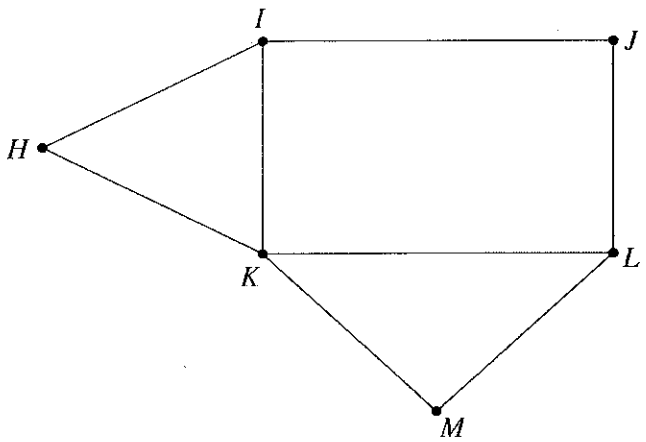
Question 1



The sum of the degrees of all the vertices shown in the network above is

A. 4
B. 6
C. 11
D. 22
E. 24

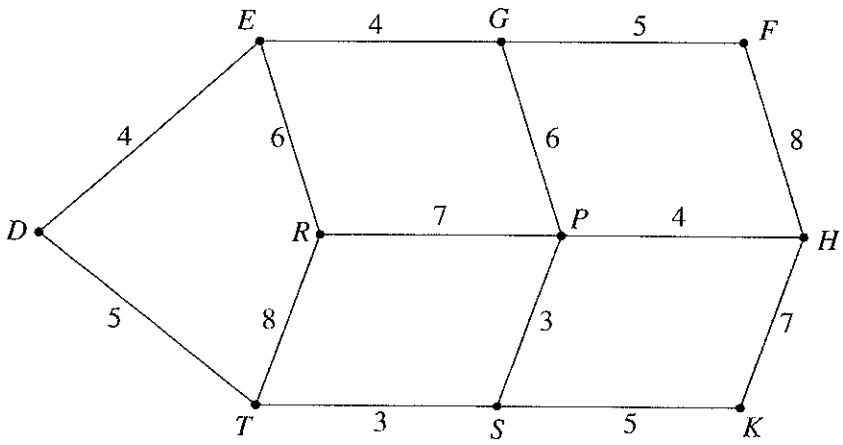
Question 2



For the network shown above, an Euler circuit can be created by adding an edge that joins

A. *H* to *I*.
B. *I* to *J*.
C. *I* to *L*.
D. *L* to *M*.
E. *L* to *K*.

The following information relates to Question 3 and 4.



Question 3

The minimal spanning tree for the network shown above would contain

A. seven edges.
B. eight edges.
C. nine edges.
D. thirteen edges.
E. fourteen edges.

Question 4

Which one of the following statements is **true** regarding the network above?

A. There are five faces.
B. The sum of the vertices is ten.
C. There are exactly two vertices with even degree.
D. *DEGFHKSTD* is a Hamilton circuit.
E. An Euler path is not possible.

Question 5

Newspaper deliveries are made to seven country towns. The delivery driver begins at one town and visits each other town exactly once then returns to the original town.

To minimise the total length of the journey, the driver must use the

A. minimal spanning tree.
B. shortest Euler circuit.
C. shortest Euler path.
D. shortest Hamilton circuit
E. critical path.

Question 6

Four employees are used by a company to complete four different tasks (*L*, *M*, *N*, *O*). No employee is able to complete more than one task. The table below shows the time, in minutes, that it takes for each employee to complete each task.

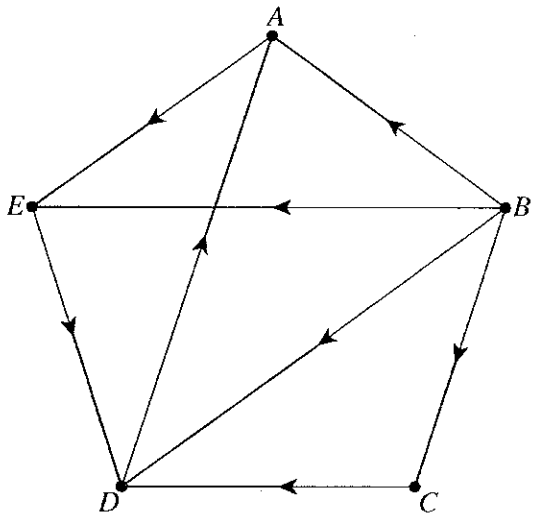
	Tasks			
	L	*M*	*N*	*O*
Doris	30	35	40	70
Ritsa	15	35	20	80
Annmarie	30	50	30	45
Ian	60	55	45	40

The company aims to allocate each employee to one task so that the completion time of all tasks is minimised.

To achieve this, Doris must be allocated

A. task *L*.
B. task *M*.
C. task *N*.
D. task *O*.
E. cannot be determined as not enough information is provided.

Question 7



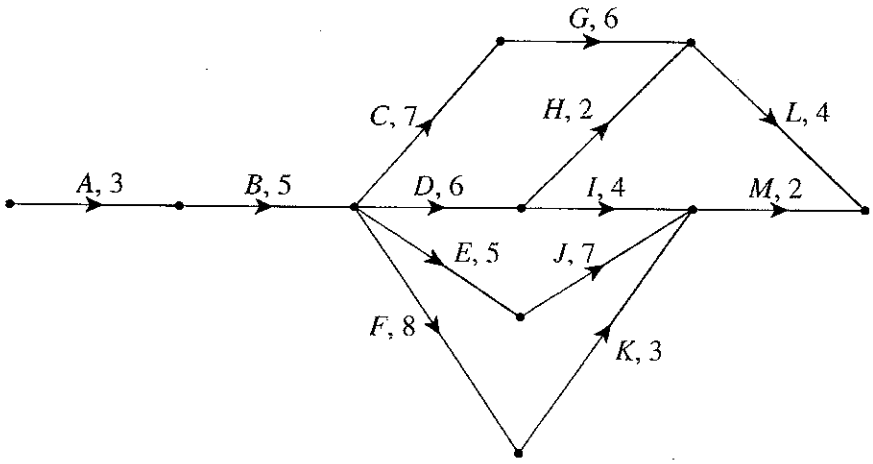
Five teams are part way through a tennis tournament. The result of each game played so far is represented in the dominance graph shown above.

Which of the following statements is **true**?

A. Team *E* defeated a team who defeated team *C*.
B. Team *C* had more two-step dominances than team *E*.
C. Team *B* had the highest number of three-step dominances.
D. Team *A* had exactly one three-step dominance.
E. After one more game, each team will have played every other team.

The following information relates to Question 8 and 9.

The activities and their completion times (hours) are shown in the diagram below.



Question 8

The length of the critical path, in hours, is

A. 19
B. 21
C. 23
D. 25
E. 27

Question 9

The company has decided to pay extra to allow activity C to be completed in a reduced amount of time. The decision allows activity C to be reduced by five hours.

The reduction in the overall project completion time associated with reducing the duration of activity C by five hours is

A. 1 hour.
B. 2 hours.
C. 3 hours.
D. 4 hours.
E. 5 hours.

END OF MODULE 5

Module 6: Matrices

Before answering these questions you must **shade** the "Matrices" box on the answer sheet for multiple-choice questions.

Question 1

The matrix addition $\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 2 & 3 & -2 \end{bmatrix}$ is

A. defined and has a result $\begin{bmatrix} 3 & -1 & -1 \\ 5 & 5 & -2 \end{bmatrix}$.

B. defined and has a result $\begin{bmatrix} 3 & -1 \\ 5 & 5 \end{bmatrix}$.

C. defined and has a result $\begin{bmatrix} -2 & -5 & 3 \\ 10 & 9 & -7 \end{bmatrix}$.

D. undefined since the column number of the left matrix does not match the row number of the right matrix.

E. undefined as the matrices have different order.

Question 2

For the matrix $A = \begin{bmatrix} 2 & 1 & 3 & -1 \\ 4 & 2 & 3 & -2 \\ 3 & 2 & 3 & 4 \end{bmatrix}$, it is **not** true that

A. element $A_{2,4}$ has a unique value.

B. $A_{2,2} + A_{3,2} = 4$.

C. $A_{2,2}$ is a 2×2 matrix.

D. $A_{4,3}$ is undefined.

E. $A_{2,2} - A_{3,2} = 0$.

Question 3

For the matrix equation $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix} X = \begin{bmatrix} 1 & 5 \\ 1 & 11 \\ -2 & 3 \end{bmatrix}$, the matrix X is

A. undefined since the orders of the matrices involved do not match.

B. $\begin{bmatrix} 5 & 18 \\ 3 & 40 \\ -6 & 15 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} -1 & -2 \\ 1 & -2 \\ 0 & -1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$

Question 4

The system of simultaneous equations

$$2x + 3y = 7$$

$$x + y + z = 5$$

$$3x + 2y - z = 6$$

can be solved by calculation of the product

A. $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 7 & 5 & 6 \end{bmatrix}$

Question 5

Matrix P gives the selling price per kilogram of three different types of fish in the form

$$\begin{bmatrix} bream \\ flounder \\ whiting \end{bmatrix}.$$

Matrix S contains the number of each of these three types of fish caught at locations V, W and X.

What would need to be calculated to obtain a matrix with the total value of the fish caught at the three locations in the form $\begin{bmatrix} V \\ W \\ X \end{bmatrix}$?

A. SP, where each row of S represents a different location and each column a different fish type.

B. SP, where each column of S represents a different location and each row a different fish type.

C. PS, where each row of S represents a different location and each column a different fish type.

D. PS, where each column of S represents a different location and each row a different fish type.

E. $S^{-1}P$, where each row of S represents a different location and each column a different fish type.

Question 6

Camilla determines that the time she spends studying each of Accounting, Environmental Science and Hungarian each week should depend on that of the preceding week according to the transition matrix

$$\begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.3 & 0.1 & 0.5 \\ 0.5 & 0.4 & 0.2 \end{bmatrix}.$$

The state matrix S_n should be in the form

$$\begin{bmatrix} Accounting \\ Environmental\ Science \\ Hungarian \end{bmatrix}.$$

In the first week, Camilla spends 20 hours on each subject.

If this process were to continue indefinitely, Camilla would spend

A. more time studying all three subjects.

B. more time on Hungarian but less on the other two subjects.

C. more than 60 hours combined on all three subjects but with equal time for each subject.

D. the same time on each of the three subjects as before.

E. less time on all three subjects.

Question 7

Arthur is aware of four brands of ice cream that members of his family enjoy and decides to vary the quantities of each purchased each week to reflect their different preferences. The transition matrix describing how each week's ice cream purchases depend on those of the preceding week is

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}.$$

The week one purchase (in kilograms) of brands A, B, C and D (in that order) is given by the initial state matrix

$$S_0 = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.8 \\ 0.7 \end{bmatrix}.$$

A correct statement relating to the transition matrix would be that

A. the quantity of brand B for any week is equal to that of brand A in the preceding week.

B. the total amount of ice cream purchased varies from week to week.

C. the quantity purchased of brand B is half that of brand A from the preceding week.

D. the quantity of brand A purchased each week is equal to that of brand B in the preceding week.

E. the quantities of brands B and C each week are equal.

Question 8

Princes Valley Council have analysed their park works projects. Of the parks requiring work in any year, 27% require further work the next year. Of the parks not requiring work in any year, 63% do require work the next year.

A possible transition matrix to describe this situation would be.

A. $\begin{bmatrix} 0.27 \\ 0.63 \end{bmatrix}$

B. $\begin{bmatrix} 0.27 \\ 0.37 \end{bmatrix}$

C. $\begin{bmatrix} 0.27 & 0.73 \\ 0.63 & 0.37 \end{bmatrix}$

D. $\begin{bmatrix} 0.27 & 0.63 \\ 0.73 & 0.37 \end{bmatrix}$

E. $\begin{bmatrix} 0.27 & 0.37 \\ 0.73 & 0.63 \end{bmatrix}$

Question 9

Matrix Y is a transition matrix. X is a 3×4 matrix.

If it is true that $P = X(YZ)$, then it could also be true that

A. Y is 4×3.

B. P and Z are both 4×2.

C. P is 3×2 and Z is 4×2.

D. P, Y and Z are the same order.

E. P and Z are the same order, but Y is different order.

END OF MULTIPLE-CHOICE QUESTION BOOKLET

Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

FURTHER MATHEMATICS FORMULAS

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \; r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, \; |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

Neap:

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 1

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like **this** example: | A | B (shaded) | C | D | E |

Show each **Module** chosen by marking the appropriate box.

Use pencil only

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E

Module:

- ☐ Number patterns
- ☐ Geometry and trigonometry
- ☐ Graphs and relations
- ☐ Business-related mathematics
- ☐ Networks and decision mathematics
- ☐ Matrices

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E

Module:

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ONE ANSWER PER LINE

1	A	B	C	D	E
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Module:

- ☐ Number patterns
- ☐ Geometry and trigonometry
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- ☐ Matrices

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
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7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E

Neap:

Trial Examination 2008

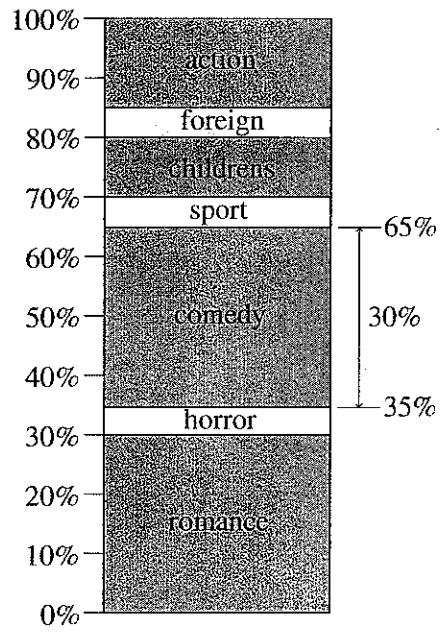
VCE Further Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

SECTION A – DATA ANALYSIS – CORE MATERIAL

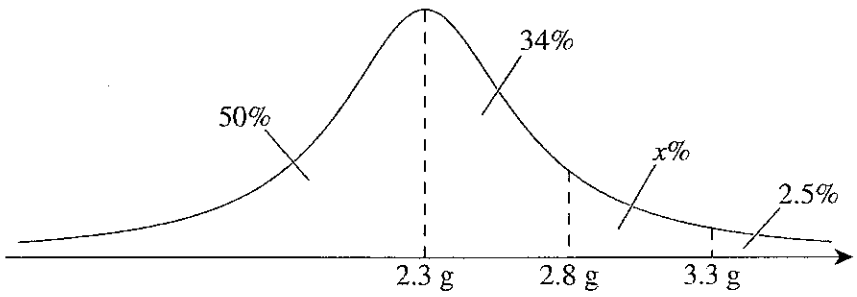
Question 1 **A**



Question 2 **C**

I True, the IQR is 20 for SAC *A* and SAC *B*.
II False. The data is positively skewed.
III False. $Q_3 + 1.5 \times \text{IQR} = 40 + 1.5 \times 20 = 70$
IV True.

Question 3 **A**



$x = 100 - (50 + 34 + 2.5)$
$= 100 - 86.5$
$= 13.5$

Question 4 **D**

Using a graphics calculator, press stat, edit, enter the numbers into the L1 column and the frequency data into the L2 column.

Using 1: 1-var stats L1, L2, find:

$\bar{x} = 4.8$

$s_x = 1.9$

$\text{IQR} = Q_3 - Q_1 = 7 - 3 = 4$

$\text{median} = 4.5$

$\text{mode} = 7$

$\text{range} = 8 - 2 = 6$

The IQR is 4 and the mode is 7.

Question 5 **A**

In correcting her mistake, the median will remain the same, the mode will become 4 and both the mean and spread will increase.

Question 6 **E**

Answer **A** is incorrect because the relationship is linear. Answer **B** is incorrect because the temperature is independent. A box plot is not appropriate for bivariate data (answer **C**). Answer **D** is incorrect because sales are dependent.

Question 7 **E**

$r = 0.7$

$r^2 = 0.49$

Height is the independent value.

The relationship is positive.

Question 8 **B**

All indices must add to 6.

$6 - (0.8 + 1.2 + 1.35 + 0.95 + 1.2) = 0.5$

Question 9 **D**

Using a graphics calculator, enter the ages into the L1 column and shoe sizes into the L2 column.

Use $\text{stat} \rightarrow \text{calc} \rightarrow 4\text{LinReg}$ to find $r = 0.944$.

Question 10 **D**

$y = ax + b$

$a = 0.66$

$b = 3.44$

Age is the independent variable,

$\therefore$ shoe size $= 0.66 \times$ age $+ 3.44$

Question 11 **D**

$y = ax + b$

$a = 13.58 = 13.6$

$b = -2.45$

Using a graphics calculator, L3 becomes log(L1). Redo stat → calc → 4LinReg using L3, L2.

Question 12 **C**

$$b = r \times \frac{s_y}{s_x}$$

$$= -0.75 \times \frac{3.8}{10}$$

$$= -0.285$$

$$a = \bar{y} - b\bar{x}$$

$$= 4.8 - (-0.285) \times 2.3$$

$$= 5.455$$

$$\therefore y = -0.29 + 5.46x$$

Question 13 **E**

The figures in club *A* are negatively skewed and have a higher median than club *B* (62 compared to 54). Therefore **A**, **B** and **C** are incorrect. Club *B*'s statistics are symmetrical, so **D** is incorrect.

SECTION B – MODULES

Module 1: Number patterns

Question 1 **B**

Subtracting successive terms must give a constant result for the sequence to be arithmetic. This is not the case for the sequence given. However, dividing successive terms does give a constant result of –0.5. Thus the sequence is geometric. The common ratio (–0.5) is negative.

Question 2 **E**

Two common differences $(2d) = 24 - 6 = 18$. So each common difference is 9. Therefore, the middle term, t_2, is 15.

Algebraically,

$a + 2d = 24$, where $a = 6$

$2d = 14 - a$

$2d = 18$

$d = 9$

$t_2 = a + d$

$= 6 + 9 = 15$

Question 3 **B**

This can be solved involving an element of trial and error. a is most likely to be either 1 or 2, since ratios of successive terms are all between 1 and 2. Using $a = 2$ finds a consistent value of $b = -2$. Using $a = 1$ does not give a consistent value of b.

Algebraically,

$t_2 = a.t_1 + b$

$6 = 4a + b$ ...1

$t_3 = a.t_2 + b$

$10 = 6a + b$ ...2

$(2) - (1)$:

$\therefore\ 4 = 2a$

$a = 2$

$6 = 4(2) + b$

$\therefore\ 6 = 8 + b$

$b = -2$

Question 4 **C**

Each year's revenue is a set constant proportion of that for the preceding year, so this is a geometric sequence. Since the rate of increase is 5% p.a., each year's revenue will be 105% of that preceding year. Thus $r = 1.05$. $a = 2.4$ since it is the first term. 2007 is t_4 in the geometric sequence.

$$t_4 = ar^3$$
$$= 2.4(1.05)^3$$
$$= 2.7783$$

Question 5 **C**

Prior to the \$0.2 million extra revenue being added, the difference equation $R_{n+1} = 1.05R_n$ would apply. The new difference equation is $R_{n+1} = 1.05R_n + 0.2$.

Question 6 **E**

This is a geometric sequence with a common ratio of 0.6. The sum to infinity needs to be found.

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{0.5}{1-0.6}$$
$$= 1.25$$

Question 7 **E**

The sum of the first four terms differs from the sum of the first three by t_4. Therefore t_4 is 6.

Algebraically,

$S_4 = S_3 + 6$

$S_3 + t_4 = S_3 + 6$

$t_4 = 6$.

Question 8 **D**

A and **E** are quite correct but lack sufficient initial values. In **A**, t_1 to t_6 would need to be provided for this sequence to be usable. Of these, only t_1 is given. **E** lacks a value for t_3. **B** and **C** are fully specified and match the graphed sequence t_1, but they both fail from t_2 onwards. **D** matches the sequence graphed and it is fully specified.

Question 9 **C**

$t_{n+2} = 2t_{n+1} - t_n$

If arithmetic, $t_n = a$

$$t_{n+1} = a + d$$
$$t_{n+2} = a + 2d$$
$$t_{n+2} = 2(a+d) - a$$
$$= a + 2d$$

The equation is always arithmetic.

Module 2: Geometry and trigonometry

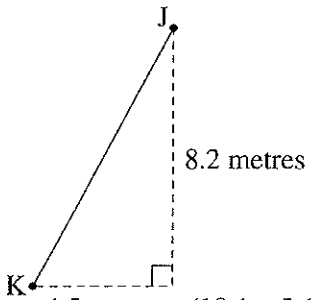
Question 1 **A**

Angle *NIJ* and angle *NLM* are corresponding angles. Corresponding angles are equal, so the angle *NIJ* is 34°.

Question 2 **D**

Angle *NIJ* and angle *HIN* are supplementary angles. Supplementary angles add to 180°, so angle *NIH* is 146°.

Question 3 **D**



4.5 metres $(10.1 - 5.6 = 4.5)$

$h^2 = a^2 + b^2$

$h^2 = 8.2^2 + 4.5^2$

$h^2 = 87.39$

$h = 9.3536...$

distance $JK \cong 9.4$ metres

Question 4 **C**

volume = end area × height

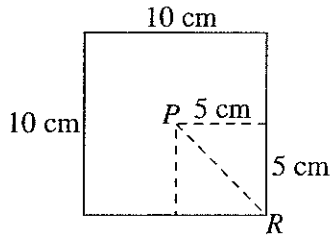
$= (\pi r^2) \times$ height

$= (\pi \times 3^2) \times 13.5$

$= 381.7035$

$\cong 382 \text{ cm}^2$

Question 5 **D**



For distance PR,

$h^2 = a^2 + b^2$

$h^2 = 5^2 + 5^2$

$h^2 = 50$

$h = 7.071067...$

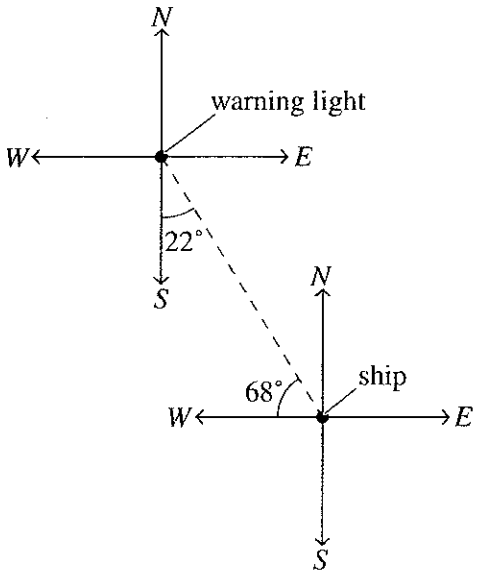
For distance from cylinder to corner,

distance $= 7.071067 - 3$

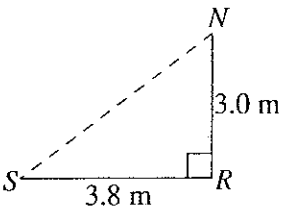
$= 4.071067...$

$\cong 4.1$ cm

Question 6 **A**



Question 7 **A**



$\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan(\theta) = \dfrac{3.0}{3.8}$

$\theta = 38.3°$

Question 8 **B**

$$\text{area of trapezium} = \frac{\text{top} + \text{bottom}}{2} \times \text{height}$$

$$= \frac{MN + SQ}{2} \times MS$$

$$11.925 = \frac{3.8 + SQ}{2} \times 3.0$$

$$\frac{11.925 \times 2}{3.0} = 3.8 + SQ$$

$$SQ = \frac{11.925 \times 2}{3.0} - 3.8$$

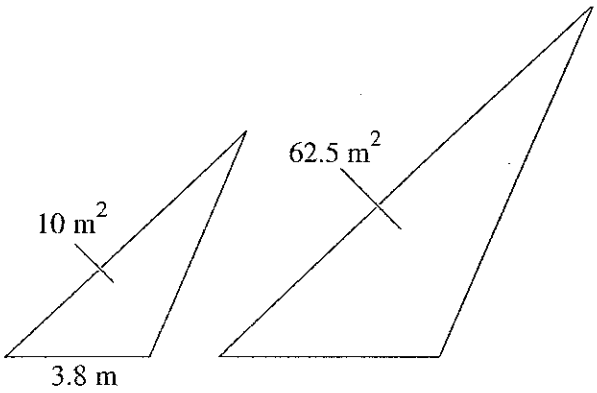
$$SQ = 4.15 \text{ m}$$

$$RQ = 4.15 - 3.8$$

$$= 0.35 \text{ m}$$

$$= 35 \text{ cm}$$

Question 9 **C**



$$\frac{\text{area of large}}{\text{area of small}} = k^2$$

$$\frac{62.5}{10} = k^2$$

$$6.25 = k^2$$

$$k = 2.5$$

$$\text{base length of large} = k \times \text{base length of small}$$

$$= 2.5 \times 3.8$$

$$= 9.5 \text{ m}$$

Module 3: Graphs and relations

Question 1 **B**

The gradient, $m = \frac{0-2}{2-0} = -1$. The y-intercept is 2, so the equation is $y = -x + 2$.

Question 2 **B**

The rule $C = 5n + 20$ specifies that a cost of \$20 exists merely to conduct the journey regardless of the number of items transported. An additional charge of \$5 for each item must be added to this.

Question 3 **C**

It does not matter how the items are distributed among the courier's journeys, so **E** is incorrect. So any distribution will achieve a valid result.
One journey with ten items would cost \$70.
Two journeys with five items each would cost \$45 + \$45 = \$90.
Three with 3, 3 and 4 items would cost \$35 + \$35 + \$40 = \$110.
Four journeys with 2, 3, 2 and 3 items would cost \$30 + \$35 + \$30 + \$35 = \$130.
Thus three journeys is the maximum possible.

Question 4 **D**

The only two incomes listed that come close to being valid are \$3500 and \$6000 (**D** and **E**). The graph shows that \$3500 *is* included in the values with 3.5% tax while \$6000 is *not included*.

Question 5 **E**

The statement being transformed into an inequation is that 'fiction books are at least 140% of non-fiction', so $x \geq 1.4y$.

Question 6 **A**

The function is a hyperbola. **A** is the only graph of this type shown. **B** and **D** are linear. **C** and **E** are parabolic (x^2).

Question 7 **E**

$6x - 4y = 10$ is the same as the line $3x - 2y = 5$ (so **A** is not the required answer). The gradient of the $3x + 2y = 5$ is different to $3x - 2y = 5$, so the two lines must intersect at some point (so **B** is not the required answer). The gradient of $3x - 2y = 7$ is the same as $3x - 2y = 5$ (so **C** is not the required response). In considering **D** and **E**, the y-intercept can be found for the equation given.

$3x - 2y = 5$

$-2y = 5 - x$

$y = -2.5 + 1.5x$

The y-intercept is –2.5 (so **D** is not the required response). **E** is an incorrect statement since the y-intercept is not 5.

Question 8 **E**

Calculate the objective function C for each of the four points.

P: $C = 0 + 12 = 12$

Q: $C = 2 + 7 = 9$

R: $C = 4 + 4 = 8$

S: $C = 8 + 0 = 8$

There are multiple points.

Question 9 **C**

Three constraints are shown. The line with x-intercept and y-intercept a is $y = a - x$. The region required is above this, so $y > a - x$ (answer **C**) is correct. **A** is incorrect as it is below the line discussed. While we don't know the x-intercept of the line that has y-intercept b, we do know that it exceeds a. Thus the gradient of this line is closer to zero. It does not have a gradient of -1 (not **B** or **D**). **E** would be valid except for the fact that the corresponding line on the graph is solid which would require a '$\geq$' symbol rather than a '$>$' symbol.

Module 4: Business-related mathematics

Question 1 **D**

$83\,000 \times 1.07^6 = 124\,560.62$

$= \$124\,561$

Question 2 **D**

deposit = 10% of \$500 = \$50

$23 \times 24 = \$552$

$\therefore\ 552 - 450 = \$102$ interest

$$r = \frac{102}{450} \times 100$$

$$= \frac{22.66\%}{2}$$

$$= 11.3\%$$

$$r_e = 11.3 \times \frac{2 \times 24}{(24+1)}$$

$$= 21.8\%$$

Question 3 **D**

$7600 \times 0.85^4 = 3967.25$

Note that the question asks for the value at the end of the fourth year.

Question 4 **A**

$P = 3200$

$r = 8.3$

$$t = \frac{300}{365}$$

$$I = \frac{3200 \times 8.3 \times \frac{300}{365}}{100}$$

$$= 218.30$$

Question 5 **E**

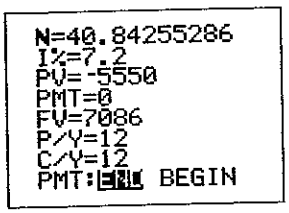
\$68 is 0.85 of price, so

$$\text{price} = \frac{68}{0.85}$$

$= \$80$

Question 6 **D**

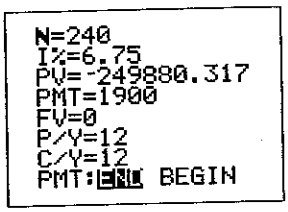
Using a graphics calculator, use the TVM Solver:



40.8 months = 41 months

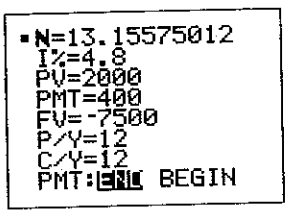
Question 7 **D**

Using a graphics calculator, use the TVM Solver:



Question 8 **C**

Using a graphics calculator, use the TVM Solver:



N = 13.2

$\therefore$ since she needs at least \$7500, must be 14 months.

Question 9 **C**

$P = 240\ 000$

$n = 15 \text{ years} = 15 \times 12 = 180$

$$R = \left(1 + \frac{\left(\frac{7.3}{12}\right)}{100}\right)$$

$I = 7.3\%$ compounding monthly

repayments made monthly

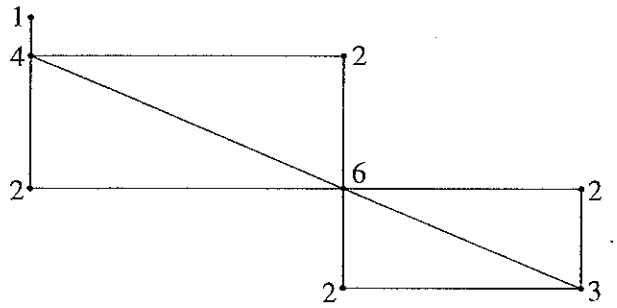
$$Q = \frac{PR^n(R-1)}{R^n - 1}$$

$$= \frac{240\ 000\left(1 + \frac{(7.3/12)}{100}\right)^{180} \times \left(1 + \frac{(7.3/12)}{100} - 1\right)}{\left(1 + \frac{(7.3/12)}{100}\right)^{180} - 1}$$

$$= \frac{240\ 000\left(1 + \frac{(7.3/12)}{100}\right)^{180} \times \left(\frac{7.3/12}{100}\right)}{\left(1 + \frac{(7.3/12)}{100}\right)^{180} - 1}$$

Module 5: Networks and decision mathematics

Question 1 **D**

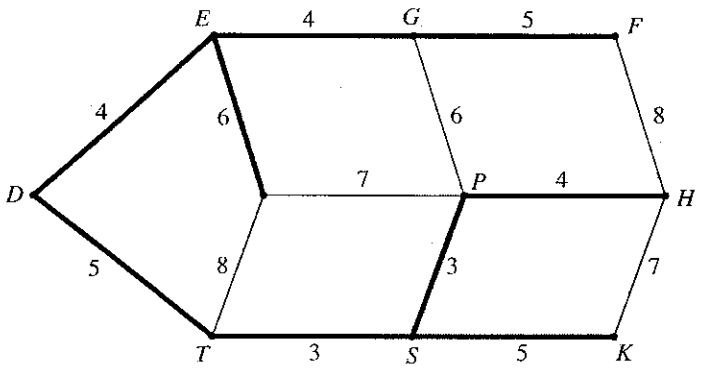


$(1 + 4 + 2) + (2 + 6 + 2) + (2 + 3) = 22$

Question 2 **C**

Every vertex of an Euler circuit must have an 'even' degree. Exactly two vertices (I and L) have an 'odd' degree, so they must be joined.

Question 3 **C**



The minimal spanning tree is shown above. Nine edges form the minimal spanning tree.

Question 4 **E**

There are six faces (so **A** is incorrect). The sum of the vertices equals 28 (so **B** is incorrect). D, P, F and K all have 'even' degrees (so **C** is incorrect). R and P are not included (so **D** is incorrect). An Euler path is not possible because more than two vertices have and 'odd' degree (so **E** is correct).

Question 5 **D**

The delivery driver visits towns which are represented by vertices. Vertices are represented by Hamilton paths or circuits. Vertices are not represented by Euler paths or circuits. The delivery driver returns to the original town, i.e. travels in a circuit.

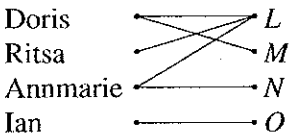
Question 6 **B**

	L	*M*	*N*	*O*
Doris	**30**	35	40	70
Ritsa	**15**	35	20	80
Annmarie	**30**	50	**30**	45
Ian	60	55	45	**40**

$$\begin{bmatrix} 0 & \mathbf{5} & 10 & 40 \\ 0 & 20 & 5 & 65 \\ 0 & 20 & 0 & 15 \\ 20 & 15 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 10 & 40 \\ 0 & 15 & 5 & 65 \\ 0 & 15 & 0 & 15 \\ 20 & 10 & 5 & 0 \end{bmatrix}$$

An optimal matching exists

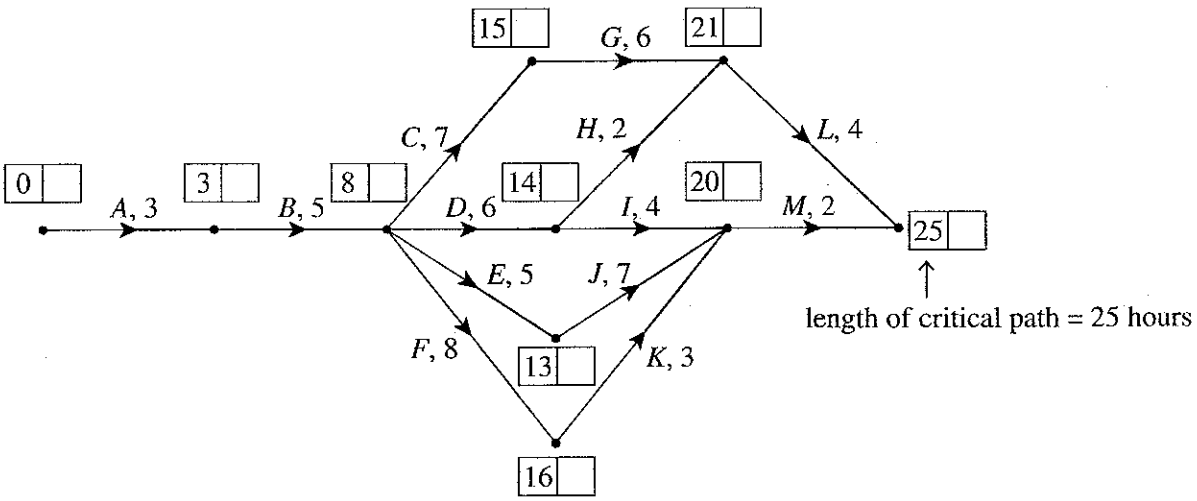


The bipartite graph shows that Doris must be allocated to task *L* or task *M*. Since Ritsa can only be allocated to task *L*, Doris must be allocated to task *M*.

Question 7 **C**

Team *E* did not defeat team *B* (so **A** is incorrect). Both teams *C* and *E* had one two-step dominance (so **B** is incorrect). Team *B* did have the highest number of three-step dominances (so **C** is correct). Team *A* had no three-step dominance (so **D** is incorrect). Two more games are needed before each team has played every other team (so **E** is incorrect).

Question 8 **D**



Question 9 **C**

original critical path – *ABCGL* = 25 hours

revised critical path – *ABEJM* = 22 hours

reduction in project completion time = 3 hours

Module 6: Matrices

Question 1 **E**

It is not possible to add matrices of different order (so **A** and **B** are incorrect). **C** is an attempt to multiply the matrices. **D** correctly states that the sum is undefined, but gives a reason that would be valid if an attempt to multiply rather than add the matrices was made.

Question 2 **C**

$A_{2,4}$ is unique (**A**): its value (–2) is not found anywhere else in the matrix (so **A** is not the required response). **B** and **E** are calculations involving individual elements in the matrix (so neither of these is the correct response). $A_{4,3}$ refers to the element in row four, column three of the matrix and since there is no fourth column, this is clearly undefined (so **D** is not the required response). $A_{2,2}$ refers to an element of the matrix A, so it is not a matrix of any type. Therefore **C** is the incorrect statement.

Question 3 **C**

$$X = \begin{bmatrix} 2 & 2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 5 \\ 1 & 11 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Question 4 **C**

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}$$

Question 5 **A**

It is not possible to find PS (so **C** and **D** are incorrect). There is no reason to find the inverse of S (so **E** is incorrect). Matrix S should be arranged in such a way that each of its rows has the same meaning as the corresponding row of the matrix resulting from the multiplication. Thus row one must relate to location V and row two to location W (so **A** is correct).

Question 6 **B**

$T^{40}S_0$ will determine the allocation of hours 40 weeks later.

$$T^{40}S_0 = \begin{bmatrix} 19.75 \\ 18.61 \\ 21.65 \end{bmatrix}$$

Question 7 **D**

The simplest element in the transition matrix is the '1' in row one, column two. Brand *B* purchases in any week will become brand *A* purchases in the following week. **A** has this process in reverse and is thus incorrect. It is true that 50% of the brand *A* purchases in any week will become brand *B* purchases in the following week, but there will also be a further 50% of brand *C* purchases that will also become brand *B* purchases (so **C** is incorrect).

Question 8 **D**

The format of the state matrix is not given so there are two transition matrices possible – for the state matrices $\begin{bmatrix} work \\ no\ work \end{bmatrix}$ and $\begin{bmatrix} no\ work \\ work \end{bmatrix}$.

The matrix required must be 2×2 for which the columns sum to 1 (eliminating **A**, **B**, and **C**). The proportion requiring work is 0.27 of those which required work last year and 0.63 of the others. Thus the first row could be 0.27 0.63 (so **D** is correct). The alternative is 0.37 0.73 in the first row which is not true of any of the options.

Question 9 **C**

Since *X* has four columns, *YZ* must have four rows and *Y* must also have four rows (so **A** is incorrect) (a definition of matrix multiplication order). Since a transition matrix is square, *X* is 4×4, and so it follows that *Z* must also have four rows. *P* and *Z* must have the same number of columns and *P* must have three rows (so **B** is incorrect). We cannot uniquely determine the order of *P* and *Z*, but some combinations are impossible. For **D** to be true, *Z* must have order 4×4, but this gives order 3×4 when multiplied by *X* which contradicts the statement that *P* and *Z* are the same order. *P* must have three rows and *Z* must have four rows – they are not same order (so **E** is incorrect). Thus **C** is correct.