2008

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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2008 Further Mathematics Trial Examination 1 Suggested Solutions

Core: Data analysis

Question 1 C

$\frac{10}{30} \times \frac{100}{1} = 33.3\%$

Question 2 C

The median is 34

The lower quartile is 22

The upper quartile is 62

The interquartile range is $62 - 22 = 40$

Question 3 B

This graph has an outlier from 90 to 100, and it has a tail to the left. This means it is negatively skewed with an outlier.

Question 4 D

8 + 10 + 2 = 20

Question 5 E

Both variables are numerical $\therefore$ scatter plot. One variable is time so the special type of scatter plot is a time series plot. Stem and leaf and box plot are used when one variable is numerical and one is categorical. A bar graph is used when both variables are categorical.

Question 6 D

68% would lie between 600 + 80 and $600 - 80$

$\therefore$ not A

The % greater than 520 would have to be greater than 50% since the mean is 600 $\therefore$ not B

440 is 2 standard deviations below the mean so 2.5% are less than 440 $\therefore$ not C

680 is 1 standard deviation above the mean so about 16% are greater than 680 $\therefore$ not E

360 is 3 standard deviations below the mean so about 0.15% are less than 360 $\therefore$ D

Question 7 B

$Z = \frac{x - \bar{x}}{s}$

$-2 = \frac{60 - \bar{x}}{5}$

$-10 = 60 - \bar{x}$

$\bar{x} = 70$

Question 8 A

When the residual plot has no clear pattern, then a linear relationship exists.

Core: Data analysis

Question 9 E

$r^2 = 0.04$

$r = \pm 0.2$

But r is negative because the gradient of the regression line is negative $\therefore\ r = -0.2$

Question 10 A

English mark $= 7.25 - 0.18 \times$ Maths mark

$7 = 7.25 - 0.18 \times M$

$-0.25 = -0.18M$

$$M = \frac{-0.25}{-0.18} = 1.4$$

Question 11 D

There is only a seasonal pattern here, the graph following a similar pattern in the 4 quarters of each year.

Question 12 A

Coordinates of median of lower 4 points is (2.5,40)

Coordinates of median of upper 4 points is (10.5,35)

$$m = \frac{35 - 40}{10.5 - 2.5} = -0.625$$

Question 13 D

Using May, June, July and August

$$\text{Mean} = \frac{16 + 15 + 14 + 13}{4} = 14.5$$

Using June, July, August and September

$$\text{Mean} = \frac{15 + 14 + 13 + 12}{4} = 13.5$$

Centring these two means that would be either side of July, August and September

4 – mean smoothed value centred at July

$$= \frac{14.5 + 13.5}{2} = 14$$

Module 1 Number patterns

Question 1 D

Each term in the sequence is found by adding 6 to the term before it. Hence it is an arithmetic sequence with a common difference of 6

Question 2 B

This is a geometric sequence with a common ratio of $-\frac{1}{2}$

$a = 8, n = 12$

$t_n = ar^{n-1}$

$t_{12} = 8\left(-\frac{1}{2}\right)^{11} = -\frac{1}{256}$

Question 3 C

$t_7 = 3 \times t_6 + 4$

$8017 = 3 \times t_6 + 4$

$8013 = 3 \times t_6$

$2671 = t_6$

$t_6 = 3 \times t_5 + 4$

$2671 = 3 \times t_5 + 4$

$2667 = 3 \times t_5$

$889 = t_5$

Question 4 C

$t_4 = t_{1+3} = t_{1+2} + t_{1+1} + t_1 = t_3 + t_2 + t_1 = 1+1+1 = 3$

$t_5 = t_{2+3} = t_{2+2} + t_{2+1} + t_2 = t_4 + t_3 + t_2 = 3+1+1 = 5$

Question 5 C

$4 + 6 + 8 + \ldots$

Sum of Arithmetic Sequence

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[8 + 9 \times 2] = 5 \times 26 = 130\text{m}$

Question 6 E

Value at beginning of first year,2008 = 32,000

Value at beginning of second year = 32,000(0.85)

Value at beginning of third year = $32{,}000(0.85)^2$

Following the pattern,

Value at beginning of fifth year = $32{,}000(0.85)^4$

= \$16704.20

This is closest to \$16704

Module 1 Number patterns

Question 7 D

Arithmetic Sequence.

$a = 85$

$d = -7$

$t_n = a + (n-1)d = 85 + (n-1)(-7)$

Use sequence mode on graphics calculator and press $y =$

$n(min) = 1$

$\mu(n) = 85 + (n-1)(-7)$

$\mu(n \text{ min}) = 85$

Press second table and scroll down to get -48

This corresponds to $n = 20$

Question 8 A

$$\frac{72}{x} = \frac{x}{8}$$

$$x^2 = 576$$

$$x = \pm\sqrt{576} = \pm 24$$

$$\text{Common ratio} = \frac{\pm 24}{8} = \pm 3$$

Question 9 E

An arithmetic sequence is linear $\therefore$ not A

The geometric sequence must have a negative common ratio because the terms go from positive to negative to positive $\therefore$ not B or D

Because the terms are getting closer to the x axis as n increases, then the common ratio must be a fraction.

Module 2 Geometry and trigonometry

Question 1 D

$\tan\theta = \frac{8}{4} = 2$

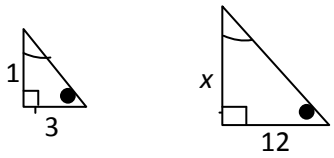
$\theta = \tan^{-1}(2) = 63^0$

Question 2 D

$\angle SOA = 90 - 25 = 65$

True bearing of $OA = 180 + 65 = 245^0\text{T}$

Question 3 B



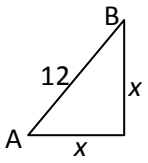
The triangles are similar,AAA

$\therefore$ sides are in the same ratio

$\frac{x}{1} = \frac{12}{3}$

$x = 4$ m.

Question 4 D



$x^2 + x^2 = 144$

$2x^2 = 144$

$x^2 = 72$

$x = \sqrt{72} = 8.485$

Perimeter = $8.485 \times 2 + 12$

Perimeter = 28.97

Question 5 C

$\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2} \times 6 \times 6 \times \sin 50^0$

Area =13.8

Question 6 E

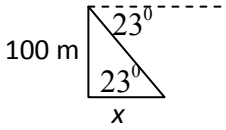
$\angle BAC = 180 - (135 + 26) = 19^0$

$\frac{AC}{\sin 135^0} = \frac{16}{\sin 19^0}$

$AC = \frac{16 \times \sin 135^0}{\sin 19^0} = 34.8$

Module 2 Geometry and trigonometry

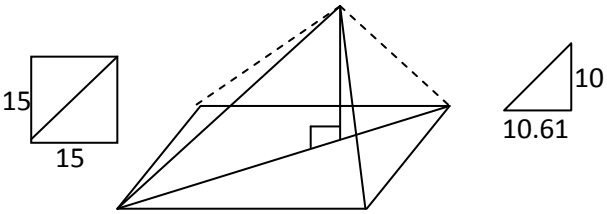
Question 7 D



$$\tan 23^0 = \frac{100}{x}$$

$$x = \frac{100}{\tan 23^0} = 235.6 \text{ m}$$

Question 8 B



Length of diagonal of square base = $\sqrt{15^2 + 15^2}$

Length of diagonal of square base =21.213

Half 1ength of diagonal of square base =10.61

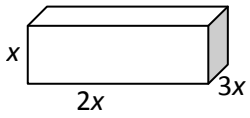
Length of sloping sides, $x = \sqrt{10.61^2 + 10^2}$

Length of sloping sides, $x = 14.58$

Wire = 4 sloping sides + 4 sides to square base

Wire = $4 \times 14.58 + 4 \times 15 = 118.32$

Question 9 E



Surface area = $2x^2 \times 2 + 6x^2 \times 2 + 3x^2 \times 2$

Surface area = $4x^2 + 12x^2 + 6x^2 = 22x^2 = 7942$

$$x^2 = \frac{7942}{22} = 361$$

$x = 19$

Width = $3x = 57$

Module 3 Graphs and relations

Question 1 B

Points are (3,0) and (0,-4)

$$m = \frac{-4-0}{0-3} = \frac{-4}{-3} = \frac{4}{3}$$

Question 2 E

When $x = 3800$, y is between 4000 and 4500

$\therefore$ \$4250

Question 3 B

Original means $t = 0$

$P = 0 + 50 = 50$

Question 4 C

When Meg stops for lunch her speed would be 0 $\therefore$ not A or D

Meg rides home faster than she goes at first, $\therefore$ not B

The distance travelled is not 0 when Meg stops for lunch $\therefore$ not E

Question 5 A

The equation of the line is $y = x - 3$

Area under or on this line is $y \le x - 3$

This is the same as $y - x \le -3$

This is the same as $-y + x \ge +3$

This is the same as $x - y \ge 3$

Question 6 B

Using points (0,6) and (4,5)

$$m = \frac{5-6}{4-0} = -\frac{1}{4}$$

Equation of this part of the function is

$$y = -\frac{1}{4}x + 6 \quad 0 \le x \le 4$$

Using points (14,0) and (4,5)

$$m = \frac{5-0}{4-14} = -\frac{5}{10} = -\frac{1}{2}$$

Equation of this part of the function is

$$y = -\frac{1}{2}x + c$$

When $x = 14, y = 0$

$0 = -7 + c$

$c = 7$

$$y = -\frac{1}{2}x + 7 \quad 4 < x \le 14$$

Module 3 Graphs and relations

Question 7 C

When $x^2 = 16, y = 4$

$\therefore y = \frac{1}{4}x^2$

Question 8 D

In A, one of the lines has a gradient of 2 and the other line has a gradient of -2, $\therefore$ not parallel

In B, one of the lines has a gradient of 3 and the other line has a gradient of -3, $\therefore$ not parallel

In C, both lines are the identical same line, not two separate lines.

In A, one of the lines has a gradient of 2 and the other line has a gradient of -2, $\therefore$ not parallel

In E, one of the lines has a gradient of $-\frac{1}{2}$ and the other line has a gradient of $-\frac{1}{7}$, $\therefore$ not parallel

In D, both lines have the same gradient of 3 $\therefore$ parallel.

Question 9 D

Move line $Z = 0$ or $3x - y = 0$ keeping the movement parallel to the line $Z = 0$.

Moving to the left passes through P last.

Moving to the right passes through S last.

P would give the minimum and S would give the maximum.

Module 4 Business-related mathematics

Question 1 A

$0.95^5 = 0.77$

$50000 - 0.77$ is just a little less than 50000

$\therefore$ not C

$50000 + 0.77$ is just a little more than 50000

$\therefore$ not D

$50000 \times (1.05)^5$ is larger than 50000

$\therefore$ not B

$0.95^5 = 0.77$

$50000 - (1.05)^5$ is just a little less than 49000

$\therefore$ not E

Question 2 E

$$\text{GST} = \frac{118.75}{950} \times \frac{100}{1} = 12.5\%$$

Question 3 D

Total Amount Paid $= 66 \times 30 + 396 = 2376$

Interest $= 2376 - 1200 = \$1176$

Question 4 C

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 9400\left(1 + \frac{7.2}{100}\right)^2 = 10802.33$$

$$I = 10802.33 - 9400 = \$1402.33$$

Question 5 E

Amount repaid $= 139 \times 6 = 834$

Interest $= 834 - 800 = 34$

$$R = \frac{100I}{PT} = \frac{100 \times 34}{800 \times 0.5} = 8.5$$

$$\text{Effective interest rate} = \frac{2 \times 6}{6 + 1} \times 8.5 = 14.6\%$$

Question 6 C

Let amount invested $= x$

$$I = \frac{PRT}{100}$$

$$740 - x = \frac{x \times 9.6 \times 5}{100}$$

$$74000 - 100x = 48x$$

$$74000 = 148x$$

$$x = \frac{74000}{148} = \$500$$

Question 7 E

$$\text{Annual Depreciation} = \frac{13500 - 920}{10} = 1258$$

$$\%\text{Depreciation} = \frac{1258}{13500} \times 100 = 9.3\%$$

Module 4 Business-related mathematics

Question 8 A

Use TVM solver

$N = 120$

$I = 9.2$

$PV = 250000$

$PMT =$

$FV = -250000$

$P/Y = 12$

$C/Y = 12$

End

This gives $PMT = 1916.67$

Total interest repayments = 120×1916.67

Total interest repayments = 230000.40

Total amount paid for property

$= 230000.40 + 250000$

=480000.40

Profit = $485000 - 480000.40 = \$4999.60$

Question 9 B

Let original deposit be x

After 5 years = $x \times (1.08)^5$

After 7 years = $x \times (1.08)^5 \times (1.06)^2 = 12000$

$$x = \frac{12000}{(1.08)^5 \times (1.06)^2} = \$7268.60$$

Module 5 Networks and decision mathematics

Question 1 C
This is the only graph where you cannot get to the bottom left point from any other point.

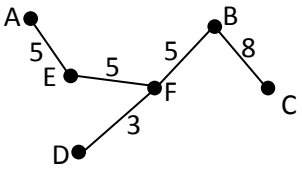
Question 2 D
Number of edges $= \frac{n\times(n-1)}{2} = \frac{20\times19}{2} = 190$

Question 3 D
Sum of vertices $= 6\times3+6 = 24$
Number of edges $= \frac{24}{2} = 12$

Question 4 A
P and R have more than 2 odd vertices ∴ no Euler path and no Euler circuit.
T has all even vertices ∴ it has an Euler circuit.
Q and S have 2 odd vertices ∴ they have an Euler path and no Euler circuit.

Question 5 B
Only E goes to E so the number in the number in the 5^{th} row and 5^{th} column must be 1. A to A is 0, so the number in the first row and first column is 0

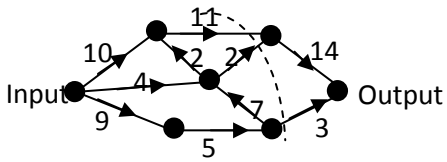
Question 6 D



3 + 5 + 5 + 5 + 8 = 26

Question 7 C
11 + 2 + 5 = 18

Question 8 A



Maximum flow = minimum cut
= 11 + 2 + 3 = 16

Question 9 E
The critical path is ADFI ∴ not A
C can be delayed for 5 hours without delaying the finishing time ∴ not B
C can be delayed for 5 hours without delaying the finishing time so delaying it by 6 hours will only delay the project by 1 hour ∴ not C
E can be delayed for 1 hour without delaying the finishing time so delaying it by 6 hours will only delay the project by 5 hours ∴ not D
Statement E is true.

Module 6 Matrices

Question 1 A

Use calculator or

$$-3\begin{bmatrix}2 & 6\\ -4 & 1\end{bmatrix}=\begin{bmatrix}-6 & -18\\ 12 & -3\end{bmatrix}$$

$$\begin{bmatrix}-6 & -18\\ 12 & -3\end{bmatrix}-\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}-9 & -14\\ 11 & -5\end{bmatrix}$$

Question 2 C

If the determinant = 0 then there is no unique solution.

For A, determinant = 12– -12 = 24
For B, determinant = -12– 12 = -24
For D, determinant = -32– -10 = 22
For E, determinant = 3– -12 = 15
For C, determinant =24– 24 = 0

Question 3 D

$$X=\begin{bmatrix}-2 & 0\\ 4 & -3\end{bmatrix}-\begin{bmatrix}a & b\\ -b & a\end{bmatrix}=\begin{bmatrix}-2-a & -b\\ 4+b & -3-a\end{bmatrix}$$

Question 4 D

From

$$\text{To}\begin{array}{c} \\ A\\ B\\ C\end{array}\begin{array}{c}\begin{array}{ccc}A & B & C\end{array}\\ \begin{bmatrix}0.5 & 0.25 & 0.1\\ 0.2 & 0.6 & 0.35\\ 0.3 & 0.15 & 0.55\end{bmatrix}\end{array}$$

Question 5 E

Steady State = $T^{30}S_o$

$$=\begin{bmatrix}0.4 & 0.8\\ 0.6 & 0.2\end{bmatrix}^{30}\begin{bmatrix}200\\ 300\end{bmatrix}$$

Use calculator to get $\begin{bmatrix}286\\ 214\end{bmatrix}$

Question 6 A

$\det A$

$=3p\times s--2q\times 4r$

$=3ps+8qr$

$$\frac{1}{\det A}=\frac{1}{3ps+8qr}$$

$$A^{-1}=\frac{1}{3ps+8qr}\begin{bmatrix}s & 2q\\ -4r & 3p\end{bmatrix}$$

Question 7 C

$2x+2y+2z=1\equiv x+y+z=0.5$

$1x+0y+1z=-1$

$0x-5y+4z=5$

Matrix is

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 1 & -5 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0.5\\ -1\\ 5\end{bmatrix}$$

Question 8 E

Columns must add to 1 ∴ not A, B or C

0.1 must be in the umbrella column and row.

Question 9 E

B and C must be of the same order which would have to be 4×2

A multiplied by a 4×2 matrix would have to have 4 columns, so it must be a 3×4 matrix.

End of suggested solutions 2008 Further Mathematics VCE Trial Examination 1

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