INSIGHT
Trial Exam Paper

2008

FURTHER MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
	Total	60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator or approved CAS calculator or CAS software (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 35 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the box provided.
- You must answer the questions in English.
- Remove the data sheet during reading time.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** question within the modules selected. You do not need to give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

TURN OVER

Core

Table 1 below shows the average daily price (in cents per litre) of unleaded petrol in Melbourne, Brisbane and Adelaide respectively over 15 randomly chosen days in February.

Table 1

Average Petrol Prices (cents per litre) in February		
Melbourne	Brisbane	Adelaide
139	132	129
138	129	129
133	128	130
144	125	129
141	126	129
139	132	128
137	130	128
134	126	128
140	123	127
136	124	128
134	130	128
132	126	128
130	127	134
134	138	147
144	136	131

Source: AIP Research www.aip.com.au/pricing/retail.htm

Question 1

a. Complete Table 2 below by calculating the standard deviation of the average daily petrol price for Adelaide during February. Write your answer correct to one decimal place.

Table 2

City	Melbourne	Brisbane	Adelaide
Mean	137.0	128.8	130.2
Standard deviation	4.2	4.3	

1 mark

CORE – continued

On a particular day in Melbourne the average petrol price is 141 cents per litre.

b.   Calculate the standard price (z score) relative to this sample of petrol prices. Write your answer correct to two decimal places.

_____

_____

1 mark

The average petrol prices of Melbourne during February are normally distributed. On a particular day in February the standardised petrol price is -1.

c.   Approximately what percentage of days in February will the petrol price be more than this day?

Percentage =

1 mark

Using the data from Table 1, boxplots have been constructed to display the distributions of average daily petrol prices in February, 2008 for Melbourne and Brisbane as shown below.
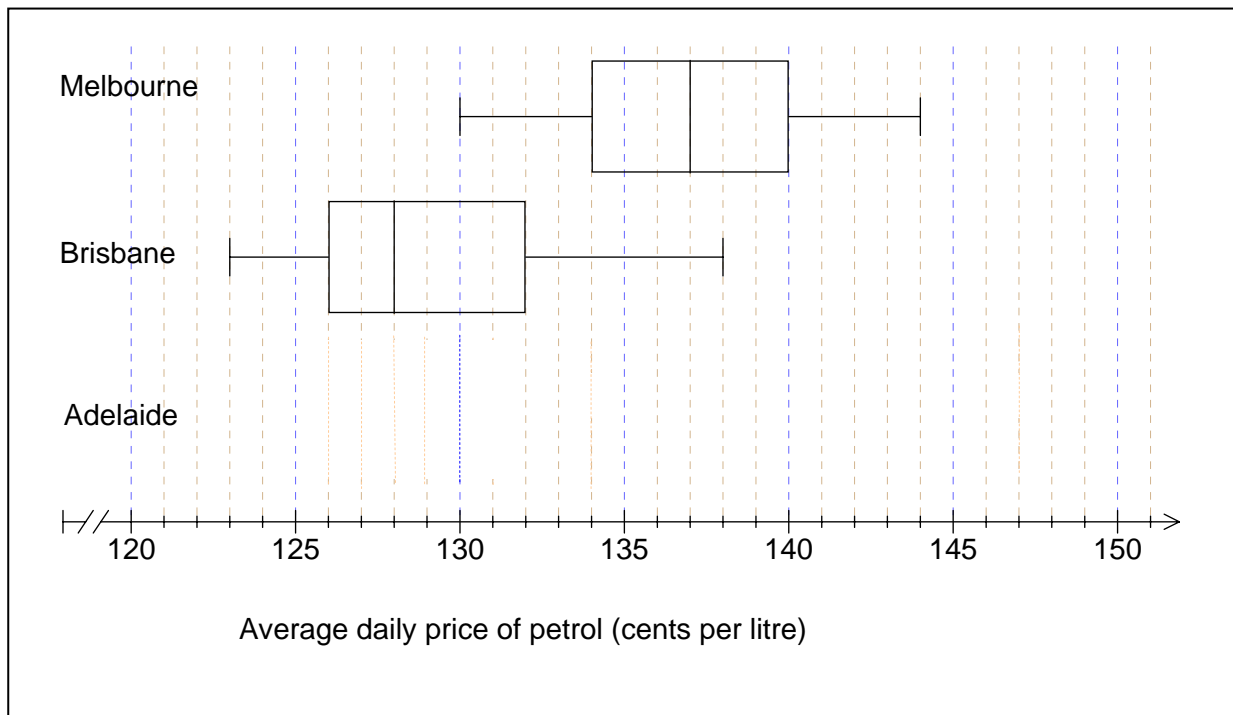
d.   Complete the display by constructing and drawing a boxplot that shows the distribution of unleaded petrol prices in Adelaide during February.

2 marks

e. *Compare* the distribution of petrol prices in the three cities in terms of shape, centre and spread.

Shape _____

_____

_____

Centre _____

_____

_____

Spread_____

_____

_____

3 marks

Question 2

The graph below shows the daily petrol prices for Melbourne in February 2008.

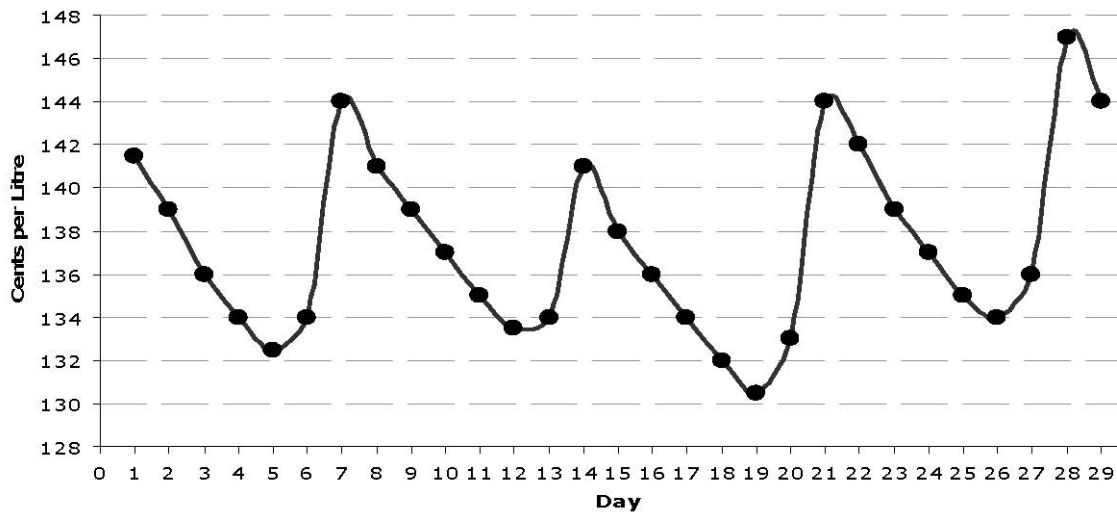**Figure 1:** *Melbourne average daily petrol price, February 2008*

a. Comment on the features of the graph.

_____

_____

_____

1 mark

Table 3 below shows the averaged daily price of unleaded petrol in Melbourne during February

Table 3

Week	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
1	141.5	139	136	134	132.5	134	144
2	141	139	137	135	133.5	134	141
3	138	136	134	132	130.5	133	144
4	142	139	137	135	134	136	147

b. The seasonal indices for this data are shown below.
Calculate the missing seasonal index figure and complete the table below.

	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Seasonal Index		1.01	1.00	0.98	0.97	0.98	1.05

1 mark

c. Use the appropriate seasonal indices and the actual petrol prices in Table 3 to complete the table of deseasonalised petrol prices for February 2008 below.

Table 4

	Deseasonalised Petrol Prices (cents per litre)						
Week	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
1	138	138	136	137	137	137	137
2	137	138	137	138	138	137	134
3	135	135	134	135	135	136	137
4	138	138	137	138	139	139	

1 mark

The equation of the least squares regression line for the deseasonalised data is given by

$$\textbf{\textit{Deseasonalised Petrol Price}} = 0.05 \times Day + 136.34$$

(where $Day = 1$ is February 1st, 2008 which is a leap year)

d. Use this equation to draw the line of the deseasonalised petrol prices on the graph shown in Figure 1.

1 mark

e. Complete the following sentence by filling in the box.

From the regression equation we can conclude that the petrol price increases on average by

cents per litre every day.

1 mark

f. Predict the deseasonalised petrol price, correct to two decimal places, for 10th March using this equation.

_____

_____

_____

1 mark

g. Hence, use the appropriate seasonal index to obtain a forecast for Monday 10th March. Give your answer correct to two decimal places.

_____

_____

_____

1 mark
Total 15 marks

END OF CORE

Module 1: Number Patterns

Andrew's swim time is recorded during a rigorous training program. The time to the nearest second for the first three laps is shown in the table below.

Lap	1	2	3
Time (seconds)	50	48.5	47

Question 1

Andrew's trainer believes that the swim time will form a decreasing arithmetic sequence.

a. Show that Andrew's trainer is correct.

_____

_____

1 mark

b. An expression for Andrew's swim time in the n th lap can be written as $A_n = b - 1.5n$. Determine the value of b.

_____

_____

_____

1 mark

c. Andrew's fastest swim time is 17 seconds for one lap of the pool. If he continues in this sequence, how many laps does he swim to achieve his fastest time?

_____

_____

_____

1 mark

d. Find the total time he swam to complete his fastest and final lap.
Give your answer in seconds correct to one decimal place.

_____

_____

_____

1 mark

Module 1: Number Patterns – continued
TURN OVER

Question 2

Betty's swim time for each lap follows a geometric sequence with a common ratio of 0.94.

Betty swam the first lap of the pool in 65 seconds.

a.   By what percentage does Betty's swim time decrease for each lap of the pool?

_____

<div align="right">1 mark</div>

b.   Determine the time it takes, to the nearest second, for Betty to swim the 4th lap.

_____

<div align="right">1 mark</div>

c.   Write an equation that gives Betty's swim time B_n for the n th lap of the pool.

_____

<div align="right">1 mark</div>

d.   How much faster did Betty swim her 10th lap of the pool in comparison to her 9th lap?
        Give your answer in seconds correct to two decimal places.

_____

_____

<div align="right">1 mark</div>

e.   How many laps did Betty complete in the first 5 minutes of her swim?

_____

_____

_____

<div align="right">1 mark</div>

f.   If Betty swims 25 laps of the pool, calculate the time it takes her to complete the last
        10 laps. Give your answer in seconds correct to two decimal places.

_____

_____

_____

<div align="right">2 marks</div>

<div align="right">**Module 1: Number Patterns** – continued</div>

Question 3

The drink machine at the swim centre contains 400 drinks. Each day 8% of the drinks are sold and at the end of each day the machine is stocked with 20 new drinks.

The number of drinks in the machine, D_n, at the beginning of the n th day is modelled by the difference equation $\qquad D_{n+1} = 0.92D_n + 20$, where $D_1 = 400$

a. Find the number of drinks, to the nearest whole number, at the beginning of day 3.

_____

_____

1 mark

b. Show that the number of drinks at the beginning of each day does not follow an arithmetic or a geometric sequence.

_____

_____

1 mark

c. For many days 8% of the drinks in the machine are sold and 20 drinks are restocked. Show a calculation explaining why there will never be fewer than 250 drinks in the machine.

_____

_____

1 mark

d. How many drinks should be restocked in the machine each day so that the number remains stable, that is, so that there are 400 drinks in the machine each day?
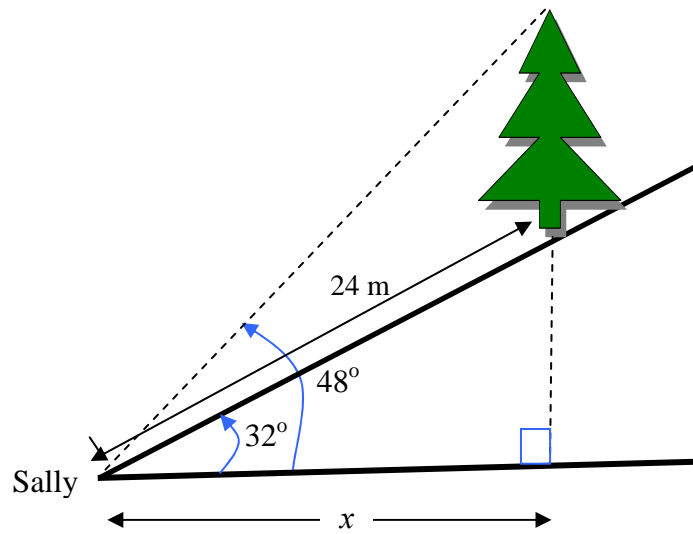
_____

_____

_____

1 mark
Total 15 marks

END OF MODULE 1
TURN OVER

Working space

Module 2: Geometry and Trigonometry

A tree stands on a hillside of slope 32° (from the horizontal). Sally stands at the bottom of the hill 24 m from the tree and measures the angle of elevation to the top of the tree to be 48° as shown in the diagram.

Question 1

a.   Show that the horizontal distance from Sally to the base of the tree, x, is 20.35 metres.

_____

_____

_____

_____

1 mark

b.   Find the height of the tree, in metres, correct to two decimal places.

_____

_____

_____

_____

_____

_____

2 marks

Module 2: Geometry and trigonometry – continued
TURN OVER

Sally walks to a river that flows due East. She stops and looks across to the opposite river bank to see a windmill that has a bearing of 023°T. After walking downstream 40 m, Sally stops to find that the windmill is now on a bearing of 345°T.

DIAGRAM NOT DRAWN TO SCALE

River

Windmill (W)

Original Position (O)
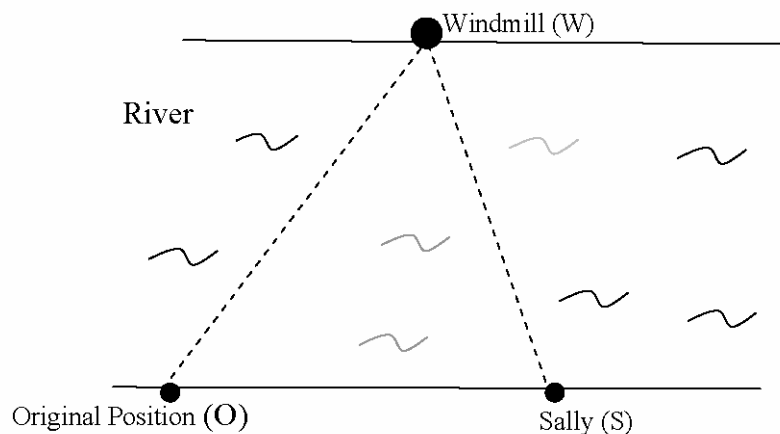
Sally (S)

Question 2

a. Show that the magnitude of angle OWS is 38°.

_____

_____

1 mark

b. Find the distance, in metres, correct to one decimal place from Sally to the windmill, SW.

_____

_____

_____

_____

_____

2 marks

c. Hence, find the width of the river.
Give your answer in metres correct to one decimal place.

_____

_____

_____

_____

_____

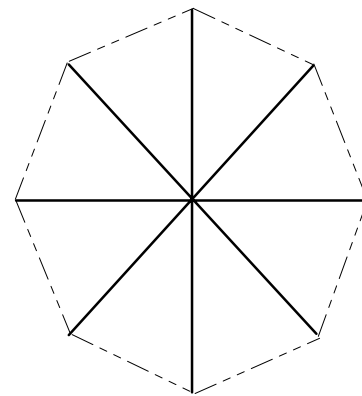2 marks

Module 2: Geometry and trigonometry – continued

Question 3

The windmill has 8 blades.

The ends of the blades form a regular octagon as shown in the diagram.

Each blade is 2 metres long.

**a.** Show that the angle at the centre, between the blades, is 45°.

_____

_____

<div align="right">1 mark</div>

b. Determine the area of the octagon correct to one decimal place.

_____

_____

_____

_____

<div align="right">2 marks</div>

The structure that holds the windmill is made of a square based pyramid.
Each side is triangular with three horizontal supporting struts as shown
on the diagram.

The horizontal struts are in the ratio 2: 3: 4

The longest horizontal strut is measured to be 3.5 metres.
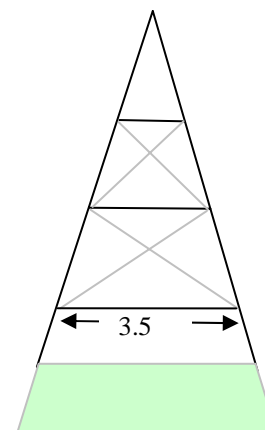
c. Find the length of the middle strut in metres, correct to 3 decimal places.

_____

_____

<div align="right">1 mark</div>

<div align="right">**Module 2: Geometry and trigonometry** – continued
TURN OVER</div>

The windmill is supported by a structure in the shape of a square based pyramid.

It is reinforced with concrete 0.6 m deep.

The upper surface of the concrete has a length of 4 metres and the base of the concrete has a length of 4.5 metres.

h m

ground level

0.6 m

4 m

4.5 m
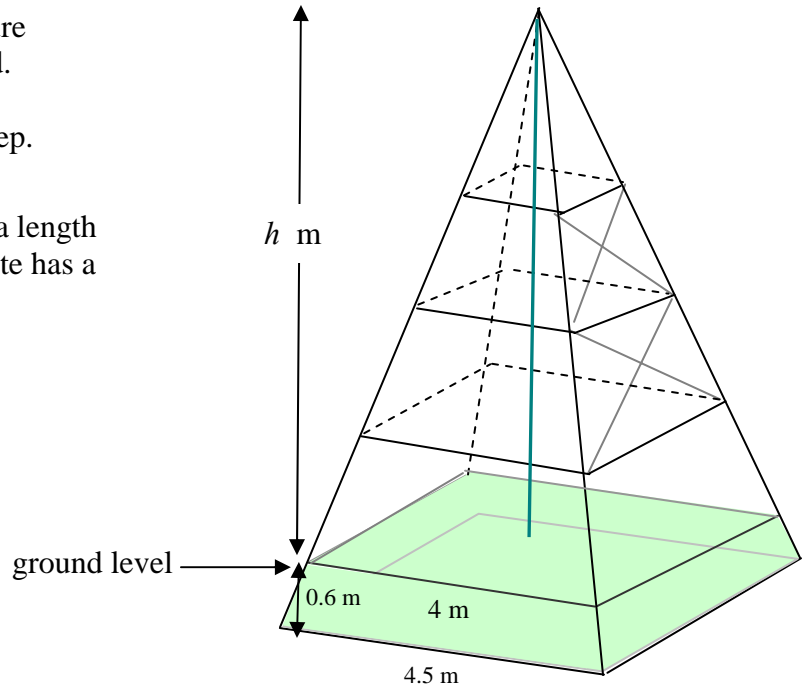
d. Use similar triangles to show that the height, *h*, of the structure above ground level is 4.8 metres.

1 mark

e. Determine the volume of the concrete, in cubic metres, correct to two decimal places.

_____

_____

_____

_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and Relations

Question 1

A company, Cleanozone, designs and manufactures various models of rainwater tanks.
The new *Slimline* model requires $400 worth of materials to make each tank.

It costs $12 000 per year to provide the manufacturing facilities, regardless of the number of
tanks that are produced. It is possible for the facilities to make up to 150 tanks per year.

The total cost of manufacturing x tanks per year is given by the equation

$$C = 400x + 12\,000, \quad 0 \le x \le 150$$

a.   Find the total cost of manufacturing 100 tanks.

_____

1 mark

b.   Sketch the graph of the cost equation on the set of axes below.

_____

1 mark

Cost ($)

112000
104000
96000
88000
80000
72000
64000
56000
48000
40000
32000
24000
16000
8000

10   20   30   40   50   60   70   80   90   100   110   120   130   140   150   160   x

Module 3: Graphs and relations – continued
TURN OVER
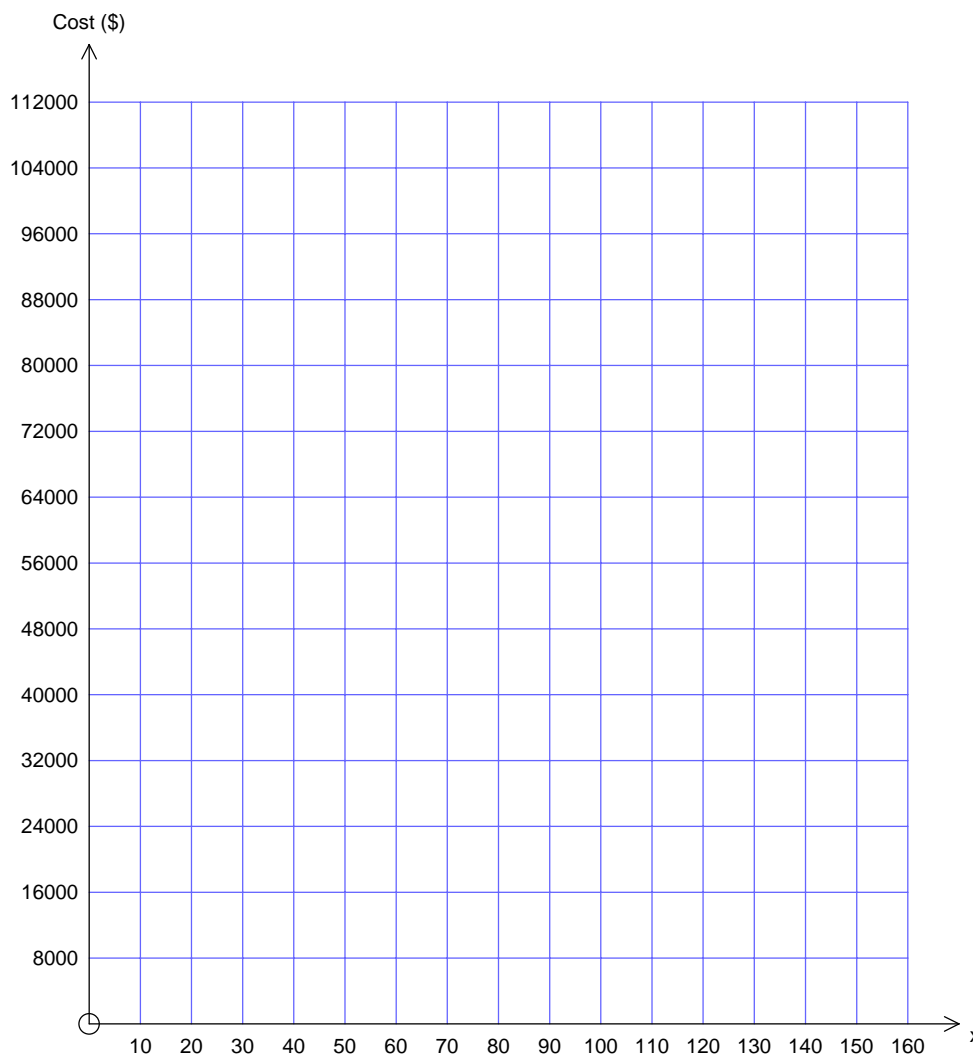
Cleanozone are able to sell the tanks to retailers. The first 40 tanks sell for $500 each but the remaining 110 bring in $700 each.

c. The revenue made from selling 40 tanks is $20 000. Calculate the revenue made from selling 100 tanks.

_____

_____

1 mark

d. Sketch the revenue on the above axes.

1 mark

The revenue, R, dollars, from selling x *Slimline* tanks is given by the function:

$$R = \begin{cases} 500x & ; 0 \le x \le 40 \\ \\ 700x + k & ; 40 \le x \le 150 \end{cases}$$

e. Show that the value for k is –8000.

_____

_____

_____

1 mark

f. Find the least number of *Slimline* tanks that need to be sold for Cleanozone to make a profit.

_____

_____

_____

1 mark

Module 3: Graphs and relations – continued

Question 2

Manufacturing a tank involves two main processes: welding and testing. The table below shows the time available in a week to manufacture two types of water tanks.

	Domestic (hours)	Garden (hours)	Time available (hours)
Welding	4	5	97
Testing	2	4	62

Let x be the number of domestic tanks and

        y be the number of garden tanks are made each week.
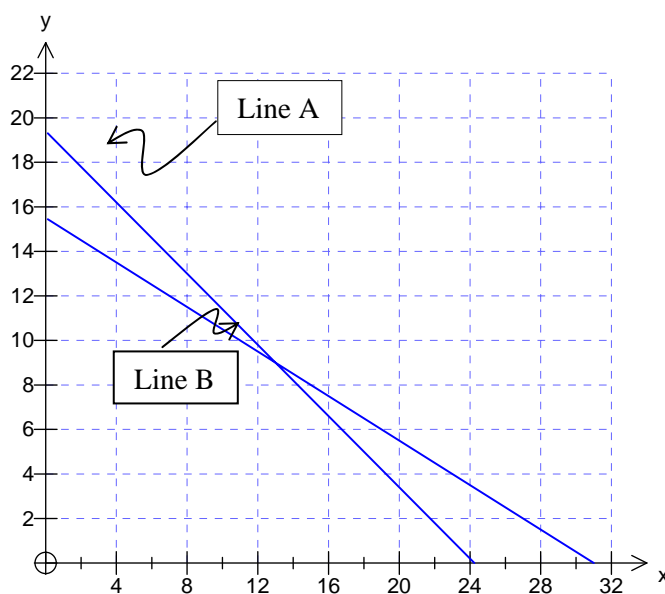
This information can be expressed as Inequalities 1 and 2.

- Inequality 1: $4x + 5y \leq 97$

- Inequality 2: $2x + 4y \leq 62$

a.   Which line (Line A or Line B) in Graph 1 below forms the boundary of the region defined by inequality 1?

_____

                                                                                                            1 mark

b.   Write down the co-ordinates of the point of intersection of Line A and Line B in Graph 1

_____

                                                                                                            1 mark

Graph 1

Module 3: Graphs and relations – continued
TURN OVER

c. Due to demand, the company must produce at least 7 domestic tanks and at least 5 garden tanks in a week.
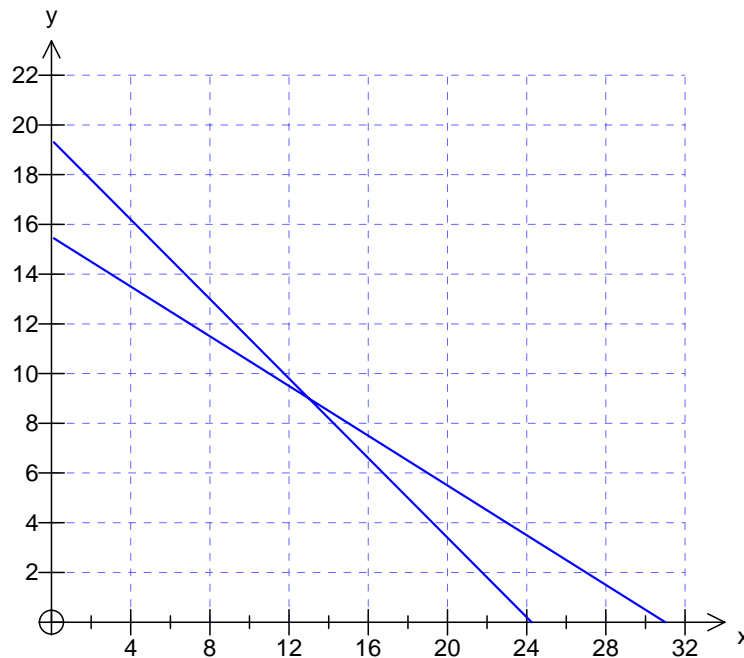
Write the two corresponding inequalities,

- Inequality 3:

- Inequality 4:

1 mark

d. Using inequalities 1 to 4, construct and shade the feasible region for the production of the two types of tanks for one week on Graph 2 below.

Graph 2

3 marks

e. The company is able to make a profit of $140 on each domestic tank and $280 on each garden tank. Write an expression for the profit, P, in terms of x and y.

_____

_____

_____

_____

1 mark

Module 3: Graphs and relations – continued

f. Find the combination of domestic tanks and garden tanks the company should produce in a week to maximise their profit.

_____

_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

Wendy wants to buy a commercial oven for her pizza restaurant. Ovens Galore normally sells them for $18 000, but they have a discounted price of $17 280.

a.   What is the percentage discount? Write you answer correct to one decimal place.

$\boxed{}$ %

1 mark

b.   Ovens Galore offers to sell the oven for the discount price of $17 280. The terms of the sale are $1 200 deposit and $515 per month for 36 months.

    i.   What is the total cost of the oven on these terms?

_____

_____

_____

1 mark

    ii. Show that the annual flat rate of interest charged is 5.1%.

_____

_____

_____

1 mark

    iii. Determine the effective rate of interest per annum.

        Write your answer correct to one decimal place.

_____

_____

_____

1 mark

    iv. Explain why an effective interest rate differs from a flat interest rate.

_____

_____

_____

1 mark

Module 4: Business-related mathematics – continued

c. Wendy sees the same oven for sale at Hot Ovens Discount Store, also for $17 280. The terms of the sale there require no deposit and monthly repayments over three years at an interest rate of 6.4% per annum, calculated monthly on a reducing balance.

The monthly repayments can be determined using the annuities formula:

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1} .$$

The loan is paid out in three years.

i. What are the values of n, P and A?

_____

_____

_____

2 marks

ii. What is the monthly repayment for this loan?
Write your answer in dollars, correct to two decimal places.

_____

_____

_____

_____

1 mark

iii. What is the total cost of the oven from Hot Ovens Discount store on these terms?
Write your answer correct to the nearest dollar.

_____

_____

_____

_____

1 mark

d. Whose terms, Ovens Galore or Hot Ovens Discount Store, offer the lowest total cost for the oven? Justify your answer.

_____

_____

_____

_____

1 mark
Module 4: Business-related mathematics – continued
TURN OVER

Question 2

Wendy purchases the oven with an initial value of $17 280.

For tax purposes Wendy considers two methods of depreciating the value of the oven.

a. Suppose the value of the oven is depreciated using the reducing balance method over five years and reducing at a rate of 14% per annum. What is the depreciated value after five years? Write your answer correct to the nearest dollar.

_____

_____

_____

_____

2 marks

b. Alternatively, suppose that the machine is depreciated using the unit cost of depreciation method. Wendy sells 25 000 pizzas per year and the unit cost per pizza is 8 cents. Determine the depreciated value of the oven after five years. Write your answer correct to the nearest dollar.

_____

_____

_____

_____

1 mark

c. Wendy wants the depreciated value of the oven after five years to be the same when calculated by both methods of depreciation. What would the unit cost per pizza have to be for this to occur? Write your answer in cents correct to two decimal places.
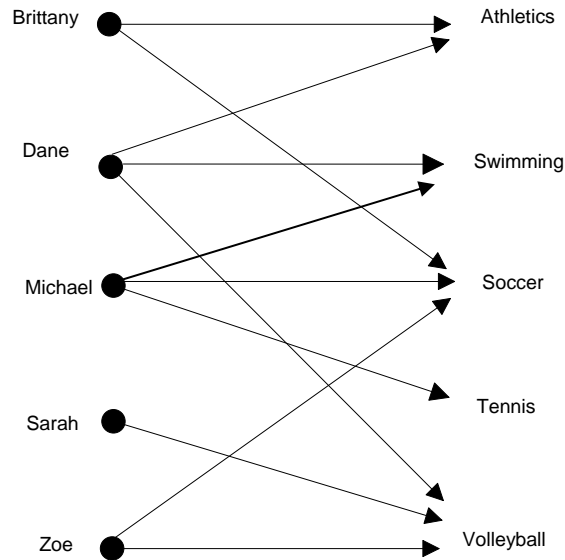
_____

_____

_____

_____

2 marks
Total 15 marks

END OF MODULE 4

Module 5: Networks and Decision Mathematics

Question 1

A group of five school friends represent their school in five different sports. The information is displayed in the following bipartite graph.

Each sport must be represented by one student at a school assembly.

a.   Which student **must** represent Swimming?

_____

1 mark

b.   Complete the table showing the sport that each student **must** represent.

Student	Sport
Brittany	
Sarah	
Zoe	

2 marks

Question 2

The five students decide to play a game of *one on one* basketball.
Each student competes against each of the other four students one at a time.
For each game there is a winner and a loser.

The results are shown in the **incomplete** dominance matrix.

On the directed graph an arrow from Dane to Brittany shows that Dane won against Brittany.
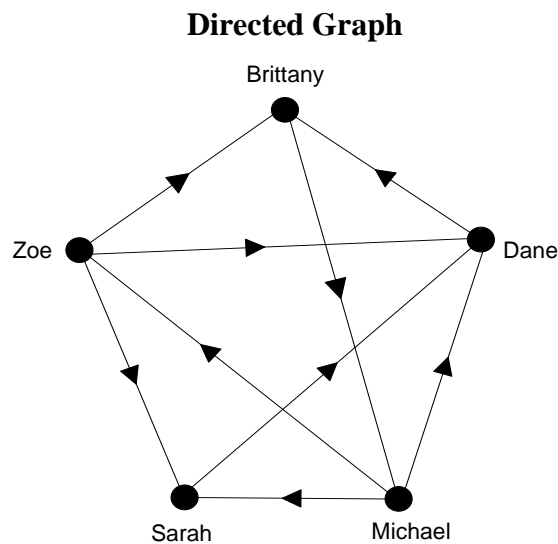
Matrix 1

loser

$$
\text{winner} \quad
\begin{array}{c}
\\ B \\ D \\ M \\ S \\ Z
\end{array}
\begin{array}{ccccc}
B & D & M & S & Z \\
\left[\begin{array}{ccccc}
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 \\
 & & & & 0
\end{array}\right]
\end{array}
$$

Zoe lost to Michael, but won all the other games.

a. Complete the dominance Matrix 1, above, showing Zoe's results.

1 mark

The results of the game are also represented in the **incomplete** directed graph below.

Directed Graph

One of the edges of the graph is missing.

b. Using the information in Matrix 1, **draw** in the missing edge on the directed graph above and clearly show its **direction**.

1 mark

Module 5: Networks and decision mathematics – continued

The results of each one on one basketball contest (one-step dominances) are summarized as follows.

Student	Dominance value (wins)
Brittany	2
Dane	1
Michael	3
Sarah	1
Zoe	3

c. Which two students are ranked equal first in this contest?

_____

1 mark

In order to rank the students from first to last in the basketball contest, two-step (two-edge) dominances will be considered.

The following **incomplete** matrix, Matrix 2, shows two-step dominances.

Matrix 2

$$
\begin{array}{c}
 \\ B \\ D \\ M \\ S \\ Z
\end{array}
\begin{array}{c}
\begin{array}{ccccc} B & D & M & S & Z \end{array} \\
\left[
\begin{array}{ccccc}
0 & 2 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 \\
2 & 2 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & x & 1 & 1 & 0
\end{array}
\right]
\end{array}
$$

d. Explain the two-step dominance of Brittany (B) over Dane (D).

_____

_____

1 mark

e. Determine the value of *x* in Matrix 2.

_____

1 mark

f. Taking into consideration both the one-step and two-step dominances, determine which student was ranked first and which was ranked last in the *one-on-one* basketball competition.
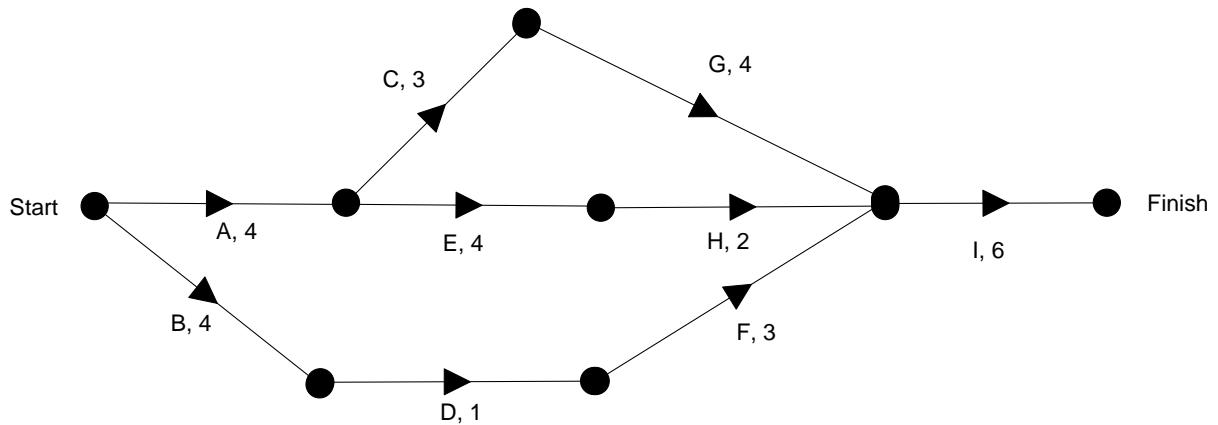
First

Last

2 marks

Question 3

A new gym is to be built at the school.

Nine activities have been identified for this building project.

The directed network below shows the activities and their completion times in weeks.

a. Determine the minimum time, in weeks, to complete this project.

_____

_____

1 mark

b. Determine the slack time, in weeks, for activity D.

_____

_____

1 mark

Module 5: Networks and decision mathematics – continued

The builders of the gym are able to speed up the project.

Some of the activities can be reduced at an additional cost.

The activities that can be reduced in time are B C E F and G.

c. Which of these activities, if reduced in time individually, would not result in an earlier completion of the project?

_____

_____

1 mark

The school council is prepared to pay an additional cost to achieve early completion.
The cost of reducing the time for each activity is $3 000 per week.
The maximum reduction in time for each one of the five activities B, C, E, F, and G is 2 weeks.

d. Determine the minimum time, in weeks, for the project to be completed now that certain activities can be reduced in time.

_____

_____

1 mark

e. Determine the minimum additional cost of completing the project in this reduced time.

_____

_____

1 mark
Total 15 marks

END OF MODULE 5
TURN OVER

Working space

Module 6: Matrices

Frank's Fruit Mart sells the following fruit packs:

	Type of Fruit		
	Apples	**Mangos**	**Bananas**
Standard Pack (S)	6	4	6
Family Fruit Pack (F)	12	12	24
Bulk Fruit Pack (B)	20	15	40

The **cost price** is:
- $7.10 for the Standard Pack,
- $22.20 for the Family Fruit Pack and
- $33.00 for the Bulk Fruit Pack.

Question 1

a. The cost price of each apple, mango and banana is x, y and z dollars respectively.
Write a matrix equation, of the form below, that you can solve to find the value of x, y and z.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

b. Write down an inverse matrix that can be used to solve these equations.

Write the elements as fractions.

1 mark

c. Solve the equation and hence write down the cost price of an apple, a mango and a banana.

Question 2

The **selling price** of each type of pack is calculated by multiplying the cost price by a factor.

These factors are different for each pack.
For the Standard pack the selling price is 1.6 times the cost price
For the Family pack the selling price is 1.5 times the cost price and
For the Bulk pack the selling price is 1.4 times the cost price.

To calculate the selling price Frank's Fruit Mart have set-up a matrix equation of the form:

$$\begin{array}{cc} Cost\ price & Selling\ price \end{array}$$
$$M \times \begin{bmatrix} 7.10 \\ 22.20 \\ 33.00 \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix} = \begin{bmatrix} 11.36 \\ 33.30 \\ 52.80 \end{bmatrix} \begin{matrix} S \\ F \\ B \end{matrix}$$

a. State the order of matrix M.

1 mark

b. Write down the matrix M.

2 marks

Question 3

Frank's Fruit Mart has three outlets for their fruit packs; stores P, Q and R.
The table below shows the number of fruit packs that were sold during one week at each of these outlets.

	Outlet		
	P	**Q**	**R**
Standard pack	25	15	8
Family pack	38	23	5
Bulk pack	11	20	12

Given that Profit = selling price – cost price

a. Set up a matrix equation that will enable Frank's Fruit to find the matrix P that represents the profit made on each of the types of fruit packs for this week.

2 marks

b. Calculate the profit made by Frank's Fruit Mart on each of the types of fruit packs for this week.

1 mark

Module 6: Matrices – continued
TURN OVER

Question 4

Frank's Fruit Mart is investigating the purchasing habits of its retail customers.
Records show that:

Of the customers who purchased the standard pack this week

50% will purchase the standard pack next week
30% will purchase the family pack next week and
20% will purchase the bulk pack next week.

Of the customers who purchased the family pack this week

20% will purchase the standard pack next week
70% will purchase the family pack next week and
10% will purchase the bulk pack next week

Of the customers who purchased the bulk pack this week

30% will purchase the standard pack next week
40% will purchase the family pack next week and
30% will purchase the bulk pack next week.

a.   Enter this information into transition matrix T as indicated below, expressing percentages as proportions.

$$
\begin{array}{c}
\textit{this week} \\
\begin{array}{ccc} S & F & B \end{array} \\
T = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{array}{l} S \\ F \ \textit{next week} \\ B \end{array}
\end{array}
$$

2 marks

During the first week of monitoring Frank's Fruit Mart there were 875 packs purchased in total. In the same week, 215 standard fruit packs and 512 family fruit packs were purchased.

b.   Write this information in the form of an initial state column matrix, I_0.

1 mark

Module 6: Matrices – continued

Assume that each customer purchases one pack each week and the pack they purchase depends entirely on their purchase in the previous week.

c. Determine the expected number of Standard, Family and Bulk fruit packs purchased in the **third** week. Give your answers to the nearest whole number.

2 marks

d. Of the 875 customers determine, in the long term, the number of bulk fruit packs that are purchased in a particular week.

1 mark
Total 15 marks

END OF QUESTION AND ANSWER BOOK