Insight Publications

INSIGHT

Trial Exam Paper

2008

FURTHER MATHEMATICS

Written examination 1

MULTIPLE-CHOICE QUESTION BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 39 pages, with an answer sheet for the multiple-choice questions.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions
- Write your **name** in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Working space

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.

Core – Data Analysis

Questions 1 and 2 refer to the following information.

A dice was rolled 20 times with the following results shown on a frequency table.

Number on die	Frequency
1	2
2	5
3	3
4	2
5	4
6	4

Question 1

The percentage of rolls that resulted in an even number was

A. 9%
B. 11%
C. 24%
D. 50%
E. 55%

Question 2

The mode and mean respectively for this data was

A. 4 and 3.65
B. 4 and 3.5
C. 2 and 3.65
D. 2 and 3.5
E. 3.5 and 4

Question 3

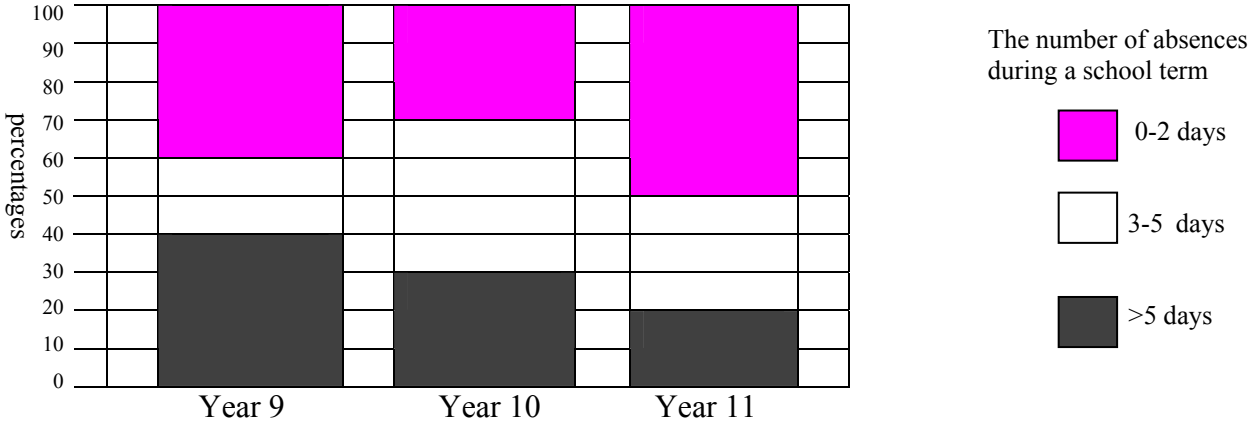
A statistically minded athlete has evaluated her 400 metre efforts. She has noticed that the data shows a bell shaped distribution with a mean of 65.2 seconds and a standard deviation of 1.3 seconds.

If she then runs a time of 66.8 seconds, which of the following statements is closest to summarising her effort?

A. This time is in the top 16% of her efforts.
B. This time is in the bottom 16% of her efforts.
C. This time is close to the average of her efforts.
D. This time is in the top 2% of her efforts.
E. This time is in the bottom 2% of her efforts.

Question 4

The segmented bar chart below shows the distribution of the number of days absent over one term for secondary students at particular year levels.



From these results, the percentage of Year 11 students who were absent for at least 3 days in the term is

A. 20
B. 30
C. 50
D. 70
E. 100

Question 5

Joan received a test score of 72. The class standard deviation for this test was 12. If Joan's z-score was 1, the class average was

A. 13
B. 73
C. 84
D. 60
E. 71

Questions 6 and 7 refer to the following information.

A group of 14 people were given a page to type and the number of errors they had was recorded in the PRE column below.

After an extensive two hour program on keyboard skills the group were given another page to type, similarly these errors were also recorded, in the POST column below.

PRE	Stem	POST
7, 6, 3	0	1, 4, 5, 8, 8, 9
7, 7, 3, 0	1	0, 0, 5, 8
8, 5, 1, 1, 0	2	1, 2, 4
4, 0	3	0

Question 6

From this back to back stem leaf plot summary, which of the following observations is true?

	PRE		POST	
	Median	Inter-quartile range	Median	Inter-quartile range
A.	13	17	10	17
B.	18.5	15	10	13
C.	17	31	8	29
D.	17	17	13.2	13
E.	18.5	15	8	17

Question 7

The data for PRE and POST results respectively can be summarised as

A. both are continuous variables with symmetrical and negative skewing.
B. both are continuous variables with symmetrical and positive skewing.
C. both are discrete variables with symmetrical and negative skewing.
D. both are discrete variables with symmetrical and positive skewing.
E. both are categorical data sets with symmetrical and positive skewing.

Question 8

The goals scored by a soccer team in a month are shown in the table below:

Month	May	June	July	August	Sept	Oct	Nov	Dec
Total Goals	12	15	12	13	8	19	11	14

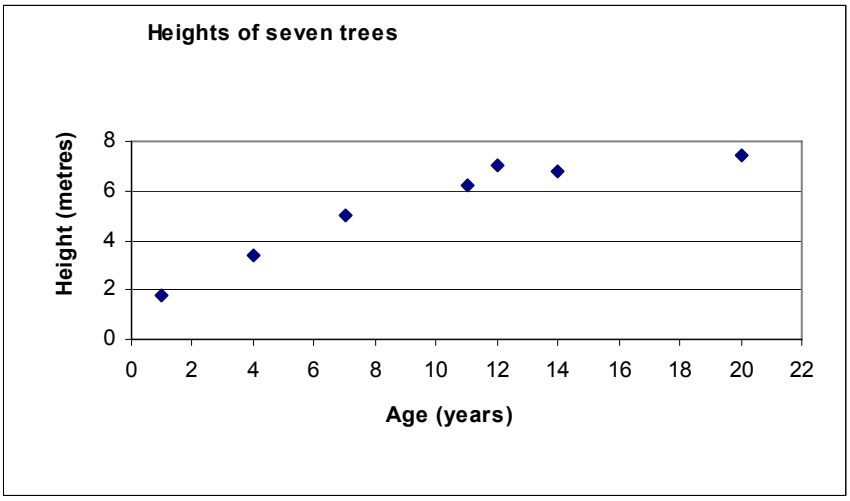
Using 4 point moving medians, the smoothed centred value for Sept is

A. 8
B. 12.875
C. 12
D. 12.5
E. 12.25

Question 9

The ages and heights of seven exotic species of trees were recorded in the table and shown on the scatterplot below.

Age (years)	1	4	7	11	12	14	20
Height (metres)	1.8	3.4	5.0	6.2	7.0	6.8	7.4



It was found that an error was transcribed for the **11 year old** tree. The actual height of the 11 year old tree is more than 7.5 metres.

Knowing that it had the **largest** height of all these trees what effect would this new value have on the previously calculated values of the 3-median regression line?

A. The regression line would not be affected.

B. The gradient will increase but the vertical intercept remains the same.

C. The gradient and the vertical intercept both increase.

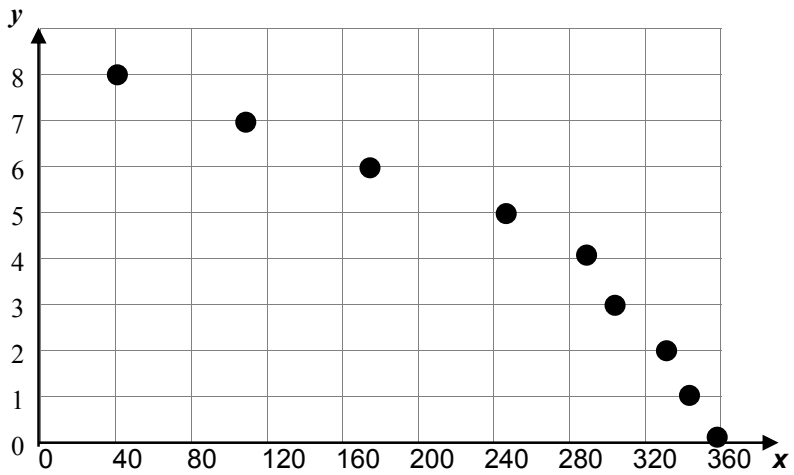
D. The gradient remains the same but the vertical intercept increases.

E. Impossible to calculate because of missing data.

Questions 10 and 11 refer to the following data.

Nathan uses the following data to construct a scatterplot as shown below.

x	360	345	330	304	285	245	172	110	40
y	0	1	2	3	4	5	6	7	8



Question 10

To linearise the data a y^2 transformation is applied. The least squares regression line is closest to

A. $y^2 = 71.88 - 0.20x$

B. $y^2 = 9.68 - 0.02x$

C. $y^2 = 8.04 - 0.01x$

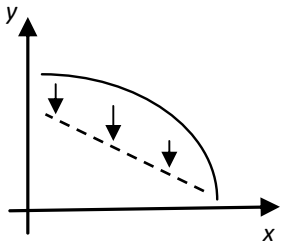
D. $y^2 = 400.78 - 35.33x$

E. $y^2 = 355.11 - 4.53x$

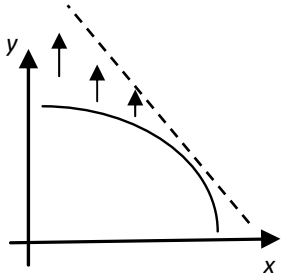
Question 11

Which of the following best describes why Nathan chose to use the y^2 transformation?

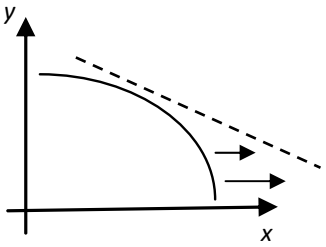
A. y^2 compresses the vertical scale, intending to linearise the data.



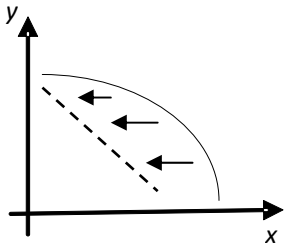
B. y^2 expands the vertical scale, intending to linearise the data.



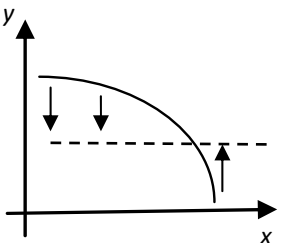
C. y^2 expands the horizontal scale, intending to linearise the data.



D. y^2 compresses the horizontal scale, intending to linearise the data.

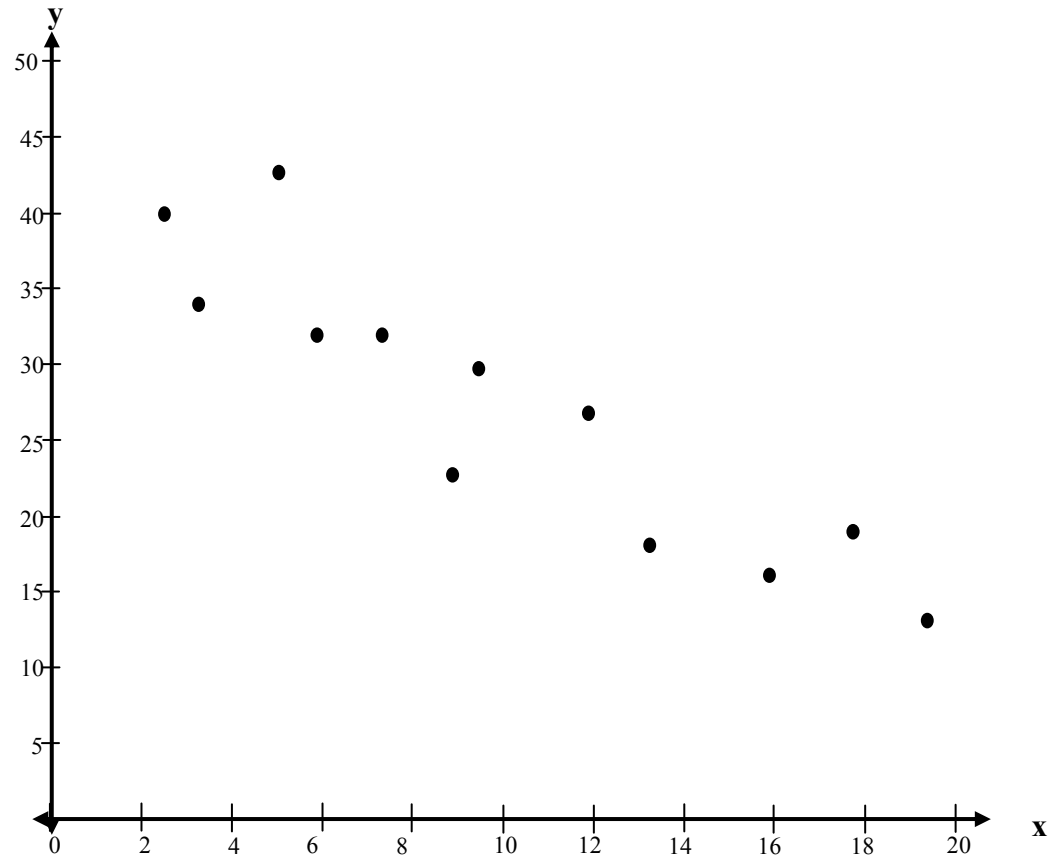


E. y^2 both expands and compresses along the median of the vertical scale, intending to linearise the data.



Question 12

Data collected has been summarised on the following graph.



The coefficient of determination for the above data is 0.8118. This would indicate that the linear association between both variables has a Pearson coefficient value of

A. 0.9010
B. -0.9010
C. 0.6590
D. -0.6590
E. 2.0000

Question 13

The table below shows the number of sales for a particular firm during 2007.

Quarter	1	2	3	4
Sales for 2007	630	840	1 005	980
Seasonal Index	0.67	0.95	1.18	1.20

Using all data including previous years` sales, the regression line was found to be:

$y = 560 + 120x$ where y is the Number of Sales
and x is the Number of Quarters from end of 2006

(Note: Quarter 1 in 2007 will mean $x = 1$)

The predicted deseasonalised figure for Quarter 1 in 2008 would be

A. 1 160
B. 780
C. 1 164
D. 3 400
E. 1 731

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected on the answer sheet provided.
Indicate the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
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SECTION B

Module 1: Number Patterns

Before you answer these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

An arithmetic sequence is

36 32 28 …

Which term would give a value of -4?

A. 7
B. 9
C. 10
D. 11
E. 12

Question 2

The first four terms of a geometric sequence are { -12, x, -3, 1.5 … }.

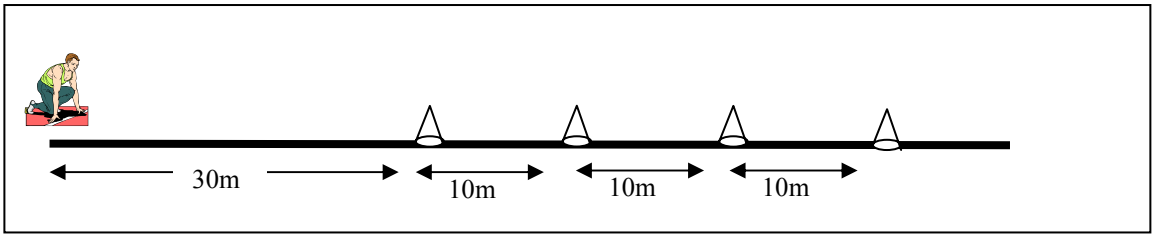
The value of x for this sequence is

A. 7.5
B. -7.5
C. 9
D. 6 or -6
E. 6

The following information relates to questions 3, 4 and 5.

As part of his stamina training, Nic places 4 cones in a straight line, each 10 metres apart. Nic starts 30 metres from the first cone.

He runs to the first cone then returns back to the start before then running to the second, then third and finally fourth. Each time he returns to the start before running to the next cone.



Question 3

The sequence to show the distance to and from the n^{th} cone would be best described as

A. An arithmetic sequence with a = 60 and d = 20
B. An arithmetic sequence with a = 60 and d = 10
C. An arithmetic sequence with a = 30 and d = 10
D. A geometric sequence with a = 60 and r = 4/3
E. A geometric sequence with a = 60 and r = 3/4

Question 4

If A_n is the distance from the start to the n^{th} cone and back to the start, a difference equation that can be used to model the distance is

A. $A_{n+1} = 40 + 20n$ where $A_1 = 60$
B. $A_{n+1} = 2A_n + 20$ where $A_1 = 30$
C. $A_{n+1} = 2A_n + 10$ where $A_1 = 30$
D. $A_n+1 = A_n + 20$ where $A_1 = 60$
E. $A_{n+1} = A_n + 10n$ where $A_1 = 60$

Question 5

More cones are added at intervals of 10 metres. Find the minimum number of cones needed so that the total distance he runs is at least 2 kilometres.

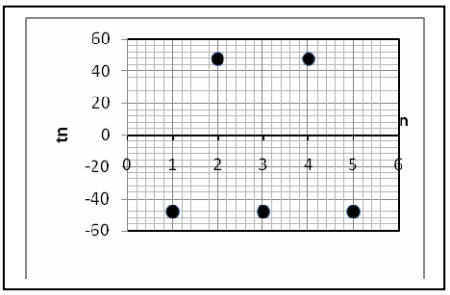
A. 9
B. 10
C. 11
D. 12
E. 13

Question 6

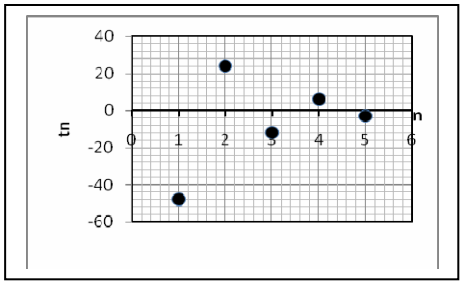
A geometric sequence follows the rule $t_n = 96\left(\frac{-1}{2}\right)^n$

A graph that represents the first five terms of this sequence is

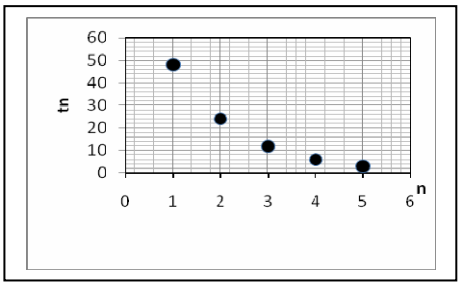
A.



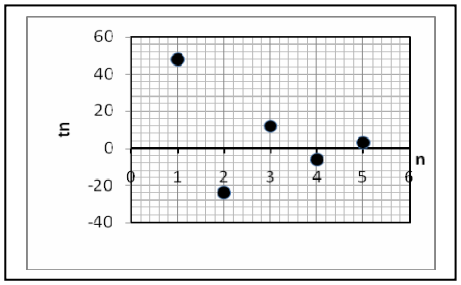
B.



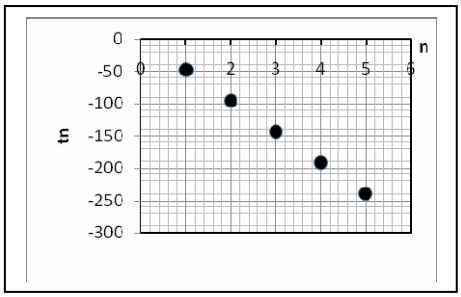
C.



D.



E.



Question 7

The sum of an infinite geometric series is $243/5$.

If the first term is 81, the common ratio value is

A. $2/3$

B. $-2/3$

C. $2/5$

D. $-8/3$

E. $-2/5$

Question 8

In the first year of its operation, an investment company has five properties on its portfolio. The aim of the company each year is to double the previous year's number of property investments minus one.

Let P_n represent the number of investment properties the company has in the *n*th year. The difference equation that describes the company's aims is

A. $P_n = 2P_{n+1} + 5$ where $P_1 = 5$

B. $2P_n = P_{n-1} - 1$ where $P_1 = 5$

C. $P_{n+1} = 2P_n - 1$ where $P_1 = 5$

D. $2P_{n+1} = 5P_n - 1$ where $P_1 = 5$

E. $P_{n+1} = 2P_n + 1$ where $P_1 = 5$

Question 9

A difference equation is defined by

$$t_n = t_{n-2} + 2t_{n-1}$$

Given that $t_9 = 746$ and $t_{10} = 1081$, the twelfth term (t_{12}) of this sequence is

A. 6897

B. 4344

C. 7332

D. 3293

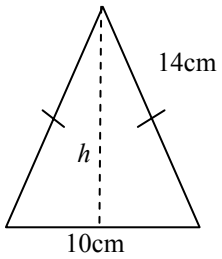
E. 2908

Module 2: Geometry and trigonometry

Before you answer these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

Question 1

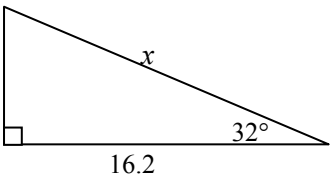
The isosceles triangle beside has a slant height of 14cm with a base of 10 cm.



The vertical height, h, to two decimal places is

A. 14.87 cm
B. 14.86 cm
C. 13.07 cm
D. 13.08 cm
E. 9.80 cm

Question 2



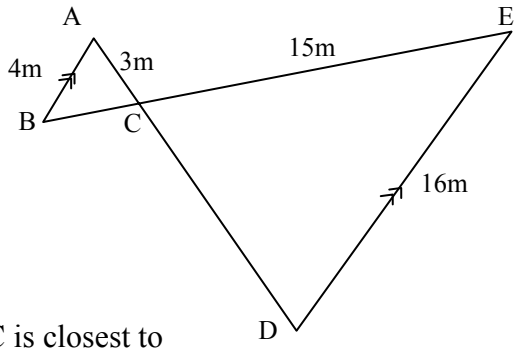
The value of x to one decimal place is

A. 8.5 m
B. 8.6 m
C. 13.7 m
D. 19.1 m
E. 30.6 m

Question 3

Beside are two similar triangles:

ABC and CDE

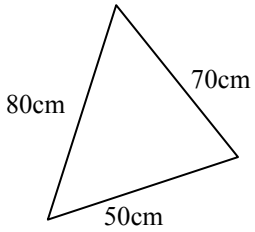


The area of ΔCDE is 84 m^2, the area of ΔABC is closest to

A. 5.25 m^2
B. 9.33 m^2
C. 21 m^2
D. 336 m^2
E. 1344 m^2

Question 4

A piece of carpet has the form of a scalene triangle with dimensions as shown beside.

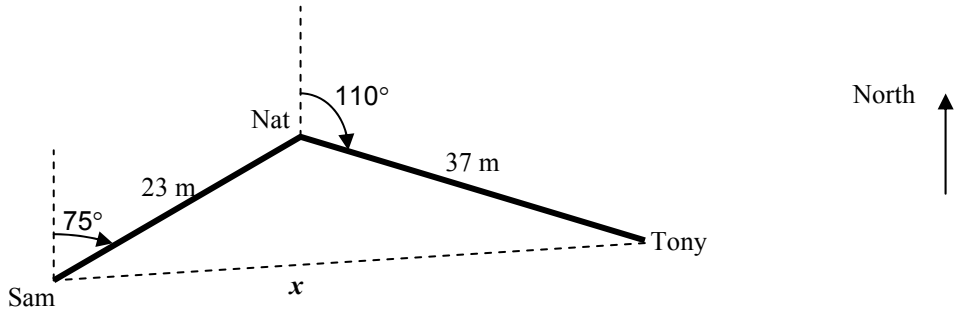


The area in square metres correct to two decimal places is

A. 1732.05 m^2
B. 0.17 m^2
C. 2.16 m^2
D. 17.32 m^2
E. 216.33 m^2

The following information relates to questions 5 and 6.

Sam, the hockey goalie, hits the ball to Nat 23 metres away at a bearing of 075°T. Nat then hits the ball downfield to Tony who is 37 metres away at 110°T.



Question 5

The true bearing of Sam from Nat is

A. 075°T
B. 075°T
C. 115°T
D. 075°T
E. 255°T

Question 6

The distance from Tony to Sam, x, can be found correctly by using which formula?

A. $x^2 = 23^2 + 37^2 - 2 \times 23 \times 37 \cos(250^\circ)$

B. $x^2 = 23^2 + 37^2 - 2 \times 23 \times 37 \cos(145^\circ)$

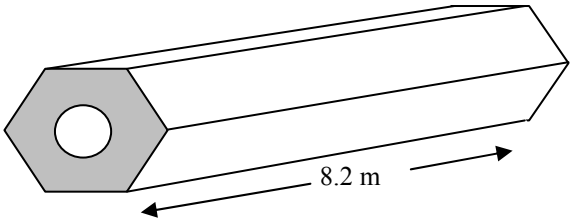
C. $x = 60 \times \cos(15^\circ)$

D. $x = \dfrac{\sin(145)}{\sin(15)} \times 37$

E. $x = \dfrac{\sin(145)}{\sin(75)} \times 23$

Question 7

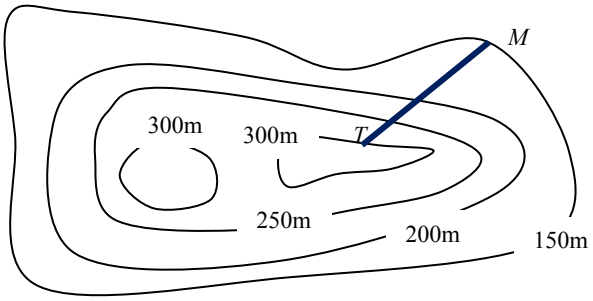
A steel hexagonal pipe is used as part of a modern art construction. The **volume** of this prism is 2.05 m^3, and it is 8.2 m long.



In square metres, the shaded cross-sectional area is closest to

A. 4.1 m^2
B. 0.0625 m^2
C. 0.25 m^2
D. 16.81 m^2
E. 70.644 m^2

Question 8



Scale 1: 20000

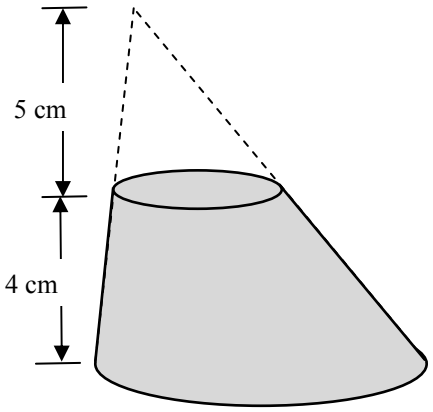
For the contour map above, the segment *MT* has a length of 2.5 cm on the map.

With aid of the scale the average gradient of the segment is

A. $\frac{2}{5}$
B. $\frac{1}{60}$
C. $\frac{3}{1000}$
D. $\frac{3}{10}$
E. $\frac{10}{3}$

Question 9

A skewed cone has the top sliced off in such a way that the top is parallel to the base.



The perpendicular height of the removed section is 5cm and the perpendicular height of the remaining section is 4cm.

The ratio of the **volumes** of the removed section to the remaining section is

A. 125 : 729
B. 125 : 604
C. 125 : 64
D. 5 : 9
E. 5 : 4

Module 3: Graphs and relations

Before you answer these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

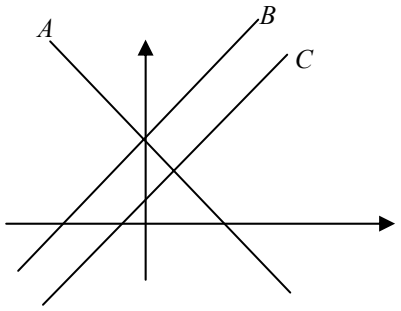
Question 1

The equation $2y - 3x = -12$ has intercept(s) of

A. (-6, 0) and (0, 4)
B. (2, 0) and (0, -3)
C. (4, 0) and (0, -6)
D. (-4, 0) and (0, -6)
E. (4, -6)

Question 2

Below is a graph with three lines *A, B* and *C* sketched. Unfortunately most labels and numerals have been omitted.



It is known that these lines belong to the following three equations:

i. $y = 2x + 4$
ii. $y - 2x = 1$
iii. $y + 2x = 4$

The correct match-up with equations and lines is

A. i $\rightarrow$ A; ii $\rightarrow$ B; iii $\rightarrow$ C
B. i $\rightarrow$ B; ii $\rightarrow$ C; iii $\rightarrow$ A
C. i $\rightarrow$ B; ii $\rightarrow$ A; iii $\rightarrow$ C
D. i $\rightarrow$ C; ii $\rightarrow$ B; iii $\rightarrow$ A
E. i $\rightarrow$ C; ii $\rightarrow$ A; iii $\rightarrow$ B

Question 3

Nic, an endurance athlete runner, starts at 200 km from his destination. He maintains a constant pace throughout his run and completes the distance in 25 hours of running.

The distance, D km, that Nic is from his destination at anytime t hours can be found according to the rule

A. $D = 8t$

B. $D = 200 - 25t$

C. $D = 200 - 8t$

D. $D = 25t$

E. $D = 200 + 25t$

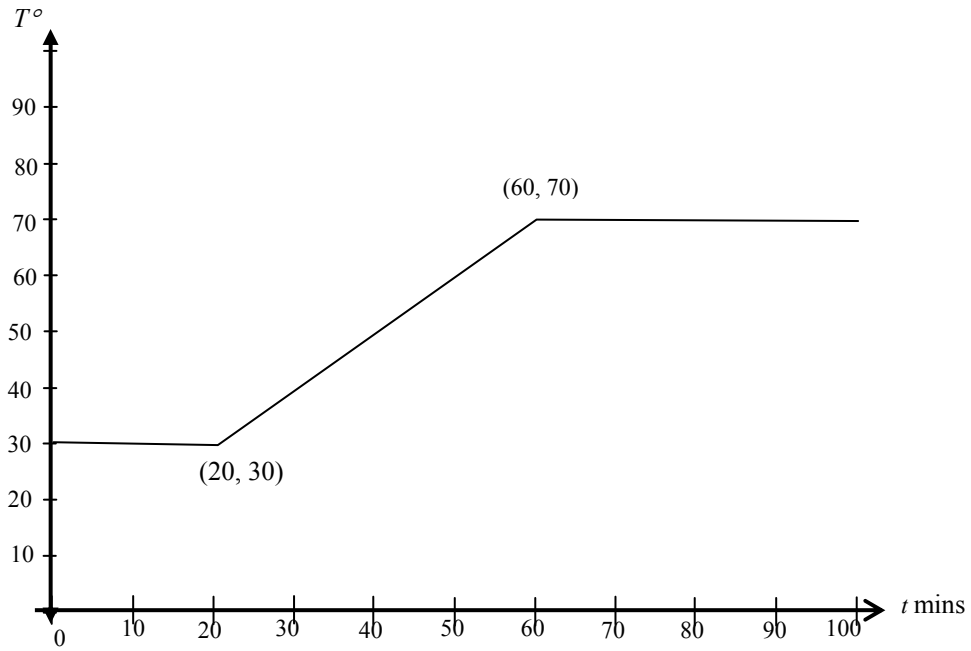
The following graph relates to Questions 4 and 5.

The temperature, $T°$ *Celsius*, of an oven is regulated over a normal working day.

For the first 20 minutes it is maintained at a constant temperature of 30°. For the next forty minutes it is gradually heated until it reaches a maximum temperature of 70°.

It maintains this temperature for the remainder of the working day.

The graph of temperature, T, versus time, t, is shown below.



Question 4

Which of the following statements is NOT true?

A. The rate of heating from 20 to 60 mins is 1° per min.

B. The temperature after 1.5 hours is 70°.

C. The maximum temperature for the working day is 70°.

D. In the first 80 minutes, the average gradient is 0.5° per min.

E. After 100 mins the oven is cooled.

Question 5

Which of the following rules best describes the graph above?

A. $T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ t + 10 & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}$

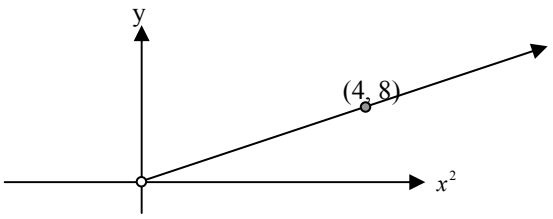
B. $T = \begin{cases} 20 & \text{for } 0 \le t \le 30 \\ t + 10 & \text{for } 30 < t \le 70 \\ 60 & \text{for } t > 70 \end{cases}$

C. $T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ t - 10 & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}$

D. $T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ \frac{7}{6}t & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}$

E. $T = \begin{cases} 30 & \text{for } 0 \le t \le 30 \\ t + 10 & \text{for } 30 < t \le 70 \\ 70 & \text{for } t > 70 \end{cases}$

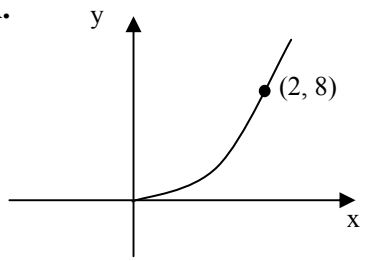
Question 6



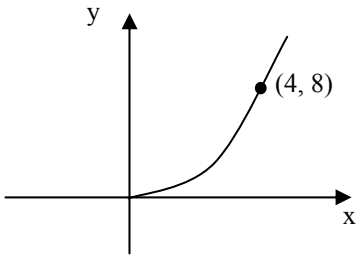
The graph connecting y and x^2 is shown above.

The graph that shows the same relationship between y and x is

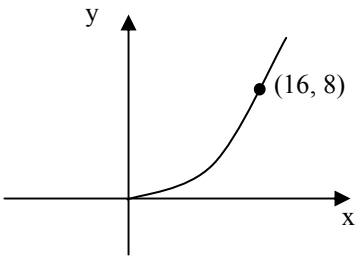
A.



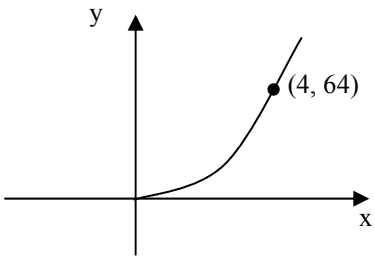
B.



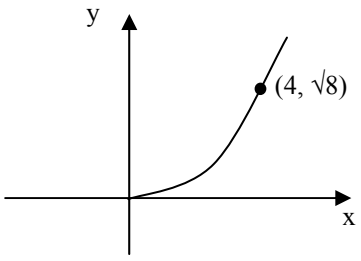
C.



D.



E.



Question 7

A freight container can take a maximum weight of 600 kg. It is to be loaded with two varieties of boxes. Box A weighs 20 kg and box B weighs 35 kg.

Due to the size of the container the maximum number of boxes it can hold is 25.

If A is the number of Box A, and B is the number of box B, the inequalities that best show the constraints are

A. $A + B \leq 55$; $A + B \leq 25$

B. $A + B \leq 545$; $A + B \leq 25$

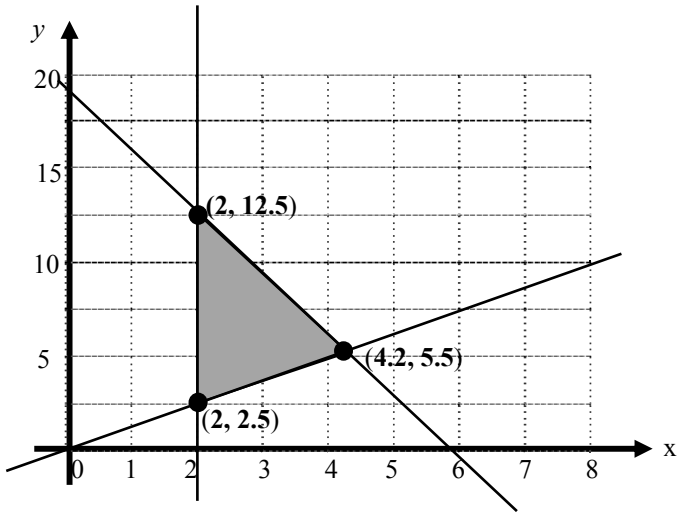
C. $A + B \leq 600$; $20A + 35B \leq 25$

D. $20A + 35B \leq 600$; $A + B \leq 25$

E. $20A + 35B \geq 600$; $A + B \geq 25$

Question 8

The shaded area below shows the feasible region.



The objective function, z, is given by the equation

$$z = 1.5x - 2y$$

Using the feasible region above the maximum value of z is

A. -22

B. -2

C. -4.7

D. 28

E. 22

Question 9

A linear programming problem has the following constraints.

$$y + 2x \leq 12$$
$$3y - 2x \geq 6$$
$$x \geq 3, y \geq 0$$

A point that lies in the feasible region is

A. (2, 5)
B. (2, 3)
C. (4, 0)
D. (3, 2)
E. (3, 5)

Module 4: Business-related mathematics

Before you answer these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

$5000 is invested at a simple interest rate of 4% per annum.

The total amount after 30 months is

A. $5 515.10
B. $5 500
C. $600
D. $500
E. $375

Question 2

In April, Lyn received the following statement from her bank showing all the transactions from her savings for the month of March.

Date	Transaction	$ Debit	$ Credit	$ Balance
1 Mar	Balance forward			985.90
9 Mar	Withdrawal	234.00		751.90
18 Mar	Deposit		400.00	1 151.90

Interest is calculated on the minimum monthly balance. If the amount of interest Lyn received for March was $1.50; the rate of interest, correct to 1 decimal place, was

A. 0.2% p.a.
B. 1.6% p.a.
C. 1.9% p.a.
D. 2.4% p.a.
E. 28.8% p.a.

Question 3

Furniture is valued at $12 000. On a hire purchase plan, it can be purchased with a deposit of $1 000.

The total paid for the furniture over 4 years on this plan is $16 000 which includes the deposit.

The flat rate of interest p.a. charged on hire purchase correct to 2 decimal places is

A. 33.33%
B. 36.36%
C. 10.00%
D. 8.33%
E. 9.09%

Question 4

Lyn invested $15 5000 in an ordinary perpetuity. The interest rate for the investment is 7.4% per annum.

The amount of pension she receives monthly from this perpetuity is closest to

A. $3 600
B. $621
C. $956
D. $11 470
E. $13 870

Question 5

Nic needs $4 500 in two years' time. He invests into an account that is advertised at 7.8%p.a. with interest compounded **daily**.

The original amount he needs to invest to the nearest dollar is

A. $3 850
B. $3 872
C. $3 889
D. $5 229
E. $5 260

Question 6

For tax purposes a person uses reducing balance depreciation of 15% to calculate the book value of a photocopier. After 2 years the book value of the copier is $4 600.

The original value of the photocopier was approximately

A. $5 980
B. $6 080
C. $6 370
D. $6 570
E. $204 440

Question 7

Stamp Duty is payable to the state government on a property transaction according to the following rate schedule.

Transfer of Real Property rates	
Range	**Rate**
\$0 – \$20 000	1.4 per cent of the price of the property
\$20 001 – \$115 000	\$280 plus 2.4 per cent of the price in excess of \$20 000
\$115 001 – \$870 000	\$2 560 plus 6 per cent of the price in excess of \$115 000
More than \$870 000	5.5 per cent of the price value

An owner pays stamp duty of \$18 000.

The price of the property lies within the range

A. \$0 – \$20 000
B. \$20 001 – \$115 000
C. \$115 001 – \$870 000
D. More than \$870 000
E. Over \$1 million

Question 8

Joan borrows \$12 000 and makes monthly repayments of \$400.

The interest rate is 8% p.a. calculated monthly on the reducing balance of the loan.

Find the total amount of interest Joan has paid, immediately after making her **tenth** payment, to the nearest dollar.

A. \$16
B. \$702
C. \$800
D. \$824
E. \$4 947

Question 9

Torie invests a sum of \$3 200 into an account earning an interest rate of 7.2%p.a. compounded monthly. Into the same account she also decides to make monthly installments of \$60.

After 2 years the **total** Torie has in her account is closest to

A. \$2 150
B. \$3 359
C. \$5 134
D. \$5 178
E. \$5 238

Module 5: Networks and decision mathematics

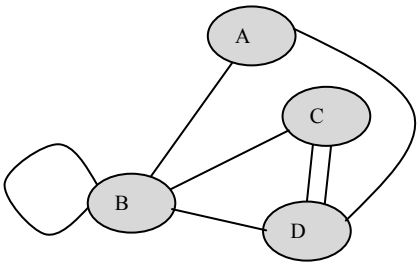
Before you answer these questions you must **shade** the Networks and decisions mathematics box on the answer sheet for multiple-choice questions.

Question 1

The number of edges on a COMPLETE network with six vertices is

A. 5
B. 12
C. 15
D. 21
E. 25

Question 2



An adjacency matrix to represent the network is:

A.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

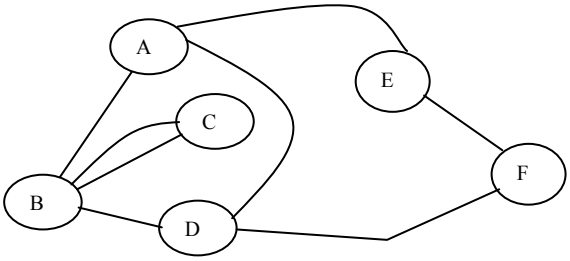
B.
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

D.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

E.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Question 3

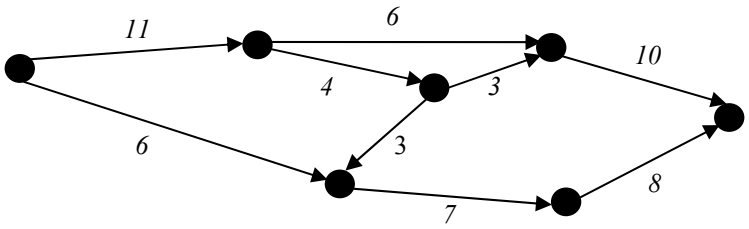


To make this Network contain an Euler circuit, an edge needs to be added connecting the following vertices:

A. A to D
B. A to F
C. B to D
D. B to F
E. C to D

Question 4

The following directed graph shows the weight of each edge.

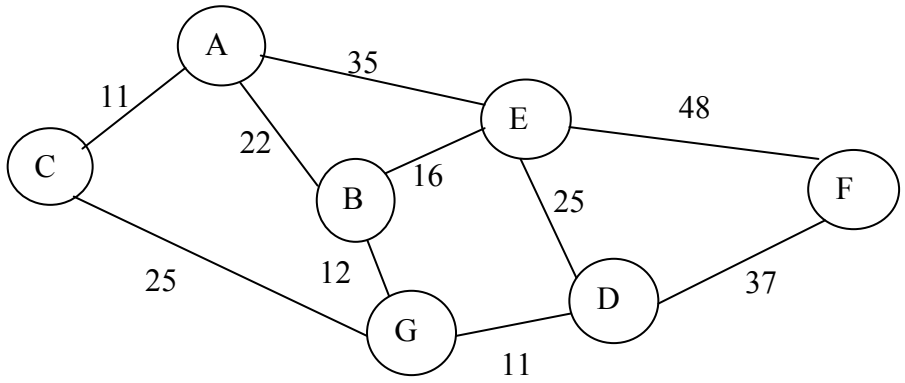


The minimum cut for this directed graph is

A. 18
B. 17
C. 16
D. 15
E. 14

Questions 5 and 6 refer to the following network.

For a shire, the graph below shows the major towns A, B…G connected by the main roads. All distances are in kilometres.



Question 5

The sporting board for this shire wishes to organise a bike road race to be completed in a circuit. The minimum distance a circuit could be raced is

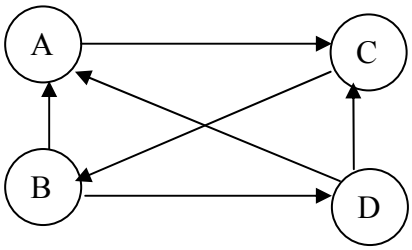
A. 73 km
B. 58 km
C. 71 km
D. 69 km
E. 64 km

Question 6

The distance for the minimum spanning tree for this network is

A. 109 km
B. 112 km
C. 118 km
D. 93 km
E. 73 km

Question 7



In a round robin competition 4 teams A, B, C and D play each other once.

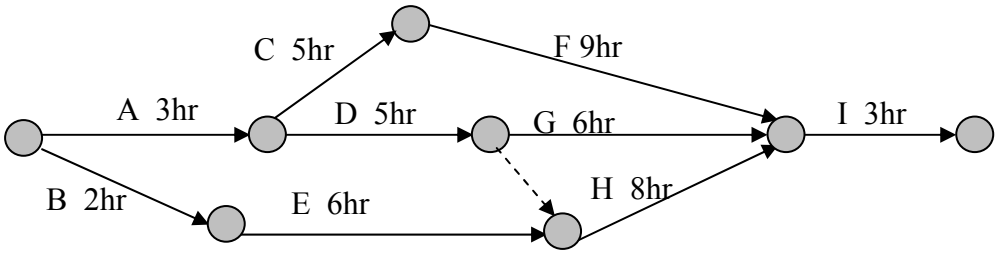
An arrow from A to C indicates that A defeated C.

Based on 1-step and 2-step reachability matrix for this network, state the winning team and their dominance value.

A. Team B, 4
B. Team B, 5
C. Team D, 4
D. Team D, 5
E. Team B and D, both 4

Questions 8 and 9 refer to the following critical path.

For a particular project there are nine activities to be completed and the time taken to complete each activity is shown in hours.



Note: GH is a dummy activity

Question 8

The critical path and completion time for this project is

A. BEHI, 19 hours
B. ADGI, 17 hours
C. ACFI, 20 hours
D. ADGHI, 19 hours
E. AEHF, 26 hours

Question 9

The project is to be crashed by reducing the completion time to a time of 18 hours.

This can be done in the minimum number of hours by reducing the following activities:

A. F by 2 hours

B. F by 2 hours and H by 1 hour

C. A by 2 hours

D. A by 2 hours and F by 1 hour

E. No reduction but increase D by 1 hour making it the quickest time

Module 6: Matrices

Before you answer these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions.

Question 1

Given that $A = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

The value of $A^2 + 2B$ is

A. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ **B.** $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 10 \\ -6 \end{bmatrix}$ **D.** $\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

E. Impossible, no solutions

Question 2

Let $C = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Which of the following is **NOT** true?

A. I raised to any power is always I.
B. There is no inverse matrix of C.
C. $C + I = I + C$
D. $CI = C$
E. $2C - I$ gives a 2×2 matrix

Question 3

Let $R = \begin{bmatrix} 1 & m \\ 0 & n \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

If the matrix $RT = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$,

the values of m and n respectively must be

A. 1 and -1
B. 2 and 1
C. 2 and $5/4$
D. -1 and 2
E. -1 and -2

Question 4

Nathan decided to plan a course for his class to visit five locations A, B, C, D and E in the area near his school, by using the following transition matrix:

Present Location

Next Location	A	B	C	D	E
A	0	0	1	0	0
B	1	0	0	0	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	1	0	0	0

If the class know that they start from location C, the correct order of the course to visit each location using the transition matrix is

A.

C	D	E	B	A

B.

C	A	B	D	E

C.

C	A	B	E	D

D.

C	D	E	A	B

E.

C	A	C	A	B

Question 5

A company produces a four cylinder car (F) and a six cylinder car (X). Each type of car comes in three styles: Economy (E), Sport (S) and Luxury (L).

The price of each make of car, in thousands of dollars, is listed in a price matrix P, where

$$P = \begin{matrix} \\ E \\ S \\ L \end{matrix} \begin{matrix} \begin{matrix} F & X \end{matrix} \\ \begin{bmatrix} 21 & 31 \\ 28 & 34 \\ 32 & 40 \end{bmatrix} \end{matrix}$$

Due to inflation and extra taxes, a new price matrix generating the new price of cars can be found by the product PQ where Q is the matrix:

$$\begin{bmatrix} 1.05 & 0 \\ 0 & 1.08 \end{bmatrix}$$

This new pricing scale is best summarised as

A. Economy cars increase by 5%, Sport cars increase by 8%, Luxury stay fixed

B. Economy cars increase by 5%, Luxury cars increase by 8%, Sport stay fixed

C. Four cylinder cars increase by 5% and six cylinder cars increase by 8%

D. Four cylinder cars increase by 1.05% and six cylinder cars increase by 1.08%

E. All cars increase by 5% and a further 8% on top of this

Question 6

How many of the following four sets of simultaneous linear equations have a unique solution?

Set A	Set B	Set C	Set D
$x - y = 3$ $x + y = 1$	$2x + 2y = 12$ $2x - y = 6$	$x - 2y = 0$ $-2x + 4y = 10$	$x + y = 5$ $5x = 10$

A. 0
B. 1
C. 2
D. 3
E. 4

Question 7

The order of matrix A is (3×2); and the number of columns of matrix B is 3.

If the result of A(B + C) is a 3×3 matrix, then the order of matrix C must be

A. (2×3)
B. (3×2)
C. (2×2)
D. (3×3)
E. (1×3)

Question 8

The solution to the following simultaneous equations

$$-2x + y = -7$$
$$x - 2y = 5$$

is:

A. $x = 19$ and $y = -17$
B. $x = 3$ and $y = -1$
C. $x = 2$ and $y = -3$
D. $x = 1$ and $y = -2$
E. $x = 9$ and $y = -3$

Question 9

A snack company specialises in making a chocolate bar (*B*) and a herbal flavoured bag of chips (*C*).

Research has found that 85% of the people that buy the chocolate bar (*B*) return to buy it again the following day and the rest decide to buy the bag of chips (*C*).

Of those who buy the bag of chips (*C*), 80% of people return to buy it again the following day and the rest decide to buy the chocolate bar (*B*).

Originally the company made 300 chocolate bars and 400 bags of chips daily.

If the research is correct and the people's buying patterns are maintained, the number of each product that the company should make in the **long term** is

A. 300 of *B* and 400 of *C*
B. 335 of *B* and 365 of *C*
C. 350 of *B* and 350 of *C*
D. 380 of *B* and 320 of *C*
E. 400 of *B* and 300 of *C*

END OF QUESTION AND ANSWER BOOK

Insight Publications

INSIGHT

Trial Exam Paper

2008

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines

SECTION A

Core – Data Analysis

Questions 1 and 2 refer to the following information.

A dice was rolled 20 times with the following results shown on a frequency table.

Number on die	Frequency
1	2
2	5
3	3
4	2
5	4
6	4

Question 1

The percentage of rolls that resulted in an even number was

A. 9%
B. 11%
C. 24%
D. 50%
E. **55%**

Answer is E

Worked solution

- The total times an even number occurred using freq is 5 + 2 + 4 = 11.

$$\frac{11}{20} \times 100 = 55\%$$

(total items)

Question 2

The mode and mean respectively for this data was

A. 4 and 3.65
B. 4 and 3.5
C. **2 and 3.65**
D. 2 and 3.5
E. 3.5 and 4

Answer is C

Worked solution

- Mode: The score with the highest frequency is 2; occurring 5 times
- Mean: Using calc STATS 1VarStats L_1,L_2
  (*Where L_1 is number on the dice and L_2 is frequency)*
  $\bar{x} = 3.65$
- Note: If you obtained answer of 3.5 for the mean, then the scores have a frequency of one only – a common error, hence the need for L_2.

Question 3

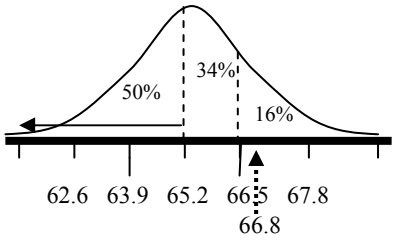
A statistically minded athlete has evaluated her 400 metre efforts. She has noticed that the data shows a bell shaped distribution with a mean of 65.2 seconds and a standard deviation of 1.3 seconds.

If she then runs a time of 66.8 seconds, which of the following statements is closest to summarising her effort?

A. **This time is in the top 16% of her efforts.**
B. This time is in the bottom 16% of her efforts.
C. This time is close to the average of her efforts.
D. This time is in the top 2% of her efforts.
E. This time is in the bottom 2% of her efforts.

Answer is A

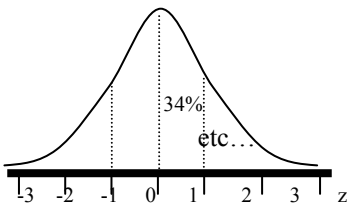
Worked solution



- 68% lies within $\bar{x} \pm s$, i.e. 63.9 to 66.5 s .Of the remaining 32%, 16% lies above this area, 16% below. The time of 66.8 is within the top 16%
- (Note 95% lie within $\bar{x} \pm 2s$, that is, from 62.6 up to 67.8).

Tip

- *You should include in your notes a bell shaped curve with appropriate % as follows. It will assist with questions as above but with 'standardising' questions as well (Like Qn5 below).*



standard deviations from the mean.

Alternatively for Q3 using

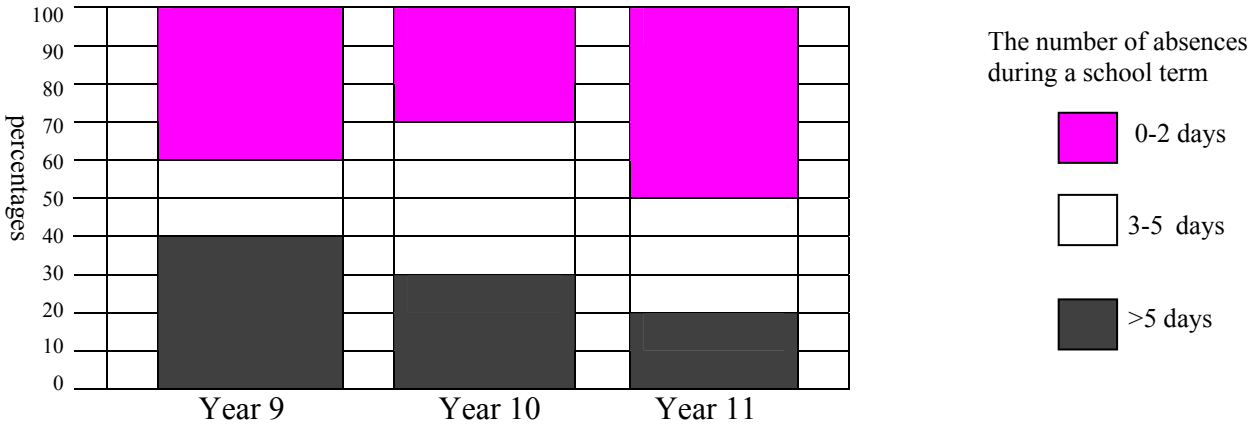
Use $z = \dfrac{x - \bar{x}}{sd}$

$z = \dfrac{66.8 - 65.2}{1.3} = 1.23$

$\therefore z \geq 1$ which means the top 16%

Question 4

The segmented bar chart below shows the distribution of the number of days absent over one term for secondary students at particular year levels.



From these results, the percentage of Year 11 students who were absent for at least 3 days in the term is

A. 20
B. 30
C. **50**
D. 70
E. 100

Answer is C

Worked solution

- At least 3 days means combining the two segments 3-5 and >5 days for the Year 11 bar chart
  i.e. 30% + 20% = 50%.

Tip

- *There can be some confusion between 'more than' and 'at least', so have an example of each wording to help clarify the difference.*

Question 5

Joan received a test score of 72. The class standard deviation for this test was 12. If Joan's z-score was 1, the class average was

A. 13
B. 73
C. 84
D. **60**
E. 71

Answer is D

Worked solution

- Use $z = \frac{x - \bar{x}}{sd}$ to standardise marks

$$\frac{72 = \bar{x}}{12} = 1$$

$\Rightarrow \quad 72 - \bar{x} = 12$ (multiply 12 both sides)
$\Rightarrow \quad -\bar{x} = -60$ (subtract 72 both sides)
$\Rightarrow \quad \bar{x} = 60$ (divide by -1 both sides)

Tip

- *These z values are the standard deviations from the class average. They will give you an idea of their 'ranking' compared to the whole class, e.g. something like this table may assist if the question is trying to rate a mark in comparison with the class results.*

Z values	-2	-1	0	1	2
% rank in class	2.5%	16%	50% Class average	84% (top 16%)	97.5% (top 2.5%)

Questions 6 and 7 refer to the following information.

A group of 14 people were given a page to type and the number of errors they had was recorded in the PRE column below.

After an extensive two hour program on keyboard skills the group were given another page to type, similarly these errors were also recorded, in the POST column below.

PRE	Stem	POST
7, 6, 3	0	1, 4, 5, 8, 8, 9
7, 7, 3, 0	1	0, 0, 5, 8
8, 5, 1, 1, 0	2	1, 2, 4
4, 0	3	0

Question 6

From this back to back stem leaf plot summary, which of the following observations is true?

	PRE		POST	
	Median	Inter-quartile range	Median	Inter-quartile range
A.	13	17	10	17
B.	**18.5**	**15**	**10**	**13**
C.	17	31	8	29
D.	17	17	13.2	13
E.	18.5	15	8	17

Answer is B

Worked solution

- Median: There are 14 values, use the ($n+1/2$ rule) $\therefore$ median = $14+1/2 = 7.5th$ value. This means there are 7 in each half. The 7th score for PRE is 17 (*POST* 10) and the 8th score is 20 (10), $\therefore$ median is 18.5 (10) (halfway between the 7th and 8th scores).
- Likewise to find the IQR, you need the quartiles which would be the 4th and 11th terms of each set.
- Note: The use of calculator could be handy to answer this question but be wary of keeping a 2 min check.
- You may need to have an example to clarify the differences between **mean**, **mode** and **median**.

Question 7

The data for PRE and POST results respectively can be summarised as

A. both are continuous variables with symmetrical and negative skewing.

B. both are continuous variables with symmetrical and positive skewing.

C. both are discrete variables with symmetrical and negative skewing.

D. **both are discrete variables with symmetrical and positive skewing.**

E. both are categorical data sets with symmetrical and positive skewing.

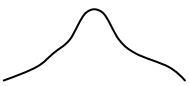
Answer is D

Worked solution

- All data is in whole numbers meaning they are ***discrete*** variables.
- PRE data is almost symmetrical whereas the number of POST data errors decline as they increase.

SHAPES of PLOTS SKEWNESS

Symmetrical Positively Skewed Negatively Skewed



Question 8

The goals scored by a soccer team in a month are shown in the table below:

Month	May	June	July	August	Sept	Oct	Nov	Dec
Total Goals	12	15	12	13	8	19	11	14

Using 4 point moving medians, the smoothed centred value for Sept is

A. 8

B. 12.875

C. 12

D. 12.5

E. **12.25**

Answer is E

Worked solution

July	August	Sept	Oct	Nov
12	13	8	19	11

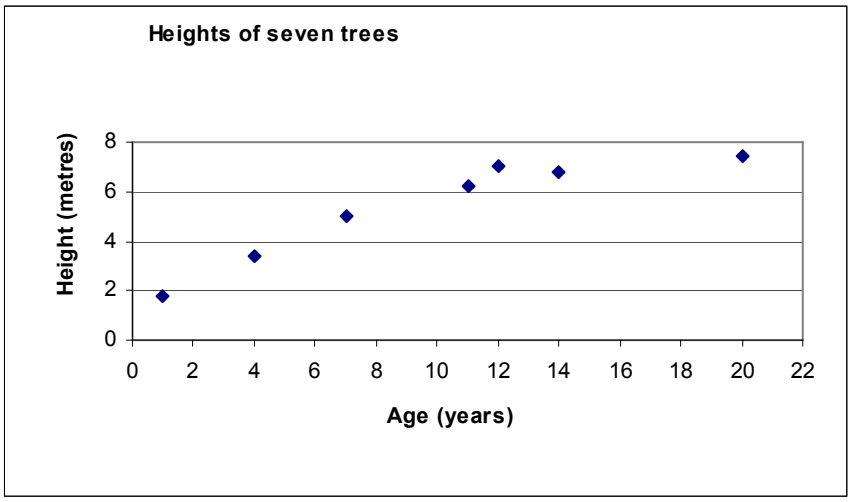
In order 8 12 13 19 8 11 13 19
Median 12.5 12
Centred 12.25

- When calculating moving medians or means using an even number of points requires **two** steps.
- Note: If you obtained an answer of B, the **mean** was used not the **median**.

Question 9

The ages and heights of seven exotic species of trees were recorded in the table and shown on the scatterplot below.

Age (years)	1	4	7	11	12	14	20
Height (metres)	1.8	3.4	5.0	6.2	7.0	6.8	7.4



It was found that an error was transcribed for the **11 year old** tree. The actual height of the 11 year old tree is more than 7.5 metres.

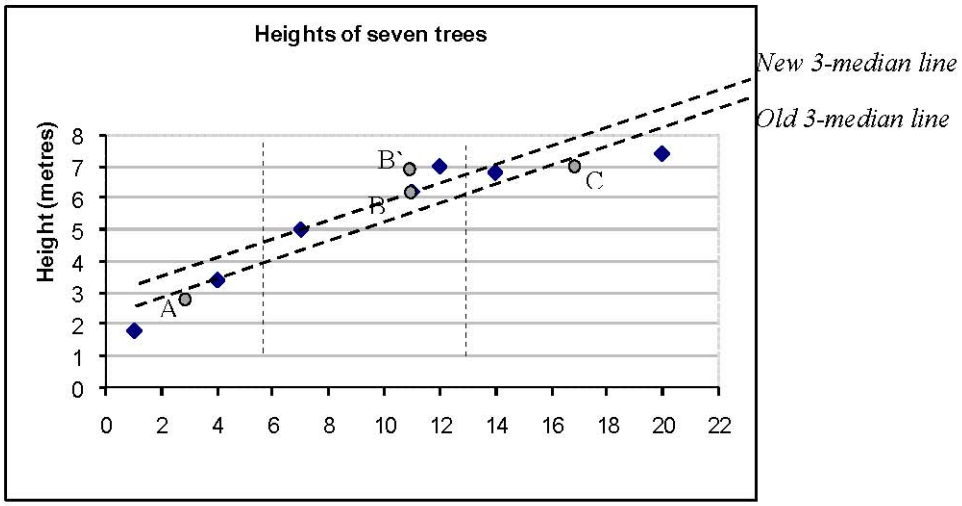
Knowing that it had the **largest** height of all these trees what effect would this new value have on the previously calculated values of the 3-median regression line?

A. The regression line would not be affected.
B. The gradient will increase but the vertical intercept remains the same.
C. The gradient and the vertical intercept both increase.
D. **The gradient remains the same but the vertical intercept increases.**
E. Impossible to calculate because of missing data.

Answer is D

Worked solution

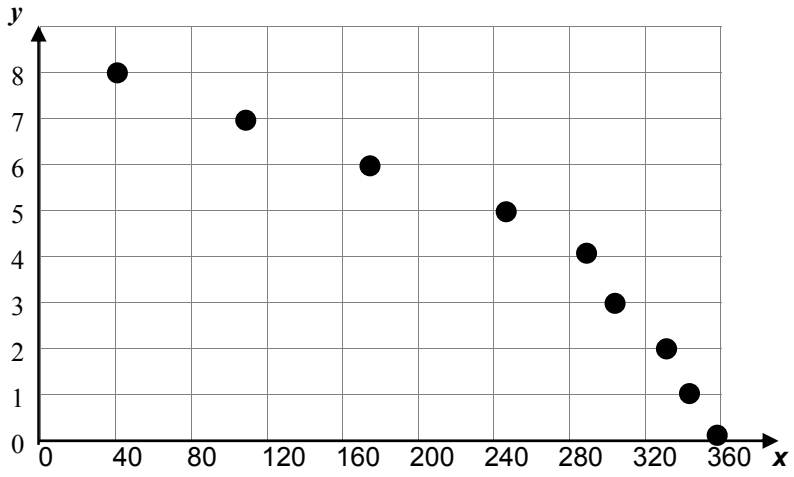
- Split the data up into three groups: using 2-3-2 points
- The median of these three groups is found but note the **gradient** is determined by the end median groups of points (Points A and C below).
- The error occurred in the middle group so this will **not** affect the gradient. However, the median '*y*' value of the middle group of three will increase from 6.2 to 7 (Point B to B` below) since we know the new height will be the maximum value. This has the effect of increasing the vertical intercept since the gradient line will need to be shifted further upward.



Questions 10 and 11 refer to the following data.

Nathan uses the following data to construct a scatterplot as shown below.

x	360	345	330	304	285	245	172	110	40
y	0	1	2	3	4	5	6	7	8



Question 10

To linearise the data a y^2 transformation is applied. The least squares regression line is closest to

A. **$y^2 = 71.88 – 0.20x$**

B. $y^2 = 9.68 – 0.02x$

C. $y^2 = 8.04 – 0.01x$

D. $y^2 = 400.78 – 35.33x$

E. $y^2 = 355.11 – 4.53x$

Answer is A

Worked solution

- The regression model $y = a + bx$
- A calculator exercise. It must be noted that *x* is the independent term and that *y* values must be squared before you perform LinReg a + bx on the calculator.
  e.g. If x values are in L_1, y values in L_2 then let $L_3 = L_2^2$ (i.e. y^2)
  then use Stat Calc: 8 L_1, L_3

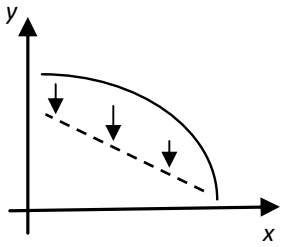
Question 11

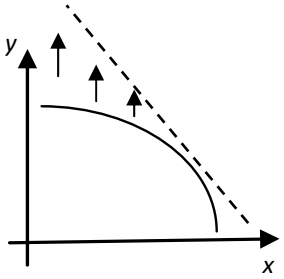
Which of the following best describes why Nathan chose to use the y^2 transformation?

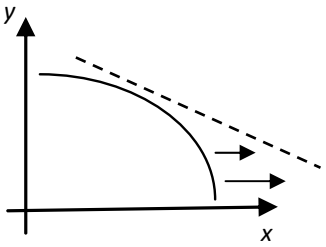
A. y^2 compresses the vertical scale, intending to linearise the data.

B. **y^2 expands the vertical scale, intending to linearise the data.**

C. y^2 expands the horizontal scale, intending to linearise the data.

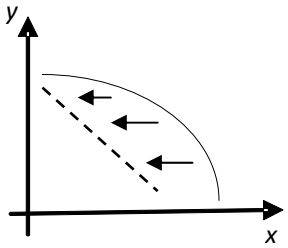


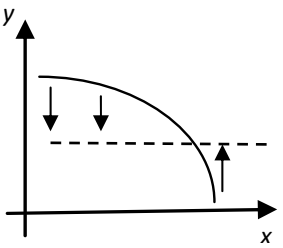




D. y^2 compresses the horizontal scale, intending to linearise the data.

E. y^2 both expands and compresses along the median of the vertical scale, intending to linearise the data.





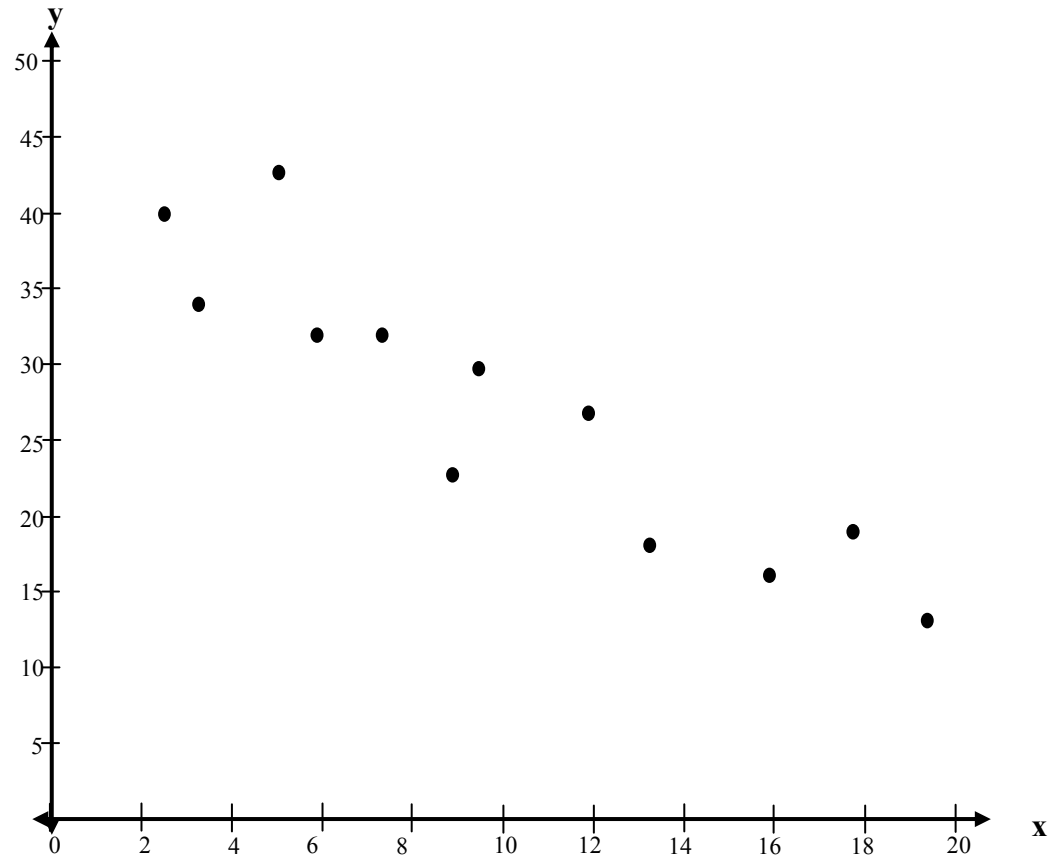
Answer is B

Worked solution

- y^2 will only affect the VERTICAL scale (hence eliminating C and D options). Apart from y= 0, 1 (which remain the same) y^2 values have expanded, becoming 0 1 4 9 16 25 36 49 64. This will tend to 'straighten' the original curve as depicted in option B.

Question 12

Data collected has been summarised on the following graph.



The coefficient of determination for the above data is 0.8118. This would indicate that the linear association between both variables has a Pearson coefficient value of

A. 0.9010
B. **-0.9010**
C. 0.6590
D. -0.6590
E. 2.0000

Answer is B

Worked solution

- The coefficient of determination is 'r^2'. We need to find the value of 'r', the Pearson product moment correlation coefficient.
- Using calculator*:* $r = \sqrt{0.8118}$, which is 0.9010. **However**, the calculator only calculates the positive square root value. You need to recognise that it is a negative association (i.e. as x increases generally the value of y decreases) so therefore $r = -0.9010$
- (Note: The answer C is 0.8118^2 showing confusion between the ***correlation coefficient*** and the ***coefficient of determination***.)

Question 13

The table below shows the number of sales for a particular firm during 2007.

Quarter	1	2	3	4
Sales for 2007	630	840	1 005	980
Seasonal Index	0.67	0.95	1.18	1.20

Using all data including previous years` sales, the regression line was found to be:

$y = 560 + 120x$ where y is the Number of Sales
and x is the Number of Quarters from end of 2006

(Note: Quarter 1 in 2007 will mean $x = 1$)

The predicted deseasonalised figure for Quarter 1 in 2008 would be

A. 1 160
B. 780
C. 1 164
D. 3 400
E. **1 731**

Answer is E

Worked solution

- To find the Predicted sales Q1 2008 (i.e. 5 quarters) use the regression line:
  Sales $= 560 + 120 \times 5$
  $= 1\ 160$
- Deseasonalised figure = Actual (predicted)/Index (This is on the formula sheet)
  $= 1\ 160/0.67$
  $= 1\ 731$ (must be whole number)

END OF SECTION A

SECTION B

Module 1: Number Patterns

Question 1

An arithmetic sequence is

36 32 28 …

Which term would give a value of -4?

A. 7
B. 9
C. 10
D. **11**
E. 12

Answer is D

Worked solution

- Arithmetic term use $t_n = a + (n - 1)\,d$
  This gives
  $-4 = 36 + (n - 1)(-4)$ (note: Use of Calculator **solver** could be used as well)
  $-40 = -4n + 4$ (subtract 36 both sides and expand bracket)
  $-44 = -4n$ (subtract 4 both sides)
  $n = 11$ (divide -4 both sides)

Question 2

The first four terms of a geometric sequence are { -12, ***x***, -3, 1.5 … }.

The value of ***x*** for this sequence is

A. 7.5
B. -7.5
C. 9
D. 6 or -6
E. **6**

Answer is E

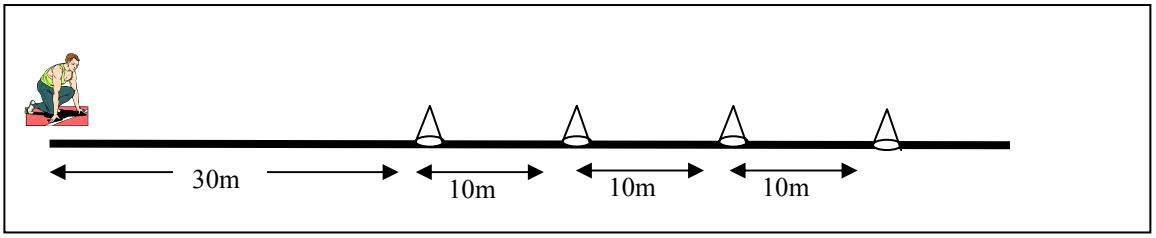
Worked solution

- It is a Geometric Sequence (has a common ratio): Using 3rd and 4th terms r = 1.5/-3
  $\therefore$ r = -0.5
  $\boldsymbol{x} = -12 \times -0.5 = 6$

The following information relates to questions 3, 4 and 5.

As part of his stamina training, Nic places 4 cones in a straight line, each 10 metres apart. Nic starts 30 metres from the first cone.

He runs to the first cone then returns back to the start before then running to the second, then third and finally fourth. Each time he returns to the start before running to the next cone.



Question 3

The sequence to show the distance to and from the n^{th} cone would be best described as

A. **An arithmetic sequence with a = 60 and d = 20**
B. An arithmetic sequence with a = 60 and d = 10
C. An arithmetic sequence with a = 30 and d = 10
D. A geometric sequence with a = 60 and r = 4/3
E. A geometric sequence with a = 60 and r = 3/4

Answer is A

Worked solution

- Continuing the sequence for all 4 cones we would get {60, 80, 100, 120}
  It shows a constant (*d*) increase of 20, hence it is arithmetic with the first term (*a*) being 60 metres.

Question 4

If A_n is the distance from the start to the n^{th} cone and back to the start, a difference equation that can be used to model the distance is

A. $A_{n+1} = 40 + 20n$ where $A_1 = 60$
B. $A_{n+1} = 2A_n + 20$ where $A_1 = 30$
C. $A_{n+1} = 2A_n + 10$ where $A_1 = 30$
D. **$A_n+1 = A_n + 20$ where $A_1 = 60$**
E. $A_{n+1} = A_n + 10n$ where $A_1 = 60$

Answer is D

Worked solution

- There is a constant increase of 20 from the previous term:
  $\therefore$ The next term $A_{n+1} = A_n + 20$, the previous term plus 20
- Options B and C can be eliminated as $A_1 = 30$, which is only half way.

Question 5

More cones are added at intervals of 10 metres. Find the minimum number of cones needed so that the total distance he runs is at least 2 kilometres.

A. 9
B. 10
C. 11
D. **12**
E. 13

Answer is D

Worked solution

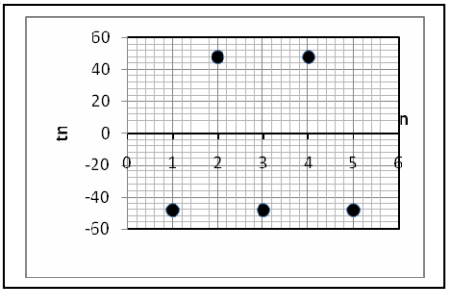
- Using trial and error approach
  Use $S_{20} = \frac{n}{2}(2a + (n-1)d)$
  $S_{12} = \frac{12}{2}(2 \times 60 + 11 \times 20) = 2040 metres$
  Therefore he needs at least 12 cones to run over 2000 metres.
- Note: solver on calculator could be used: $0 = \frac{x}{2}(2 \times 60 + (x-1) \times 20) - 2000$

Question 6

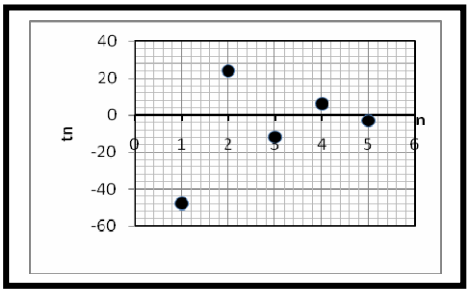
A geometric sequence follows the rule $t_n = 96\left(\frac{-1}{2}\right)^n$

A graph that represents the first five terms of this sequence is

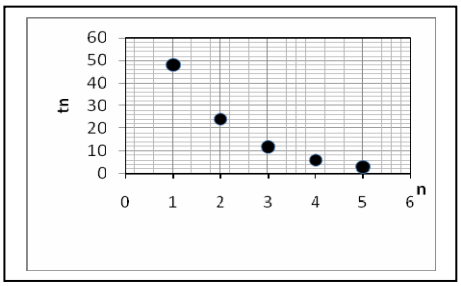
A.



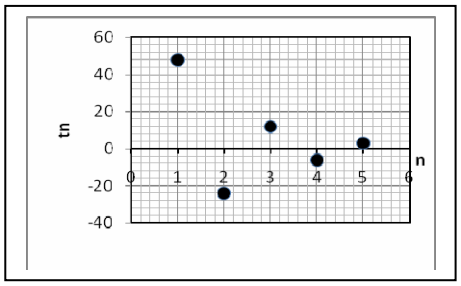
B.



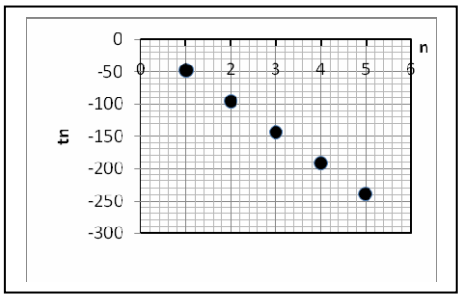
C.



D.



E.



Answer is B

Worked solution

$$t_n = 96\left(\frac{-1}{2}\right)^n$$

- Using a table for various n values gives us:

n	1	2	3	4	5
t_n	$96\left(\frac{-1}{2}\right)^1 = 96\times\frac{-1}{2}$ = -48	$96\left(\frac{-1}{2}\right)^2 = 96\times\frac{1}{4}$ = 24	$96\left(\frac{-1}{2}\right)^3 = 96\times\frac{-1}{8}$ = -12	$96\left(\frac{-1}{2}\right)^4 = 96\times\frac{1}{16}$ = 6	$96\left(\frac{-1}{2}\right)^5 = 96\times\frac{-1}{32}$ = -3

B is the only graph that fluctuates this way.

- Also note that if n is an odd number, t_n is **negative** and if n is an even number, t_n is **positive** hence giving this fluctuation.

Question 7

The sum of an infinite geometric series is $243/5$.

If the first term is 81, the common ratio value is

A. $2/3$

B. $\mathbf{-2/3}$

C. $2/5$

D. $-8/3$

E. $-2/5$

Answer is B

Worked solution

- For an infinite geometric series

$$s_\infty = \frac{a}{1-r}$$

$$\therefore \quad \frac{243}{5} = \frac{81}{1-r}$$

$$\frac{1-r}{81} = \frac{5}{243} \qquad \text{(flip both sides so that the unknown is a numerator)}$$

$$1-r = \frac{5}{243} \times 81 \qquad \text{(multiply 81 both sides)}$$

$$1-r = \frac{5}{3} \qquad \text{(cancel down)}$$

$$-r = \frac{2}{3} \qquad \text{(subtract 1 from both sides)}$$

$$r = \frac{-2}{3} \qquad \text{(divide by -1 both sides)}$$

Question 8

In the first year of its operation, an investment company has five properties on its portfolio. The aim of the company each year is to double the previous year's number of property investments minus one.

Let P_n represent the number of investment properties the company has in the *n*th year. The difference equation that describes the company's aims is

A. $P_n = 2P_{n+1} + 5$ where $P_1 = 5$

B. $2P_n = P_{n-1} - 1$ where $P_1 = 5$

C. $\mathbf{P_{n+1} = 2P_n - 1}$ **where** $\mathbf{P_1 = 5}$

D. $2P_{n+1} = 5P_n - 1$ where $P_1 = 5$

E. $P_{n+1} = 2P_n + 1$ where $P_1 = 5$

Answer is C

Worked solution

- 1^{st} year is 5; the next year is $2 \times 5 - 1 = 9$ … then $2 \times 9 - 1 = 17$
  The sequence generated by this first order difference equation is:
  {5, 9, 17, 33 …} e.g. $P_2 = 2 \times P_1 - 1$ $P_3 = 2 \times P_2 - 1$
  Hence using this pattern
  $P_{n+1} = 2P_n - 1$

Question 9

A difference equation is defined by

$$t_n = t_{n-2} + 2t_{n-1}$$

Given that $t_9 = 746$ and $t_{10} = 1081$, the twelfth term (t_{12}) of this sequence is

A. **6897**
B. 4344
C. 7332
D. 3293
E. 2908

Answer is A

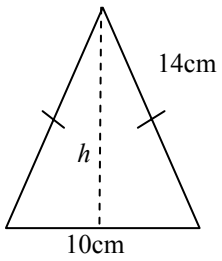
Worked solution

- The sequence for this second order difference equation:
  $t_{11} = t_9 + 2t_{10} \rightarrow 746 + 2 \times 1081 = 2908$
  $t_{12} = t_{10} + 2\, t_{11} \rightarrow 1081 + 2 \times 2908 = 6897$

Module 2: Geometry and trigonometry

Question 1

The isosceles triangle beside has a slant height of 14cm with a base of 10 cm.



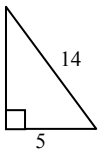
The vertical height, *h*, to two decimal places is

A. 14.87 cm
B. 14.86 cm
C. 13.07 cm
D. **13.08 cm**
E. 9.80 cm

Answer is D

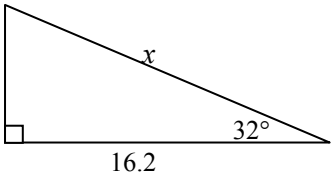
Worked solution

- Divide the triangle in half making it a right angled Δ. We can now use Pythagoras.
  $h^2 = 14^2 - 5^2$
  $= 196 - 25$
  $h = \sqrt{171} \approx 13.0766.$
  This rounds up to 13.08 (to two decimal places).



Question 2

The value of *x* to one decimal place is



A. 8.5 m
B. 8.6 m
C. 13.7 m
D. **19.1 m**
E. 30.6 m

Answer is D

Worked solution

- Trig: right angled Δ, in this case use $\cos\theta = A/H$.

  Rearranged gives $H = A/\cos\theta$

  $\therefore \quad x = 16.2/\cos 32°$
  $= 19.1$

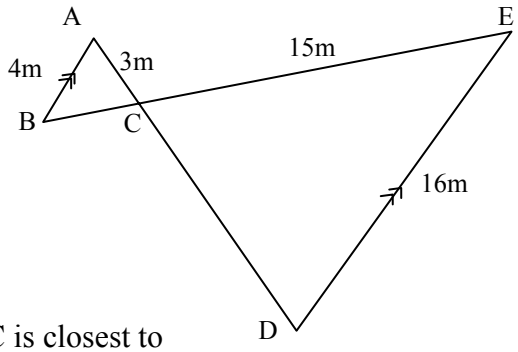
Tip

- *It is vital that the calculator mode is on DEGREES. All Further Mathematics students **must** have their calculator mode set to degrees.*

Question 3

Beside are two similar triangles:

ABC and CDE



The area of ΔCDE is 84 m^2, the area of ΔABC is closest to

A. **5.25 m^2**
B. 9.33 m^2
C. 21 m^2
D. 336 m^2
E. 1344 m^2

Answer is A

Worked solution

- The ratio of ABC : CDE is 4 : 16 which simplifies to 1 : 4 (Note AC and CE are not corresponding sides)

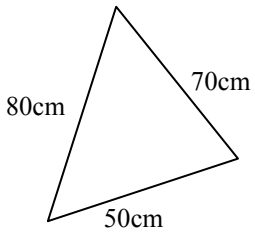
  Area ratio is $1^2 : 4^2$ i.e. 1 : 16

  Scale factor for the smaller triangle is 1/16

  Area of ABC = $84 \times 1/16$
  $= 5.25$

Question 4

A piece of carpet has the form of a scalene triangle with dimensions as shown beside.



The area in square metres correct to two decimal places is

A. 1732.05 m^2
B. **0.17 m^2**
C. 2.16 m^2
D. 17.32 m^2
E. 216.33 m^2

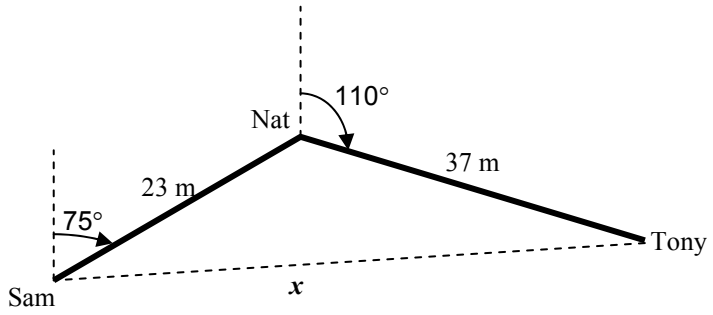
Answer is B

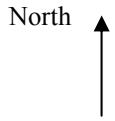
Worked solution

- Use Heron's formula with measurements changed to metres (although this conversion can be done later).
- Note: Heron's formula is on formula sheet)
  Area, A = $\sqrt{1(1-0.8)(1-0.7)(1-0.5)}$ where s = 0.5(0.8 + 0.7 + 0.5) = 1m
  $= 0.17$ m^2

The following information relates to questions 5 and 6.

Sam, the hockey goalie, hits the ball to Nat 23 metres away at a bearing of 075°T. Nat then hits the ball downfield to Tony who is 37 metres away at 110°T.





Question 5

The true bearing of Sam from Nat is

A. 075°T

B. 075°T

C. 115°T

D. 075°T

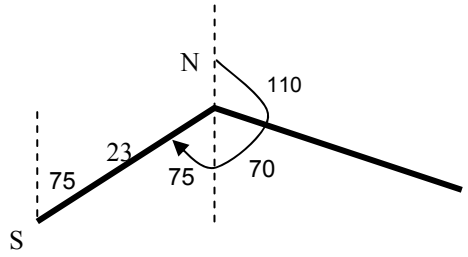
E. **255°T**

Answer is E

Worked solution

- Using alternate angles and straight line angles gives
  A total bearing of 110 + 70 + 75 = 255

  Remember a true bearing is clockwise from North.



Tip

- *A handy note is that the bearing from A to B compared with B to A is always 180° apart. In the case above it would be 75 + 180 = 255*

Question 6

The distance from Tony to Sam, ***x***, can be found correctly by using which formula?

A. $x^2 = 23^2 + 37^2 - 2 \times 23 \times 37 \cos(250^\circ)$

B. $\mathbf{x^2 = 23^2 + 37^2 - 2 \times 23 \times 37 \cos(145^o)}$

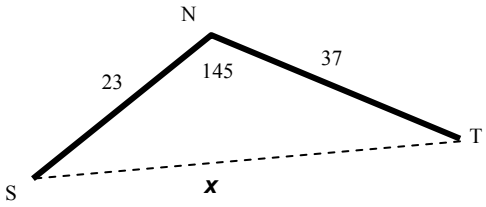
C. $x = 60 \times \cos(15^\circ)$

D. $x = \dfrac{\sin(145)}{\sin(15)} \times 37$

E. $x = \dfrac{\sin(145)}{\sin(75)} \times 23$

Answer is B

Worked solution



- The difficulty with this question is calculating ∠SNT = 145
  This question involves 3 sides and an angle
  Use cosine rule to find side ***x***

  $x^2 = 23^2 + 37^2 - 2 \times 23 \times 37\cos(145°)$

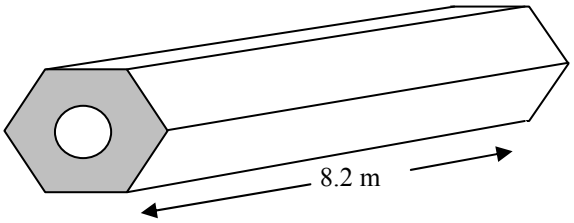
The SINE rule is not possible so options D and E can be eliminated.

Tip

- *It is important that students are familiar with use of formulas instead of relying solely on calculator programs like TRISOLV or the like.*

Question 7

A steel hexagonal pipe is used as part of a modern art construction. The **volume** of this prism is 2.05 m^3, and it is 8.2 m long.



In square metres, the shaded cross-sectional area is closest to

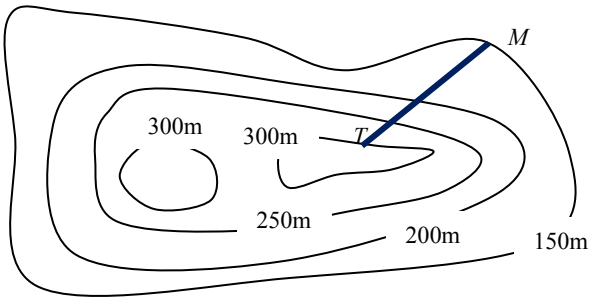
A. 4.1 m^2
B. 0.0625 m^2
C. **0.25 m^2**
D. 16.81 m^2
E. 70.644 m^2

Answer is C

Worked solution

- Volume of Prism, V = Area of cross-section (A) × Length of Prism (L)
  i.e. $V = A \times L$
  $\Rightarrow$ $A = V/L$
  $\therefore$ $A = 2.05/8.2 = 0.25$

Question 8



Scale 1: 20000

For the contour map above, the segment *MT* has a length of 2.5 cm on the map.

With aid of the scale the average gradient of the segment is

A. $\frac{2}{5}$

B. $\frac{1}{60}$

C. $\frac{3}{1000}$

D. $\mathbf{\frac{3}{10}}$

E. $\frac{10}{3}$

Answer is D

Worked solution

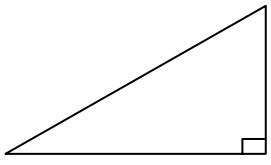
- Gradient = $\frac{height(contours)}{run(horizontal)}$ Difference in contours from M to T is 300 – 150 = 150m

  Using the scale, the actual horizontal distance is 2.5 × 20000 = 50000cm or 500m

  $\therefore$ gradient $= \frac{150}{500} = \frac{3}{10}$
- It is vital that the same units are used i.e. metres in this case.

Tip

- *Generally there could be a number of questions based on this triangle when applying contours, such as elevation (depression), average gradient (rate of slope), distance between points, etc.*

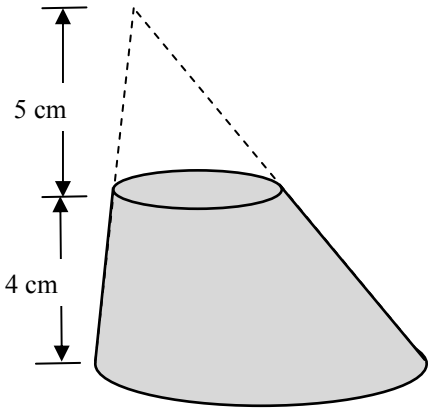


Rise : Difference in height, subtract the contours

Run : Horizontal distance
(In some cases you may be told the actual horizontal distance)

Question 9

A skewed cone has the top sliced off in such a way that the top is parallel to the base.



The perpendicular height of the removed section is 5cm and the perpendicular height of the remaining section is 4cm.

The ratio of the **volumes** of the removed section to the remaining section is

A. 125 : 729

B. **125 : 604**

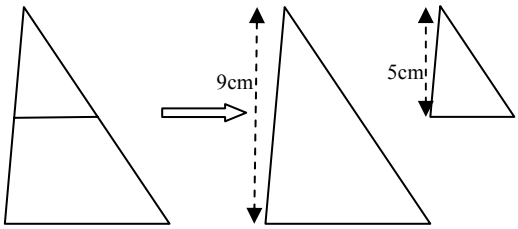
C. 125 : 64

D. 5 : 9

E. 5 : 4

Answer is B

Worked solution



- Since the cut is parallel we have the cross-section of two similar triangles.

  Note: To do this problem they MUST be similar triangles; Not top and bottom sections

  Length ratio of triangles is 5 : 9
  Volume ratio of the corresponding cones is $5^3 : 9^3$ i.e. 125 : 729.

  $\therefore$ Volume of removed : remaining sections is
  125 : 729 – 125
  125 : 604

Module 3: Graphs and relations

Question 1

The equation $2y - 3x = -12$ has intercept(s) of

A. (-6, 0) and (0, 4)

B. (2, 0) and (0, -3)

C. **(4, 0) and (0, -6)**

D. (-4, 0) and (0, -6)

E. (4, -6)

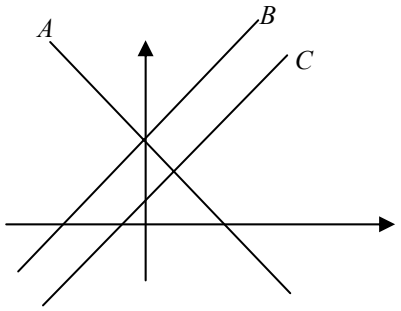
Answer is C

Worked solution

- To find x-intercept put y = 0
  this gives $-3x = -12$
  $x = 4$ ($\div -3$ both sides) This is the coord (4, 0)
- To find y-intercept put x = 0
  this gives $2y = -12$
  $y = -6$ ($\div 2$ both sides) This is the coord (0, -6)
- Note: Option E is not an intercept and can be eliminated.

Question 2

Below is a graph with three lines *A, B* and *C* sketched. Unfortunately most labels and numerals have been omitted.



It is known that these lines belong to the following three equations:

i. $y = 2x + 4$
ii. $y - 2x = 1$
iii. $y + 2x = 4$

The correct match-up with equations and lines is

A. i $\rightarrow$ A; ii $\rightarrow$ B; iii $\rightarrow$ C

B. **i $\rightarrow$ B; ii $\rightarrow$ C; iii $\rightarrow$ A**

C. i $\rightarrow$ B; ii $\rightarrow$ A; iii $\rightarrow$ C

D. i $\rightarrow$ C; ii $\rightarrow$ B; iii $\rightarrow$ A

E. i $\rightarrow$ C; ii $\rightarrow$ A; iii $\rightarrow$ B

Answer is B

Worked solution

- On observation lines B and C have **positive** gradients with B having a larger y-intercept. Line A is the only one with a negative gradient.
  Firstly re-write all equations in the form y = mx + c:
  i. (no change) y = 2x + 4 ii. becomes y = 2x + 1 iii. becomes y = -2x + 4

  'iii' is the only negative gradient (-2) so this must be line A
  'i' has a larger y-intercept(4) than 'ii' so this must be line B

Question 3

Nic, an endurance athlete runner, starts at 200 km from his destination. He maintains a constant pace throughout his run and completes the distance in 25 hours of running.

The distance, *D* km, that Nic is from his destination at anytime *t* hours can be found according to the rule

A. $D = 8t$

B. $D = 200 - 25t$

C. $\boldsymbol{D = 200 - 8t}$

D. $D = 25t$

E. $D = 200 + 25t$

Answer is C

Worked solution

- 200 km is completed in 25 hours – this is 8 km/hr.
  ∴ Gradient (rate) = -8 (using this you can eliminate all options B, D and E).
  At t = 0 D = 200. Only remaining option C will satisfy this.

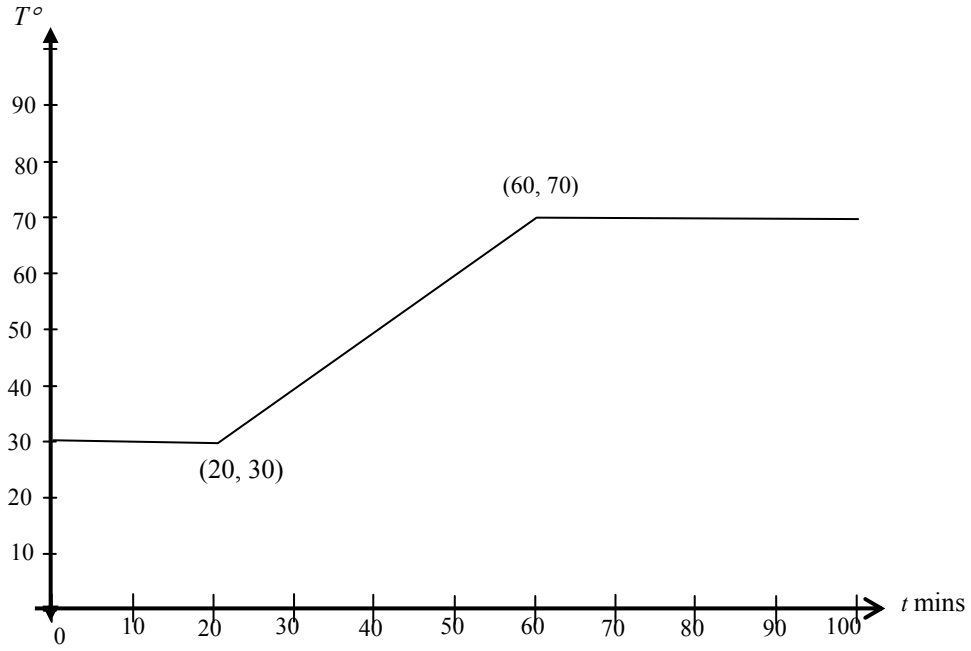
The following graph relates to Questions 4 and 5.

The temperature, *T° Celsius*, of an oven is regulated over a normal working day.

For the first 20 minutes it is maintained at a constant temperature of 30°. For the next forty minutes it is gradually heated until it reaches a maximum temperature of 70°.

It maintains this temperature for the remainder of the working day.

The graph of temperature, T, versus time, t, is shown below.



Question 4

Which of the following statements is NOT true?

A. The rate of heating from 20 to 60 mins is 1° per min.

B. The temperature after 1.5 hours is 70°.

C. The maximum temperature for the working day is 70°.

D. In the first 80 minutes, the average gradient is 0.5° per min.

E. **After 100 mins the oven is cooled.**

Answer is E

Worked solution

- We are not shown any information after 100 mins, apart from the fact that it maintains this max temp for the remainder of the working day – hence option E is inaccurate.
- Note for option D
  At the start it is 30° and rises to 70° after 60 mins; this is a rate of 40°/60min or simply 2/3.

Question 5

Which of the following rules best describes the graph above?

A. $\boldsymbol{T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ t + 10 & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}}$

B. $T = \begin{cases} 20 & \text{for } 0 \le t \le 30 \\ t + 10 & \text{for } 30 < t \le 70 \\ 60 & \text{for } t > 70 \end{cases}$

C. $T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ t - 10 & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}$

D. $T = \begin{cases} 30 & \text{for } 0 \le t \le 20 \\ \frac{7}{6}t & \text{for } 20 < t \le 60 \\ 70 & \text{for } t > 60 \end{cases}$

E. $T = \begin{cases} 30 & \text{for } 0 \le t \le 30 \\ t + 10 & \text{for } 30 < t \le 70 \\ 70 & \text{for } t > 70 \end{cases}$

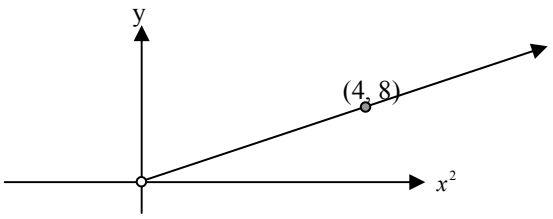
Answer is A

Worked solution

- 1st Stage Using T = mt + c c = 30, m = 0
  *T = 30
  2nd Stage m = (70 – 30)/(60 – 20) = 1
  T = 1t + c
  Sub in the point (20, 30) to find c
  30 = 20 + c $\Rightarrow$ c = 10
  * T = t + 10

  Only Option A has this solution, Option E has the wrong domain; Option B has wrong temp values.
- Students need to watch domain of solutions as this will also help eliminate some options.

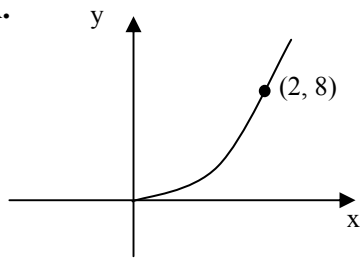
Question 6



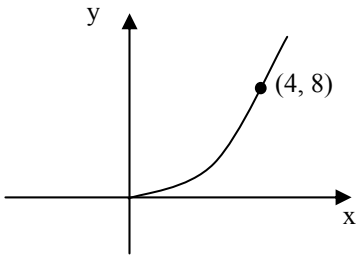
The graph connecting y and x^2 is shown above.

The graph that shows the same relationship between y and x is

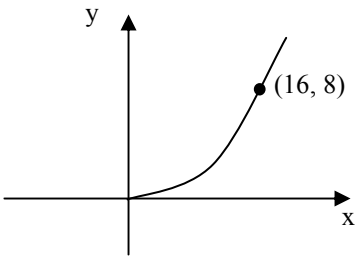
A.



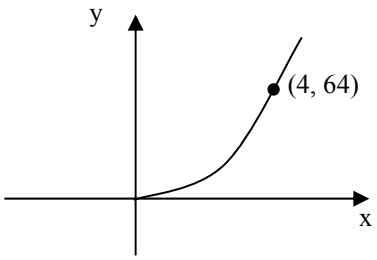
B.



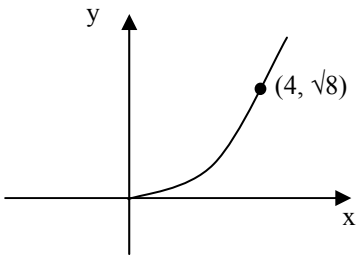
C.



D.



E.



Answer is A

Worked solution

- Linear: $y = k x^2$
  value of k is 8/4 = 2 being the gradient of the line
  $\therefore y = 2x^2$ finding x when y = 8
  $8 = 2x^2$ giving $4 = x^2$ square root BS results in $x = 2$ (since $x \geq 0$)
  $\therefore$ At x = 2 y = 8 i.e. (2, 8)

Question 7

A freight container can take a maximum weight of 600 kg. It is to be loaded with two varieties of boxes. Box A weighs 20 kg and box B weighs 35 kg.

Due to the size of the container the maximum number of boxes it can hold is 25.

If A is the number of Box A, and B is the number of box B, the inequalities that best show the constraints are

A. $A + B \leq 55$; $A + B \leq 25$

B. $A + B \leq 545$; $A + B \leq 25$

C. $A + B \leq 600$; $20A + 35B \leq 25$

D. $\mathbf{20A + 35B \leq 600}$ **;** $\mathbf{A + B \leq 25}$

E. $20A + 35B \geq 600$; $A + B \geq 25$

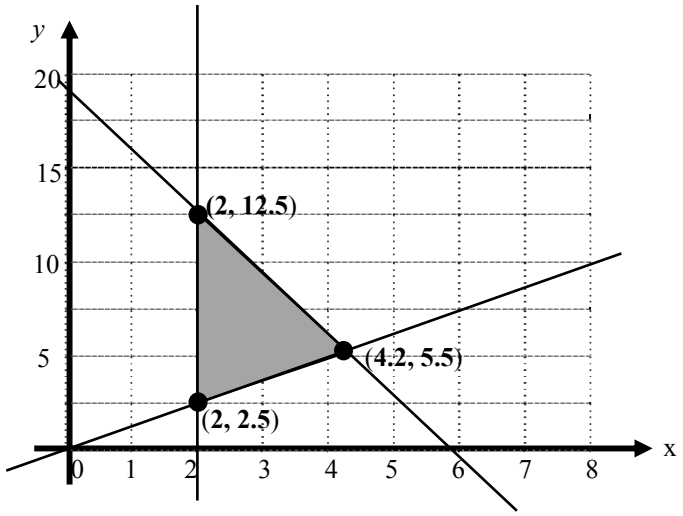
Answer is D

Worked solution

- The number of boxes cannot exceed 25 $\therefore$ A + B $\leq$ 25
  The max weight is 600. The total weight of A boxes is 20A, the weight of B boxes is 35B
  $\therefore$ 20A + 35B $\leq$ 600

Question 8

The shaded area below shows the feasible region.



The objective function, z, is given by the equation

$$z = 1.5x - 2y$$

Using the feasible region above the maximum value of z is

A. -22

B. **-2**

C. -4.7

D. 28

E. 22

Answer is B

Worked solution

Feasible end points	Value of $z = 1.5x - 2y$
(2, 12.5)	$1.5 \times 2 - 2 \times 12.5 = -22$
*(2, 2.5)	$1.5 \times 2 - 2 \times 2.5 = -2$*
(4.2, 5.5)	$1.5 \times 4.2 - 2 \times 5.5 = -4.7$

* maximum

Question 9

A linear programming problem has the following constraints.

$y + 2x \leq 12$
$3y - 2x \geq 6$
$x \geq 3$, $y \geq 0$

A point that lies in the feasible region is

A. (2, 5)
B. (2, 3)
C. (4, 0)
D. (3, 2)
E. **(3, 5)**

Answer is E

Worked solution

- The constraint $x \geq 3$ eliminates A and B as these coordinate points have an x value of 2
- Option C (4, 0) is incorrect since $3y - 2x \geq 6$ becomes $3 \times 0 - 2 \times 8 \geq 6$ $\therefore$ $-16 \geq 6$ which is false
- Option D (3, 2) is incorrect since $3y - 2x \geq 6$ becomes $3 \times 2 - 2 \times 3 \geq 6$ $\therefore$ $0 \geq 6$ which is false
- Option E satisfies all inequalities

Module 4: Business-related mathematics

Question 1

$5000 is invested at a simple interest rate of 4% per annum.

The total amount after 30 months is

A. $5 515.10
B. **$5 500**
C. $600
D. $500
E. $375

Answer is B

Worked solution

- Amount of interest I = 5 000 × 4/100 × 2.5 (where 30 months = 2.5 years)
  I = 500
  Total amount A = P + I ∴ A = 5 000 + 500 = 5 500

Question 2

In April, Lyn received the following statement from her bank showing all the transactions from her savings for the month of March.

Date	Transaction	$ Debit	$ Credit	$ Balance
1 Mar	Balance forward			985.90
9 Mar	Withdrawal	234.00		751.90
18 Mar	Deposit		400.00	1 151.90

Interest is calculated on the minimum monthly balance. If the amount of interest Lyn received for March was $1.50; the rate of interest, correct to 1 decimal place, was

A. 0.2% p.a.
B. 1.6% p.a.
C. 1.9% p.a.
D. **2.4% p.a.**
E. 28.8% p.a.

Answer is D

Worked solution

- Minimum monthly balance, P = 751.90 Using I=PrT/100 which can be rearranged so that the rate, r is:
  r = 100I/(PT) where I = 1.50, T = 1/12 and P = 751.90
  ∴ Rate is r = 100 × 1.50/ (751.90 × 1/12)
  = 2.3939
  ≈ 2.4 to 1 decimal place

Question 3

Furniture is valued at $12 000. On a hire purchase plan, it can be purchased with a deposit of $1 000.

The total paid for the furniture over 4 years on this plan is $16 000 which includes the deposit.

The flat rate of interest p.a. charged on hire purchase correct to 2 decimal places is

A. 33.33%
B. 36.36%
C. 10.00%
D. 8.33%
E. **9.09%**

Answer is E

Worked solution

- Deposit is $1 000
  Principal = 12 000 – 1 000
  = 11 000
  Amount of interest = 16 000 – 12 000
  = 4 000
  $\therefore$ Flat rate, R = $\frac{100 \times 4000}{11000 \times 4}$ using R= $\frac{100I}{PT}$
  = 9.09%

Question 4

Lyn invested $15 5000 in an ordinary perpetuity. The interest rate for the investment is 7.4% per annum.

The amount of pension she receives monthly from this perpetuity is closest to

A. $3 600
B. $621
C. **$956**
D. $11 470
E. $13 870

Answer is C

Worked solution

- Use Q = $\frac{\text{Pr}}{100}$ where Q is yearly amount of payment
  Q = $\frac{155000 \times 7.4}{100}$
  = $11470

  Monthly it is $11470/12 = 955.83

Alternatively use TVM solver
N= 1 (monthly payment)
I%= 7.4
PV= -15 5000
*PMT= 955.83 (alpha solve)
FV= 15 5000
P/Y= 12
C/Y=12

Question 5

Nic needs $4500 in two years' time. He invests into an account that is advertised at 7.8%p.a. with interest compounded **daily**.

The original amount he needs to invest to the nearest dollar is

A. **$3 850**
B. $3 872
C. $3 889
D. $5 229
E. $5 260

Answer is A

Worked solution

- Compound Interest
  Rate = 7.8%p.a. ÷ 365 = 0.0214% per day
  The number of terms, n = 365 × 2 = 730
  ∴ A = 4 500/(1.000214….)^{730}
  = $3 850.08

Alternatively use TVM solver
N= 730 (daily over 2 yrs)
I%= 7.8
*PV= 3 850 (alpha solve)
PMT= 0 (no regular repayments)
FV= 4 500
P/Y= 365 (daily)
C/Y= 365

Tip

- *Watch for key words **compound** and periods of interest i.e. **monthly, quarterly**, etc…: it indicates to use the compound interest formula, and the rate and number of terms may need attention.*
- *Do not round off too early!*

Question 6

For tax purposes a person uses reducing balance depreciation of 15% to calculate the book value of a photocopier. After 2 years the book value of the copier is $4 600.

The original value of the photocopier was approximately

A. $5 980
B. $6 080
C. **$6 370**
D. $6 570
E. $204 440

Answer is C

Worked solution

- As stated, need to use reducing balance depreciation formula:

  Book Value, $BV = P(1-\frac{r}{100})^n$,

  $4600 = P(1-\frac{15}{100})^2$

  4 600 = P (0.7225)

  ∴ P = 4 600/0.7225 = 6 366.78

Alternatively use TVM solver
N= 2 (over 2 yrs)
I%= -15 (negative due to depreciation)
*PV= -6 366.78 (alpha solve)
PMT= 0 (no regular repayments)
FV= 4 600
P/Y= 1 (yearly)
C/Y=1

Question 7

Stamp Duty is payable to the state government on a property transaction according to the following rate schedule.

Transfer of Real Property rates	
Range	**Rate**
\$0 – \$20 000	1.4 per cent of the price of the property
\$20 001 – \$115 000	\$280 plus 2.4 per cent of the price in excess of \$20 000
\$115 001 – \$870 000	\$2 560 plus 6 per cent of the price in excess of \$115 000
More than \$870 000	5.5 per cent of the price value

An owner pays stamp duty of \$18 000.

The price of the property lies within the range

A. \$0 – \$20 000
B. \$20 001 – \$115 000
C. **\$115 001 – \$870 000**
D. More than \$870 000
E. Over \$1 million

Answer is C

Worked solution

- For each segment the MAXIMUM stamp duty payable is shown in the table below.

Range	MAXIMUM stamp duty payable
\$0 – \$20 000	1.4/100 × 20 000 = \$280
\$20,001 – \$115 000	\$2560
\$115,001 – \$870 000	2560 + 0.06 × (870 000 – 115 000) = \$4 7860
More than \$870 000	≥0.055 × 870 000 ≥ \$4 7850

Going by the table, the property must be valued within \$115 000 – \$870 000 because stamp duty payable varies from \$2 560 up to \$47 860. \$18 000 is within this range.

Question 8

Joan borrows $12 000 and makes monthly repayments of $400.

The interest rate is 8% p.a. calculated monthly on the reducing balance of the loan.

Find the total amount of interest Joan has paid, immediately after making her **tenth** payment, to the nearest dollar.

A. $16
B. **$702**
C. $800
D. $824
E. $4 947

Answer is B

Worked solution

- Total paid is 10 × 400 = $4 000

  The amount owing has reduced from 12 000 to 8 702.27
  (without interest it would be 12 000 – 4 000 = 8 000)
  Meaning an extra $702.27 is due to interest.

N= 10
I%= 8
PV= 12 000
PMT= -400
FV=(-8702.27)
P/Y= 12
C/Y= 12

Question 9

Torie invests a sum of $3 200 into an account earning an interest rate of 7.2%p.a. compounded monthly. Into the same account she also decides to make monthly installments of $60.

After 2 years the **total** Torie has in her account is closest to

A. $2 150
B. $3 359
C. $5 134
D. $5 178
E. **$5 238**

Answer is E

Worked solution

Using TVM solver

N= 24
I%= 7.2
PV= -3 200
PMT= -60
FV= (5237.91)
P/Y= 12
C/Y=12

Monthly for two years i.e. 2×12

Amount of each repayment

This is the value we need to find after two years (Use ALPHA solve)
Answer is $5 237.91 ≈ $5 238 when rounded off

Module 5: Networks and decision mathematics

Question 1

The number of edges on a **complete** network with six vertices is

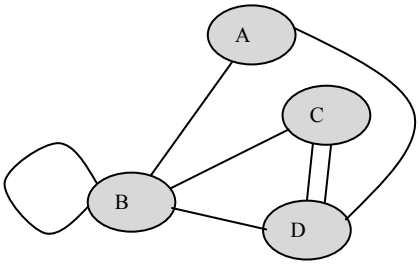
A. 5
B. 12
C. **15**
D. 21
E. 25

Answer is C

Worked solution

- $5 + 4 + 3 + 2 + 1 = 15$ (The 6^{th} vertex must join to **5** others, 5^{th} vertex joins to **4** others since it is already connect to the 6^{th} vertex … etc for the other vertices).
- Otherwise you can use the formula $\boldsymbol{n(n-1)/2}$ for $\boldsymbol{n}$ vertices

Question 2



An adjacency matrix to represent the network is

A.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

D.
$$\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{0} \end{bmatrix}$$

E.
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Answer is D

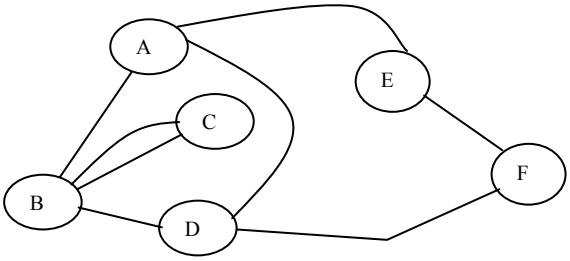
Worked solution

- D is correct

Vertex A has 1 edge going to B and D Vertex B has 1 edge going to A, B, C and D
Vertex C has 1 edge to B and 2 edges to D Vertex D has 1 edge going to A, B and C

- Note: An Adjacency matrix is symmetrical along the diagonal; this leaves only A and D as possibilities.

Question 3



To make this Network contain an Euler circuit, an edge needs to be added connecting the following vertices:

A. **A to D**
B. A to F
C. B to D
D. B to F
E. C to D

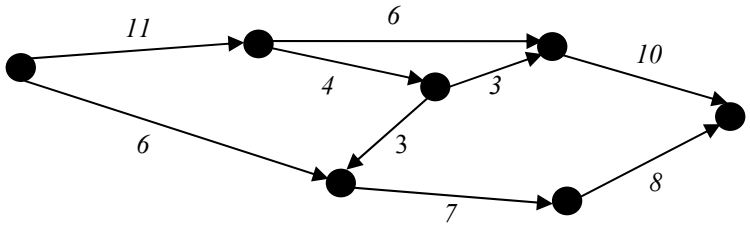
Answer is A

Worked solution

- To have an Euler circuit all degrees must be even. This network has two odd degrees at A(3) and D(3).
- Adding an edge between A and D will increase both degrees by one.

Question 4

The following directed graph shows the weight of each edge.



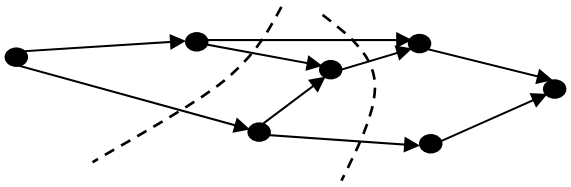
The minimum cut for this directed graph is

A. 18
B. 17
C. **16**
D. 15
E. 14

Answer is C

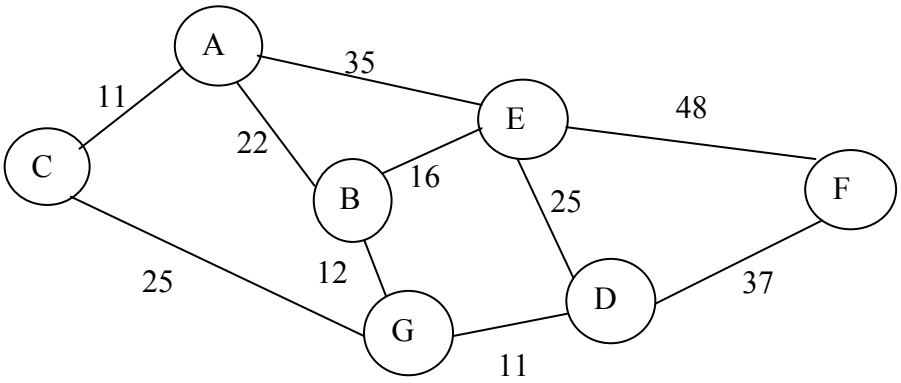
Worked solution

- There are two cuts that give 16



Questions 5 and 6 refer to the following network.

For a shire, the graph below shows the major towns A, B…G connected by the main roads. All distances are in kilometres.



Question 5

The sporting board for this shire wishes to organise a bike road race to be completed in a circuit. The minimum distance a circuit could be raced is

A. 73 km
B. 58 km
C. 71 km
D. 69 km
E. **64 km**

Answer is E

Worked solution

- Towns BEDG will give a total distance 12 + 16 + 25 + 11 = 64 km
- Please note: if all towns were to be considered, the words "Hamiltonian Circuit" would be required.

Question 6

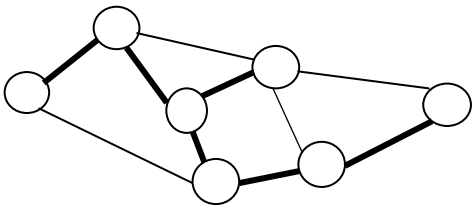
The distance for the minimum spanning tree for this network is

A. **109 km**
B. 112 km
C. 118 km
D. 93 km
E. 73 km

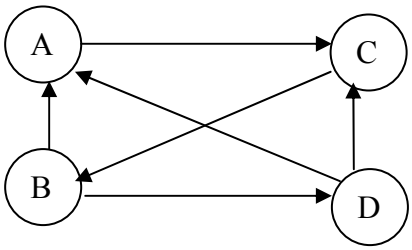
Answer is A

Worked solution

- 11 + 22 + 12 + 11 + 16 + 37 = 109
- Using Prim's algorithm (i.e. start with lowest edge, AC with 11, **from** either of these vertices select the lowest edge, AB with 22 etc., taking care not to form any circuits). The minimum tree is highlighted below:



Question 7



In a round robin competition 4 teams A, B, C and D play each other once.

An arrow from A to C indicates that A defeated C.

Based on 1-step and 2-step reachability matrix for this network, state the winning team and their dominance value:

A. Team B, 4
B. **Team B, 5**
C. Team D, 4
D. Team D, 5
E. Team B and D, both 4

Answer is B

Worked solution

$D^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ $D^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ where D^1 and D^2 are 1 and 2 step adjacency matrices
Then add along each row gives

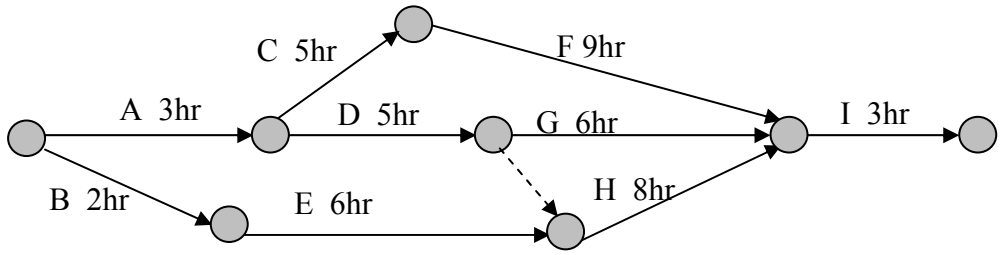
Alternatively

TEAM	1-step	2-step	Dominance
A	1 (AC)	1(ACB)	2
B	2(BA,BD)	3(BAC,BDA,BDC)	5*
C	1(CB)	2(CBA, CBD)	3
D	2(DA,DC)	2(DAC,DCB)	4

With a dominance value of 5, Team B is the winner.

Questions 8 and 9 refer to the following critical path.

For a particular project there are nine activities to be completed and the time taken to complete each activity is shown in hours.



Note: GH is a dummy activity

Question 8

The critical path and completion time for this project is

A. BEHI, 19 hours
B. ADGI, 17 hours
C. **ACFI, 20 hours**
D. ADGHI, 19 hours
E. AEHF, 26 hours

Answer is C

Worked solution

- The critical path is the 'longest' path because all activities must be completed before you start the next one.
- ACFI, 3 + 5 + 9 + 3 = 20 hrs

Question 9

The project is to be crashed by reducing the completion time to a time of 18 hours.

This can be done in the minimum number of hours by reducing the following activities:

A. F by 2 hours

B. **F by 2 hours and H by 1 hour**

C. A by 2 hours

D. A by 2 hours and F by 1 hour

E. No reduction but increase D by 1 hour making it the quickest time

Answer is B

Worked solution

- Need to look along all activities on the critical path.
- Option A reduces ACFI to 18 hours but now BEHI and ADGHI are 19 (new critical path)
- Option B reduces ACFI to 18 hours but now BEHI and ADGHI are reduced to 18* (satisfies project)
- Option C reduces ACFI to 18 hours but now BEHI with 19 is new critical path
- Option D reduces ACFI to 17 hours but now BEHI with 19 is new critical path
- Option E ignores the fact that we are looking at the longest paths not shortest

Module 6: Matrices

Question 1

Given that $A = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$, and $B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

The value of $A^2 + 2B$ is

A. $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ **B.** $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 10 \\ -6 \end{bmatrix}$ **D.** $\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

E. Impossible, no solutions

Answer is E

Worked solution

- A^2 can't be done.
- When multiplying, the number of columns on the 1st matrix must equal the number of rows on the 2nd matrix. This eliminates all but E.

Question 2

Let $C = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Which of the following is **NOT** true?

A. I raised to any power is always I.

B. **There is no inverse matrix of C.**

C. C + I = I + C

D. CI = C

E. 2C – I gives a 2 × 2 matrix

Answer is B

Worked solution

- Inverse of C is $C^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$
- All others are true

Question 3

Let $R = \begin{bmatrix} 1 & m \\ 0 & n \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

If the matrix $RT = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$,

the values of *m* and *n* respectively must be

A. 1 and -1

B. 2 and 1

C. 2 and $5/4$

D. **-1 and 2**

E. -1 and -2

Answer is D

Worked solution

$$RT = \begin{bmatrix} 1+2m & 4-m \\ 2n & -n \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$$

$\therefore$ $1+2m = -1$

$2m = -2$ (-1 BS)

$\mathbf{m = -1}$

$\therefore$ $2n = 4$

$\mathbf{n = 2}$ ($\div 2$ BS)

ALTERNATIVELY using calculator let $A = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

$RT = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$ $\therefore$ $R = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix} T^{-1}$

Using calculator $R = AB^{-1}$

$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ (m, n)

Question 4

Nathan decided to plan a course for his class to visit five locations A, B, C, D and E in the area near his school by using the following transition matrix:

Present Location

Next Location

	A	B	C	D	E
A	0	0	1	0	0
B	1	0	0	0	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	1	0	0	0

If the class know that they start from location C, the correct order of the course to visit each location using the transition matrix is

A.	C	D	E	B	A
B.	C	A	B	D	E
C.	**C**	**A**	**B**	**E**	**D**
D.	C	D	E	A	B
E.	C	A	C	A	B

Answer is C

Worked solution

- Starting at C
  Order is C to A to B to E then D

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Question 5

A company produces a four cylinder car (F) and a six cylinder car (X). Each type of car comes in three styles: Economy (E), Sport (S) and Luxury (L).

The price of each make of car, in thousands of dollars, is listed in a price matrix P, where

$$P = \begin{matrix} \\ E \\ S \\ L \end{matrix} \begin{matrix} F & X \\ \end{matrix} \begin{bmatrix} 21 & 31 \\ 28 & 34 \\ 32 & 40 \end{bmatrix}$$

Due to inflation and extra taxes, a new price matrix generating the new price of cars can be found by the product PQ where Q is the matrix:

$$\begin{bmatrix} 1.05 & 0 \\ 0 & 1.08 \end{bmatrix}$$

This new pricing scale is best summarised as

A. Economy cars increase by 5%, Sport cars increase by 8%, Luxury stay fixed

B. Economy cars increase by 5%, Luxury cars increase by 8%, Sport stay fixed

C. **Four cylinder cars increase by 5% and six cylinder cars increase by 8%**

D. Four cylinder cars increase by 1.05% and six cylinder cars increase by 1.08%

E. All cars increase by 5% and a further 8% on top of this

Answer is C

Worked solution

- Options A and B can't be multiplied.
  When multiplying, the value 1.05 increases only the F values: making F increase by 5% and the value 1.08 increases only the X values: equivalent of making X increase by 8%.

Question 6

How many of the following four sets of simultaneous linear equations have a unique solution?

Set A	Set B	Set C	Set D
x – y = 3 x + y = 1	2x + 2y = 12 2x – y = 6	x – 2y = 0 -2x + 4y = 10	x + y = 5 5x = 10

A. 0
B. 1
C. 2
D. **3**
E. 4

Answer is D

Worked solution

- When the determinant is zero, there are no unique solutions. This means that the two equations are either parallel (no solution) OR they are collinear (infinite solutions)
- In this question set C has a determinant of zero, hence no solution.
- The other 3 sets will have a unique solution.

Question 7

The order of matrix *A* is (3 × 2); and the number of columns of matrix B is 3.

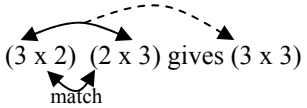
If the result of A(B + C) is a 3 × 3 matrix, then the order of matrix C must be

A. **(2 × 3)**
B. (3 × 2)
C. (2 × 2)
D. (3 × 3)
E. (1 × 3)

Answer is A

Worked solution

- When multiplying, the number of columns on the 1st matrix must equal the number of rows on the 2nd matrix.
- When adding matrices must have identical dimensions



If A is 3 × 2 then B + C must be a 2 × 3 matrix to give a result of 3 × 3

Question 8

The solution to the following simultaneous equations

$$-2x + y = -7$$
$$x - 2y = 5$$ is

A. $x = 19$ and $y = -17$

B. $\boldsymbol{x = 3}$ **and** $\boldsymbol{y = -1}$

C. $x = 2$ and $y = -3$

D. $x = 1$ and $y = -2$

E. $x = 9$ and $y = -3$

Answer is B

Worked solution

- Calculator easiest way to solve although it can be done by hand.
- Using matrix, this simultaneous equation can be written in the form:

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$ where $\frac{1}{3} \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$ is the inverse of $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

$$= \frac{1}{3} \begin{bmatrix} -2 \times -7 + -1 \times 5 \\ -1 \times -7 + -2 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Question 9

A snack company specialises in making a chocolate bar (*B*) and an herbal flavoured bag of chips (*C*).

Research has found that 85% of the people that buy the chocolate bar (*B*) return to buy it again the following day and the rest decide to buy the bag of chips (*C*).

Of those who buy the bag of chips (*C*), 80% of people return to buy it again the following day and the rest decide to buy the chocolate bar (*B*).

Originally the company made 300 chocolate bars and 400 bags of chips daily.

If the research is correct and the people's buying patterns are maintained, the number of each product that the company should make in the **long term** is

A. 300 of *B* and 400 of *C*

B. 335 of *B* and 365 of *C*

C. 350 of *B* and 350 of *C*

D. 380 of *B* and 320 of *C*

E. **400 of *B* and 300 of *C***

Answer is E

Worked solution

- The transition matrix is

$$T = \begin{bmatrix} 0.85 & 0.20 \\ 0.15 & 0.80 \end{bmatrix}$$

Next Day	Current Day	
	BAR	CHIPS
BAR	85%	20%
CHIPS	15%	80%

This shows 85% of those who buy a bar on the current day also buy a bar the next day.

This shows 80% of those who buy chips on the current day also buy chips the next day.

- Initial state matrix is

$$S_0 = \begin{bmatrix} 300 \\ 400 \end{bmatrix}$$

- Need to find the steady-state solution,
  i.e. $T^n S_0$ for a large n
  (Using n = 20, 30, 40) gave

$$\begin{bmatrix} 400 \\ 300 \end{bmatrix}$$, which is option E.

END OF WORKED SOLUTIONS