Student Name: ____________________

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2007 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied
- Question book of 31 pages.
- Working space provided throughout the book.

Instructions
- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

TURN OVER

Core

Matilda is collecting information for a documentary on pollution and greenhouse gases. From her research she has been able to develop a table which shows Australia's greenhouse emissions for the years 1990 – 1997 from a variety of sources.

Table 1
Australian Greenhouse Emissions –carbon dioxide equivalent - megatonnes

Sector	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
Stationary Energy	205.8	208.7	209.9	212.4	214.3	223.3	231.5	237.0
Transport	61.5	61.0	62.8	63.8	65.6	68.4	70.7	72.5
Fugitive	29.4	28.7	29.8	28.9	28.7	29.6	29.7	29.5
Industrial Processes	12.1	11.7	10.4	10.2	9.9	9.0	9.2	9.0
Agriculture	92.1	92.5	91.2	92.0	92.0	93.0	92.9	94.2
Forestry and Other	-27.0	-26.5	-26.9	-26.5	-26.5	-25.0	-24.9	-26.5
Waste	14.8	15.1	15.3	15.6	15.5	15.2	15.3	15.6
Net Emissions from Land Clearing	102.7	73.0	70.1	67.4	64.8	62.7	66.6	64.8
Total	491.4	464.2	462.6	463.8	464.3	476.2	491.0	496.1

Source data: National Greenhouse Calculator

Question 1
Matilda is particularly interested in *Transport, Industrial Processes* and *Waste*. She calculates the mean and standard deviation over the 8 year period for these three sectors.

a. Complete Table 2 by calculating the mean of the *Transport* sector. Write your answer correct to one decimal place.

Table 2

Sector	Transport	Industrial processes	Waste
Mean		10.2	15.3
Standard deviation	4.3	1.2	0.3

1 mark

In 1998 the *Industrial processes* sector produced carbon dioxide equivalent emissions of 9.1 megatonnes.

Core – continued

b.

i. Calculate the standardised carbon dioxide equivalent (z score) relative to the results for the previous 8 years. Write your answer correct to one decimal place.

____

ii. Complete the following sentence by circling the correct alternatives given.

This z score indicates that in 1998 the *Industrial Processes* sector carbon dioxide equivalent emissions were **greater than or less than** one standard deviation **above or below** the mean.

1 + 1 = 2 marks

Matilda calculates the five figure summary for *Agriculture* as shown below:

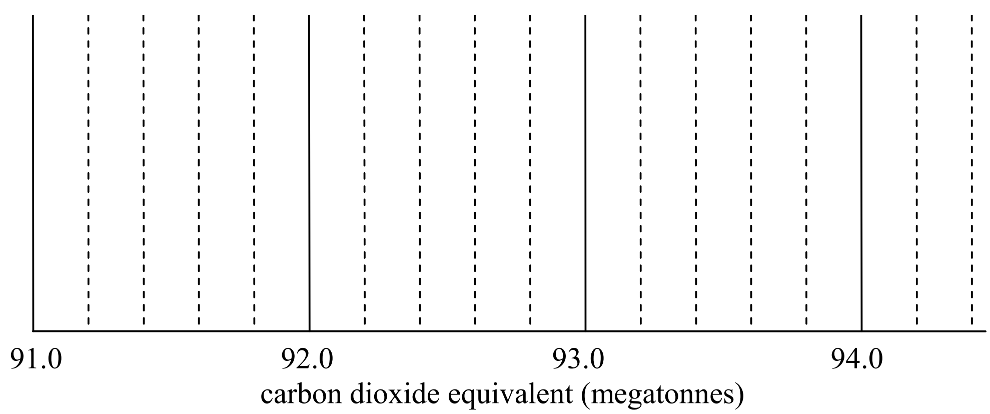
Lowest Score	91.2
Lower Quartile	
Median	92.3
Upper Quartile	92.95
Highest Score	94.2

c. Complete the calculations. Write your answer correct to one decimal place in the space provided above.

1 mark

d. Matilda uses this data to construct a box plot of the information. Complete the box plot using the scale provided.

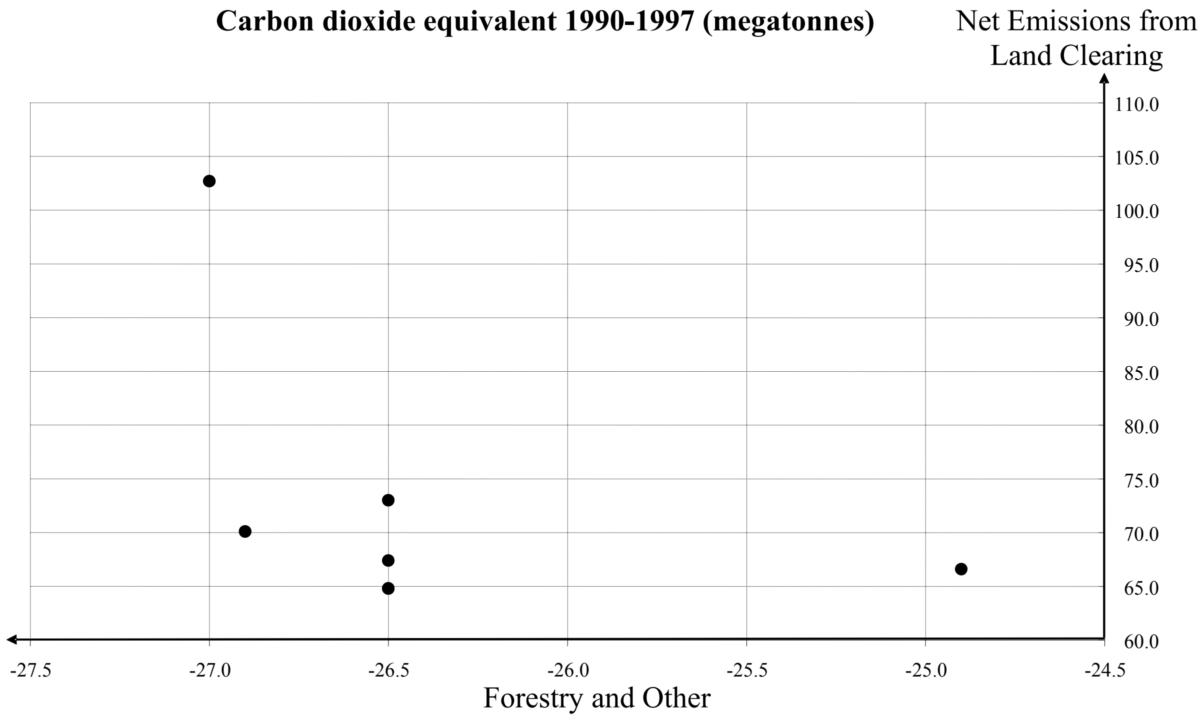
Agriculture (Box Plot 1990 – 1997)



2 marks

Core – continued
TURN OVER

Matilda next prepares a scatterplot of *Net emissions from Land clearing* versus *Forestry and other*.



e. The point for 1995 has been left off the graph. Mark it in the appropriate place on the graph.

1 mark

The equation for the least square regression line has been calculated. E has been used to represent *Net Emissions from Land Clearing* and F has been used to represent *Forestry and Other*.

f.

i. Complete the equation. Write your answers to one decimal place.

$$E = [\quad\quad] + [\quad\quad]F$$

ii. What value would the *Net Emissions from Land Clearing* be when this line cut the E axis?

---

1 + 1 = 2 marks

The correlation coefficient for this graph is $r = -0.49$

g. Complete the following sentence. Write your answer to one decimal place.

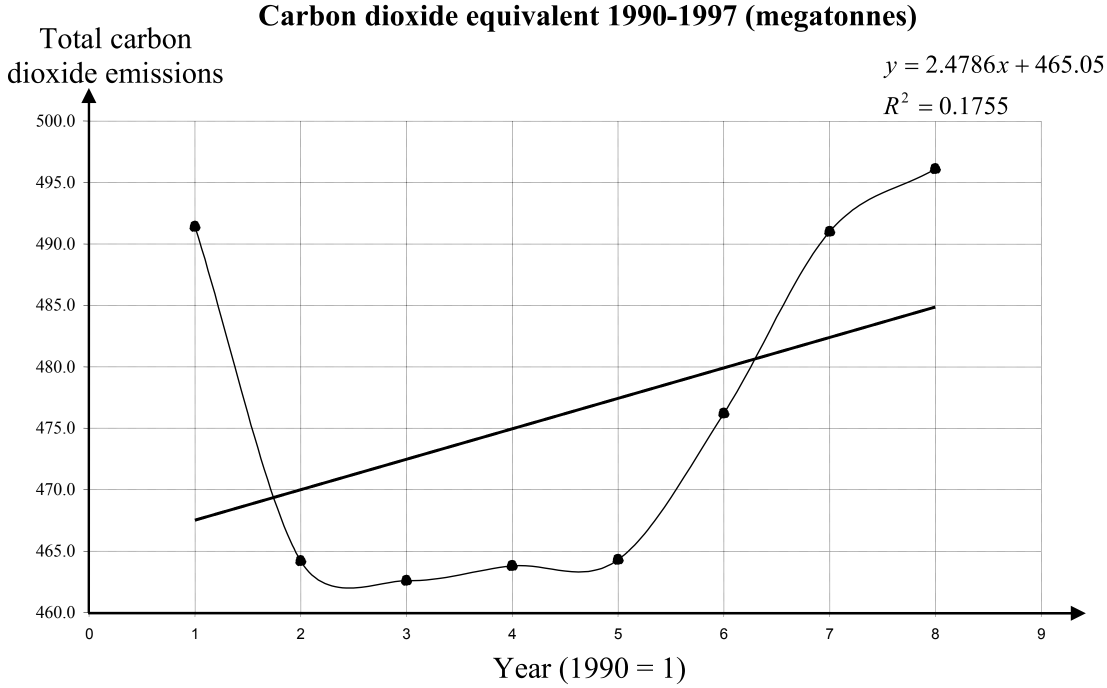
[ ] of the variation in the *Net Emissions from Land Clearing* can be explained by the variation in the *Forestry and Other* emissions.

1 mark

Core- continued

Question 2

Matilda completed a scatterplot of the total carbon dioxide equivalent emissions verses time. She altered the time values so that year 1 = 1990, year 2 = 1991 and so on.



Matilda decided to perform a residual analysis on her data using the least squares regression line equation given on the graph:

$$y = 2.4786x + 465.05$$

a. Complete the residual analysis table below. Write your answer to one decimal place.

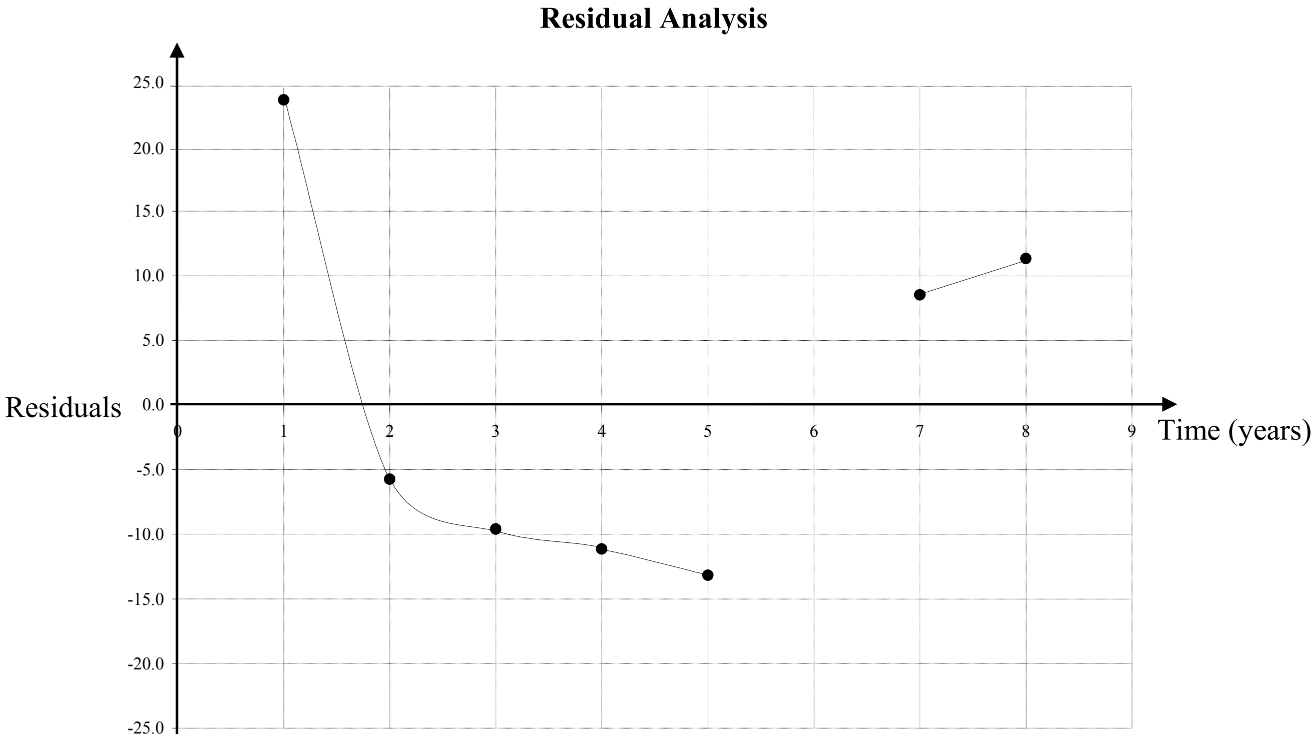
Time	*y* actual	*y* predicted	residual
1	491.4	467.5	23.9
2	464.2	470.0	-5.8
3	462.6	472.5	-9.9
4	463.8	475.0	-11.2
5	464.3	477.4	-13.1
6	476.2		
7	491.0	482.4	8.6
8	496.1	484.9	11.2

1 mark

Core - continued

TURN OVER

Matilda plotted the residual graph she obtained below.



b. Add your calculated residual result from **part a**, to the above graph.

1 mark

c. As a result of this analysis, what transformation should Matilda apply to her original data?

____

1 mark

Matilda applied an x^2 transformation. The new trend line she calculated was $y = 0.3775x^2 + 466.57$ and the new r value was 0.588.

d. Use this equation to calculate the predicted total carbon dioxide equivalent emissions for 1998. Write your answer to one decimal place.

____

1 mark

e. What does the r value tell you about the accuracy of Matilda's prediction?

____

1 mark

Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1
In 2001 the number of visitors to the Burrton Bowling club was 200. In 2002 the number of visitors was 280. For each subsequent year the number of visitors to the Burrton Bowling Club can be summarised by the difference equation

$$V_n = V_{n-1} + 0.5V_{n-2}\,.$$

a. How many visitors were there to the club in 2007?

1 mark

b. If this trend continues, in what year will the number of visitors exceed 3000?

1 mark

Question 2
Brian and Barry have only been playing bowls for a few weeks. In singles matches Brian has noticed that his scores have followed a particular pattern, shown below:

1, 4, 7, 10, …

To win at bowls you have to score 21.

a. How many singles games will Brian have to play before he will win assuming this pattern continues?

1 mark

b. Barry wins his first game of bowls (score of 21) in his seventh match. In his fourth match Barry had a score of 12. If his scoring pattern strictly follows an arithmetic sequence, what is the total score of all his games in the seven matches he has played?

2 marks

Module 1: Number patterns-continued
TURN OVER

Question 3

The Burrton Bowling Club is part of the Burrton Sport Complex. Bernadette Burrton and her family have invested heavily in the sports complex during the first five years.

a. Bernadette invested $25000 in the first year of the complex. In the next four years she invested the same amount plus 3.5% to allow for inflation.

i. How much did Bernadette invest in the fourth year of the complex? Write your answer correct to the nearest dollar.

ii. What was the total amount of Bernadette's investment over the five years? Write your answer correct to the nearest dollar.

1 + 1 = 2 marks

b. The insurance value of the facilities at the sports complex over the first three years is given in the table below:

Year	2001	2002	2003
Insurance Value	$1.53 million	$1.5606 million	$1.591812 million

i. A difference equation that gives the insurance value (in millions of dollars) in a particular year is in the form of

$$V_{n+1} = aV_n + b, \quad V_1 = 1.53$$

Determine the value of the constants a and b.

ii. By what fixed percentage is the insurance value increasing each year?

iii. If this trend in the insurance value continues, in what year will the value reach $2 million?

2 + 1 + 2 = 5 marks

Module 1: Number patterns- continued

Question 4
Blake, Byron, Billy and Bob Burrton have been training at the Burrton Athletics Centre for the 4×100 metre relay. Their average time to the nearest hundredth of a second during each month of training has been calculated for the first three months and follows a geometric progression.

300.00, 294.00, 288.12

a. Write the difference equation which describes their progress.

1 mark

b. If this improvement trend continues, calculate their time in the sixth month. Write your answer correct to the nearest hundredth of a second.

1 mark

c. In order to compete in the All High Final the boys want to break the 240 second barrier. At their current rate of progress, how many months training will they need to break this record?

1 mark
Total 15 marks

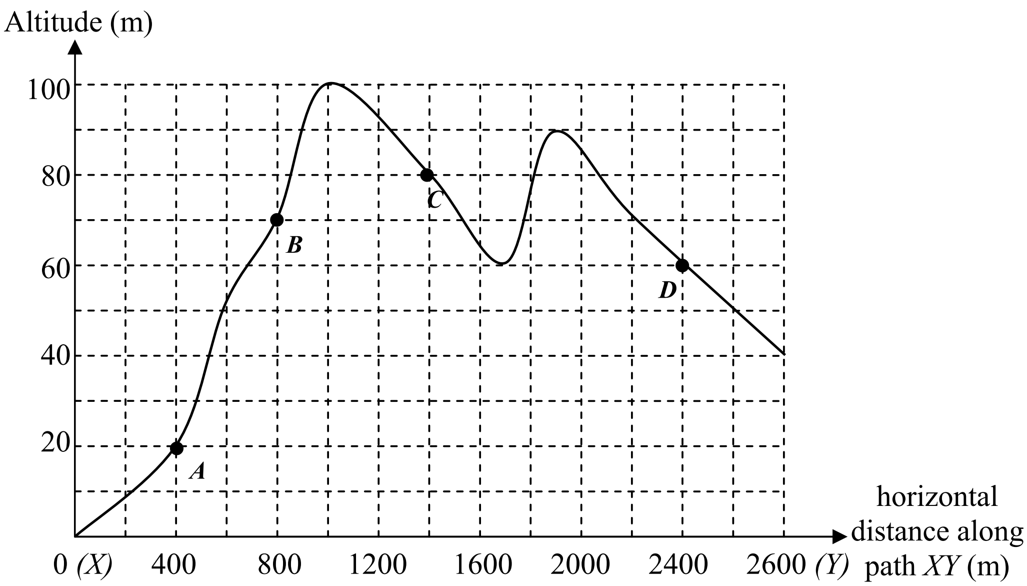
END OF MODULE 1
TURN OVER

Working space

TURN OVER

Module 2: Geometry and trigonometry

Question 1
Gary has decided to do some cross country hiking. The cross section of his route, *XY* is given in the graph below. *A, B, C* and *D* are four places of interest along route *XY*.



a. Draw a possible contour map with contours at intervals of 20 m.

1 mark

Module 2: Geometry and trigonometry-Question 1-continued
TURN OVER

b. If the distance *XY* on the graph is 10 cm what is the ratio scale for horizontal distances on the contour map you have just drawn?

1 mark

c. The track from *A* to *B* on the map follows a stream. What is the average slope of the stream between *A* and *B*? Write your answer to three decimal places.

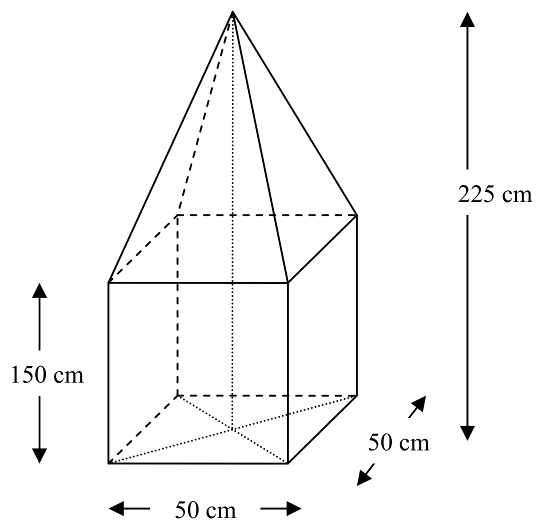
1 mark

d. At point *C* on the map Gary stops to take a photograph. He decides to stop for a snack when he gets to point *D*. Using right-angled triangles, estimate the distance Gary still has to hike until he stops for his snack? Write your answer, in metres, to the nearest metre.

1 mark

Module 2: Geometry and trigonometry- continued

Question 2



At point *C* on the map Gary notices a triangulation point. It is designed in the shape of a right pyramid sitting on top of a rectangular box. He measures the lengths and discovers that the base is a $50\text{cm} \times 50\text{cm}$ square. The height of the base is 150 cm and the overall height is 225cm.

a. Gary decides that he would like to make the structure out of wood panels. Calculate the total area of the 9 wood panels that Gary needs. Write your answer to the nearest cm^2.

2 marks

b. To make the four isosceles triangles for the pyramid Gary needs to know the angles at the base and the angle at the peak of each of the triangles.

i. Calculate the base angle of the triangle of the pyramid. Write your answer in degrees correct to one decimal place.

ii. Calculate the angle at the peak of each triangle. Write your answer in degrees correct to one decimal place.

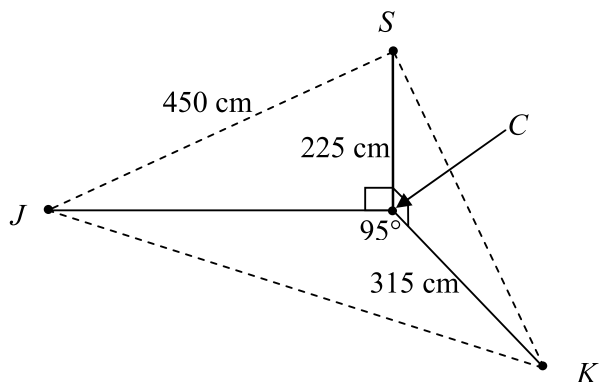
1 + 1 = 2 marks

Module 2: Geometry and trigonometry-Question 2-continued

TURN OVER

c. Gary wants to place his structure (*SC*) in his back yard as part of an ornamental feature.

This feature has an unusual triangular shape and it is not quite square, having an angle of 95° in the back corner (Point *C*). Gary has decided to stabilise the structure with guy ropes attached to the ground at points *J* and *K* as indicated.
The length of his structure *SC* = 225 cm.
The length of guy rope *JS* = 450 cm.
The distance along the ground *CK* = 315 cm.



i. Calculate the angle *SJC* that this guy rope makes with the horizontal. Write your answer in degrees correct to one decimal place.

____

____

ii. Calculate the total length of guy rope needed by Gary for both *JS* and *KS*. Write your answer, in centimetres, correct to one decimal place.

____

____

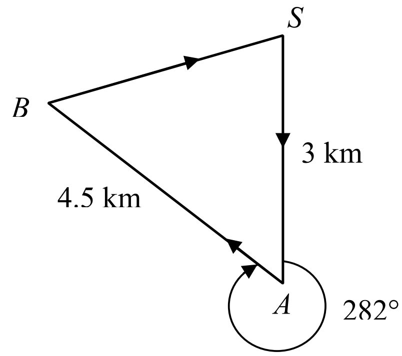
iii. Find the length *JK* to see how far apart the guy ropes have been set. Write your answer, in centimetres, correct to one decimal place.

____

____

1 + 1 + 2 = 4 marks

Question 3



Gary's next hiking trip followed a triangular course around the edge of a park. For the first leg of the course he walked 3 km due south to point *A*. Then he changed direction and walked 4.5 km on a true bearing of 282°. On the last leg of his hike he returned to his starting point *S*.

a. Determine the distance Gary walked on the third leg of his journey. Write your answer, in kilometres, correct to two decimal places.

1 mark

b. Calculate the true bearing of the starting point *S* from point *B*. Write your answer, in degrees, correct to one decimal place.

1 mark

c. What was the area of the park? Write your answer, in square kilometres, correct to one decimal place.

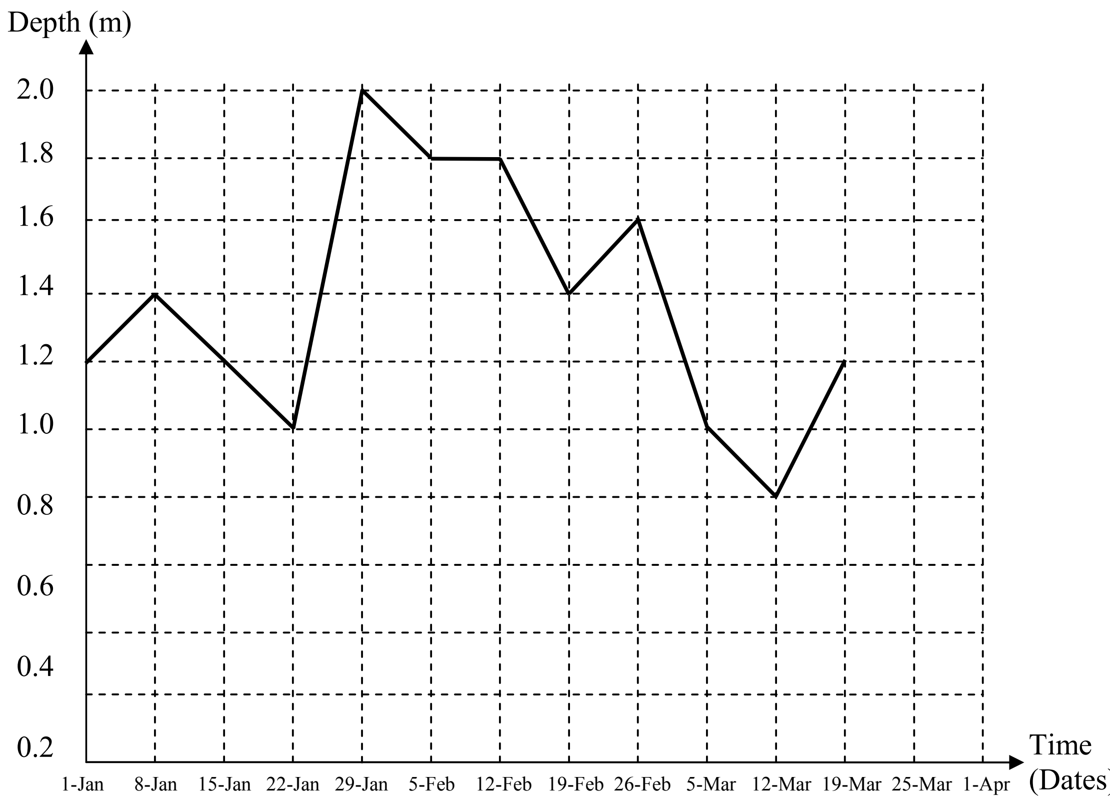
1 mark
Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Living on a farm the water in Steve's tank provides for all of his water needs. Steve monitors this volume carefully especially as he has had very low rainfall recently. The following graph shows the depth of water in Steve's tank every week for a three month period from the 1st of January to the 1st of April.



a. Give the dates between which the following events happened.

i. Steve was away for one week when there was no rainfall.

____

ii. During another week without rain Steve had guests staying over and they consumed water at the most rapid rate.

____

iii. It rained all week in an incredible downpour.

____

1 + 1 + 1 = 3 marks

b. In an average week Steve consumes a 20 cm depth of water in his tank. There was no rainfall during the last two weeks in March. Complete the graph to show the depth of water in the tank over that time.

1 mark

Module 3: Graphs and relations-Question 1-continued

Question 2
Nora makes cards as a hobby. She has decided to sell her cards at a local market. She has calculated that the raw materials for making each card average out at $0.75 per card. It also costs her $5.50 for electricity and other overheads for each batch of cards she makes. She decides to sell her cards for $2.00.

a. Calculate:

i. the cost of producing a batch of x number of cards.

---

ii. the revenue gained in selling a batch of x number of cards.

---

1 + 1 = 2 marks

b. Write an equation to calculate Nora's profit $\$P$ in terms of x.

---

1 mark

c. What is the minimum number of cards that Nora must sell to make a profit?

---

1 mark

d. Nora has been asked to make and sell some cards for charity. Her raw materials have increased to $1.00 per card and her overheads are now $8.20 for the special batch of cards she has made.

i. How much should she charge if she wishes to break-even with 10 cards?

---

ii. How much profit will the charity make if Nora sells 60 cards at a price of $2.50 each?

---

1 + 1 = 2 marks

Module 3: Graphs and relations - continued
TURN OVER

Question 3
Neville and Natasha just inherited some money from their uncle Nicholas. Nicholas left them \$1 500 000. He also left some advice on how this money should be invested. Neville and Natasha have decided to follow the advice offered.

They may invest in property trusts (x) or shares (y) subject to the following constraints:

1. The value of the property trust must be less than or equal to three times the value of the shares.
2. The value of the property trust must be greater than the value of the shares.

In recent times, property trusts have delivered returns of around 6.8% (this is a 6.8% increase in value) and shares have increased in value by 7.5%.

a. Write the constraints as linear equations.

1 mark

b. Graph the constraints and indicate the solution region.

2 marks

c. State the objective function, P.

1 mark

d. Determine the value of property trust and shares purchased to maximise the profit.

1 mark

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Mark purchased a block of land for \$230000 at the start of 2004. It has increased in value since then at an average rate of 4.5% per annum compounded annually.

a. Calculate the value of the land at the start of 2010 to the nearest whole dollar.

1 mark

b. At the start of 2010 Mark sells the land to Hilary for \$425000. Calculate the compounded growth rate earned by Mark. Write your answer correct to two decimal places.

2 marks

Question 2

Leanne and Mark have just had a baby and have decided to start investing to pay for their daughter's secondary and tertiary education. They would like to have \$14 000 per year interest available in 12 years time. They estimate the interest rate then will be about 4% per annum.

a. Calculate the principal that they will need in order to have a return of \$14 000 a year.

1 mark

Module 4: Business-related mathematics-Question 2-continued
TURN OVER

Leanne and Mark receive the Federal Government's $3000 baby bonus and a further $1500 in gifts from family and friends. They decided to use this as the initial deposit for their investment. The best interest rate available is 3.75% per annum compounded monthly.

b. Calculate the monthly deposit (to the nearest whole number of dollars) they will need to make in order to have enough money for their daughter's education.

2 marks

c. Leanne and Mark have discovered that they can afford to deposit $1800 each month and they decide to do this for 14 years rather than 12 years. Calculate the value of the investment (to the nearest whole number of dollars) at the end of the 14 years.

2 marks

d. How much money will they now receive each year to pay for their daughter's education, assuming the interest rate is 5.0%? Write your answer to the nearest whole number of dollars.

1 mark

Module 4: Business-related mathematics- continued

Question 3

Leanne is the manager of a computer sales and repairs store. She recently purchased a box of 250 rewritable CD's. The wholesale price is normally \$325 but Leanne was given a discount of 6% because she is a regular customer.

a. Calculate the amount of discount in dollars and cents.

1 mark

b. Leanne repackages the CD's into lots of 25. She wants to make a 35% profit (before GST) on the packages of CD's. Calculate the amount she should sell each pack of 25 CD's for (before GST). Write your answer to the nearest 25 cents.

1 mark

c. Leanne needs to add 10% GST onto the cost of her CD's. Calculate the selling price of the CD's. Give your answer to the nearest 25 cents.

1 mark

Leanne signs a three year lease on a delivery van to collect and deliver computers and components for repair. The van is valued at \$28500 new. Leanne pays \$650 per month for the lease and the van depreciates at a reducing balance rate of 12.5% per annum.

d. Calculate the book value of the van at the end of the three years. Give your answer to the nearest whole number of dollars.

1 mark

Module 4: Business-related mathematics-Question 3-continued
TURN OVER

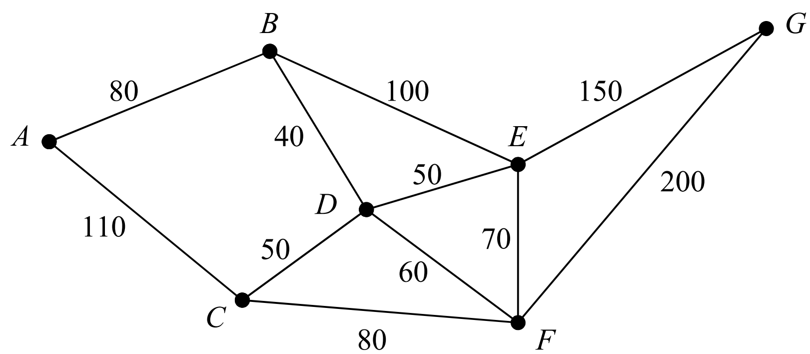
e. Calculate how much extra Leanne has paid for the van compared to its book value after three years. Give your answer to the nearest whole number of dollars.

2 marks
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1
A part of a suburban rail, bus and tram network is shown in the diagram below. The letters represent the places where passengers can get on and off the network. All distances are in metres.



a. What is the degree of each of the vertices in the network?

A ________ *B* ________ *C* ________ *D* ________

E ________ *F* ________ *G* ________

3 marks

b.

i. Mary wishes to visit each of the stops on the network to check passenger safety conditions. List a route that she could take to visit each stop once without repeating any leg of the journey.

____

1 mark

ii. List the edges in order, for the minimum spanning tree for this network.

____

1 mark

iii. What is the distance of the minimum spanning tree?

____

1 mark

iv. What is the minimum distance Mary must travel starting from and returning to *A* so that she visits each stop in the network. (Mary may repeat some legs of the network if this is the shortest distance).

____

1 mark

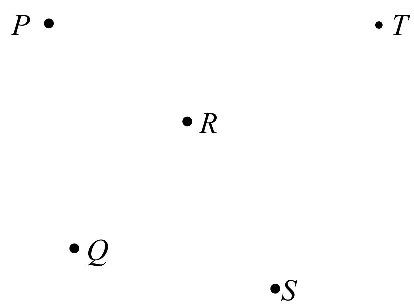
Module 5: Networks and decision mathematics–Question 1-continued
TURN OVER

Question 2

A directed graph (digraph *PQRST*) has the following adjacency matrix:

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a. Construct the diagraph using the points indicated in the diagram below.



1 mark

b. This network actually represents the results in a series of tennis games. In the adjacency matrix 1 represents a win and 0 a loss. We can treat this matrix as a dominance matrix *D*.

i. Find $D^2 + D$.

1 mark

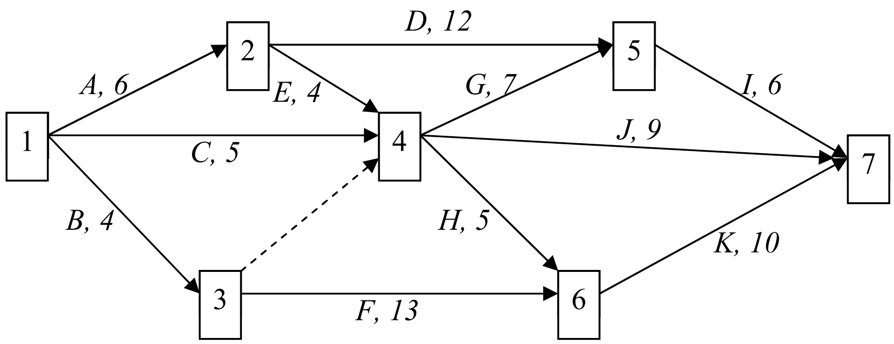
ii. Rank the five players in the tournament from highest to lowest.

____

1 mark

Module 5: Networks and decision mathematics- continued

Question 3
A small construction project has the activity network shown below. The numbers on the edges represent time in days for the completion of the project.



a. What does the dotted arrow between nodes 3 and 4 represent?

____

1 mark

b. Calculate the project completion time.

____

1 mark

c. What is the greatest float time and to which activity does it belong?

____

1 mark

Module 5: Networks and decision mathematics-Question 3-continued
TURN OVER

The cost of completion for each of the activities is given in the table below. Some activities also have a crash time and a consequent **addition** to the associated costs.

Activity	Current Duration	Crash Duration	Current Cost	Crash Cost
A	6		\$30	
B	4		\$20	
C	5	4	\$25	\$35
D	12	10	\$72	\$100
E	4		\$24	
F	13	10	\$78	\$120
G	7	6	\$49	\$13
H	5		\$35	
I	6		\$49	
J	9		\$72	
K	10		\$90	

d. What is the current cost of the project?

____

1 mark

e. What will be the effect on the project in terms of both project time and cost if Activity *F* is crashed?

____

1 mark
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

An electrical warehouse stores microwave ovens (*M*), refrigerators (*R*), televisions (*T*) and washing machines (*W*). It delivers to retail outlets in Allanstown, Bensford and Colinswood.

In February the number of each item delivered to the stores was written as a matrix *D*, where

$$D = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 32 & 14 & 20 \\ 12 & 15 & 11 \\ 6 & 17 & 5 \\ 15 & 11 & 8 \end{bmatrix} \end{array} \begin{array}{c} \\ M \\ R \\ T \\ W \end{array}$$

a. What is the order of matrix *D*.

1 mark

The transporting costs for delivery per item to each of the retail outlets is given in matrix *T*

$$T = \begin{bmatrix} \$3.20 \\ \$2.75 \\ \$4.16 \end{bmatrix} \begin{array}{c} A \\ B \\ C \end{array}$$

b.

i. In which order should *D* and *T* be multiplied?

ii. Calculate this multiplication matrix.

iii. What does this information tell you?

1 + 1 + 1 = 3 marks

Module 6: Matrices- continued
TURN OVER

Question 2
Don and his children Maria and John run three jewellery shops in Chelsea, Bentleigh and Pakenham. Their business is organised into three sections, Jewellery, Giftware and Repairs. In July this year Don collected the quarterly figures for the total number of items sold or repaired from each of the three stores.

In Don's Chelsea store these were: Jewellery – 250, Giftware – 375 and Repairs – 82.
In Maria's Bentleigh store these were: Jewellery – 450, Giftware – 300 and Repairs – 50.
In John's Pakenham store these were: Jewellery – 340, Giftware – 270 and Repairs – 160.

a. Set this information up in a matrix with each row representing a different store.

1 mark

Don has calculated the profit for the quarter from each store.
The Chelsea store made a profit of $23 194.
The Bentleigh store made a profit of $24 150.
The Pakenham store made a profit of $24 940.

b.
i. Set this information up in a 3×1 profit matrix.

ii. Use the two matrices to find the profit per item of Jewellery, Giftware and Repairs from all three stores.

1 + 2 = 3 marks

Module 6: Matrices- continued

Question 3

A new supermarket Pricelow has opened in John's town. In its first week of operation it received 32% of the custom while the remaining percentage of customers went to the established supermarket Davies.

a. Represent this information as an initial state matrix C_0.

$$C_0 = \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

In the following week it was noticed that 40% of those who had shopped at Pricelow returned to Davies and 25% of those who had shopped at Davies changed to Pricelow.

b. Represent this information as a Transition matrix T.

$$T = \begin{bmatrix} & \\ & \end{bmatrix}$$

2 marks

c. Use T and C_0 to **evaluate** and **write** the percentage of customers who shop at each of the two supermarkets in the following week.

2 marks

d. There are 5000 people who use the supermarket in John's town each week. Calculate the number of people who shop at the Pricelow supermarket six weeks (T^5) after opening.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK