FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2007 Trial Examination

SOLUTIONS

Core

Question 1

a.

Table 2

Sector	Transport	Industrial processes	Waste
Mean	$\frac{526.3}{8} = 65.8$	10.2	15.3
Standard deviation	4.3	1.2	0.3

A1

b.

i. $z = \frac{9.1 - 10.2}{1.2} = -0.9$

M1

ii. This z score means that in 1998 the *Industrial Processes* sector carbon dioxide equivalent emissions were *less than* one standard deviation *below* the mean.

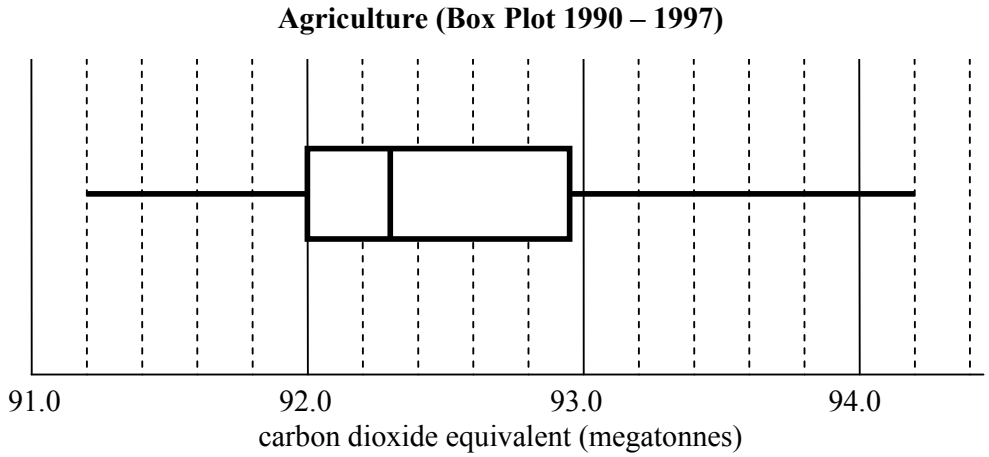
A1

c. Five figure summary:

Lowest Score	91.2
Lower Quartile	92.0
Median	92.3
Upper Quartile	92.95
Highest Score	94.2

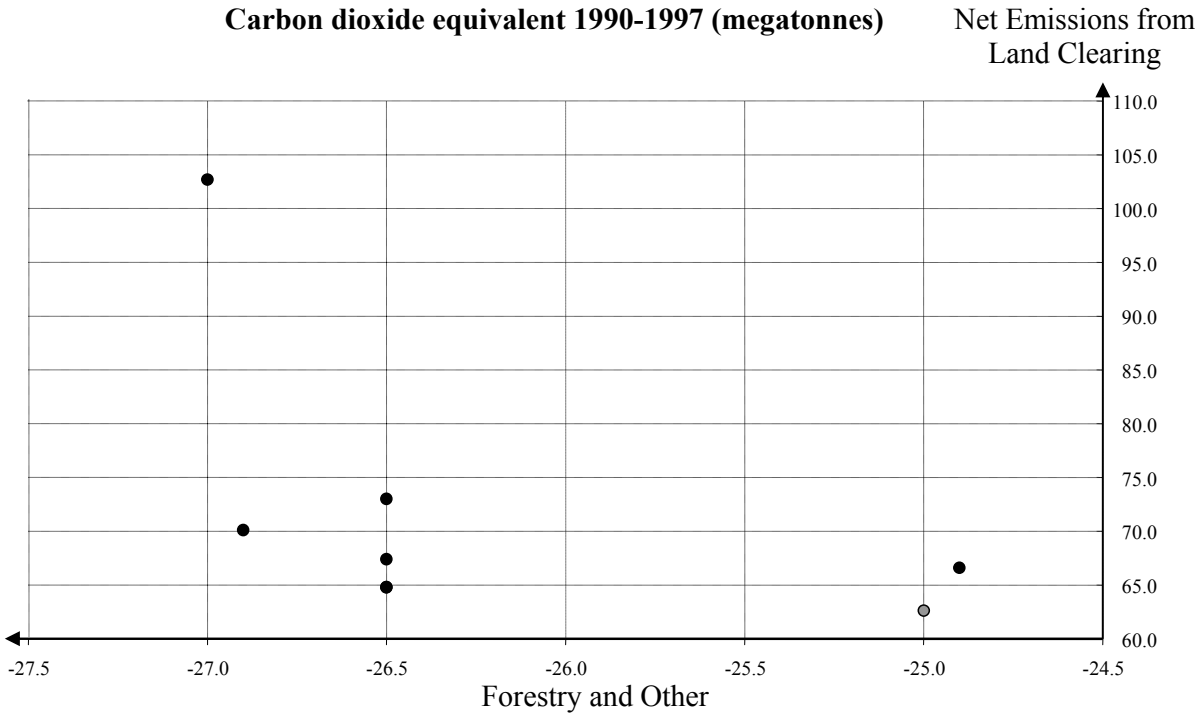
A1

d.



Correctly graphed A2

e.



Correctly labelled A1

f.

i. $E = -133.7 + -7.8F$

M1

ii. -133.7 megatonnes of carbon dioxide equivalent.

A1

g. 24.0% of the variation in the *Net Emissions from Land Clearing* emissions can be explained by the variation in the *Forestry and Other* emissions.

A1

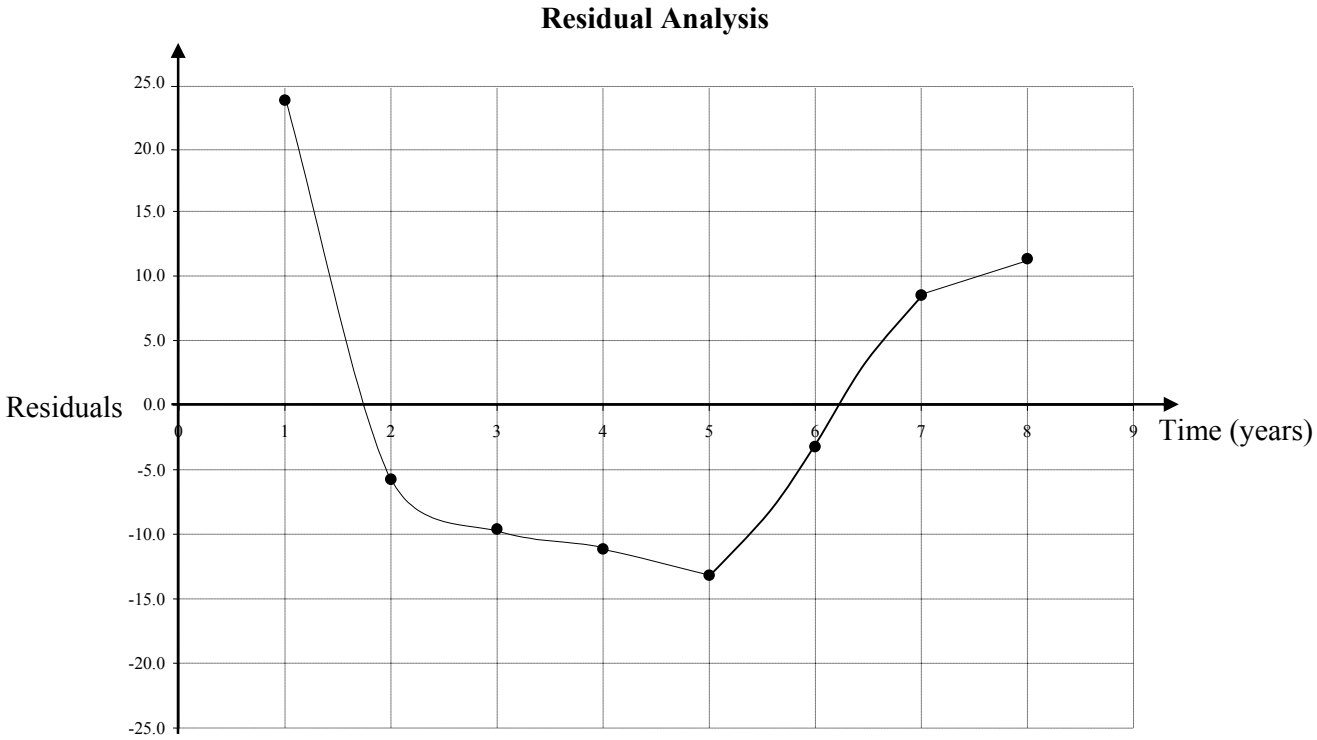
Question 2

a. Complete the residual analysis table below. Write your answer to one decimal place.

Time	*y* actual	*y* predicted	residual
1	491.4	467.5	23.9
2	464.2	470.0	-5.8
3	462.6	472.5	-9.9
4	463.8	475.0	-11.2
5	464.3	477.4	-13.1
6	476.2	$2.4786 \times 6 + 465.05 = 479.9$	$476.2 - 479.9 = -3.7$
7	491.0	482.4	8.6
8	496.1	484.9	11.2

A1

b.



Correctly graphed A1

c. The graph appears to be a parabola so an x^2 transformation should be applied.

A1

d. $y = 0.3775 \times 9^2 + 466.57 = 497.1$

A1

e. An *r* value of 0.588 indicates a moderate positive linear association. The prediction would have some validity.

M1

Total 15 marks

Module 1: Number patterns

Question 1

a.

2001	2002	2003	2004	2005	2006	2007
V_1	V_2	V_3	V_4	V_5	V_6	V_7
200	280	280 + 100 = 380	380 + 140 = 520	520 + 190 = 710	710 + 260 = 970	970 + 355 = 1325

There are 1325 visitors to the club in 2007.

A1

b.

2006	2007	2008	2009	2010
V_6	V_7	V_8	V_9	V_{10}
970	1325	1810	2472.5	3377.5

In 2010 the number of visitors will exceed 3000.

A1

Question 2

a.

$a = 1, d = 3$

$t_n = a + (n-1)d$

$21 = 1 + (n-1)3$

$n - 1 = \dfrac{21-1}{3}$

$n = 7\dfrac{2}{3}$

Brian will win in his eighth game.

A1

b.

$t_7 = 21 = a + 6d, \qquad t_4 = 12 = a + 3d$

$3d = 9, \qquad d = 3, \qquad a = 3$ M1

$S_7 = \dfrac{n}{2}(2a + (n-1)d) = 3.5(6 + 6 \times 3) = 3.5 \times 24 = 84$

Barry had a score of 84 in total.

A1

Question 3

a.

i.

$a = 25000, \qquad r = 1.035$

$I_4 = ar^3 = 25000 \times 1.035^3 = 27717.95$

Bernadette invested \$27718 in the fourth year.

A1

ii.

$S_5 = \dfrac{25000(1.035^5 - 1)}{1.035 - 1} = 134061.6$ M1

Bernadette invested \$134 062 over the five years.

b.

i. $V_{n+1} = aV_n + b, \;\; V_1 = 1.5300$

The value of the constants a and b.

$a \times 1.5606 + b = 1.591812.....(1)$

$a \times 1.53 + b = 1.5606.....(2)$

$a \times 0.0306 = 0.031212.....(1) - (2)$ M1

$a = 1.02$

$b = 1.5606 - 1.02 \times 1.53 = 0$

A1

ii. 2%

A1

iii.

$2 = 1.53(1.02)^{n-1}$

$n = \dfrac{\log\left(\dfrac{2}{1.53}\right)}{\log(1.02)} + 1 \approx 14.53 years$ M1

Or by using a graphics calculator.

The value will reach 2 million in 2015 A1

Question 4

a.

$$\frac{288.12}{294.0} = \frac{294.00}{300.00} = 0.98$$

These terms have a common ratio so the difference equation is given by

$$T_{n+1} = 0.98T_n, \qquad T_1 = 300.00$$

A1

b. $T_6 = 300 \times 0.98^5 = 271.18$

A1

c.

$$300 \times 0.98^{n-1} = 240$$

$$0.98^{n-1} = \frac{24}{30} = \frac{4}{5}$$

M1

$$n - 1 = \frac{\log\left(\frac{4}{5}\right)}{\log 0.98} \approx 11.05$$

$$n \approx 12.05$$

Or by using a graphics calculator.

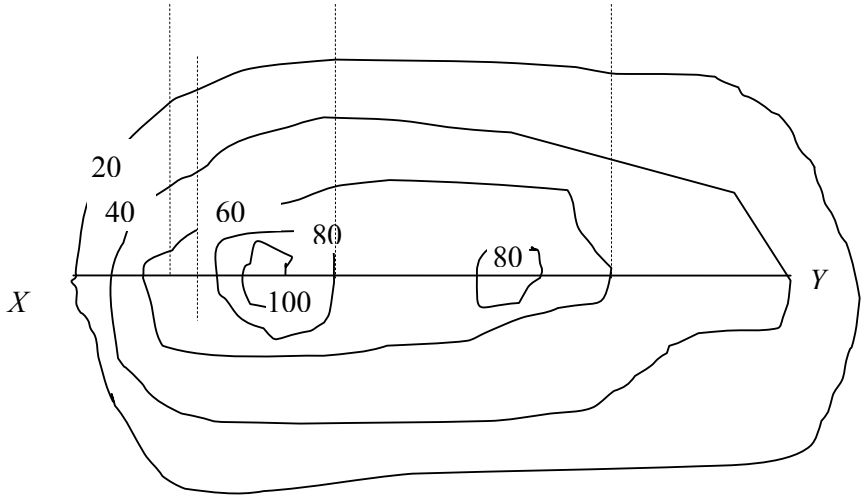
The boys will need 13 months to break the 240 second barrier.

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

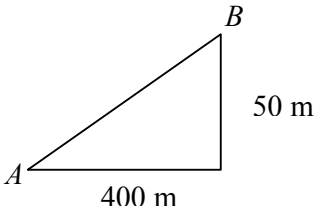
a.



Correctly drawn A1

b. 10cm = 2600m = 260000 cm $\Rightarrow$ 1 : 26000 A1

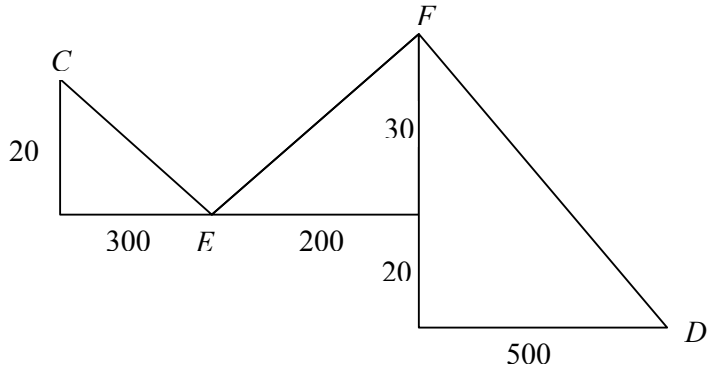
c.



$$m = \frac{50}{400} = 0.125$$

A1

d.



$CE = \sqrt{(20^2 + 300^2)} \approx 300.67, EF = \sqrt{(30^2 + 200^2)} \approx 202.24, FD = \sqrt{(30^2 + 500^2)} \approx 500.90$

$d \approx 1003.8m$

Gary needs to walk for 1004 m until he can stop. Accept estimates from 1002 m to 1006 m.

A1

Question 2

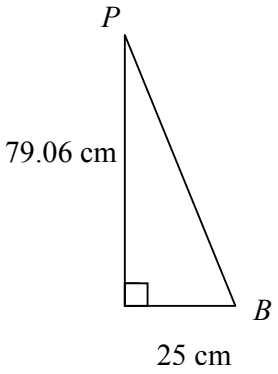
a. $TSA = 50 \times 50 + 4 \times 50 \times 150 + 4 \times 0.5 \times 50 \times \sqrt{25^2 + 75^2} \approx 40406 cm^2$ M1

Gary needs 40406 cm^2 = 4.04 m^2 of wood to make the structure.

A1

b.

i.



$B = \tan^{-1}\left(\frac{79.06}{25}\right) = 72.45^\circ$

The base angle of the triangle is 72.5°. A1

ii. $P = 180 - (90 + 72.5) = 17.5^\circ$

The peak angle of the triangle is $17.5^\circ \times 2 = 35.0^o$.

M1

c.

i. $SJC = \sin^{-1}\left(\frac{225}{450}\right) = 30.0^\circ$

A1

ii. $Length = 450 + \sqrt{\left(315^2 + 225^2\right)} = 837.1 cm$

A1

iii.

$JC = \sqrt{\left(450^2 - 225^2\right)} = 389.7 cm$

$JK = \sqrt{\left(389.7^2 + 315^2 - 2 \times 389.7 \times 315 \times \cos 95^\circ\right)}$ M1

$= 522.0 cm$

The guy ropes are 522.0 cm apart.

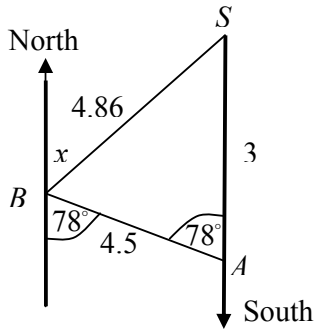
A1

Question 3

a. $BS = \sqrt{\left(3^2 + 4.5^2 - 2 \times 3 \times 4.5 \times \cos 78\right)} = 4.86km$

A1

b.



$\frac{\sin S}{4.5} = \frac{\sin 78}{4.86}$

$S = 64.9^\circ$

$x = S = 64.9^\circ$

True bearing (x) = 064.9°

A1

c. $A = 0.5 \times 4.5 \times 3 \times \sin 78 = 6.6km^2$

A1

Total 15 marks

Module 3: Graphs and relations

Question 1

a.

i. 5th February to 12th February.

A1

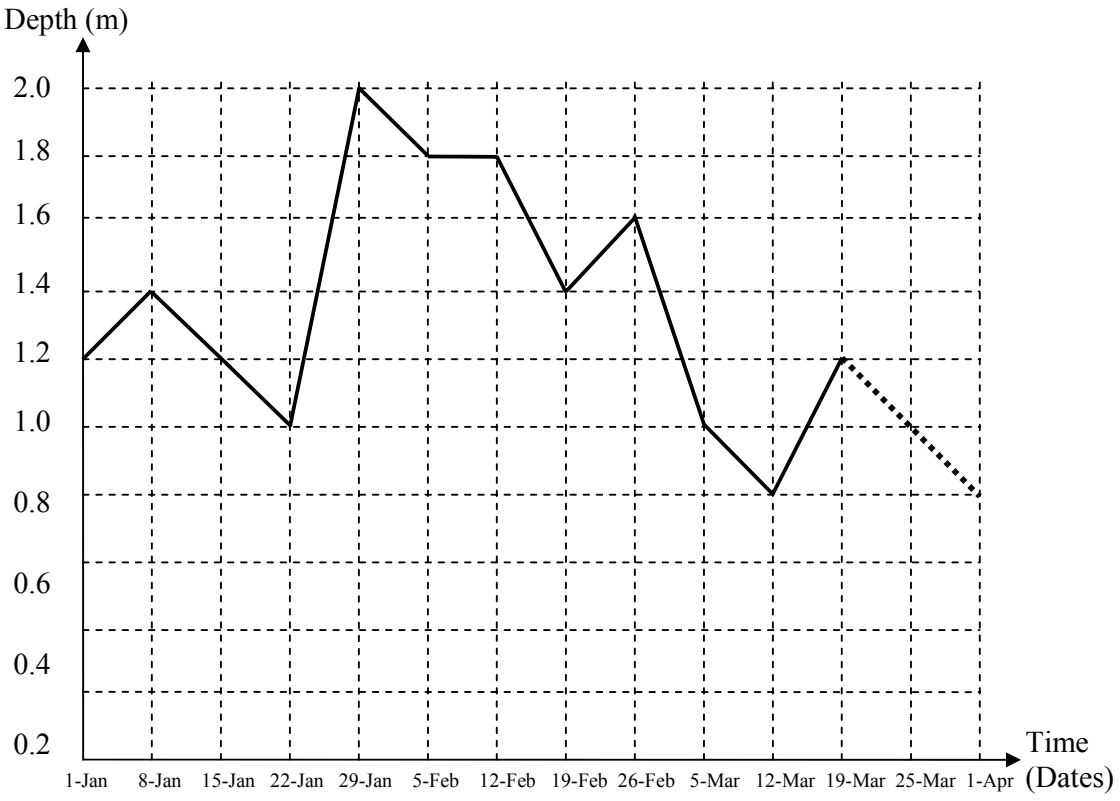
ii. 26th February to 5th March.

A1

iii. 22nd January to 29th January.

A1

b. See dotted line marked on the graph below.



Correctly graphed A1

Question 2

a.

i. $\$C = 0.75x + 5.50$

A1

ii. $\$R = 2x$

A1

b. $\$P = 1.25x - 5.5$

A1

c.

$$0.75x + 5.5 = 2x$$
$$1.25x = 5.5$$
$$x = \frac{5.5}{1.25} = 4.4$$

Nora needs to sell at least 5 cards to make a profit.

A1

d.

i.

$$10y = 10 + 8.20 = 18.20$$
$$y = \frac{18.20}{10} = 1.82$$

M1

Nora needs to sell her cards at a price of \$1.82 in order to break even.

ii. $\$P = 1.5 \times 60 - 8.2 = \$81.80$

A1

Question 3

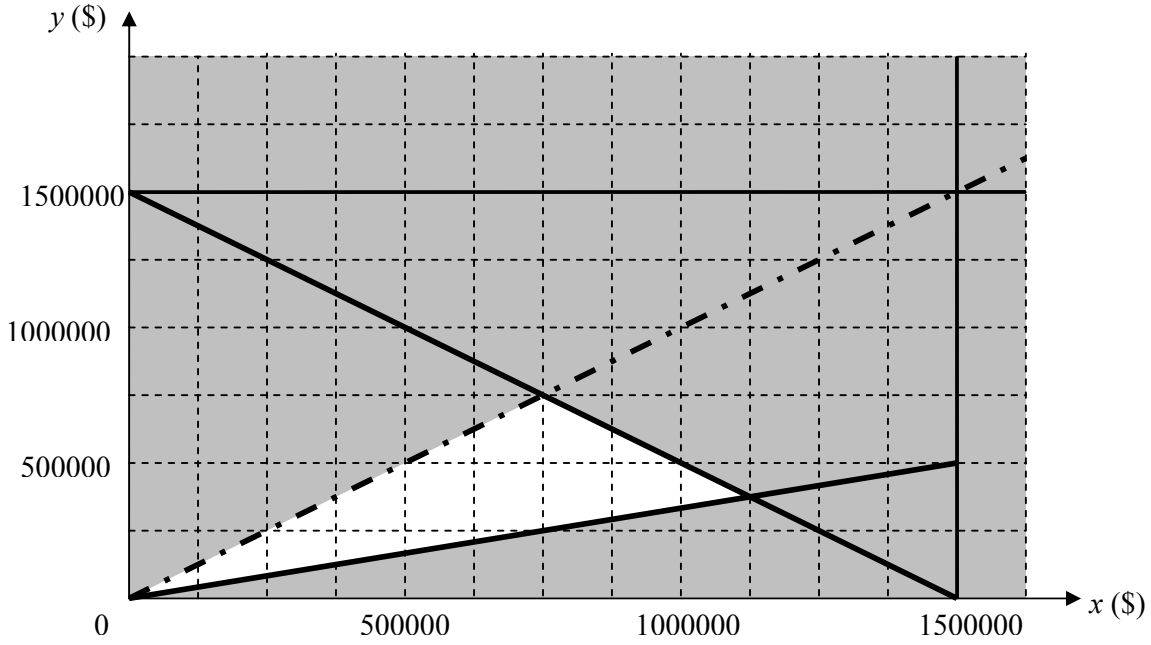
a.

$$0 \le x \le 1500000,\quad 0 \le y \le 1500000,\quad x \le 3y,\quad x > y,\quad x + y \le 1500000$$

A1

b.

The solution region is unshaded:



Correctly labelled A1
Correctly graphed A1

c. $\$P = 1.068x + 1.075y$

A1

d. The two vertices in the solution region are:

(1125000,375000)

$\$P = 1.068 \times 1125000 + 1.075 \times 375000$

$= \$1604625$

(750000,750000)

$\$P = 1.068 \times 750000 + 1.075 \times 750000$

$= \$1607250$

(There is also (0,0) but this will give zero profit.)

To maximise the profit Neville and Natasha should purchase \$0.75 million in property trusts and \$0.75 million in shares.

A1

Total 15 marks

Module 4: Business-related mathematics

Question 1

a.

$P = 230000, \qquad R = 1.045, \qquad n = 6 \quad years$

$A = PR^n = 230000 \times 1.045^6$

$= \$299520$

The land is worth \$299 520

A1

b.

$425000 = 230000 \times R^6$

$R^6 = 425000 \div 230000$

$6 \log R = \log\left(\frac{425000}{230000}\right)$

$\log R = \frac{1}{6}\log 1.84783 = 0.04444$

$R = 1.1078$

Or by using a graphics calculator.

M1

Growth rate = 10.78%

A1

Question 2

a. $P = \dfrac{14000}{0.04} = \$350000$

A1

b.

$$Q = \frac{\left(PR^n - A\right)\left(R-1\right)}{\left(R^n - 1\right)}$$

$$P = 4500, \qquad n = 144, \qquad R = 1 + \frac{3.75}{1200} = 1.003125, \qquad A = 350000$$

$$Q = \frac{\left(4500 \times 1.003125^{144} - 350000\right) \times 0.003125}{\left(1.003125^{144} - 1\right)}$$

Or by using a graphics calculator.

M1

$\approx \$1889$

A1

c.

$$A = PR^n + \frac{Q\left(R^n - 1\right)}{\left(R-1\right)}$$

$$P = 4500, \qquad n = 168, \qquad R = 1.003125, \qquad Q = 1800$$

$$A = 4500 \times 1.003125^{168} + \frac{1800\left(1.003125^{168} - 1\right)}{0.003125} = \$404508.38$$

M1

$A \approx \$404\ 508$

A1

d. $Payment = 404508 \times 0.05 = 20225.4$

Leanne and Mark will receive \$20225 each year.

A1

Question 3

a. $0.06 \times 325 = \$19.50$

A1

b.

$325 - 19.5 = 305.50 \qquad 305.50 \div 10 = 30.55$

$30.55 \times 1.35 = \$41.25$

A1

c. $41.25 + 4.125 = 45.375$ Selling price = \$45.50

A1

d.

$$\begin{aligned} Value &= PR^n \\ &= 28500 \times (1 - 0.125)^3 \\ &= 28500 \times 0.875^3 \\ &\approx \$19093 \end{aligned}$$

A1

e.

$650 \times 12 \times 3 = 23400$

A1

$23400 - 19093 = \$4307$

A1

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

a.

A	2	***B***	3	***C***	3	***D***	4
E	4	***F***	4	***G***	2		

A3

b.

i. Various solutions are possible. Some solutions are:
ABEGFDC *ABEGFCD* *ABDCFEG* *ABDCFGE* *ACFGEDB* *ACDBEFG*

A1

ii. The edges in order of the minimum spanning tree are *BD*, *DC* or *DE*, *DF*, *AB*, *EG*. Note *DC* and *DE* can be interchanged.

A1

iii. 430 m

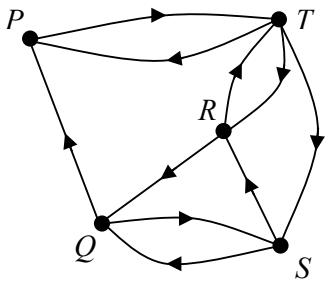
A1

iv. 710 m (route *ABDEGFCA*)

A1

Question 2

a.



Correctly drawn A1

b.

i.

$$D^2 + D = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 & 2 \end{bmatrix}$$

A1

ii. *TRSQP*

A1

Question 3

a. a dummy activity – Activities *G, J* & *H* cannot proceed before *B* is completed

A1

b. $B + F + K = 4 + 13 + 10 = 27 \quad days$

A1

c. $Activity \quad J = 18 - 10 = 8 \quad days$

A1

d. $30 + 20 + 25 + 72 + 24 + 78 + 49 + 35 + 49 + 72 + 90 = \$544$

A1

e. Activity *F* is on the critical path so both project time and cost will be altered. The project time will drop to 25 days and the cost will increase to \$586.

A2

Total 15 marks

Module 6: Matrices

Question 1

a. This matrix has 4 rows and 3 columns so it is a 4×3 matrix.

A1

b.

i. $D \times T$ because of the order of the matrices.

A1

ii.

$$\begin{bmatrix} 32 & 14 & 20 \\ 12 & 15 & 11 \\ 6 & 17 & 5 \\ 15 & 11 & 8 \end{bmatrix} \begin{bmatrix} \$3.20 \\ \$2.75 \\ \$4.16 \end{bmatrix} = \begin{bmatrix} 32 \times 3.2 + 14 \times 2.75 + 20 \times 4.16 \\ 12 \times 3.2 + 15 \times 2.75 + 11 \times 4.16 \\ 6 \times 3.2 + 17 \times 2.75 + 5 \times 4.16 \\ 15 \times 3.2 + 11 \times 2.75 + 8 \times 4.16 \end{bmatrix} = \begin{bmatrix} 224.1 \\ 125.41 \\ 86.75 \\ 111.53 \end{bmatrix}$$

A1

iii.

The cost of delivery of Microwave ovens is \$224.10 in the month of February.
The cost of delivery of Refrigerators is \$125.41 in the month of February.
The cost of delivery of Televisions is \$86.75 in the month of February.
The cost of delivery of Washing machines is \$111.53 in the month of February.
(It would be acceptable to say that this information gives the delivery costs of the different appliances. It would not be acceptable to write just delivery costs or delivery costs per store).

A1

Question 2

a.

$$\begin{bmatrix} 250 & 375 & 82 \\ 450 & 300 & 50 \\ 340 & 270 & 160 \end{bmatrix}$$

The order of the rows may be different – care should be taken with subsequent questions if a different order is given.

A1

b.

i.

$$\begin{bmatrix} 23194 \\ 24150 \\ 24940 \end{bmatrix}$$

A1

ii.

$$\begin{bmatrix} 250 & 375 & 82 \\ 450 & 300 & 50 \\ 340 & 270 & 160 \end{bmatrix} \begin{bmatrix} j \\ g \\ r \end{bmatrix} = \begin{bmatrix} 23194 \\ 24150 \\ 24940 \end{bmatrix}$$

$$\begin{bmatrix} j \\ g \\ r \end{bmatrix} = \begin{bmatrix} 250 & 375 & 82 \\ 450 & 300 & 50 \\ 340 & 270 & 160 \end{bmatrix}^{-1} \begin{bmatrix} 23194 \\ 24150 \\ 24940 \end{bmatrix}$$

$$\begin{bmatrix} j \\ g \\ r \end{bmatrix} = \begin{bmatrix} 25 \\ 36 \\ 42 \end{bmatrix}$$

M1

Jewellery makes a profit of \$25 per item,
Giftware makes a profit of \$36 per item.
Repairs make a profit of \$42 per item.

A1

Question 3

a.

$$C_0 = \begin{bmatrix} P \\ D \end{bmatrix} = \begin{bmatrix} 0.32 \\ 0.68 \end{bmatrix}$$

A1

b.

$$T = \begin{bmatrix} 0.6 & 0.25 \\ 0.4 & 0.75 \end{bmatrix}$$

A2

c.

$$\begin{bmatrix} 0.6 & 0.25 \\ 0.4 & 0.75 \end{bmatrix}\begin{bmatrix} 0.32 \\ 0.68 \end{bmatrix} = \begin{bmatrix} 0.362 \\ 0.638 \end{bmatrix}$$

M1

36.2 % of customers shop in Pricelow and 63.8 % of customers shop in Davies.

A1

d.

$$5000 \times T^5 \times C_0 = 5000\begin{bmatrix} 0.6 & 0.25 \\ 0.4 & 0.75 \end{bmatrix}^5\begin{bmatrix} 0.32 \\ 0.68 \end{bmatrix} = \begin{bmatrix} 1921 \\ 3079 \end{bmatrix}$$

M1

1921 people (to the nearest person) shop in Pricelow supermarket six weeks after opening.

A1

Total 15 marks

