The Mathematical Association of Victoria

FURTHER MATHEMATICS

Trial written examination 2

(Extended Answer)

2007

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2007 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Working space

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

CORE: DATA ANALYSIS

Question 1

A luxury car dealer sells second hand BMWs.
Table 1 gives a list of 14 BMWs with their corresponding *mileage*, in thousands of kilometres, and *price*, in thousands of dollars.

Table 1

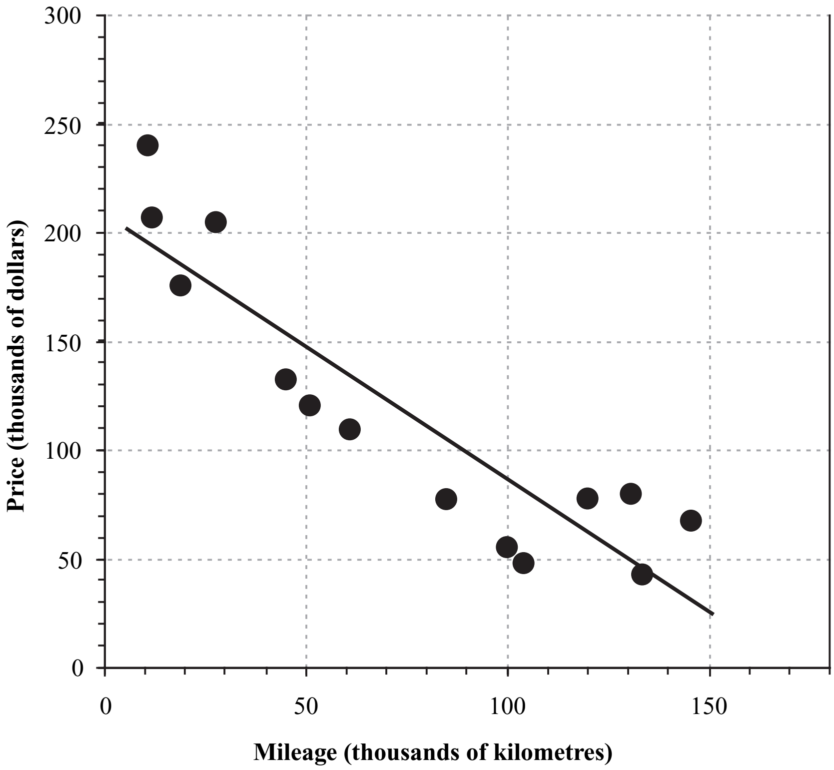
Car	Mileage (1000's km)	Price (in $000's)
A	11	239
B	12	206
C	19	175
D	28	205
E	45	132
F	51	120
G	61	109
H	85	77
I	100	55
J	104	48
K	120	77
L	131	80
M	134	42
N	146	67

a. The data in table 1 is to be used to predict the price of a BMW car from its mileage.

In this situation, the independent variable is

[1 mark]

Scatterplot 1 is constructed from the data displayed in Table 1 and a least squares regression line fitted as shown.



Scatterplot 1

b. The coefficient of determination for this data is 0.8135.

i. Find the value of the correlation coefficient correct to three decimal places.

r = ☐

ii Write down the percentage of the variation in the value of a BMW that can be accounted for by the variation in its mileage.

Percentage = ☐

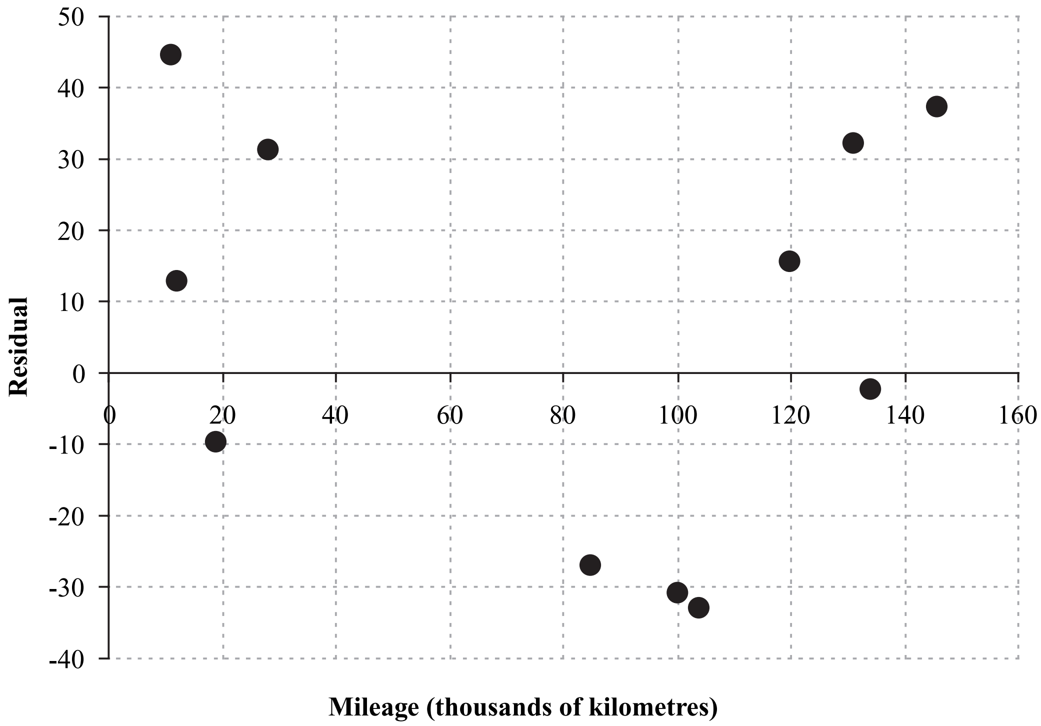
[1 + 1 = 2 marks]

c. Using the data is Table 1, determine the equation of the least squares regression line. Write the coefficients correct to two decimal places.

Price (in thousands \$) = ☐ + ☐ × Mileage

[2 marks]

Scatterplot 1 suggests that the relationship may be nonlinear. To investigate this data, Residual plot 1 is constructed. It is incomplete.



d. Using the information in Scatterplot 1, or otherwise, complete Residual plot 1 above by marking in the missing residual values for car E, F and G

[2 marks]

e. When complete does Residual plot 1 suggest that a nonlinear relationship will provide a better fit for the data? Justify your response.

[1 mark]

Scatterplot 1 indicates that a logarithmic transformation of the horizontal (mileage) axis may linearise the data. The original data has been reproduced in Table 2. An extra column has been added for the transformed variable, log(mileage). The table is incomplete.

Table 2

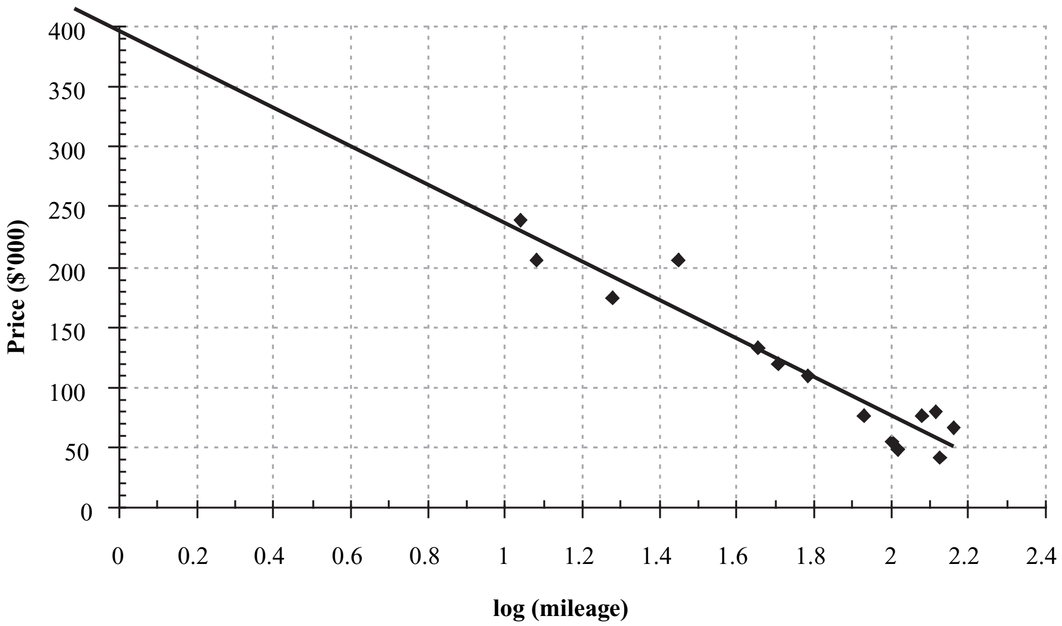
Mileage ('000)	**Log(mileage)**	**Price ($'000)**
11	1.04	239
12	1.08	206
19	1.28	175
28	1.45	205
45	1.65	132
51	1.71	120
61	1.79	109
85	1.93	77
100	2.00	55
104	2.02	48
120	2.08	77
131	2.12	80
134	2.13	42
146		67

f. Complete the table above.
Write your answer correct to two decimal places.

[1 mark]

g. In Scatterplot 2 below, the price is plotted against log (mileage).

A least squares regression line is fitted to the transformed data.



Use the information in Scatterplot 2 to describe the relationship between value and log(mileage) in terms of direction, form and strength.

[3 marks]

h. The equation of the least squares regression line is given by Price = 395 – 159log(mileage)

Use this equation to predict the price of a BMW that has travelled 125 thousand km.

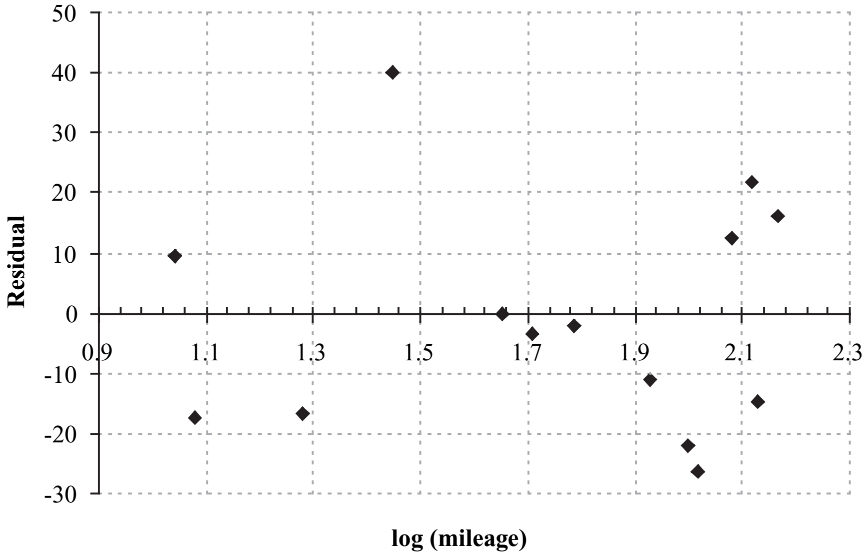
Write your answer correct to the nearest hundred dollars.

[1 mark]

i. The coefficient of determination for this relationship is 0.915. Interpret this value in terms of the variables.

[1 mark]

The residual plot for this relationship is shown in Residual plot 2



j. What feature of the residual plot shows that the transformation has been successful?

[1 mark]

END OF CORE

MODULE 1: NUMBER PATTERNS

Question 1

Jack owns a hobby farm and planted some magic beans. His thriving business produced 80 kg of beans in the first week of harvesting, 96 kg in the second week and 112 kg in the third week. If this trend continues

a. Show that the amount of beans harvested follows an arithmetic sequence.

[1 mark]

b. Find the amount (in kg) of beans harvested in the 14^{th} week.

[1 mark]

c. Determine the total amount (in kg) of beans harvested in 20 weeks.

[2 marks]

Question 2

Jack experiments with a magic fertilizer and notices that a particular bean stalk grows at an incredibly fast rate. The height of one stalk over 4 consecutive weeks is shown in the table below.

Time	Week 1	Week 2	Week 3	Week 4
Height (m)	5.2	7.8		17.55

Jack suspects that the height of this bean stalk follows a geometric sequence. If this is the case,

a. Determine the common ratio r.

[1 mark]

b. Find the height during the 3^{rd} week.

[1 mark]

c. During which week would the bean stalk first exceed a height of 200 m?

[2 marks]

Question 3

Jack notices that another particular bean stalk has a different growth pattern after it reaches a height of 15 metres. Over the next three weeks it grows 3 m, 2.55 m, 2.1675 m and so on.

a. Find the height of this beanstalk at the end of these 3 weeks.

[1 mark]

b. If this growth pattern continues indefinitely, find the maximum height of the beanstalk. Give your answer correct to two decimal places.

[2 marks]

Question 4

Jack's wife, Jill, mindful of the water restrictions, is determined to carry buckets of bath water out of the house to water the bean stalks.

She carries out 18 litres of water on the first day.

The amount of water Jill carries out, A_n, in litres on the nth day is given by the difference equation

$$A_{n+1} = 0.96\,A_n + 2,\ \ A_1 = 18$$

a. Determine the amount of water Jill will carry out during the 3rd day.
Give your answer correct to 2 decimal places.

[1 mark]

b. Determine the total amount of water Jill recycles in the first week.

[2 marks]

c. Explain with a calculation why Jill will never recycle more than 50 litres of water in a day.

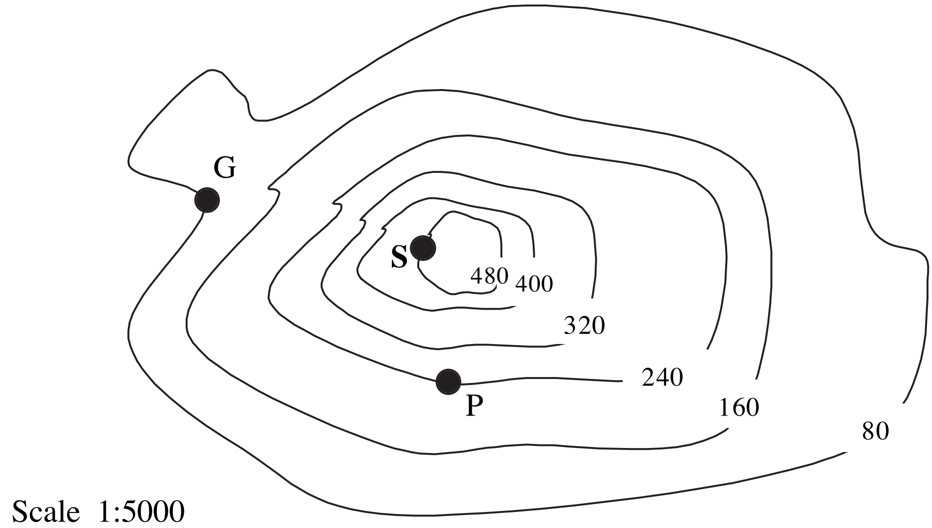
[1 mark]

END OF MODULE 1

MODULE 2: GEOMETRY AND TRIGONOMETRY

Question 1

A contour map of part of a National Park located in country Victoria, is shown below. It has contours drawn at intervals of 80 metres. The map shows the play ground, G, a picnic area, P and a scenic lookout, S.



a. What is the difference in height (in metres) between the scenic lookout S and the picnic area P?

[1 mark]

b. An electricity cable exists between the picnic area P and the playground G.
The horizontal distance between P and G is 300 m.
Find the shortest distance between P and G.
Write your answer correct to the nearest metre.

[1 mark]

c. The average slope between the playground G and the scenic lookout S is 1.3.
Find the horizontal distance between G and S correct to the nearest metre.

[1 mark]

d. The scale on the contour map is 1:5000.
The horizontal distance between S and P on the contour map is 23 mm.

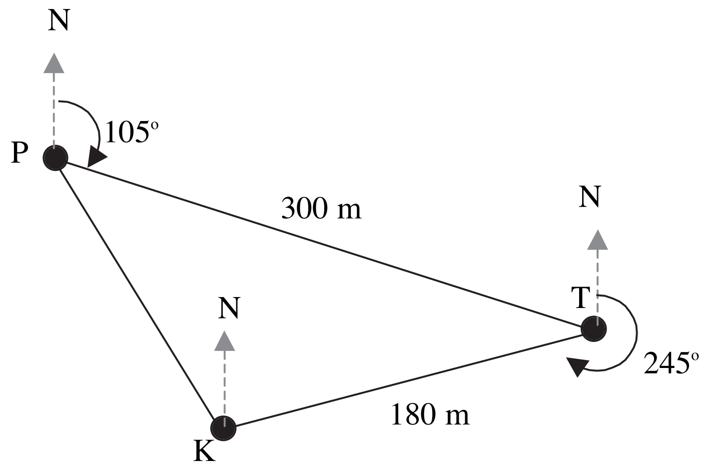
i Determine the horizontal distance between S and P to the nearest metre.

ii Hence determine the angle of inclination of the direct path between the picnic area P and the scenic lookout S. Write your answer correct to the nearest degree.

[1 + 1 = 2 marks]

Question 2

From the picnic ground, P, Michael walks 300m on a bearing 105^{o} T to the toilet block, T, and then on a bearing of 245^{o} T for 180m to the kiosk, K, before he returns to the picnic ground.



a. Show that the magnitude of angle PTK is 40^{o}.

[1 mark]

b. Determine the distance from the kiosk K to the picnic ground P.
Give your answer correct to the nearest metre.

[1 mark]

c. Calculate the bearing of the kiosk K from the picnic ground P to the nearest degree.

[2 marks]

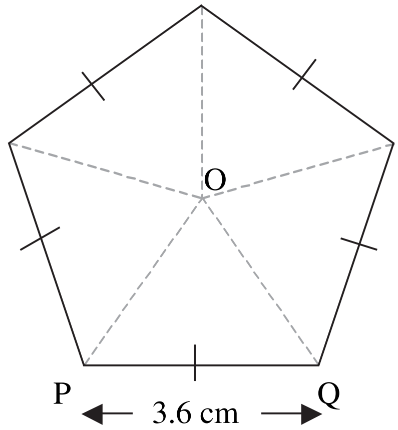
d. Calculate the area bounded by this triangular region.
Give your answer correct the nearest square metre.

[1 mark]

Question 3

Michael purchased ice-cream from the kiosk. The cross section of the ice-cream cone is in the shape of a regular pentagon.

The top of the cone shown in the diagram has side length of 3.6 cm.



a. Show by calculation that the size of angle POQ is 72°

[1 mark]

b. Determine the length PO. Give your answer correct to two decimal places.

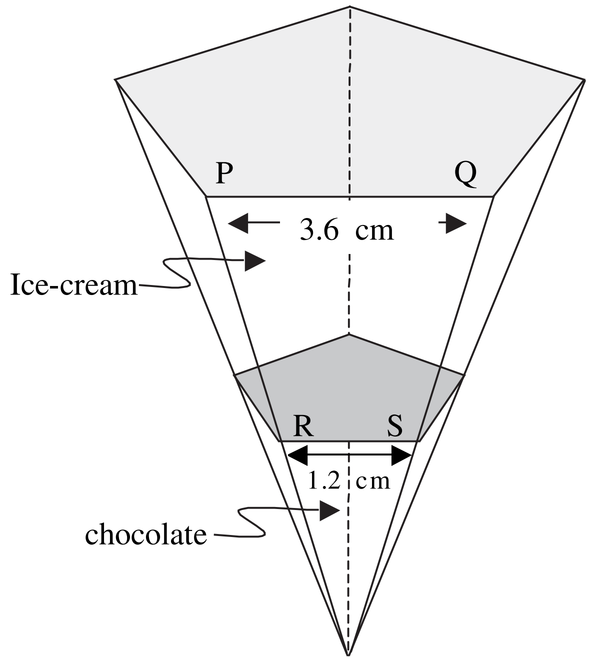
[1 mark]

c. Calculate the area of the pentagonal cross section at the top of the cone. Write your answer correct to two decimal places.

[1 mark]

d. The ice-cream cone is in the shape of an inverted pyramid with a regular pentagonal base.
The space below the bottom lining contains solid chocolate.

PQ is 3.6 centimetres and is the side length along the top of the cone.
RS is 1.2 centimetres and is the side length of the top of the chocolate inside the cone.
PQ and RS are parallel.



The total space inside the cone (including the chocolate) is V cm^3.
What fraction of V is used for the ice cream?

[2 marks]
[Total 15 marks]

END OF MODULE 2

MODULE 3: LINEAR RELATIONS AND GRAPHS

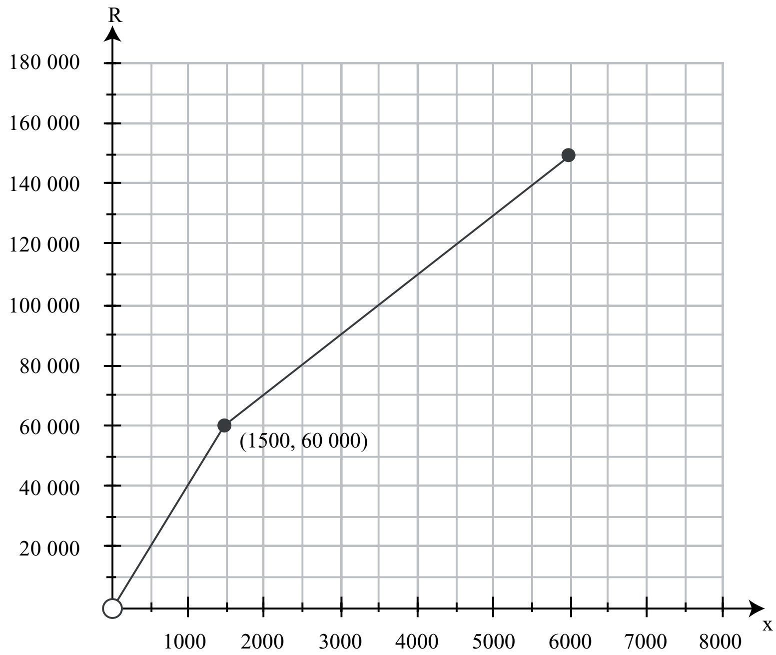
Question 1

The shoe company has begun to manufacture a new brand of shoes, the *Super Sneakers*. The company can make a maximum of 6000 pairs of shoes in the first year of production.

The revenue, R dollars, from selling x pairs of shoes in the first year is given by the function:

$$R = \begin{cases} 40x; & 0 \le x \le 1500 \\ k + mx; & 1500 < x \le b \end{cases}$$

This function is graphed below



a. State the value of b

[1 mark]

b. Use the coordinates of two points on the graph to show that the values of m, and k are 20 and 30 000 respectively

[2 marks]

c. Find the revenue from selling

i) 1000 pairs of shoes

ii) 3200 pairs of shoes

[2 marks]

The cost, C dollars, of producing x pairs of *Super Sneakers* is given by the function $C = 60000 + 7x$, where $0 \leq \times \leq 6000$.

d. Graph this function on the same set of axes as the revenue function.

[2 marks]

e. Find the minimum number of shoes that need to be produced and sold for the shoe company to make a profit.

[2 mark]

f. The 6000 pairs of *Super Sneakers* manufactured in the first year of production were sold.
Determine the maximum profit that the shoe company made during the first year of production.

[1 mark]

g. The cost, C dollars, of producing x pairs of shoes changes to $C = 30000 + 12x$.
How many pairs of shoes would the shoe company now need to produce and sell in order to break even?

[2 marks]

Question 2

The managers of the shoe factory decide to expand their business so that the number of pairs of shoes they produce per week will increase progressively until a capacity of 5000 pairs per week is reached.

The table below gives the number of pairs of shoes made during each of the first four weeks.

Week (t)	1	2	3	4
Production (p)	1000	3000	3667	4000
Deficit (d)	4000			

a. The deficit is the number of pairs of shoes that the factory is making below capacity. Complete the table.

[1 mark]

b. It has been determined that the relationship between deficit and week number is non-linear.

Assume that this data is modelled by the relation $d = \frac{k}{t}$.

Find the values of k.

[1 mark]

c. Explain by calculation why the company will never produce a capacity of 5000 pairs of shoes.

[1 mark]

END OF MODULE 3

MODULE 4: BUSINESS-RELATED MATHEMATICS

Question 1

Imogen's grandparents gave her a gift of \$5000 on her fifth birthday. Her parents invested the money in an internet account that paid 5.4% p.a. compounding monthly.

a. Show that the amount in the account after five years, if no additional amount has been invested or withdrawn, will be \$6546, to the nearest dollar. Fill in the boxes below with the figures for the calculation.

$5000 \times \boxed{}^{\boxed{}} \approx 6546$

[2 marks]

When Imogen is ten her parents decide that they will use the account to save for her living expenses when she goes to university. They estimate that \$240 a week in their present day dollars would be a workable amount for living expenses.

b. In eight years time what amount would have the buying power of \$240 in present day dollars if inflation averages 2.8% over the eight years? Give your answer correct to the nearest dollar.

[2 marks]

c. If Imogen's parents were to save for her living expenses at university how much will they need in the account when she starts university so that it provides \$1300 per month for four years. Assume the money is in an account paying 6.5% p. a. compounding monthly, no more will be added to the account once the deductions start and that nothing is left after the four years. Give your answer correct to the nearest dollar.

[2 marks]

d. If Imogen's parents decide to have \$60 000 in the account after 8 years how much will they need to add to the account each month assuming they start with \$6546 and the account earns 6.5% p. a. compounding monthly. Give your answer correct to the nearest cent.

[2 marks]

Question 2

In the first year of Christopher's university course he will accumulate \$5500 as a HECS/HELP debt.

a. If he pre-pays this debt then he will only need to pay \$4400. Calculate the percentage reduction that this represents.

[1 mark]

Christopher decides not to pre-pay his HECS/HELP debt and in his second and third years he acquires additional debts of \$5800 and \$6100 respectively. After six years from the start of his university course his level of income reaches the set amount where he is required to make repayments.

b. Complete the table below to calculate Christopher's total HECS/HELP debt after six years if inflation has averaged 2.8% over this time. Give your final answer correct to the nearest dollar.

Year of course	HECS/HELP debt	Years of inflation	Inflated amount (\$)
1	\$5500	6	6491.15
2	\$5800		
3	\$6100		
		Total	

[3 marks]

c. In the first year of making repayments Christopher has to repay 4% of his total HECS/HELP debt. Calculate the amount that he will need to repay to the nearest dollar.

[1 mark]

Some investment advisors suggest that it is better to invest the lump sums of money rather than pre-paying HECS/HELP debts.

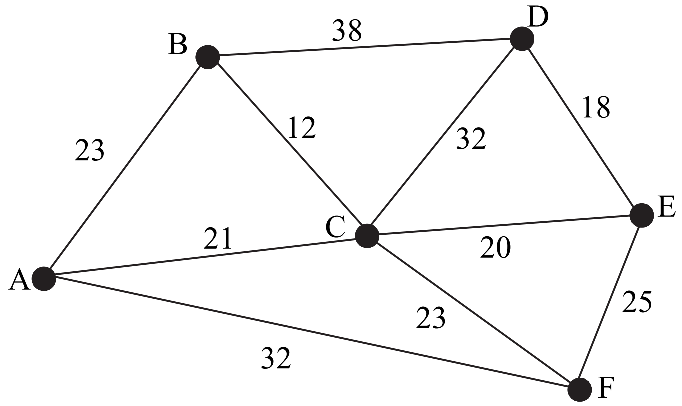
d. If Christopher has the $4400 to pre-pay his first-year debt but instead decides to invest this amount, calculate the compound interest rate per annum that the account will need to earn if the balance is to be $6491 after six years. Assume interest is credited monthly and give your answer correct to three decimal places.

[2 marks]
[Total 15 marks]

END OF MODULE 4

MODULE 5: NETWORKS AND DECISION MATHEMATICS

Steve is organizing a soccer tournament involving six teams from six country towns.
A network map of the towns is given below. The vertices on the map represent the six towns A, B, C, D, E and F and the edges represent roads connecting the towns. The values on the edges represent the distance in kilometres between the towns.



Question 1

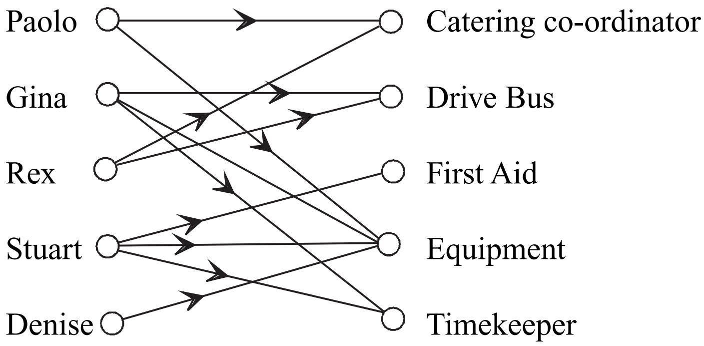
a. If Steve wants to visit each of the towns once only starting and ending at his hometown C **state the path** that he should take so that he travels the minimum distance.

[2 marks]

b. Name the type of path that Steve would have travelled in part **a.**

[1 mark]

Steve has five volunteers who are capable of doing various tasks involved in the tournament. The following bi-partite graph illustrates the tasks that each volunteer is able to do.



Question 2

Allocate one task to each of the volunteers and complete the following table of tasks for the given volunteers.

Volunteer	Task
Gina	
Rex	
Stuart	

[2 marks]

Matches are going to be played in five of the towns over five weekends with each team playing one match per weekend.
Steve has five different referees organized for each of the weekends. The distance in kilometres, of each of the referees from each of the towns is given in the table below.

	Town				
Referee	**B**	**C**	**D**	**E**	**F**
Greg	68	60	30	30	30
Harry	24	21	53	41	32
Jane	98	92	60	79	88
Ken	72	84	50	73	98
Mario	62	60	92	85	60

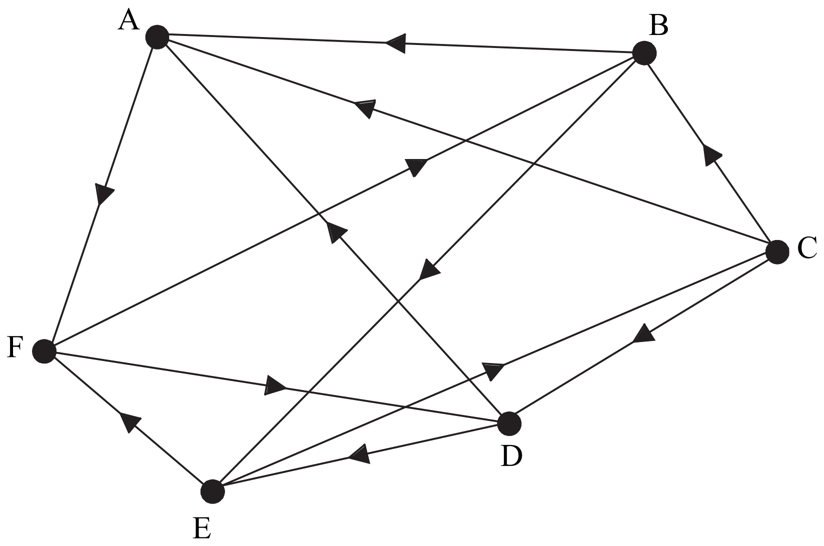
Question 3

Use the Hungarian algorithm, or otherwise, to allocate one referee to each of the towns so that the total distance that is travelled by the referees is a minimum. Complete the box below to show the allocations.

Referee	**Town**
Greg	
Harry	
Jane	
Ken	
Mario	

[3 marks]

The results of the matches for the first four weeks are given in the directed network graph below where an arrow going from B to A represents the team from town B defeating the team from town A in a match.



The dominance matrix, M, for this graph is given below. A '1' in the matrix represents a win and a '0' represents a defeat.

Defeated town

$$M = \text{Town} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & ..... \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & ..... & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

In the final matches

- the team from Town A defeated the team from Town E
- the team from Town B defeated the team from Town D
- the team from Town F defeated the team from Town C

Question 4

a. Complete the graph to show these final results.

[1 mark]

b. Complete the dominance matrix to show the results of all the matches.

[1 mark]

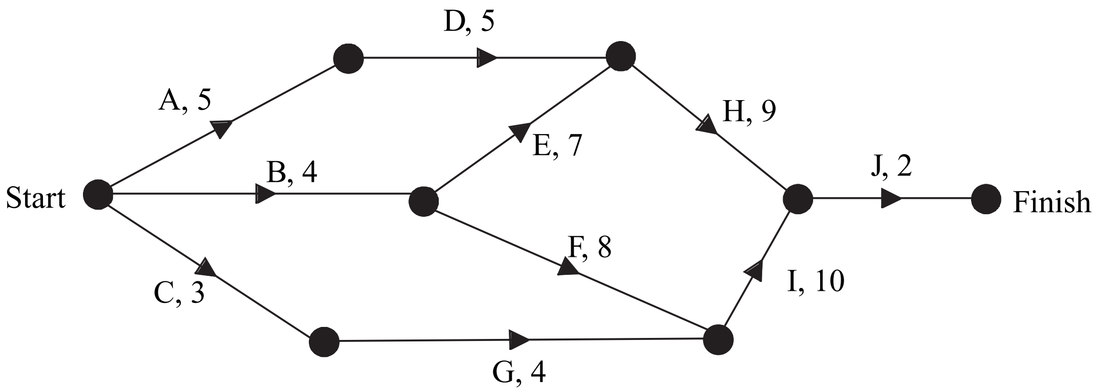
c. Use the information given to find the team that **did not** have a second-level win over the team from Town A.

[1 mark]

d. Use the first and second level wins to find the overall winner of the tournament.

[1 mark]

Steve has a project of refurbishing one of the soccer fields and he has identified 10 activities that need to be completed so that it meets the required standard.
The following network graph outlines the activities, A to J, their predecessors and the duration, in days, for each of the activities.



Question 5

a. State the minimum completion time for the project.

[1 mark]

It is possible to reduce the duration of the some of the activities. The possible reductions are given in the table below.

Activity	**Reduction (days)**
D	2
E	2
F	3

b. State the activities, and the number of days that they should be reduced by, so that the completion time of the project is reduced as much as possible.

[2 marks]
[Total 15 marks]

END OF MODULE 5

MODULE 6: MATRICES

Budget Meats offers three types of barbeque packs:

	Type of meat		
Pack	**Chops**	**Hamburgers**	**Sausages**
Family pack (F)	4	4	6
Bulk pack (B)	10	10	20
Party pack (P)	15	20	30

The **cost price** of the Family pack is \$6.70, \$18.00 for the Bulk pack and \$29.50 for the Party pack.

Question 1

a. If the cost price of each chop, hamburger and sausage is x , y and z dollars respectively write a matrix equation, of the form below, that you can solve to find the value of x, y and z.

$$\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \quad \\ \\ \end{bmatrix}$$

[1 mark]

b. Write down an inverse matrix that can be used to solve these equations.

[1 mark]

c. Solve the equation and hence write down the cost price of a chop, a hamburger and a sausage.

[1 mark]

Question 2

The **selling price** of each type of pack is calculated by multiplying the cost price by a factor. These factors are different for each pack.
For the Family pack the selling price is 1.5 times the cost price, for the Bulk pack the selling price is 1.4 times the cost price and for the Party pack the selling price is 1.3 times the cost price.
To calculate the selling price Budget Meats have set-up a matrix equation of the form

$$M \times \begin{array}{c} \text{Cost price} \\ \begin{bmatrix} 6.70 \\ 18.00 \\ 29.50 \end{bmatrix} \end{array} \begin{array}{c} \\ F \\ B \\ P \end{array} = \begin{array}{c} \text{Selling price} \\ \begin{bmatrix} 10.05 \\ 25.20 \\ 38.35 \end{bmatrix} \end{array} \begin{array}{c} \\ F \\ B \\ P \end{array}$$

a. State the order of matrix M

[1 mark]

b. Write down the matrix M

[2 marks]

Question 3

Budget Meats have three outlets for their barbeque packs; stores R, S and T.
In a particular week the following number of packs were sold at each of these outlets.

	Outlet		
	R	**S**	**T**
Family pack	21	12	7
Bulk pack	35	18	4
Party pack	9	25	15

a. Using ***profit*** = ***selling price – cost price*** set up a matrix equation that will enable Budget Meats to find the matrix P that represents the profit made **at each of the three stores** for this week.

[2 marks]

b. Calculate the profit made by Budget Meats at each of the three stores for this week.

[1 mark]

Question 4

A gymnasium is investigating the attendance habits of its members. They are looking at the weekly figures for those that attend at *least three times* (≥3), *1 or 2 times* (1, 2) and those that do *not attend* (0)
Records show that
60% of the members who attend *at least three times* in a week will attend *at least three times* in the following week, 30% will attend *1 or 2 times* and 10% will *not attend.*
80% of the members who attend *1 or 2 times* in a week will attend *1 or 2 times* in the following week, 10% will attend *three or more times* and 10% will *not attend*.
30% of the members who do *not attend* in a week will *not attend* in the following week, 40% will attend *1 or 2 times* and 30% will attend *three or more times*.

a. Enter this information into transition matrix T as indicated below, expressing percentages as proportions.

$$T = \begin{array}{c} \text{this week} \\ \begin{array}{ccc} \geq 3 & 1,2 & 0 \end{array} \\ \left[\begin{array}{ccc} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{array}\right] \end{array} \begin{array}{l} \\ \\ \geq 3 \\ 1,2 \text{ next week} \\ 0 \end{array}$$

[2 marks]

During the first week of monitoring the gymnasium had 131 members who attended *at least three times*, 386 who attended *1 or 2 times* and 203 who did *not attend*.

b. Write this information in the form of an initial state column matrix, I_0.

[1 mark]

c. Assuming that the attendance behaviour of the members depends entirely on their attendance in the previous week determine the number of members that are expected to attend *at least three times, 1 or 2 times* and *not attend* in the **fifth** week. Give your answers to the nearest whole number.

[2 marks]

d. Of the 720 members of the gymnasium determine, in the long term, the number of members who are expected to attend *at least three times* in a particular week.

[1 mark]
[Total 15 marks]

END OF MODULE 6

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

April 2006

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET