The Mathematical Association of Victoria
FURTHER MATHEMATICS

Trial written examination 1
(Facts, skills and applications)

2006

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: .....

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13	6	3	13
B	54	27			27
					Total 40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Further Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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MULTIPLE CHOICE ANSWER SHEET

Student Name: .....

Circle the letter that corresponds to each correct answer

Section A	Section B		
Compulsory	Answer three different modules. Show each module selected by ticking the appropriate box.		
	Module:	Module:	Module:
	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices
1. A B C D E	1. A B C D E	1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E	2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E	3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E	4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E	5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E	6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E	7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E	8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E	9. A B C D E	9. A B C D E
10. A B C D E			
11. A B C D E			
12. A B C D E			
13. A B C D E			

Working Space

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A **correct** answer scores 1, an **incorrect** answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core:

The following information relates to questions 1 and 2.

In a recent one-day cricket match a batsman faced 74 balls. The number of runs scored for each of these balls is shown

Runs per ball	Frequency
0	13
1	18
2	21
3	11
4	9
5	0
6	2

Question 1

A graph that would best display this information is a

A. line graph
B. bell-shaped curve
C. scatterplot
D. dot plot
E. stemplot

Question 2

The mean number of runs scored per ball for this batsman, correct to one decimal place, is

A. 1.9
B. 2.0
C. 2.1
D. 3.0
E. 3.5

The following information relates to questions 3 and 4.

The split stem and leaf plot below has been used to organize the data collected from an experiment where the height of seedlings is measured after three weeks growth using a particular seed growing mix:

Stem	Leaf
8	4
8*	8
9	1 4 4
9*	5 6 8 9
10	0 1 2 2 3 4
10*	5 5 7
11	1 3
11*	6
12	1
12*	

9 | 6 means 96 millimetres

Question 3

The percentage of plants that have a height of at least 110 mm is closest to

A. 5

B. 18

C. 32

D. 50

E. 59

Question 4

Which one of the following is **not** a true statement about the data?

A. The distribution of the data is almost symmetric

B. The range of the data is 37 mm.

C. The median height is not one of the data values.

D. There are no outliers in this data set.

E. The middle 50% of the data is spread over 10 mm.

Question 5

Jake was absent on the day the other 12 students in his class sat a Mathematics test. Jake's teacher calculated some statistics for the scores of these 12 students, noting that no two of the students scored the same mark. When Jake returned to school he sat the Mathematics test scoring the highest mark in the class. His teacher recalculated the statistics, including Jake's mark.

Which one of the following statistics would not change when Jake's mark is included?

A. the lower quartile

B. the median

C. the upper quartile

D. the mean

E. the standard deviation.

The following information relates to questions 6 and 7.

David, a teacher, is analyzing the results of two tests that he has given to his class.

For *Test 1* the mean was 62% and the standard deviation was 9%

For *Test 2* the mean was 73% and the standard deviation was 6.5%

Pearson's correlation coefficient for the two tests was 0.88

Question 6

Rebecca, one of the students in David's class, obtained a z-score of 2 on *Test 1*.

To obtain a z-score of 2 on *Test 2* Rebecca would have to score

A. 75%

B. 80%

C. 82%

D. 86%

E. 91%

Question 7

David plans to use a least-squares regression line as a linear model to predict the score on *Test 2* from the score on *Test 1*.

From the statistics given David can calculate the slope of this regression line, correct to two decimal places, as

A. −4.40

B. 0.64

C. 1.22

D. 1.63

E. 18.4

Question 8

The distribution of times for the 100 metres Womens Freestyle Swimming Heats at the Commonwealth Games Trials was bell-shaped with a mean of 58.42 seconds and a standard deviation of 1.55 seconds.

The fastest 16% of the swimmers would have swum the 100 m in

A. more than 59.97 seconds

B. less than 56.87 seconds

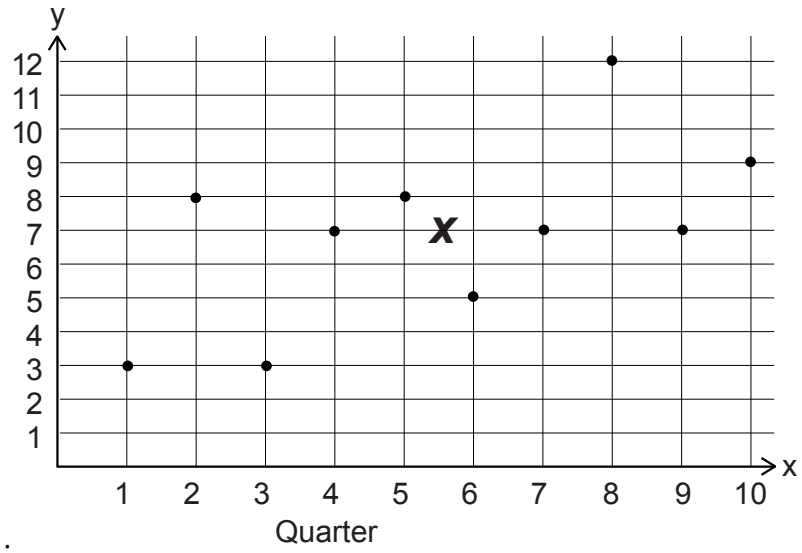
C. a time between 55.32 and 56.87 seconds

D. more than 61.52 seconds

E. less than 55.32 seconds

Question 9

A three-median regression line is to be fitted to the data graphed below. The middle median point has been marked **X** on the graph below



.

The co-ordinates of the left and right median points respectively will be

A. (2, 3) and (9, 7)

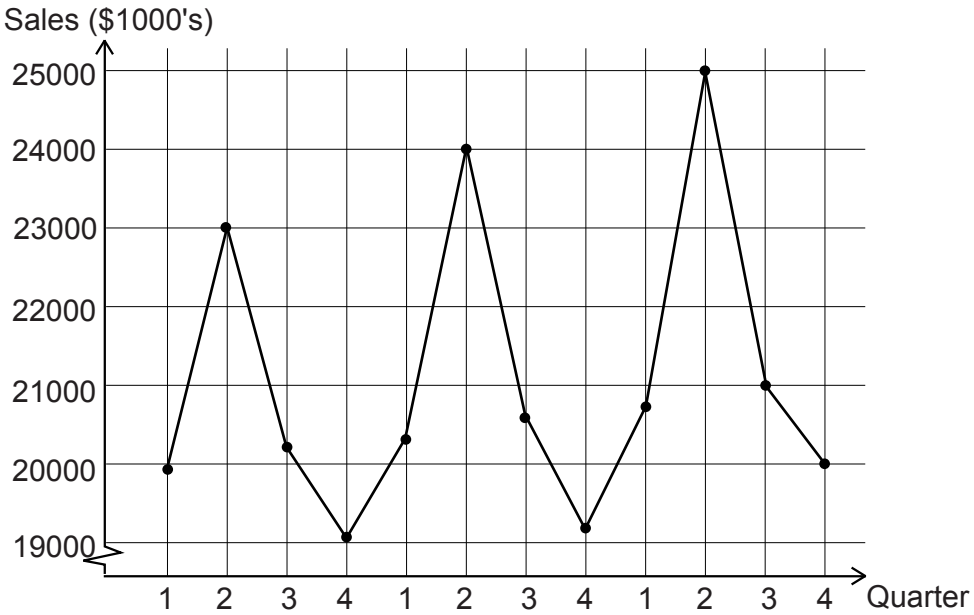
B. (3, 6) and (9, 9)

C. (2, 5) and (9, 9)

D. (2, 3) and (9, 9)

E. (2, 5) and (9, 7)

Question 10



The quarterly sales data over three years has been plotted on the graph above. This data was deseasonalised to calculate the seasonal indices.

The two quarters that would have a seasonal index closest to one would be quarters

A. 1 and 3

B. 2 and 3

C. 3 and 4

D. 1 and 4

E. 2 and 4

Question 11

A least-squares regression line, with equation $y = 3.2 - 0.45x$, is fitted to a set of numerical bivariate data. The residual for the data point (5, 1) will be

A. −0.5

B. −0.05

C. 0.05

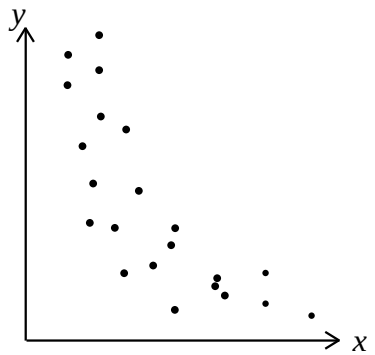
D. 0.5

E. 2.25

Question 12

The relationship between two variables y and x, as shown on the scatterplot below, is nonlinear.

Which one of the following would be **the least likely** transformation to linearise the data?



A. an x^2 tranformation

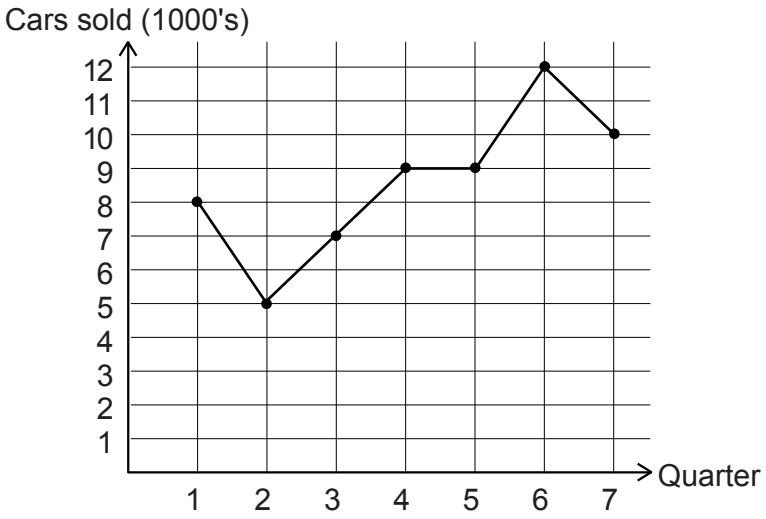
B. a $\log x$ transformation

C. a $\log y$ transformation

D. a $\frac{1}{x}$ transformation

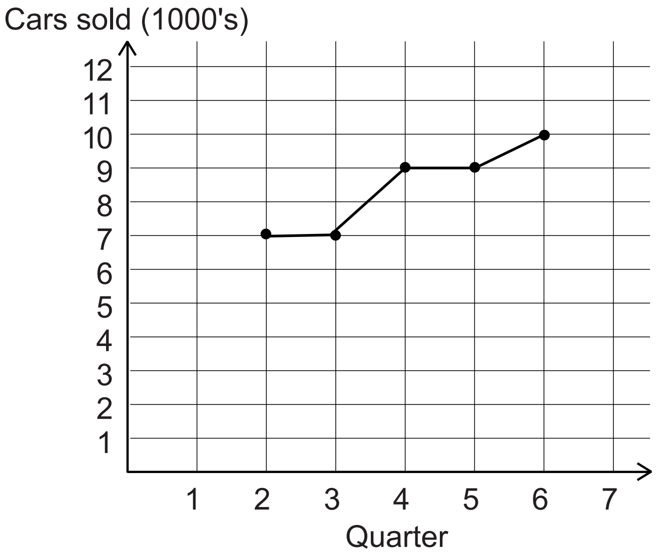
E. a $\frac{1}{y}$ transformation

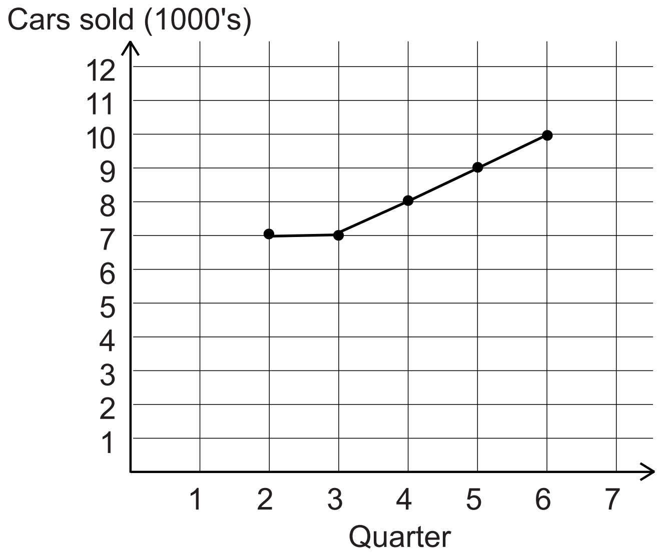
Question 13

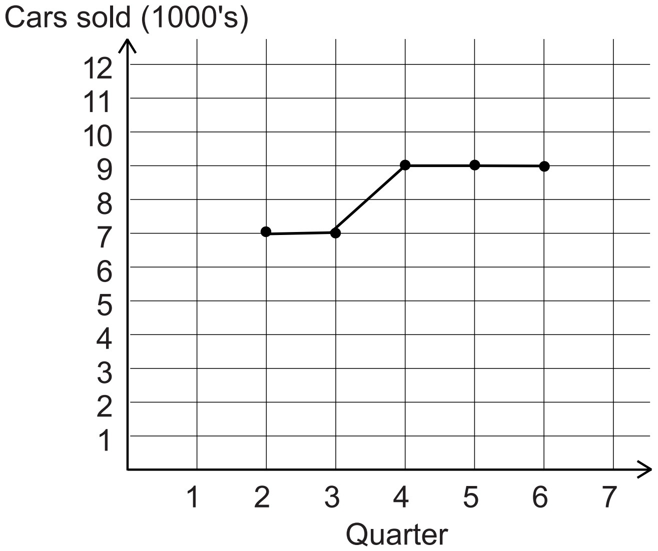


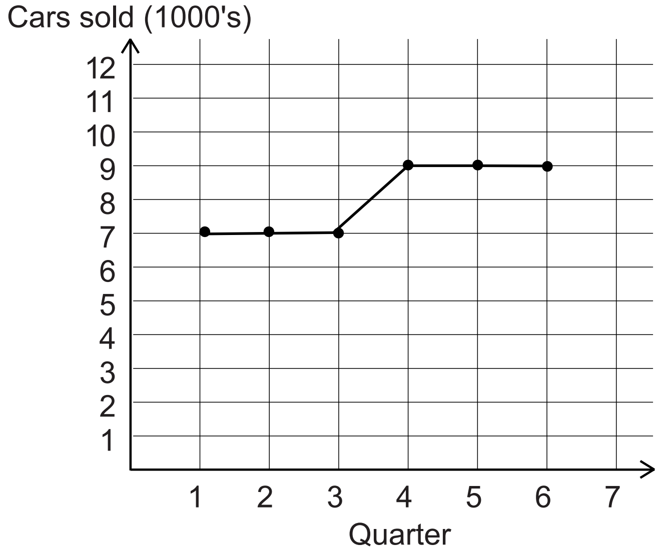
The data graphed above is to be smoothed using 3-point median smoothing.

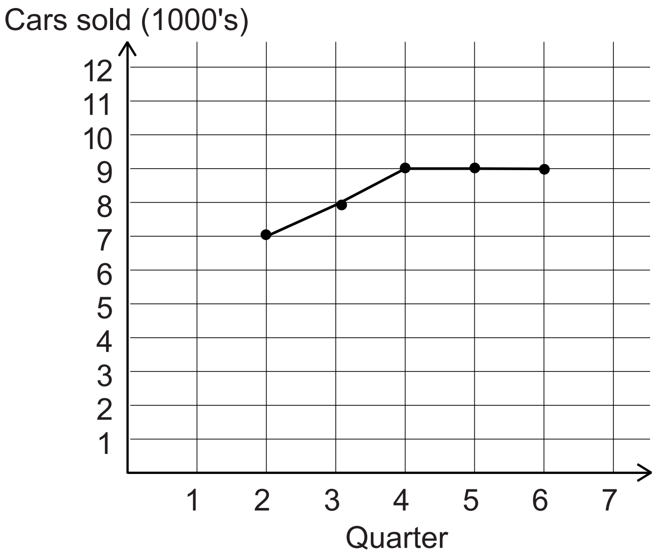
Which one of the following graphs shows the data after it has been smoothed?











END OF SECTION A
TURN OVER

Working Space

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected with pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns and applications

Before answering these questions, you must **shade** the Number patterns and applications box on the answer sheet for multiple-choice questions.

Question 1

For the arithmetic sequence $-8, -11.5, -15, -18.5, \ldots$ the values of a, the first term, and d, the common difference, are:

A. $a = 8, d = 2.5$
B. $a = -8, d = 2.5$
C. $a = -8, d = -2.5$
D. $a = -8, d = -3.5$
E. $a = -8, d = 3.5$

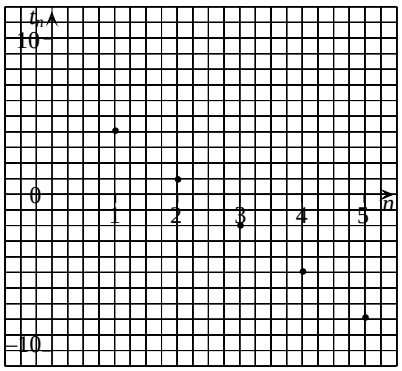
Question 2

The eighth term of the geometric sequence 3, –9, 27, –81, … is:

A. -2187
B. -6561
C. 19 683
D. -1296
E. 2592

Question 3

The first five terms of a sequence are plotted on the graph below.



Which of the following matches the sequence in the graph?

A. $t_{n+1} = t_n - 3$, $t_1 = 4$
B. $t_{n+1} = 0.5t_n - 1$, $t_1 = 4$
C. $t_{n+1} = t_n + 4$, $t_1 = 3$
D. $t_{n+1} = t_n + 3$, $t_1 = 4$
E. $t_{n+1} = -0.5t_n + 3$, $t_1 = 4$

Question 4

A sequence is given as $f_n = 2f_{n-2} + f_{n-1}$, $f_1 = 2$ and $f_2 = 3$

The first five terms are:

A. 2, 3, 5, 8,13
B. 2, 3, 8, 19, 46
C. 2, 3, 4, 5, 6
D. 2, 3, 4.5, 6.75, 10.125
E. 2, 3, 7, 13, 27

Question 5

In an arithmetic sequence, the value of the 11th term is $t_{11} = 43$, while the sum of the first 20 terms is $S_{20} = 820$. The value of a, the first term is:

A. 5
B. 4
C. 3
D. 2
E. 1

Question 6

The amount of bread sold at a grocery store increases by 5% every week. If the store sold 1000 loaves in the first week, in which week will it sell more than 1600 loaves in a week?

A. 10th week
B. 11th week
C. 12th week
D. 13th week
E. Never

Question 7

The infinite sum of a geometric series is $21\frac{1}{3}$. If the first term is 16 then the sum of the first five terms **only** is:

A. $21\frac{1}{16}$

B. $21\frac{1}{2}$

C. $21\frac{1}{4}$

D. $21\frac{5}{16}$

E. $21\frac{1}{3}$

Question 8

For the Fibonacci-type sequence given as …… 19, 31, 50, 81, 131 …….the first two terms could be given as

A. $f_1 = 1, f_2 = 5$
B. $f_1 = 2, f_2 = 3$
C. $f_1 = 3, f_2 = 2$
D. $f_1 = 4, f_2 = 3$
E. $f_1 = 5, f_2 = 1$

Question 9

Louise opens a supersaver account at the Moni Bank. She opens the account with \$100 and earns an interest rate of 4.8% per annum. She deposits \$80 per month into her account at the start of the month, while the interest is calculated and added to the balance at the end of the month. A difference equation, which would describe this situation, is:

A. $M_{n+1} = 1.048M_n + 80, M_0 = 100$

B. $M_{n+1} = 1.048M_n + 100, M_0 = 80$

C. $M_{n+1} = 1.004M_n + 80, M_0 = 100$

D. $M_{n+1} = 1.004M_n + 100.4, M_0 = 80$

E. $M_{n+1} = 1.004M_n + 80.32, M_0 = 100$

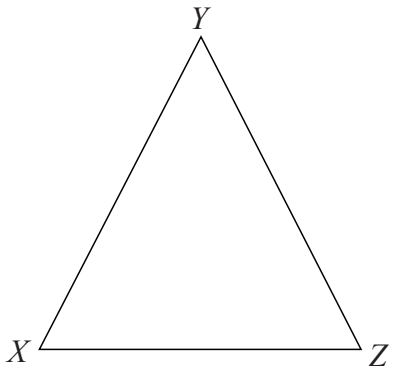
Module 2: Geometry & trigonometry

Before answering these questions, you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

Question 1

Triangle *XYZ* is an isosceles triangle with $XY = YZ = 22$cm and $XZ = 20$ cm

The magnitude of angle *YXZ*, to the nearest degree, is

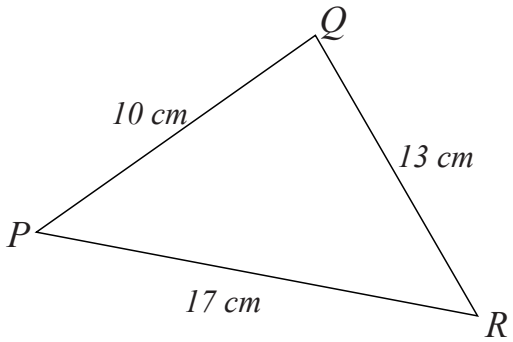


A. 25°
B. 27°
C. 57°
D. 60°
E. 63°

The following information relates to questions 2 and 3.

Question 2

Using Heron's formula to calculate the area of the triangle *PQR*, the calculation will be



A. $\sqrt{40(40-10)(40-13)(40-17)}$

B. $40\sqrt{30 \times 27 \times 23}$

C. $20\sqrt{(20-10)(20-13)(20-17)}$

D. $\sqrt{20(20-10)(20-13)(20-17)}$

E. $\frac{1}{2} \times 10 \times 13 \times 17$

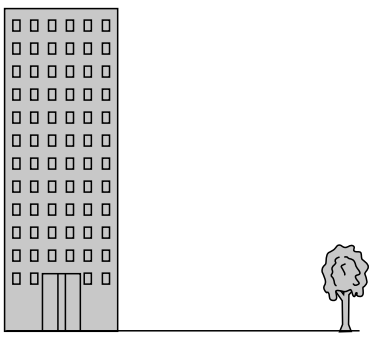
Question 3

The magnitude of the angle *QPR*, in degrees correct to two decimal places, is

A. 31.79°
B. 32.20°
C. 35.91°
D. 40.32°
E. 49.68°

Question 4

The angle of depression from the top of a building to the base of a tree in the courtyard below is 64°. If the base of the tree is 18 metres from the base of the building then the height of the building, in metres correct to one decimal place, is

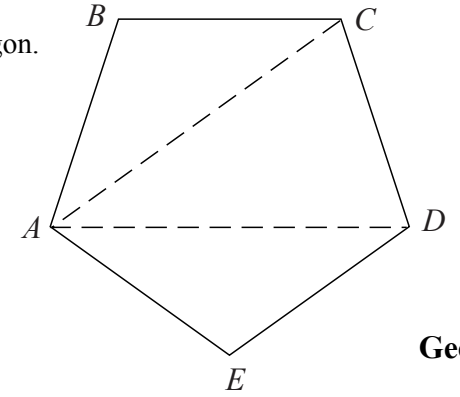


A. 7.9 m
B. 8.8 m
C. 16.2 m
D. 20.0 m
E. 36.9 m

Question 5

The polygon *ABCDE* is a regular pentagon.

The magnitude of angle *CAD* will be



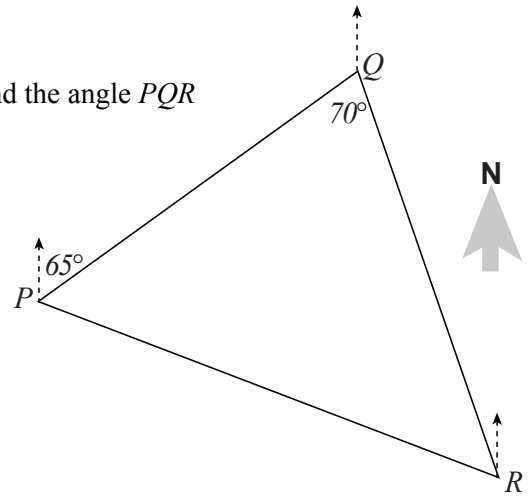
A. 30°
B. 36°
C. 54°
D. 48°
E. 72°

Question 6

On the diagram at right, the bearing of point Q from point P is 065° and the angle PQR in the triangle PQR is 70°.

The bearing of point Q from point R is

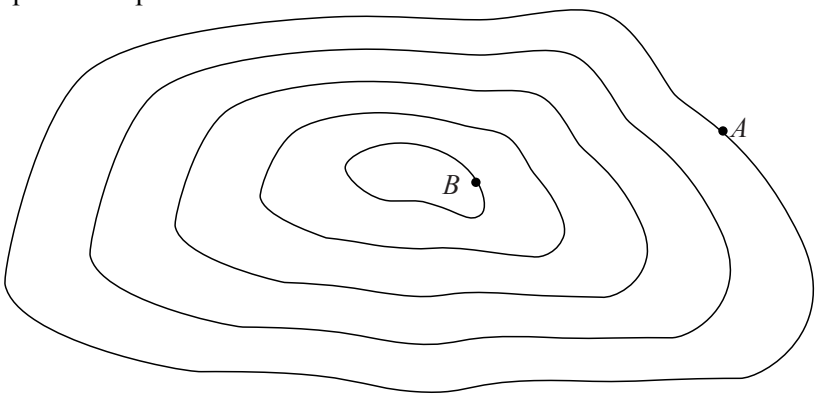
A. 065°
B. 070°
C. 175°
D. 185°
E. 355°



Question 7

The contour interval on the contour map below is 50 metres. If the actual horizontal distance from point A to point B is 1.35 kilometres then the average slope from point A to point B is closest to

A. 0.015
B. 0.148
C. 0.185
D. 0.019
E. 1.481

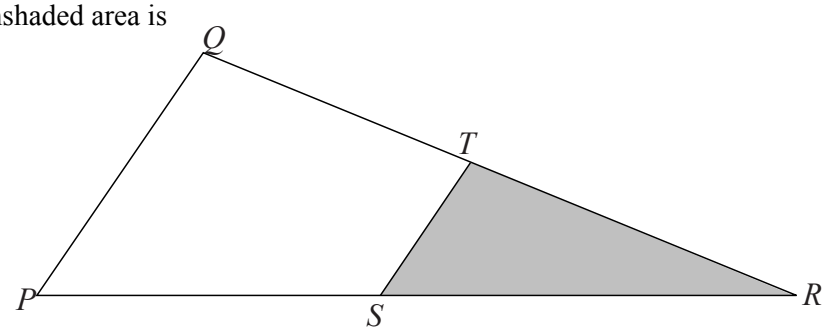


Question 8

In triangle PQR, PQ is parallel to ST, and $PS = SR$

In triangle PQR the ratio of the shaded area to the unshaded area is

A. 1 : 2
B. 1 : 4
C. 3 : 4
D. 1 : 3
E. 2 : 3

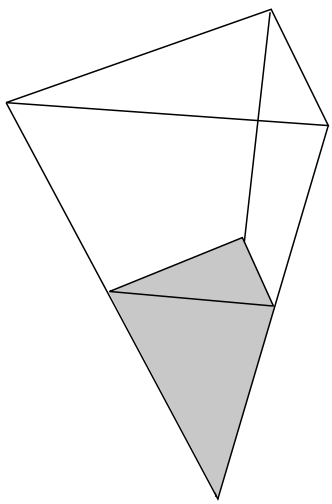


Question 9

When the container, at right, is filled to half its depth it contains 1250 millilitres of liquid.

The capacity of the container, in litres, when it is completely filled will be

A. 2.5 litres
B. 3.75 litres
C. 5 litres
D. 8.75 litres
E. 10 litres

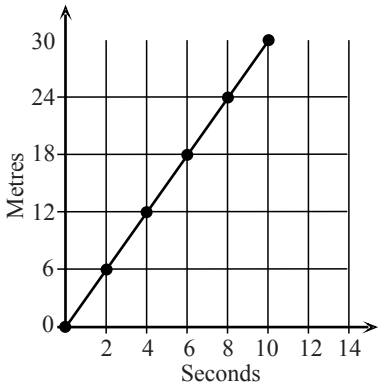


Module 3: Graphs and relations

Before answering these questions, you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1

The graph shows the distance, d metres, covered by a scooter and its rider, at time t seconds, along a flat road at a constant speed.



The speed of scooter at the 6 second mark of the recorded trip is:

A. 3 metres per second
B. 5 metres per second
C. 6 metres per second
D. 18 metres per second
E. 30 metres per second

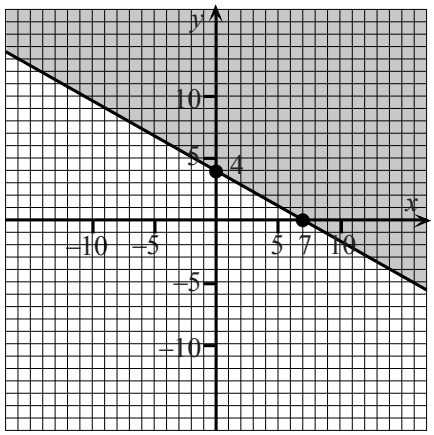
Question 2

The solution to the simultaneous equations $y = 8 – 2x$ and $y = 3x + 3$ is:

A. (6, 1)
B. (5, 18)
C. (1, 6)
D. (5, –2)
E. (–1, 0)

Question 3

The shaded region, including the boundary line, on the graph at right represents an inequation.



The inequation for the shaded region is best given by

A. $4x + 7y \leq 28$
B. $4x + 7y \geq 28$
C. $4x + 7y > 28$
D. $y \leq 4$ and $x \geq 7$
E. $4y + 7x \geq 28$

Question 4

If a linear programming problem where the solution region is restricted to the first quadrant, the region **must** have the following constraints:

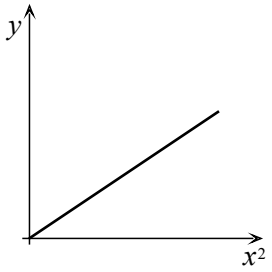
A. $x \leq 0$ and $y \leq 0$
B. $x \geq 0$ and $y \leq 0$
C. $x \leq 0$ and $y \geq 0$
D. $x \geq 0$ and $y \geq 0$
E. None of the above

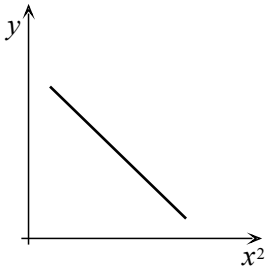
Question 5

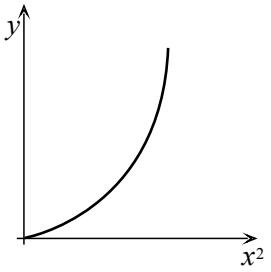
Consider the table below of values for x and y:

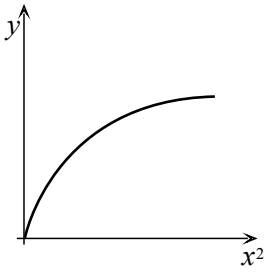
x	1	1.5	2	2.5	3	3.5	4
y	0.5	1.125	2	3.125	4.5	6.125	8

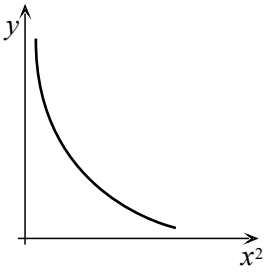
The graph of y against x^2 is best shown by:

A.


B.


C.


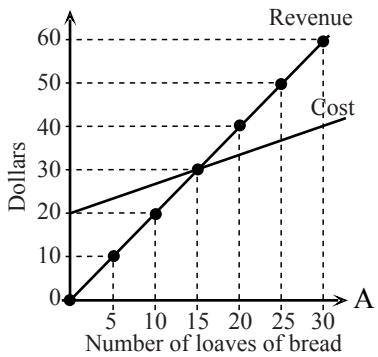
D.


E


Question 6

A small bakery owner has prepared a graph that shows the relationships for revenue and cost against the number of loaves of bread sold.

Given that the cost function ($C) is $C = \frac{2}{3}n + 20$, where n is the number of loaves, then the profit function ($P) for the graph at right is given by



A. $P = \frac{4}{3}n + 20$

B. $P = 2n$

C. $P = \text{n} - 20$

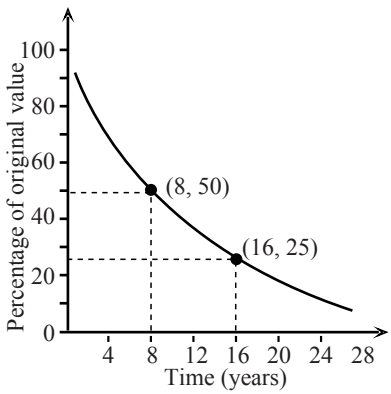
D. $P = \frac{4}{3}n - 20$

E. $P = 1\frac{1}{3}n + 20$

Question 7

The value of an industrial machine decreases over time. The graph below shows the value of the industrial machine as a percentage of its original price.

A relation that best defines this relationship is of the form:



A. $y = kx$ where $k = -5$

B. $y = \frac{k}{x^2}$ where $k = 3200$

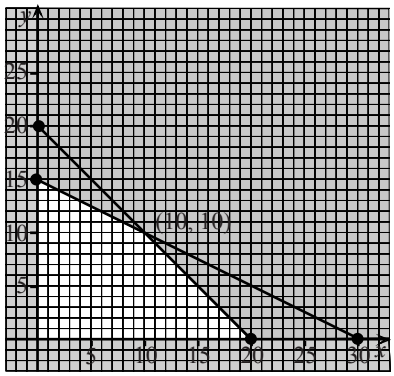
C. $y = \frac{k}{x}$ where $k = 400$

D. $y = kx^2$ where $k = \frac{1}{10}$

E. $y = kx^2$ where $k = \frac{1}{6}$

Question 8

In a linear programming problem where the feasible region is the unshaded region (including boundaries) shown in the figure at right.



If the objective function is given as $3x + 7y$, then the maximum value for this objective function is given by:

A. 140

B. 100

C. 90

D. 105

E. 60

Question 9

Creamy Krisps is a donut shop that makes two kinds of donuts. Donut X requires 20 grams of icing for each donut, while Donut Y requires 45 grams of icing. There is a total of 20 kilograms (1kg = 1000g) of icing delivered each day. The daily expected sales for the two kinds of donuts is at least 3000 in total and no more than 4000 of Donut Y.

If x = number of Donut X sold each day and y = number of Donut Y sold each day, then, in a linear programming problem, two of the constraints would be $x \geq 0$ and $0 \leq y \leq 4000$. The other two constraints are:

A. $x + y \geq 3000$ and $0.20x + 0.45y \leq 20$

B. $x + y \geq 3000$ and $20x + 45y \leq 20000$

C. $x + y \geq 4000$ and $20x + 45y \leq 20000$

D. $x + y > 4000$ and $20x + 45y \leq 20000$

E. $x + y \geq 4000$ and $0.02x + 0.045y \leq 20$

Module 4 Business-related mathematics

Before answering these questions, you must **shade** the Business related mathematics box on the answer sheet for multiple-choice questions.

Question 1

The original price of a fashion item is discounted by 20% in January. In February the item is discounted by a further 30% off the January price.

The overall discount of the original price of the item is

A. 44%

B. 50%

C. 56%

D. 60%

E. 76%

The following information relates to questions 2 and 3

Carlo has $336 000 that he wishes to invest so that he receives a monthly payment. He has found a fund that returns 6.24% p.a. interest in which he is going to invest his money.

Question 2

The amount Carlo will receive each month as a perpetuity, to the nearest cent, will be

A. $174.72

B. $1747.20

C. $2096.64

D. $4487.18

E. $20 966.40

Question 3

If Carlo wishes to receive $2400 per month from his investment then which one of the following statements will be true for his invested amount of $336 000?

A. his invested amount will increase in value each month

B. his invested amount will be less than half the original value after 10 years

C. his invested amount will accumulate an increasing amount of interest each month

D. his invested amount will last no longer than 236 months

E. his invested amount will last no longer than 252 months

Question 4

Three years ago the cost of a ticket was $2.80 but this has risen with inflation each year. If inflation has been 1.6% p.a. over the last three years then the cost of the ticket now, to the nearest cent, is

A. $2.84

B. $2.93

C. $2.94

D. $3.28

E. $4.37

Question 5

Mia has started an investment account with an initial deposit of $1000. She will add $200 to this account at the end of each month and the account is credited with 5.25% p.a. interest, compounding monthly.

If Mia makes no withdrawals then the amount accumulated in the account after three years, to the nearest dollar, will be

A. $6609
B. $8366
C. $8946
D. $8950
E. $9560

Question 6

A loan of $86 400 is being charged 8.25% p.a. interest, compounding quarterly. If no repayments are made on the loan then the interest paid on this loan in the third quarter, to the nearest dollar, will be

A. $1782
B. $1819
C. $1856
D. $1895
E. $5457

Question 7

A photocopier, originally bought for $29 500, is valued at $7000 after 5 years. The annual reducing balance rate of depreciation for this photocopier will be closest to

A. 15.3%
B. 20.0%
C. 25.0%
D. 33.3%
E. 66.5%

Question 8

A time-payment plan has been negotiated for a refrigerator that has a purchase price of $1450. The purchaser is to pay a deposit of $290 then twelve monthly payments of $115. The flat rate of interest being charged in this plan is closest to

A. 4.8%
B. 9.5%
C. 15.2%
D. 15.9%
E. 19.0%

Question 9

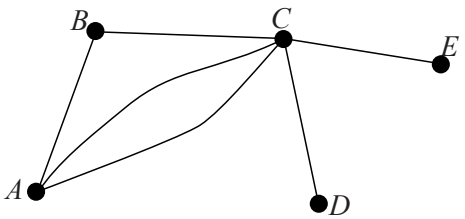
Anastasia and Boris have borrowed $156 000 to purchase their home. They are being charged 5.95% p.a. interest on the loan and it is to be repaid in ten years. If they repay the loan in 26 fortnightly repayments per year, rather than 12 monthly repayments per year, which one of the following will **not** be true?

A. They will owe slightly less after one year.
B. They will pay about $200 less in interest over the ten years.
C. The total amount paid will be less over the ten year term.
D. In the first year they will pay slightly less in total repayments.
E. The amount owing after two fortnightly payments will not be the same as the amount owing after one monthly payment.

Module 5: Networks and decision mathematics

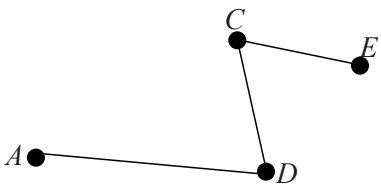
Before answering these questions, you must **shade** the Networks & decision mathematics box on the answer sheet for multiple-choice questions.

Question 1

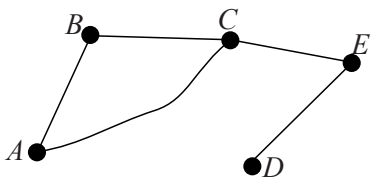


Which one of the following is a subgraph of the graph above?

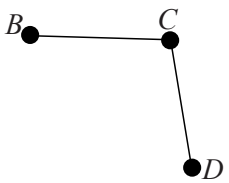
A.



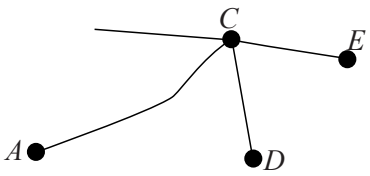
B.



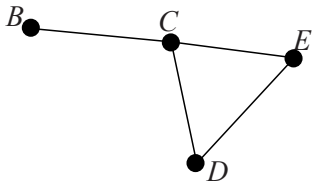
C.



D.

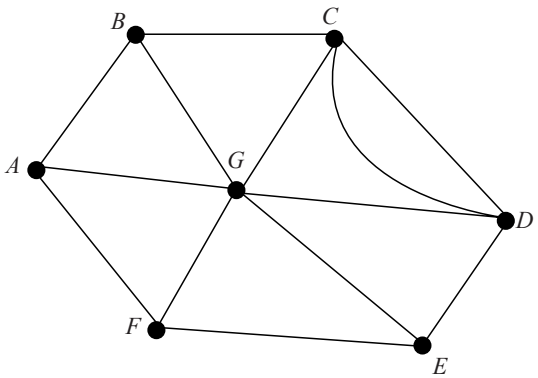


E.



The following information relates to Questions 2 and 3

A network graph is drawn below



Question 2

Which one of the following would correctly describe the network graph?

A. It is a directed graph.

B. It is a complete graph.

C. It is a simple graph.

D. It is a spanning tree.

E. It is a planar graph.

Question 3

Which one of the following would **not** be a correct statement for this graph?

A. The sum of the degrees of the vertices is 26

B. The addition of an edge from vertex *A* to vertex *F* would mean that an Euler path would be possible.

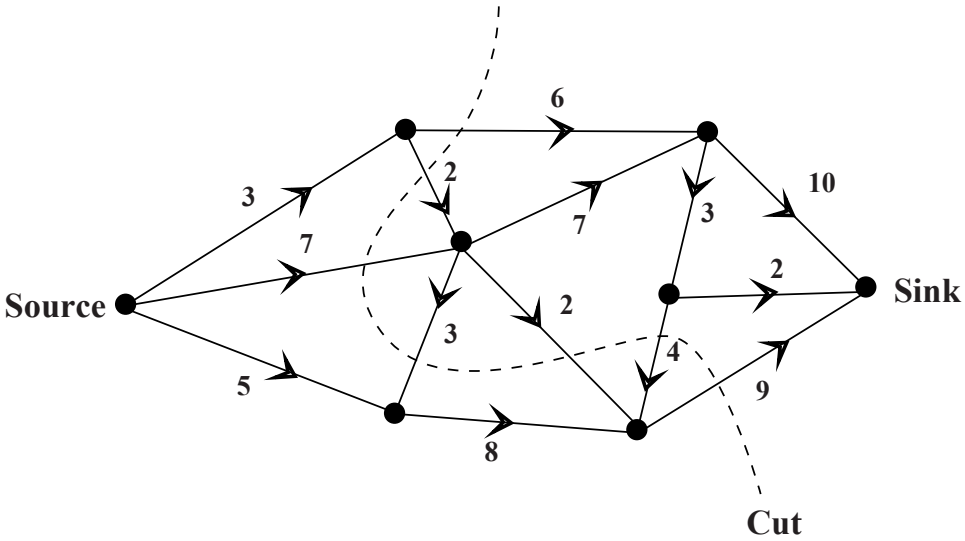
C. A possible Hamiltonian circuit is *ABCDGEFA*

D. The addition of an edge from vertex *A* to vertex *B* would mean that an Euler path is possible starting at *A* and finishing at *B*

E. The graph divides the plane into 8 regions.

Question 4

On the directed graph below, the values on the edges give the maximum flow between nodes in the direction of the arrows.



A cut that separates the source from the sink is also shown on the graph.

The capacity of this cut is

A. 15

B. 22

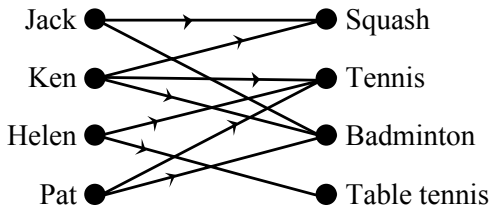
C. 24

D. 28

E. 33

Question 5

Four referees are to be allocated to one of four sports and they have been asked to nominate their preferred sports. Their preferences are summarized on the bipartite graph below.



Which one of the following statements would **not** follow from the graph above?

A. Only Helen is willing to referee table tennis.

B. There are only two possible different allocations of referees to sports.

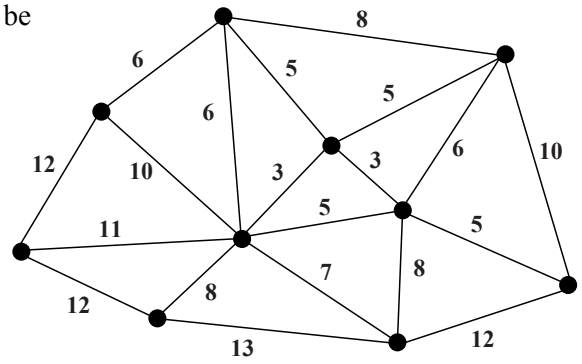
C. If Ken referees tennis then Pat can referee badminton and Jack can referee squash.

D. If Ken referees squash then Jack can referee badminton and Pat can referee tennis.

E. If Ken referees badminton then Pat can referee tennis and Jack can referee squash.

Question 6

The network diagram below shows the work stations in an office represented as nodes. The numbers on the edges represent the distances, in metres, between work stations in the office. If these work stations are to be linked by co-axial cable then the minimum length of cable needed will be



A. 45 m
B. 50 m
C. 52 m
D. 53 m
E. 54 m

Question 7

The connectivity matrix has been constructed for a network graph.

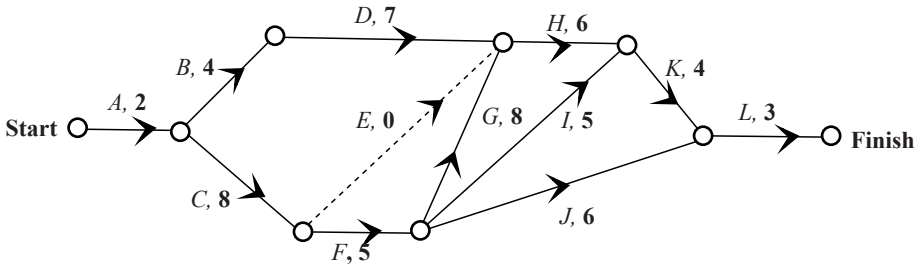
Which one of the following is **not** a correct statement about the graph?

A. It is a directed graph with four nodes.
B. There are 6 edges on the graph.
C. Node A is an isolated node.
D. There is a loop on node B.
E. Node B is unreacheable from node C.

From \ To	*A*	*B*	*C*	*D*
A	0	0	0	0
B	0	1	1	2
C	0	0	0	1
D	0	1	0	0

The following information relates to questions 8 and 9

The activities and their completion times (in days) that are needed to complete a project are shown in the network below.



Question 8

For this project the minimum time to complete the whole project would be

A. 23 days
B. 24 days
C. 26 days
D. 28 days
E. 36 days

Question 9

The slack time for activity *J* will be

A. 1 day
B. 2 days
C. 3 days
D. 4 days
E. 12 days

Module 6: Matrices

Before answering these questions, you must **shade** the Matrices mathematics box on the answer sheet for multiple-choice questions.

Question 1

For the matrix shown, the value of element $a_{2,3}$ is

$$\begin{bmatrix} 4 & 1 & 1 & 0 \\ -3 & 3 & 0 & -1 \\ -2 & 2 & 5 & -4 \end{bmatrix}$$

A. 2
B. 1
C. 0
D. –1
E. –2

The following information relates to Questions 2 and 3

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 2 & 3 \\ 5 & 4 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 3 & -1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$$

Question 2

Which one of the following products does exist?

A. AB
B. BC
C. DA
D. ED
E. CE

Question 3

The matrix product BE is:

A. $\begin{bmatrix} 4 & 0 & 0 \\ 18 & 2 & 0 \\ 13 & 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 0 \\ 18 & 2 \\ 13 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 2 \\ 18 & 12 \\ 13 & 11 \end{bmatrix}$

D. $\begin{bmatrix} 4 & -1 & 1 \\ 5 & 4 & 3 \\ 6 & 5 & -2 \end{bmatrix}$

E. undefined.

Question 4

If $\begin{bmatrix} 2 & -9 \\ c & 0 \end{bmatrix} + \begin{bmatrix} -5 & d \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$

then the values of c and d are:

A. $c = -2, d = 7$
B. $c = -2, d = 11$
C. $c = 0, d = -11$
D. $c = 5, d = 5$
E. $c = -1, d = -5$

Question 5

Which one of these matrices is singular?

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
B. $\begin{bmatrix} 3 & 6 \\ 4 & 3 \end{bmatrix}$
C. $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
D. $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix}$
E. $\begin{bmatrix} 6 & -8 \\ 3 & 4 \end{bmatrix}$

Question 6

Two linear equations were given as $y = -2x + 1$ and $y = 3x + 2$. A student in year 11 wants to use matrix techniques to find their point of intersection. The correct matrix presentation for finding the intersect point (x, y) is:

A. $\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
B. $\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$
C. $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
D. $\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$
E. $\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Question 7

The determinant of matrix $S = \begin{bmatrix} 2 & 4 & 5 \\ 1 & 7 & 3 \\ 4 & 8 & 6 \end{bmatrix}$, denoted det S, is :

A. –1
B. 0
C. 12
D. 20
E. –40

Question 8

An airline company has 60 planes that operate between Melbourne and the Gold Coast. At the end of each week it finds that 30% of the planes that started from Melbourne airport are now at the Gold Coast airport and 40% of the planes that started from Gold Coast airport are now at the Melbourne airport. If the **transition matrix** is of the form

$$\begin{array}{cc} & \text{FROM} \\ & \begin{array}{cc}\text{Melb} & \text{Gold Coast}\end{array} \\ \text{TO}\begin{array}{c}\text{Melb}\\ \text{Gold C}\end{array} & \begin{bmatrix}\cdots & \cdots \\ \cdots & \cdots\end{bmatrix}\end{array}$$

then the distribution of planes is given by:

A. $\begin{bmatrix}0.7 & 0.4\\ 0.3 & 0.6\end{bmatrix}$

B. $\begin{bmatrix}0.7 & 0.6\\ 0.3 & 0.4\end{bmatrix}$

C. $\begin{bmatrix}0.7 & 0.3\\ 0.6 & 0.4\end{bmatrix}$

D. $\begin{bmatrix}33\\ 27\end{bmatrix}$

E. $\begin{bmatrix}33 & 0\\ 0 & 27\end{bmatrix}$

Question 9

A transition analysis of the movement of voters preferred political party in a Melbourne electorate between Liberal Party and Labor Party is given by $S_n = \begin{bmatrix}0.45 & 0.4\\ 0.55 & 0.6\end{bmatrix}^n \begin{bmatrix}58000\\ 52000\end{bmatrix}$

The expected number of voters supporting each of the parties, to the nearest 1000, in the long term and represented as a matrix is

A. $\begin{bmatrix}52000\\ 48000\end{bmatrix}$

B. $\begin{bmatrix}56000\\ 54000\end{bmatrix}$

C. $\begin{bmatrix}\frac{8}{19}\\ \frac{11}{19}\end{bmatrix}$

D. $\begin{bmatrix}46000\\ 64000\end{bmatrix}$

E. $\begin{bmatrix}64000\\ 46000\end{bmatrix}$

END OF QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

April 2006

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET