2006 Further Mathematics Trial Exam 1 Solutions
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SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
B	A	B	A	A	D	C	C	E	B	A	E	C

SECTION B
Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
D	A	B	A	E	C	A	C	D

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
D	E	A	B	A	B	B	C	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
B	C	C	A	B	C	D	C	C

SECTION A Core: Data analysis

Q1 House prices are discrete and numerical; conditions are categorical.

Q2 Arrange the house prices in ascending order, the middle two are 269000 and 276000.

Median = $\dfrac{269000+276000}{2} = 272500$

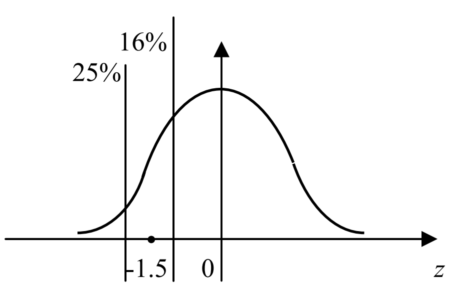
Q3 $Q_1 = 48$, $Q_3 = \dfrac{69+70}{2} = 69.5$, $IQR = 69.5 - 48 = 21.5$

Q4 $Q_1 - 1.5\times IQR = 48 - 1.5\times 21.5 = 15.75$

$Q_3 + 1.5\times IQR = 69.5 + 1.5\times 21.5 = 101.75$

All test marks are between 15.75 and 101.75, $\therefore$ no outliers

Q5



Q6 80 is 1σ higher than the mean 70, $\therefore$ 16% of students score higher than 80 in Science.

Q7 English mark $80 = 74 + 0.5\sigma$

Mathematics mark $70 = 61 + 0.5\sigma$

Science mark $80 = 70 + 1\sigma$

$\therefore$ Student has the same rank in English and Mathematics.

Q8 Negative, linear and moderate.

Q9 Residual = actual value − predicted value

$= 3 - (5.6 - 0.81\times 2) = -0.98 \approx -1.0$

Q10 $y = a + bx$. When $x = 0$,

$y = a = \bar{y} - r\dfrac{s_y}{s_x}\bar{x} = 5.28 - {}^{-}0.8913\times\dfrac{1.72}{0.243}\times 1.30 = 13.48$

Q11 $a = 13.48$, $b = r\dfrac{s_y}{s_x} = -6.31$, $\therefore y = 13.48 - 6.31x$

Q12 Fourth quarter seasonal index

$= 4 - (0.93 + 0.90 + 0.85) = 1.32$

Deseasonalised sale figure $= \dfrac{639500}{1.32} = 484470$ dollars

Q13 Annual rainfall peaked every 6/7 years, $\therefore$ cyclical. There was a gentle up trend.

SECTION B
Module 1: Number patterns and applications

Q1 There is a common difference of $-\dfrac{1}{3}$ in sequence D.

Q2 Common ratio: $\dfrac{1.04}{3.12} = \dfrac{3.12}{x}$, $x = 9.36$

Q3 The sequence is formed by adding successive odd integer (1, 3, 5, …) to a term to obtain the next term.
−1, 0, 3, 8, 15, 24, 35, 48, 63, 80, 99. There are 11 terms.

Q4 $a = -\dfrac{1}{2}$, $r = \dfrac{\frac{1}{6}}{-\frac{1}{2}} = -\dfrac{1}{3}$, $S_\infty = \dfrac{a}{1-r} = \dfrac{-\frac{1}{2}}{1-{}^{-}\frac{1}{3}} = -\dfrac{3}{8}$

Q5 For any arithmetic sequence with even number of terms, $t_1 + t_n = t_2 + t_{n-1} = t_3 + t_{n-2} = \ldots\ldots$, $\therefore$ the sum of the middle two terms equals the sum of the first and the last terms.
For any arithmetic sequence with odd number of terms, the middle term equals a half of the sum of the first and the last terms.

Q6 $T_{n+1} = 2T_n - 9$, $T_n = 12.5$, $p = 2\times 12.5 - 9 = 16$,
$a = 2\times 16 - 9 = 23$, $w = 2\times 23 - 9 = 37$

Q7 $u_n = \dfrac{1}{2}u_{n-1} - 9$, $\therefore u_{n-1} = 2(u_n + 9)$, $u_5 = 3$,

$\therefore u_4 = 2(u_5 + 9) = 2(12) = 24$, $u_3 = 2(u_4 + 9) = 2(33) = 66$

$u_2 = 2(75) = 150$, $u_1 = 2(159) = 318$

Q8 $t_{n+1} = at_n + b$, the ratio $\dfrac{t_{n+1}}{t_n} = a + \dfrac{b}{t_n}$ is a constant if $b = 0$ and $a \neq 0$. $\therefore$ The sequence is geometric if $b = 0$ and $a \neq 0$.

Q9 $t_{n+2} = t_{n+1} + t_n$ where $t_1 = t_2 = 1$. The generated sequence is 1, 1, 2, 3, 5, 8, 13, 21, 33, 54, ……

$\therefore t_8 \times t_5 - t_7 \times t_6 = 21\times 5 - 13\times 8 = 105 - 104 = 1$

Module 2: Geometry and trigonometry

Q1 L = the base of the large right angle triangle − the base of the small right angle triangle $=\sqrt{8^2-3^2}-4=3.4$

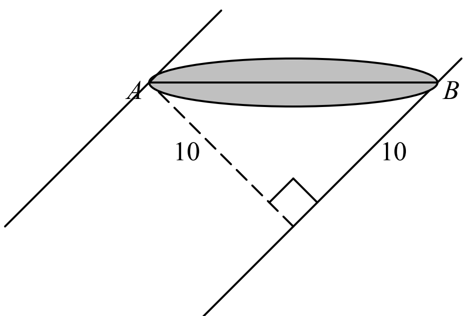
Q2 Length scale factor $=\dfrac{10cm}{0.5m}=\dfrac{10cm}{50cm}=0.2$

$\therefore$ area scale factor $=0.2^2=0.04$

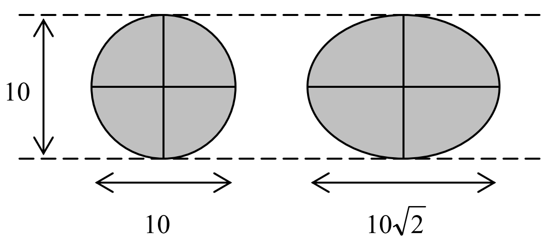
Q3 When the vase is upright, the depth of water would be $\dfrac{15+25}{2}=20$ cm,

$\therefore$ volume of water $=\pi r^2h=\pi 5^2\times 20=1570.8\ \text{cm}^3=1.6$ L

Q4



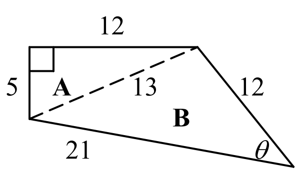
$AB=\sqrt{10^2+10^2}=\sqrt{200}=10\sqrt{2}$.



Area of circle $=\pi r^2=\pi 5^2=78.5$

Area of ellipse $=\sqrt{2}\times 78.5=111.1\,\text{cm}^2$

Q5



Area **A** $=\dfrac{1}{2}\times 5\times 12=30$.

$s=\dfrac{13+12+21}{2}=23$,

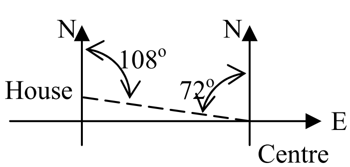
area **B** $=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{23(10)(11)(12)}=71.1$

Total area $=30+71.1=101.1\,\text{cm}^2$

Q6 $\cos\theta=\dfrac{c^2-a^2-b^2}{2ab}=\dfrac{21^2+12^2-13^2}{2(21)(12)}=0.8254$,

$\therefore \theta=\cos^{-1}(0.8254)=34.4^0$

Q7



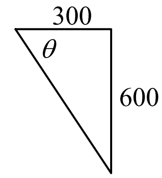
$180-72=108$

Q8 Horizontal distance from P to summit ≈ 300 m

Vertical distance from P to summit $\approx 1000-400=600$ m

Average slope $\approx\dfrac{600}{300}=2$

Q9



$\tan\theta=\dfrac{600}{300}=2$, $\theta=\tan^{-1}(2)=63.4^0$

Module 6: Matrices

Q1 It is not a transitional matrix. Transitional matrices are square matrices.

Q2 $2\begin{bmatrix}0&1\\1&2\\2&3\end{bmatrix}-\dfrac{1}{2}\begin{bmatrix}0&4\\4&8\\8&12\end{bmatrix}=\begin{bmatrix}0&2\\2&4\\4&6\end{bmatrix}-\begin{bmatrix}0&2\\2&4\\4&6\end{bmatrix}=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$,

$2\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}\begin{bmatrix}-1&1\\0&2\end{bmatrix}=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$.

Q3 $\underset{2\times 3}{A}\underset{3\times 4}{\begin{bmatrix}a&b&c&d\\e&f&g&h\\i&j&k&l\end{bmatrix}}=\underset{2\times 4}{\begin{bmatrix}m&n&o&p\\q&r&s&t\end{bmatrix}}$

Q4 $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&1\\1&0\end{bmatrix}=\begin{bmatrix}0&1\\1&1\end{bmatrix}$, $a+b=0$, $a=1$, $c+d=1$, $c=1$

$\therefore b=-1$, $d=0$

Q5 Inverse of $\begin{bmatrix}1&-1\\1&-2\end{bmatrix}$ is $\dfrac{1}{1\times{}^-2-{}^-1\times 1}\begin{bmatrix}-2&1\\-1&1\end{bmatrix}$

$={}^-1\times\begin{bmatrix}-2&1\\-1&1\end{bmatrix}=\begin{bmatrix}2&-1\\1&-1\end{bmatrix}$

Q6 $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&-2\\-3&4\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\\frac{1}{4}\end{bmatrix}=\begin{bmatrix}-1\\\frac{5}{2}\end{bmatrix}$

Q7 $\begin{bmatrix}0.95&0.30\\0.05&0.70\end{bmatrix}$

Q8 Second night: $\begin{bmatrix}0.50&0.50\\0.50&0.50\end{bmatrix}\begin{bmatrix}120\\60\end{bmatrix}=\begin{bmatrix}90\\90\end{bmatrix}$,

third night: $\begin{bmatrix}0.50&0.50\\0.50&0.50\end{bmatrix}\begin{bmatrix}90\\90\end{bmatrix}=\begin{bmatrix}90\\90\end{bmatrix}$, steady state, 90 working light globes.

Q9 $3y=2$, $y=5x-1$ $\therefore 0x+3y=2$, $5x-y=1$

$\therefore\begin{bmatrix}0&3\\5&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$

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