Year 2005

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: 03 9817 5374
Fax: 03 9817 4334
chemas@chemas.com
www.chemas.com

Question 1

a. 15 months (1 mark)

b. It is negatively skewed (1 mark)

c. The median is the middle number = 81 (1 mark)

d. Interquartile range = upper quartile – lower quartile = 87 – 77 = 10 (1 mark)

e. Use graphics calculator. Enter data in stat edit., then stat calc, 1-Var Stats, 2^{nd} L_1 = 81 to the nearest whole number (1 mark)

f. M T Hooker has an outlier (1 mark)
The outlier = 69 (1 mark)

Question 2

a. The independent variable is interest rate. (1 mark)

b. Use graphics calculator. Enter data in stat edit.,in L_1 and L_2 then stat calc, linear regression L_1, L_2 to get number of new houses = 31.1 + –1.3 × interest rate. (2 marks)

c.

$8 = 31.1 - 1.3 \times I$

$1.3I = 23.1$

$I = 17.8$

$I = 18\%$ to the nearest percent. (1 mark)

d. Use graphics calculator as in **b** to get $r = -0.97$ (1 mark)

e. There is a very strong negative linear relationship. (1 mark)

f. As the interest rate increases by one percent then the number of new houses decreases by 2000 (2 marks)

Total = 15 marks

End of Core

Question 1

a.

55 + 15 = 70cals/hr. (1 mark)

b.

This is an arithmetic sequence with

$a = 55 - 30 = 25$ at level 1 (1 mark)

$t_n = a + (n-1)d$

$370 = 25 + (n-1)15$

$345 = (n-1)15$

$n - 1 = 23$

$n = 24$

needs to set bike on level 24.(1 mark)

Question 2

a.

$C_1 = 4$

$C_2 = 1.5 \times 4 = 6$

$C_3 = 1.5 \times 6 = 9$

$4, 6, 9$ (1 mark)

b.

This is a geometric sequence with a common ratio of 1.5 (1 mark)

c.

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(1.5^n - 1)}{0.5} = 8(1.5^n - 1) = 300$$

$1.5^n = 38.5$ (1 mark)

$$n = \frac{\log_e(38.5)}{\log_e(1.5)}$$

$n = 9$ mins (1 mark)

d.

$$C_n = 1.05^{n-1} \times 5 + \frac{1.325 \times (1.05^{n-1} - 1)}{1.05 - 1}$$

$C_n = 1.05^{n-1} \times 5 + 26.5 \times (1.05^{n-1} - 1)$ (1 mark)

$C_n = 31.5 \times 1.05^{n-1} - 26.5$

$C_n = 31.5 \times 1.05^n \times 1.05^{-1} - 26.5$

$C_n = \frac{31.5}{1.05} \times 1.05^n - 26.5$

$C_n = 30 \times 1.05^n - 26.5$ (1 mark)

Question 3

a.

$A = 0.5^n$ (1 mark)

b.

$A = 0.5^3 = 0.125$ litres (1 mark)

c.

This means less than or equal to 5% sportade.

$0.5^n \leq 0.05$ (1 mark)

$n \leq \dfrac{\log_e(0.05)}{\log_e(0.5)}$

$n \geq 4.3$

$\therefore n = 5$ (1 mark)

d.

$\dfrac{150}{1400} \times 100$ (1 mark)

$= 10.7\%$ (1 mark)

Total = 15 marks

End of Module 1

Question 1

a.

$1:25,000$

$x:5.2$

$x=\dfrac{5.2}{25,000}$ km

$x=\dfrac{5.2}{25,000}\times 100,000$ cm

$x=20.8$ cm

(1 mark)

b.

$180-(35+42)$

$=180-77$

$=103^0$ (1 mark)

c.

$BC^2=2.6^2+5.2^2-2\times 2.6\times 5.2\cos 103^0$

$BC^2=39.8827$

$BC=6.3$ km

(1 mark)

d.

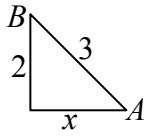
$A=\dfrac{1}{2}bc\sin\theta$

$A=\dfrac{1}{2}\times 2.6\times 5.2\sin 103^0$

$A=6.6\text{ km}^2$ (1 mark)

Question 2

a.



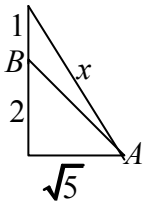
$x^2+4=9$

$x^2=5$

$x=\sqrt{5}$ (1 mark)

Average slope $=\dfrac{\text{rise}}{\text{run}}=\dfrac{2}{\sqrt{5}}=0.89$ (1 mark)

b.



(1 mark)

$x^2=5+9=14$

$x=\sqrt{14}=3.74$ (1 mark)

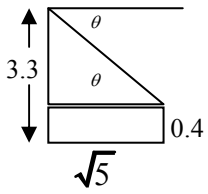
c.

$\tan\theta=\dfrac{\sqrt{5}}{3}$

$\theta=\tan^{-1}\left(\dfrac{\sqrt{5}}{3}\right)=37^0$

(1 mark)

d.

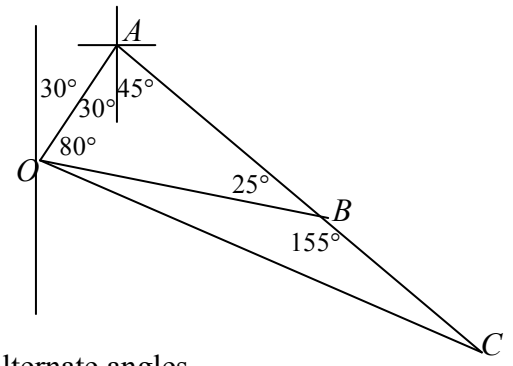


$\tan\theta=\dfrac{2.9}{\sqrt{5}}$

$\theta=52^0$

Question 3

a.



Using alternate angles,
angle $OAB = 30 + 45 = 75^0$

(1 mark)

Angle $AOB = 110 - 30 = 80$
Angle $ABO = 180 - (75 + 80)$
(angles in triangle sum to 180^0)
Angle $ABO = 180 - 155 = 25^0$ (1 mark)

b.

$$\frac{OB}{\sin 75^0} = \frac{200}{\sin 25^0}$$

$$OB = \frac{200 \sin 75^0}{\sin 25^0}$$

$OB = 457.1$ m (1 mark)

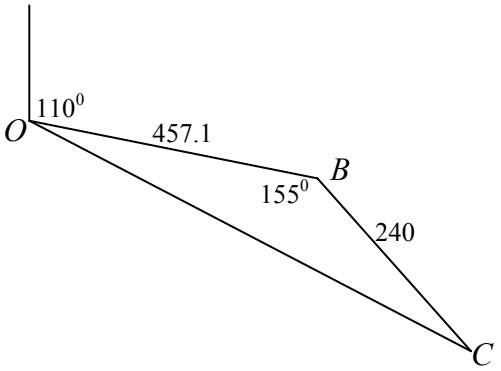
c.

$$OC^2 = 240^2 + 457.1^2 - 2 \times 457.1 \times 240 \cos 155^o$$

$OC = 682.19615$ (1 mark)

$$\frac{682.19615}{\sin 155^0} = \frac{240}{\sin BOC}$$

$$\sin BOC = \frac{240 \sin 155^0}{682.19615}$$



$BOC = 8.55^0$ (1 mark)

$\therefore$ bearing $= 30 + 80 + 9 = 119^0$ (1 mark)

Total = 15 marks

End of Module 2

Question 1

a.
7 – 5 = 2 mins. (1 mark)

b.
25 mins (1 mark)

c.
8,200 – 200 = 8000 metres (1 mark)

d.
From D to F = 8,200 – 7,200 = 1,000 metres (1 mark)

e.
7,200 metres (1 mark)

f.
25 – 19 = 6 mins. (1 mark)

g.
$\frac{8000}{11} = 727.2727$ m/min (1 mark)

$= \frac{727.2727 \times 60}{1000} = 43.6$ km/hr (1 mark)

Question 2

a.
$x + y + z = 1000$

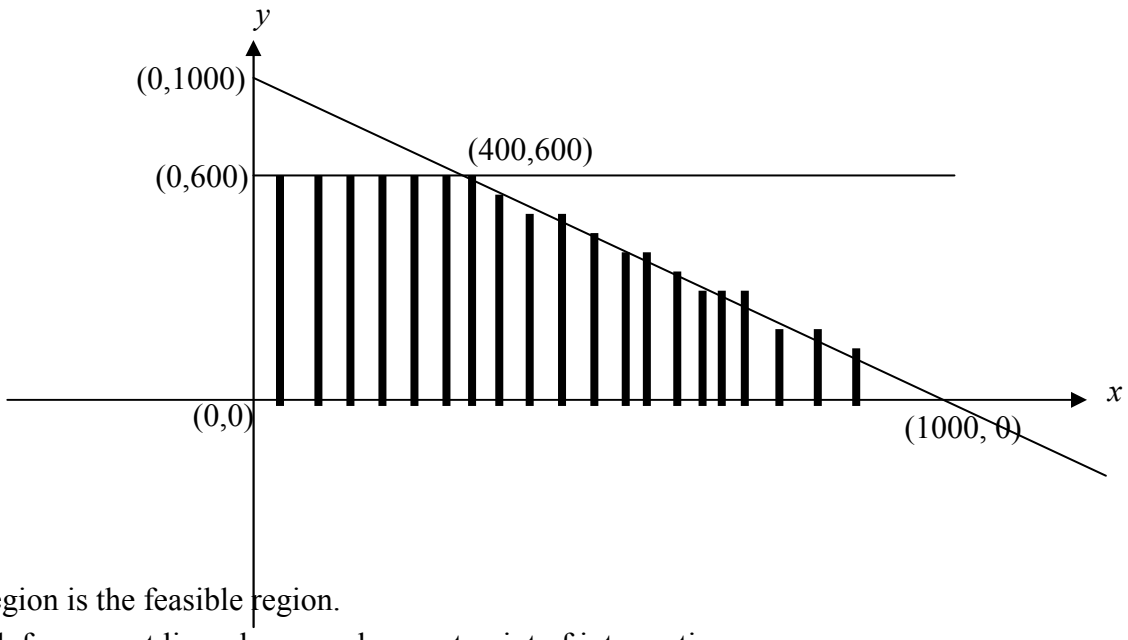
$z = 1000 - x - y$ (1 mark)

b.
$y \leq 600$

$x \geq 0$

$y \geq 0$ (1 mark)

c.



Shaded region is the feasible region.

- 1 mark for correct lines drawn and correct point of intersection
- 1 mark for correct region

Question 2 (continued)

d.

$P = 80x + 100y + 40z$

$= 80x + 100y + 40(1000 - x - y)$

$= 80x + 100y + 40,000 - 40x - 40y$

$= 40x + 60y + 40,000$ (1 mark)

e.

Profit at (0,0)
= \$40,000

Profit at (0,600)
= 0 + 36,000 + 40,000= \$76,000

Profit at (400,600)
= 16,000 + 36,000 + 40,000 = \$92,000

Profit at (1000,0)
= 40,000 + 0 + 40,000 = \$80,000

Maximum profit = \$92,000 (1 mark)

f.

Number of medals = z

$z = 1000 - x - y$

Maximum profit when $x = 400$ and $y = 600$ and $\therefore z = 0$ (1 mark)

Total = 15 marks

End of Module 3

Question 1

a.

$0.5 \times 12 = 6\%$ (1 mark)

b.

$$I = \frac{PRT}{100} = \frac{15{,}000 \times 6 \times 8}{100}$$

$I = \$7{,}200$ (1 mark)

c.

Total Amount = 15,000 + 7,200 + 5,000 = \$27,200 (1 mark)

d.

Original value = 20,000
Residual value = 4,000
Total depreciation over 8 years = \$16,000 (1 mark)
Depreciation each year = 16,000/8
Depreciation each year =\$2,000 (1 mark)

Question 2

a.

$222 \times 96 + 7{,}000 = \$28{,}312$ (1 mark)

b.

$222 \times 96 - 13{,}000 = \$8{,}312$ (1 mark)

c.

$$I = \frac{PRT}{100}$$

$$8312 = \frac{13000 \times r \times 8}{100}$$

$$r = \frac{8312 \times 100}{13000 \times 8} = 8\%$$ (1 mark)

d.

$$\text{Effective rate} = \frac{2n}{n+1} \times \text{flat rate}$$

$$= \frac{2 \times 96}{97} \times 8 = 15.8\%$$ (1 mark)

Question 3

a.

$P = 17{,}000$
$n = 96$ (1 mark)

$$R = 1 + \frac{6.5}{1200} = 1.005$$ (1 mark)

b.

$$PR^n = \frac{Q(R^n - 1)}{R - 1}$$

$$17{,}000(1.005)^{96} = \frac{Q(1.005^{96} - 1)}{0.005}$$

$Q = \$223.40$ (1 mark)

c.

$$A = 17000(1.005)^{24} - \frac{223.4(1.005^{24} - 1)}{0.005} = 13480.21739$$

$$0 = 13480.21739(1.005)^n - \frac{446.8(1.005^n - 1)}{0.005}$$ (1 mark)

$$13480.21739(1.005)^n = 89360(1.005^n - 1)$$

$$0.849147(1.005)^n = 1$$

$$(1.005)^n = 1.17765$$

$$n \log_{10} 1.005 = \log_{10} 1.17765$$

$n = 32.8$ months (1 mark)

He has paid 24 months

24 + 32.8 = 56.8 instead of 96 months.

96 – 56.8 = 39.2 months

3 years and 3 months (1 mark)

End of Module 4 **Total = 15 marks**

Question 1

a.

D and F

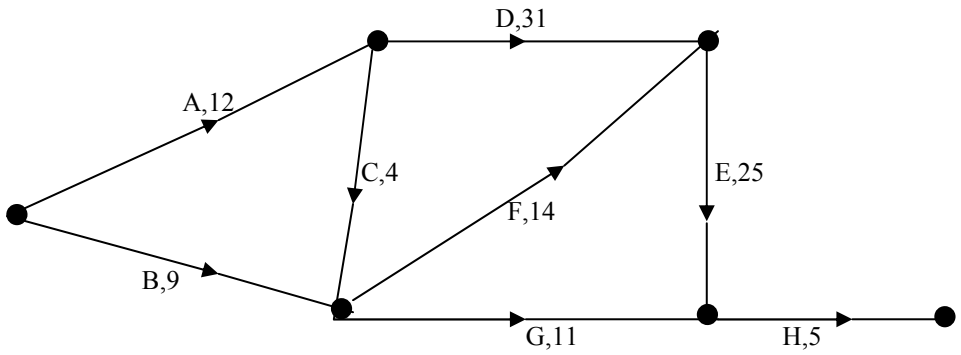
(1 mark)

b.

9 weeks

(1 mark)

c.



(1 mark)

d.

A D E H

- 1 mark for A D
- 1 mark for A D E H

e.

12 + 31 + 25 + 5 = 73 weeks (1 mark)

f.

The earliest time E can commence is when A and D are completed

12 + 31 = 43

At end of 43 weeks (1 mark)

g.

73 – 5 – 25 – 14 = 29

Latest starting time is end of 29th week

(1 mark)

h.

Latest start time = 29

Earliest start time = 16

Float time = 29 – 16 = 13 weeks (1 week)

Question 2

a.

	Cut Out	Seams	Braid	Pockets
Wayne	**16**	**17**	**10**	**5**
Xavier	**18**	**19**	**12**	**8**
Yolande	**18**	**20**	**12**	**12**
Zeta	**15**	**18**	**6**	**9**

(1 mark)

d. The length of the critical path = 12 + 2 + 4 + 3 = 21 hours (1 mark)

Question 2

a.

Row reduction

	C	S	B	P
W	11	12	5	0
X	10	11	4	0
Y	~~6~~	~~8~~	~~0~~	~~0~~
Z	~~9~~	~~12~~	~~0~~	~~3~~

Column reduction

	C	S	B	P
W	5	5	5	0
X	4	3	4	0
Y	~~0~~	~~0~~	~~0~~	~~0~~
Z	~~3~~	~~4~~	~~0~~	~~3~~

(1 mark)

	C	S	B	P
W	2	1	2	0
X	1	0	1	0
Y	~~0~~	~~0~~	~~0~~	~~1~~
Z	~~3~~	~~4~~	~~0~~	~~4~~

(1 mark)

Allocate

Wayne	Pockets
Zeta	Braid
Xavier	Seams
Yolande	Cut

(1 mark)

c.

Pockets	5 mins
Braid	6 mins
Seams	19 mins
Cutting	18 mins

Total time = 5 + 6 + 19 + 18 = 48 mins.

End of Module 5 Total = 15 marks

End of suggested solutions
2005 Further Mathematics Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** **www.chemas.com**