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FURTHER MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2005

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, "Data Analysis".
Section A is compulsory and is worth 15 marks.
Section B consists of 5 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
Section B begins on page 6 of this exam.
There is a total of 60 marks available for this exam.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.
Formula sheets can be found on pages 28 - 29 of this exam.

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Section A - Core

This section is compulsory.

Question 1

Ten Grade 6 students were randomly chosen at a school and asked to estimate how many hours a week they were involved in physical activity such as cycling or basketball.
Their responses together with their weight in kg are shown in Table 1 below.

Table 1

Student	Physical activity a week (hours)	Weight (kg)
A	7	50
B	4	55
C	12	44
D	15	41
E	10	45
F	6	49
G	9	46
H	11	43
I	5	51
J	7	48

a. Find the mean and the standard deviation of the weight of the students.
Express your answers correct to one decimal place where appropriate.

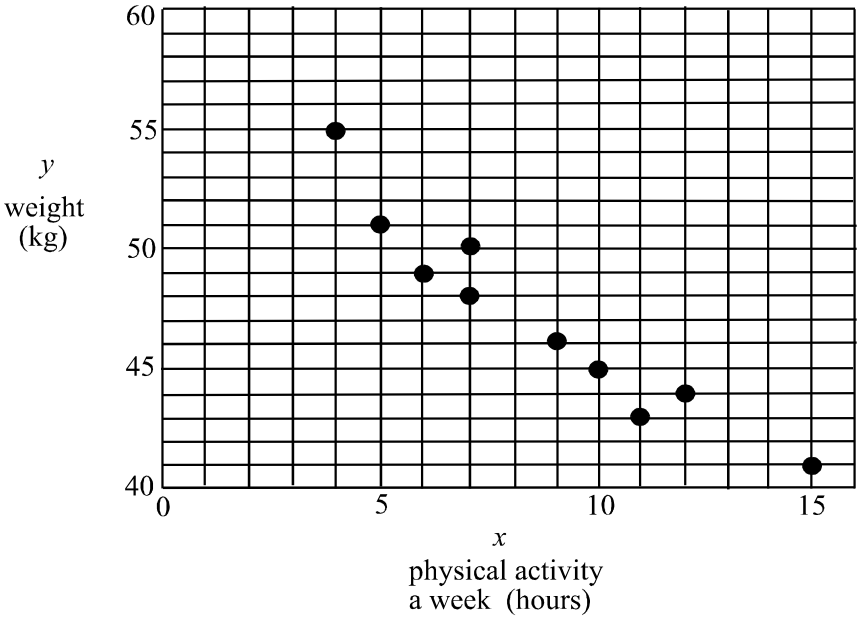
mean = ☐ kg , standard deviation = ☐ kg

2 marks

b. The interquartile range of the weight of these ten students is ☐ kg

1 mark

On the scatterplot below, the points representing the data from Table 1 have been plotted.



c. In this analysis, the independent variable is ☐

1 mark

d. The equation of the least squares regression is to be calculated for this set of data. It has been assumed therefore that the relationship between the number of hours of physical activity that a student engages in and their weight, is ☐

1 mark

e. The equation of the least squares regression line is given by

weight = ☐ − ☐ × number of hours of physical activity a week

Express answers correct to two decimal places.

2 marks

f. We know that ☐ % of the variation in the weight of these students can be explained by the number of hours a week that they engage in physical activity. Express your answer correct to the nearest whole number.

1 mark

g. How much does the equation of least squares regression overestimate or underestimate the weight of student A?

1 mark

Question 2

At the same school, the numbers of Grade 5 boys and girls who engage in 10 hours or more of physical activities a week together with the number of Grade 5 boys and girls who engage in less than 10 hours a week is shown in Table 2 below.

Table 2

	boys	girls	Total
10 hours or more a week	17	10	27
Less than 10 hours a week	6	12	18
Total	23	22	

a. Percentage Table 2 by calculating column percentages and write your answers in Table 3 below. Express your percentages correct to 2 decimal places.

Table 3

	boys	girls
10 hours or more a week		
Less than 10 hours a week		

2 marks

b. In percentaging Table 2 by calculating column percentages, which two categories of grade 5 students are we comparing?

1 mark

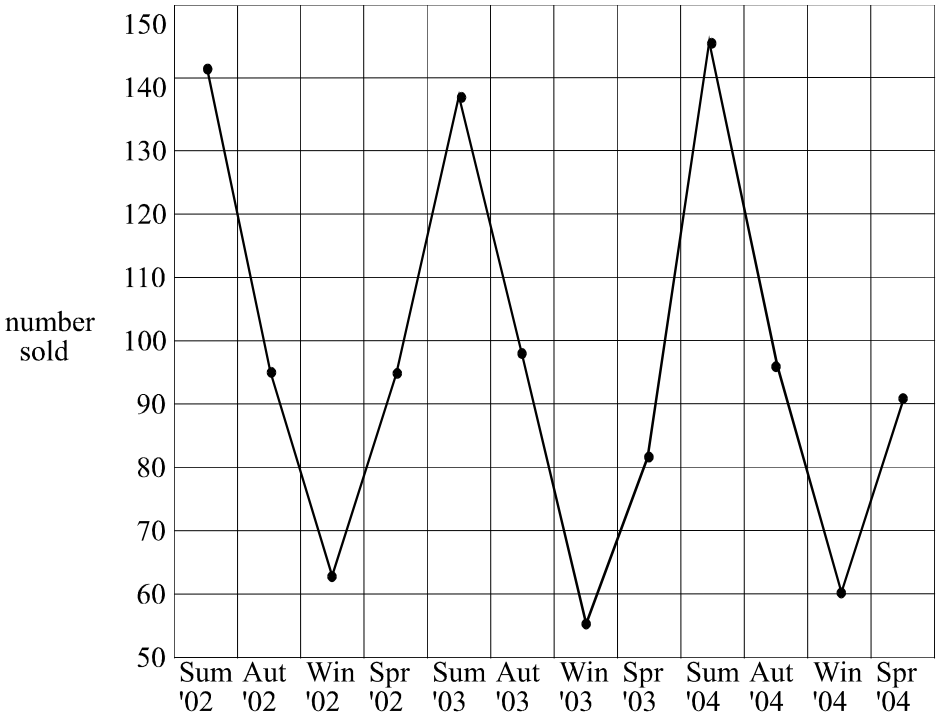
Question 3

A slimming business displays its seasonal sales in 'Slimming Plans' for 2002 – 2004 in Table 4 below.

Table 4

Season	Sum 02	Aut 02	Win 02	Spr 02	Sum 03	Aut 03	Win 03	Spr 03	Sum 04	Aut 04	Win 04	Spr 04
Number of Slimming Plans sold	142	95	63	95	137	98	55	82	145	96	60	91

A time series plot for this data is shown on the graph below.



a. Describe the data shown on the time-series plot.

1 mark

b. By using the three-median smoothing technique, smooth the time series shown on the graph above. Plot the smoothed time series on this same graph.

2 marks

Total 15 marks

Section B – Optional modules

Module 1: Number patterns and applications

If you choose this module all questions must be answered.

In a movie theatre, the horizontal distance in metres from the movie screen to the back of the seat of each of the first three rows is shown in the table below.

Row number (n)	1	2	3	4
Distance (t_n)	6.2	7.4	8.6	

The spacing of rows continues in this pattern to the back wall of the movie theatre that is located immediately behind the back row of seats.

Question 1

a. Complete the table above by finding the distance from the movie screen to the back of the 4^{th} row of seats.

1 mark

b. What is the distance from the back of one seat of one row to the back of the seat in the row immediately behind?

1 mark

c. The distances given in the second row of the table form a sequence. What type of sequence is it?

1 mark

d. The distance from the movie screen to the back wall of the movie theatre is 45.8 metres. How many rows of seats are in this movie theatre?

2 marks

e. If t_n is the distance from the movie screen to the back of the n^{th} row in the movie theatre, write down a difference equation relating t_{n+1} and t_n.

1 mark

Question 2

A particular movie was screened in this theatre, named theatre *A*, at each session each day. At the first session, 250 people were in the theatre. This number decreased by 6% each session thereafter.

a. How many people were there at the second session?

1 mark

b. How many people were there at the fifth session? (Express your answer to the nearest whole number.)

1 mark

c. What was the total number of people who saw this movie in this theatre in the first 50 sessions? Express your answer to the nearest whole number.

2 marks

d. At which session would the number of people attending first drop below 50?

2 marks

Question 3

In the adjoining movie theatre, theatre *B*, another movie is being shown at every session. The number of people, P_n, attending the n^{th} session of this movie is described by the difference equation

$$P_n = 2P_{n-1} - 50, \text{ where } P_2 = 116$$

a. How many people attended the first session?

1 mark

b. There are five screenings per day in theatres *A* and *B* of the respective films. Find which theatre had more people in total attending on the first day and by how many more. Express your two totals and hence your answer to the nearest whole number.

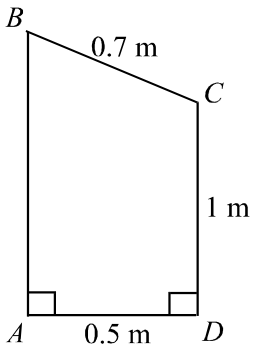
2 marks
Total 15 marks

Module 2: Geometry and trigonometry

If you choose this module all questions must be answered.

Question 1

The side view of a public rubbish bin is shown in the diagram below.



The height of the front face of the bin; indicated by *CD* is 1m. The distance *BC* is 0.7m and the distance *AD* is 0.5m.

a. Find angle *ABC*. Express your answer to the nearest minute.

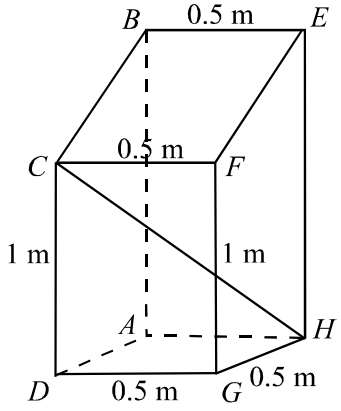
1 mark

b. Find the height *AB*, of the bin. Express your answer in metres; correct to two decimal places.

1 mark

Question 2

A "front on" view of the bin is shown in the diagram below with the lid indicated by *BCFE*. The base of the bin, indicated by *DGHA* is square.



a. Find the internal distance *CH*. Express your answer correct to 2 decimal places.

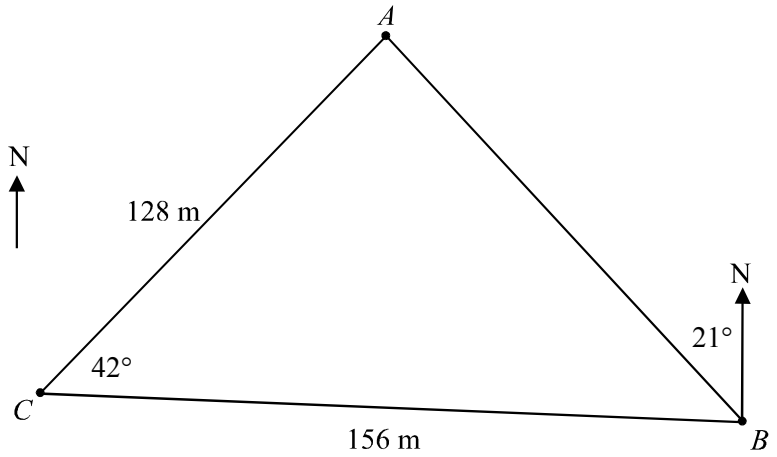
2 marks

b. On a scale diagram of the bin, the area of the base of the bin (where the base is indicated by *DGHA*), had an area of 1cm^2. Given the dimensions of the actual bin in the diagram above, find the scale used on the scale diagram.

2 marks

Question 3

Three of these bins are placed in a park at points *A*, *B* and *C* indicated in the diagram below.



a. What is the bearing of *A* from *B*?

1 mark

b. Find the distance *AB*. Express your answer to the nearest metre.

2 marks

c. Find angle *BAC* using your rounded answer to part **b.** and the sine rule. Express your answer to the nearest minute.

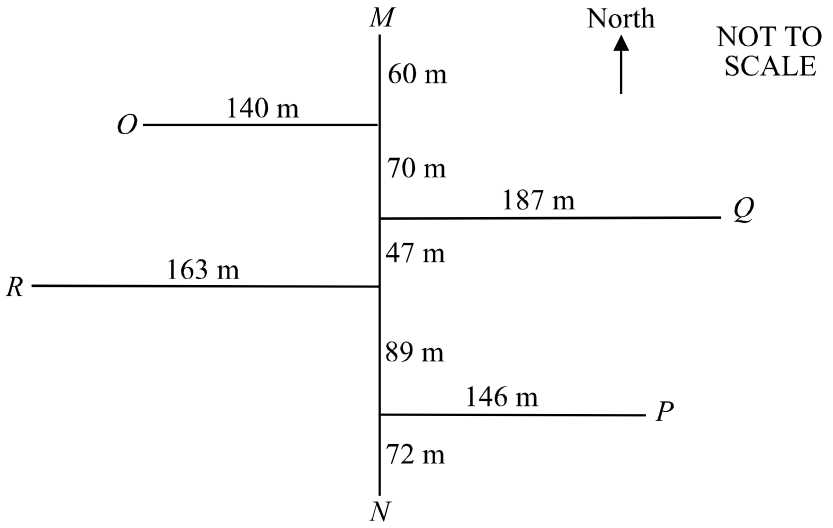
2 marks

d. What is the area of the triangle enclosed by the points *A*, *B* and *C*? Express your answer to the nearest square metre.

1 mark

Question 4

The park in Question 3 is rectangular in shape. A traverse survey of the park has been conducted and the field sketch is shown below.



The line *MN* runs north-south and the points *O* and *P* indicate gas barbecues in the park. The points *M* and *N* lie on the north and south boundaries respectively. The points *Q* and *R* lie on the east and west boundaries respectively of the park. All measurements are in metres.

a. What is the area of the park?

2 marks

b. What is the distance *OR*? Express your answer to the nearest metre.

1 mark

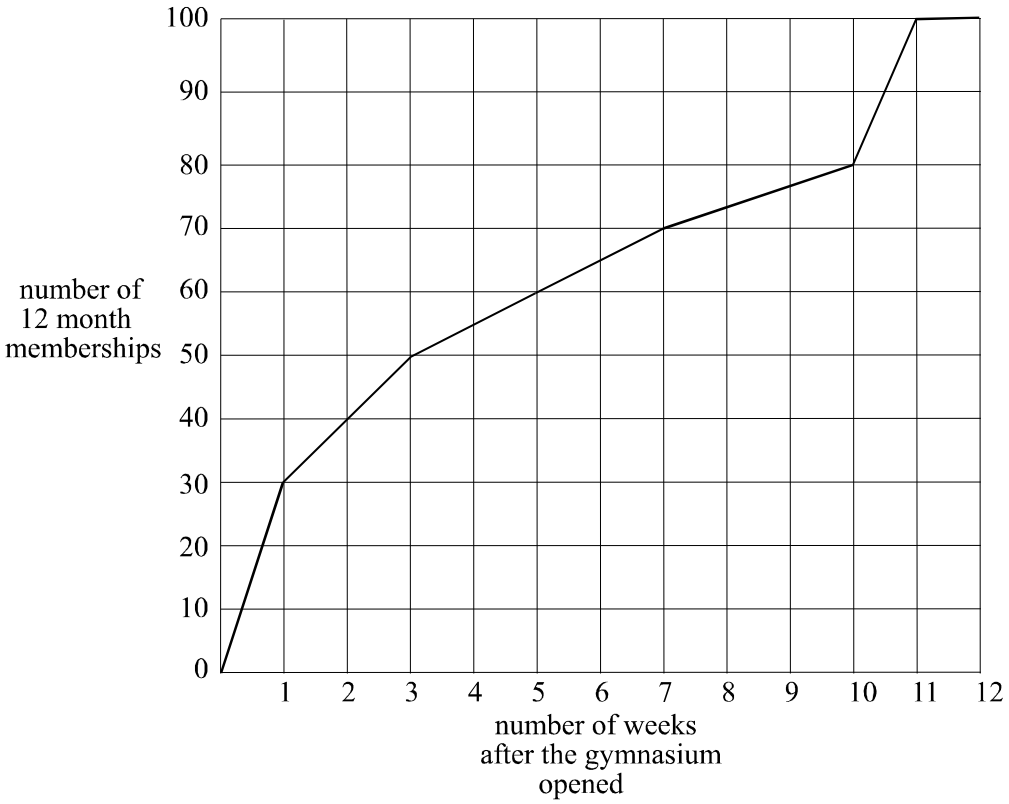
Total 15 marks

Module 3: Graphs and relations

If you choose this module all questions must be answered.

Question 1

The number of 12-month memberships that a new gymnasium has in the first 12 weeks of its operation is shown on the graph below.



a. How many 12-month memberships did the gymnasium have 3 weeks after it opened?

1 mark

b. How many 12-month memberships were taken out during the 11^{th} week after the gymnasium opened.

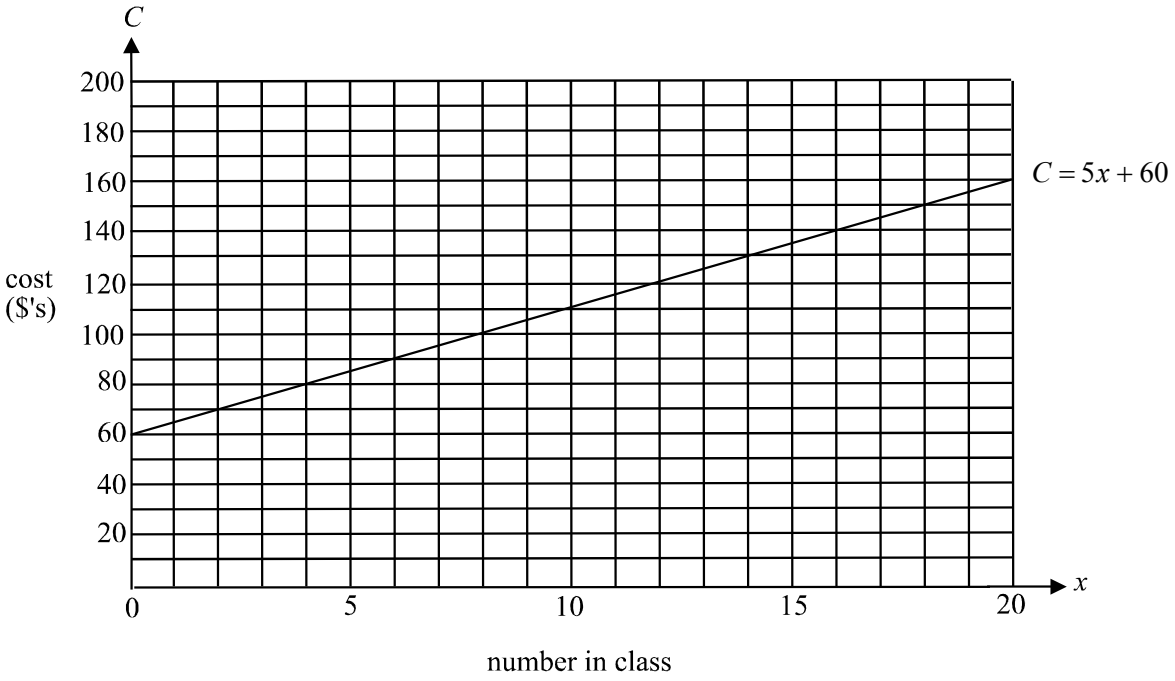
1 mark

c. During which week after the gym opened were the greatest number of 12-month memberships taken out?

1 mark

Question 2

Yoga classes are conducted at the gymnasium. The cost C, in dollars, of running a yoga class that has x class members is given by the equation $C = 5x + 60$.
The graph of this equation is shown below.



a. The revenue, R, in dollars, received by the gymnasium for running a yoga class that has x class members is given by $R = 10x$. Sketch the graph of R against x on the graph above.

2 marks

b. Find the number of class members needed in a yoga class for the gymnasium to break even by

i. using the graph

ii. solving algebraically the equations

$$R = 10x$$
$$C = 5x + 60$$

1 + 2 = 3 marks

c. How much profit would the gymnasium make on a yoga class that has 15 class members?

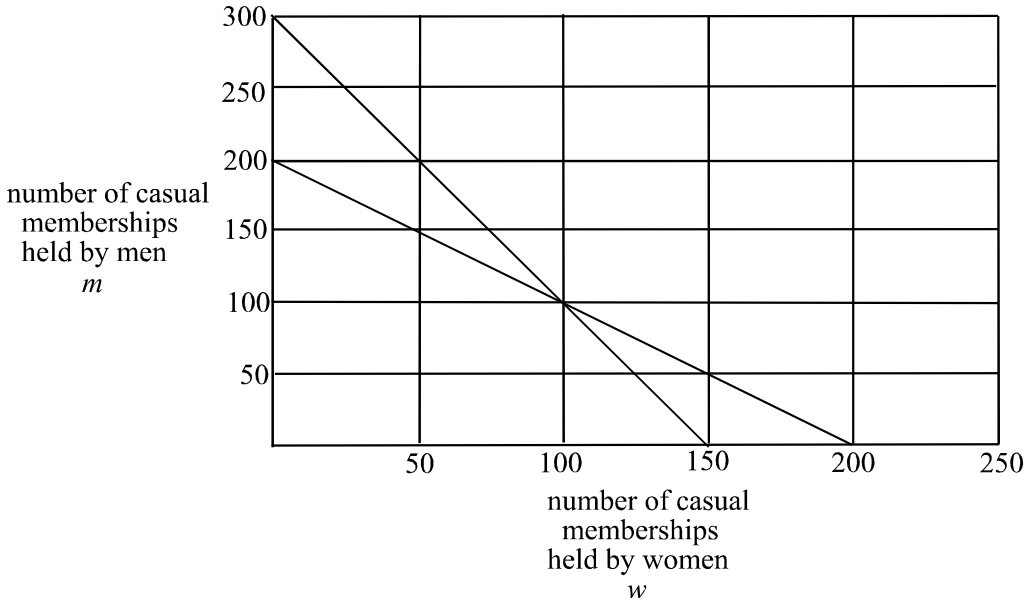
2 marks

Question 3

The gymnasium has constraints as far as the number of casual memberships it can have for men and women.
Let w equal the number of casual memberships held by women.
Let m equal the number of casual memberships held by men.
The constraints, brought about by available facilities and staff are given by

$$m + w \leq 200$$
$$\text{and } m + 2w \geq 300$$

The lines bordering the region described by these constraints are shown in the graph below.



a. On the diagram above, shade the feasible region for the number of casual memberships that the gymnasium can have for men and women.

1 mark

b. The profit, P, in dollars, for the gymnasium that comes from casual memberships held by men and women is given by

$$P = 2w + 3m$$

What is the maximum profit that can be made by the gymnasium through its casual memberships?

2 marks

c. For a short period, no more than 50 men can hold a casual membership.
Explain why this changes the maximum profit that the gymnasium can make from casual memberships and calculate the maximum profit that can now be made.

2 marks
Total 15 marks

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

The July statement for Micaela's savings account is shown below.

Date	Particulars	Deposits	Withdrawals	Balance
06July2005	Deposit	250		14 721.62
09July2005	Account keeping fee		5.00	14 716.62
15July2005	Withdrawal			14 391.62
22July2005	Withdrawal		118.50	14 273.12
31July2005	Deposit			14 453.12

a. Find the amount

i. withdrawn on 15 July 2005

ii. deposited on 31 July 2005

1 + 1 = 2 marks

b. Micaela receives 2.4% annual interest on this account that is calculated monthly on the minimum monthly balance. How much interest did Micaela receive in July from this account?

1 mark

Question 2

Micaela has $40 000 invested in an account that earns interest of 6.4% per annum compounding quarterly.

a. How much is in this account after 3 years?

2 marks

b. How long must Micaela keep this money invested until there is at least $100 000 in the account? Express your answer to the nearest quarter.

2 marks

Question 3

Micaela is going to use this money to start her own business in which she will be using computer equipment with a value of $24 000.

What will this computer equipment be worth after 5 years if it is depreciated

i. at a flat rate of 10% per annum?

ii. by 20% per annum on its reducing value?

1 + 2 = 3 marks

Question 4

Micaela has to borrow an additional $120 000 to set up her business. She takes out a reducing balance loan for this amount. The monthly repayments that Micaela will pay have been calculated using the annuities formula

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1}$$

The interest rate charged on Micaela's loan is 9% per annum and she intends fully paying the loan off over 12 years.

a. What are the values of n and R used to calculate Micaela's monthly repayments?

$n =$ ☐ $R =$ ☐

2 marks

b. Find Q, the monthly repayment that Micaela must make.

1 mark

c. Based on the monthly repayment from **(b)**, how much does Micaela still owe after 8 years?

1 mark

d. How much interest does Micaela pay in total on this loan?

1 mark

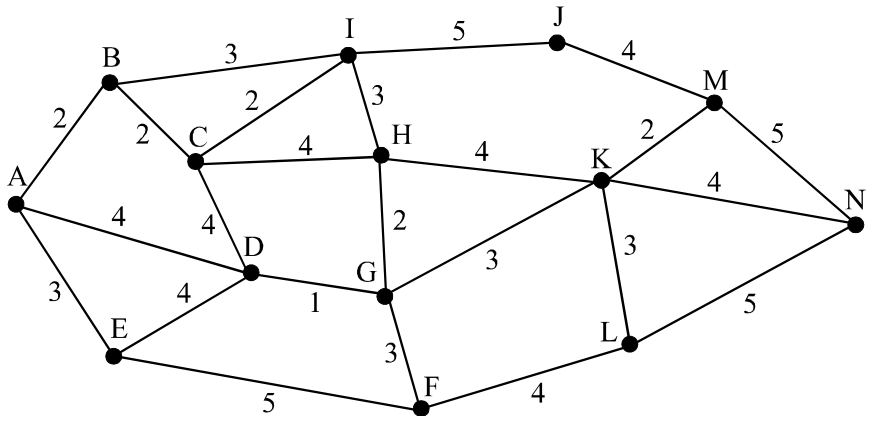
Total 15 marks

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

In an open plan office, cabling is run between workstations $A - N$ as indicated in the diagram below. The length of the cabling is given in metres.



a. What is the shortest length of cabling that runs from A to N?

_____

1 mark

b. Explain whether or not the network formed by the cabling contains an Euler circuit.

_____

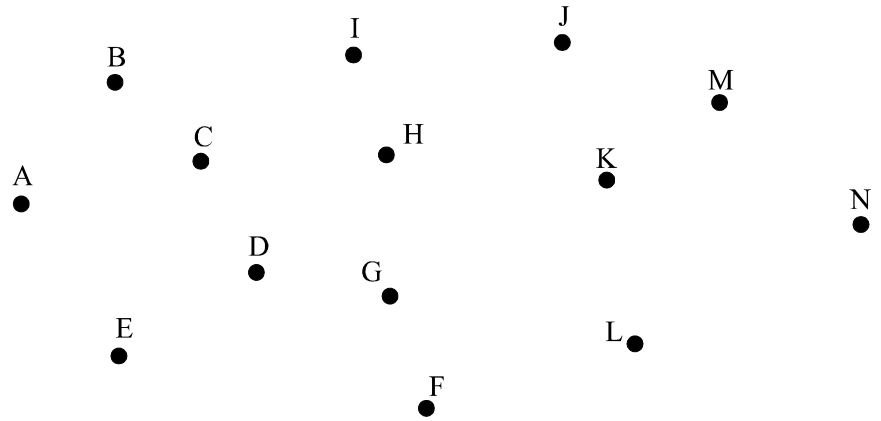
_____

_____

_____

1 mark

c. **i** On the diagram below draw the minimal-length spanning-tree for the cable network joining the workstations.



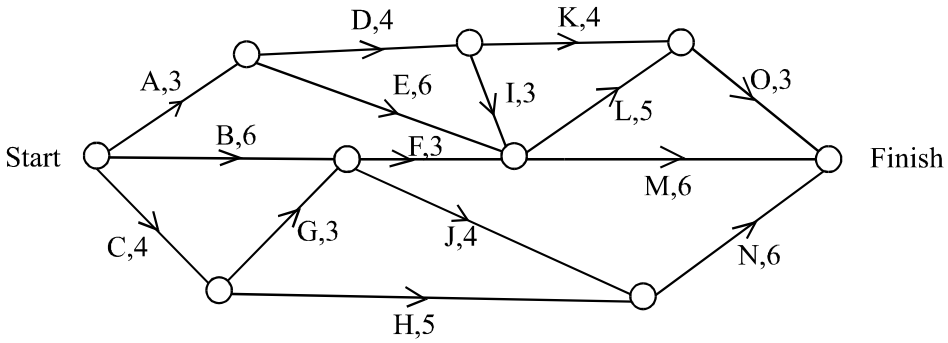
ii. What is the minimum length of cable required for this minimum spanning tree?

_____

2 + 1 = 3 marks

Question 2

An upgrade of the office was organized and involved 15 contractors performing their various activities. These activities together with the time they would take in days to complete are shown in the network diagram below.



For each of the activities required for the upgrade the table below shows the immediate predecessor(s), the earliest start time and the latest start time.

Activity	Immediate predecessor(s)	Earliest start time	Latest start time
A	-	0	1
B	-	0	1
C	-	0	0
D	A	3	4
E	A	3	5
F	B,G		8
G	C	4	4
H	C	4	6
I	D	7	8
J	B,G	7	7
K	D	7	
L	E,F,I	10	12
M		10	11
N	H,J	11	11
O	K,L	12	14

a. Use the information from the network to complete the shaded cells in the table.

3 marks

b. Write down the critical path for the project.

_____

_____

1 mark

c. What is the minimum time required to complete the project?

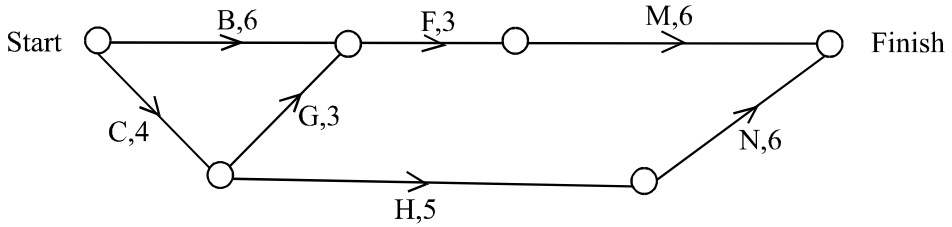
_____

1 mark

d. What is the slack or float time for activity *K*?

1 mark

e. Suppose that activities *A, D, E, I, J, K, L* and *O* are cut from the project as indicated in the network diagram.



What is the critical path for this network?

2 marks

f. Suppose that some of the remaining activities can be completed in less time with a given cost per day to achieve this as shown in the following table.

Activity	Original completion time (days)	Possible new completion time (days)	Cost per day to achieve new completion time ($'s)
B	6	4	$200
C	4	4	-
G	3	2	$400
F	3	3	-
H	5	5	-
M	6	5	$100
N	6	5	$50

Using this table together with the network showing activities *B, C*, *F, G, H*, *M* and *N*, find

i. the maximum number of days that can be cut from the time taken to complete the upgrade.

ii. the lowest cost to achieve this

1 + 1 = 2 marks
Total 15 marks

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \dfrac{r}{100}$

Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

area of circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Graphs and relations

Straight line graphs

gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a+d) + \ldots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \quad |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1}-1)}{a-1}, \quad a \neq 1$

$$= a^n t_0 + b\frac{(a^n - 1)}{a-1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$