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FURTHER MATHEMATICS - UNIT 3 & 4

TRIAL EXAMINATION 2 - Solutions

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Section A –

Core – Data Analysis Solutions

Question 1

a. Write the data is ascending order.
Then the data set x_i:= 12 14 25 28 32 36 36 43 50

Mean = $\frac{\sum_{i=1}^{8} x_i}{n}$, n = 8 (number of data) , $\sum_{i=1}^{8} x_i = 240$

Hence mean $\bar{x}$ = 240 / 8 = 30

1 mark

b. Here there are even number of data. Hence median is (n+1) / 2
Then (8+1) / 2 = 4.5 (position)
Hence median = (28 + 32) / 2 = 30

1 mark

c. The variance is $\frac{\sum_{i=1}^{8} (x_i - \bar{x})^2}{n-1}$ = 174

1 marks

d. The stranded deviation is the root of variance (s) .
s = 13.19 $\approx$ 13
Now calculate $\bar{x} - s, \bar{x} + s$ = 27 to 43
Hence percentage is 65 %

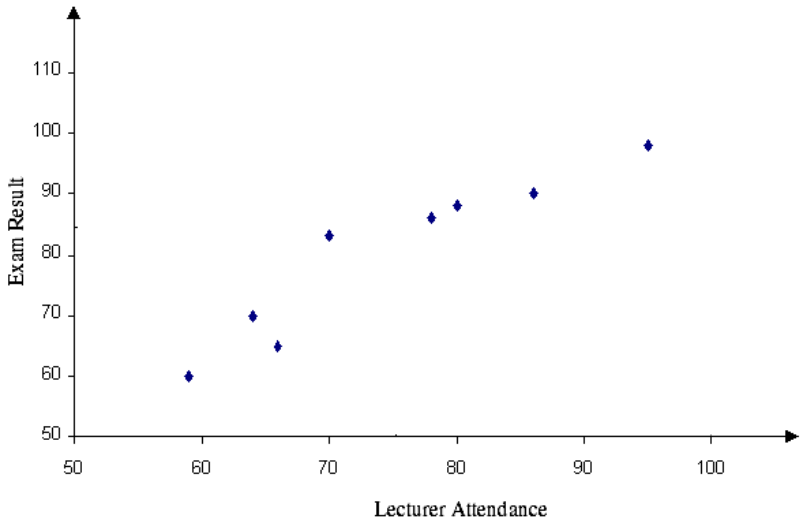
2 marks

Question 2

a. independent variable := Lecture attendance
dependent variable := exam result

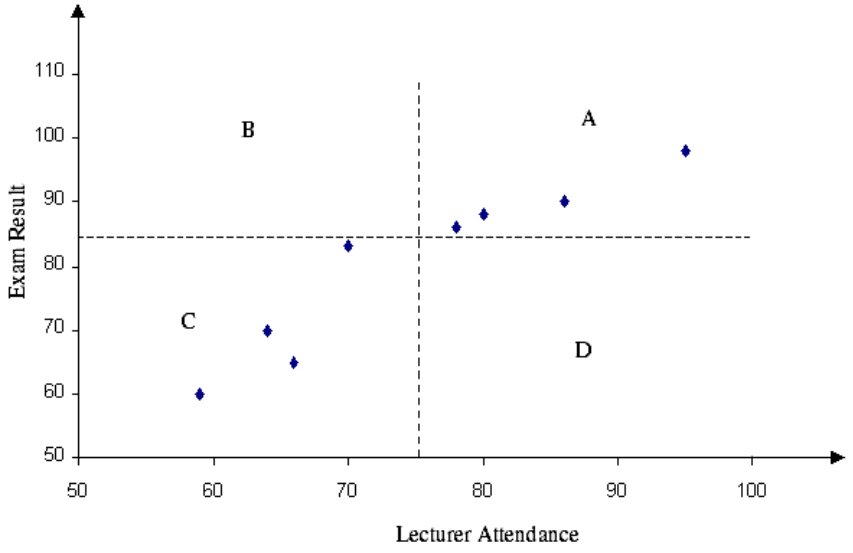
1 mark

b.



2 marks

c.



After drawing the scatter plot, locate the medians of x values and y values, then draw the vertical horizontal lines respectively the medians of x and y values.
Median of x = 74
Median of y = 85.5

Then calculate the q-correlation coefficient using the above graph and the following formula.

$$q = \frac{(a+c)-(d+d)}{a+b+c+d} = \frac{(4+4)-(0+0)}{4+4} = 1$$

1 mark

d. Moderate positive linear association.

1 mark

Question 3

a. You have to divide the points in to three groups as 3, 4, 3. Then have to find three medians

xl = 4 xm = 9 xu = 18
yl = 17 ym = 8.5 yu = 4

Then gradient is (m) = (yu – yl) / (xu – xl) = -13/14 = -0.93
Intercept is (b) = 1/3 * [(yl + ym + yu) – m * (xl + xm + xu)]
= 1/3 * (29.5 + 13/14 *31) = 19.42

y = -0.93x + 19.42

2 marks

b.

y actual	18	17	16	10	8	9	6	5	4	2
y predict	17.56	15.7	14.77	13.84	11.98	10.12	6.4	5.47	2.68	0.82
residual	0.44	1.3	1.23	-3.84	-3.98	-1.12	-0.4	-0.47	1.32	1.18

y actual means the given y values. Substituting values given as y predict.
Residual values calculated as y actual values – y predict.

3 marks

Total 15 marks

Module 1: Number patterns and applications

Question 1

a. First day = 3, second day = 3*3 = 9, third day = 9*3 = 27
Hence the geometric sequence is 3 9 27 81

2 marks

b. Here r = 3 $t_1 = a = 3$ n = 10
We have to find the total number of people who know the rumour.
For geometry sequence, the sum of the sequence is $S_n = \frac{a(r^n - 1)}{r - 1}$,(r >1)
Hence for a 10^{th} day $S_{10} = 88572$

1 mark

c. The n^{th} term of the geometric sequence is $t_n = ar^{n-1}$
Hence 5^{th} day means n = 5
Hence $t_5 = 3*3^4 = 243$

2 marks

Question 2

a. Here $t_1 = 6.4$, $t_2 - t_1 = 2.1$
Hence d = 2.1
We know $t_n = a + (n - 1) * d$
Hence $t_n = 6.4 + (n-1) * 2.1$

2 marks

b. Here n = 5
Hence $t_5 = 6.4 + (5-1)*2.1 = 15.2$ tones

1 mark

c. We know $t_n = a + (n - 1) * d$
Here $t_n = 40$
$40 = 6.4 + (n - 1) * 2.1$
n = 17

2 marks

Question 3

a. Here $t_1 = 2$, $t_2 = 6$, $t_3 = 18$ and a = 2 (since $t_1 = 2$)
We know $t_n = a\, r^{n-1}$
Hence $t_n = 2\, r^{n-1}$, $t_2/t_1 = 6 / 2 = 3$
Hence r = 3
Hence $t_n = 2 * 3^{n-1}$

2 marks

b. The form of first order difference equation for the geometry series is
$t_{n+1} = at_n$
We know $t_1 = 2$, $t_2 = 6$, $t_3 = 18$
When n =1 $t_2 = at_1$ $6 = a2$,
Hence a = 3
Hence $t_{n+1} = 3\, t_n$

1 mark

c. From part (a) $t_n = 2* 3^{n-1}$ ⟶ (1)
From part (b) $t_{n+1} = 3\ t_n$ ⟶ (2)
Substituting (1) for (2)
$t_{n+1} = 3 *2 * 3^{n-1}$
Hence $t_{n+1} = 2 * 3^{n}$

2 marks

Total 15 marks

Module 2: Geometry and Trigonometry

Question 1

a. Volume of the cylinder = area of uniform cross section * height
$= \pi r^2 h$
$= (22 / 7) * 25 * 30 = 2357.14\ cm^3$

1 mark

b. We know 1000 cm3 = 1 litre
Hence 2.35714 litre

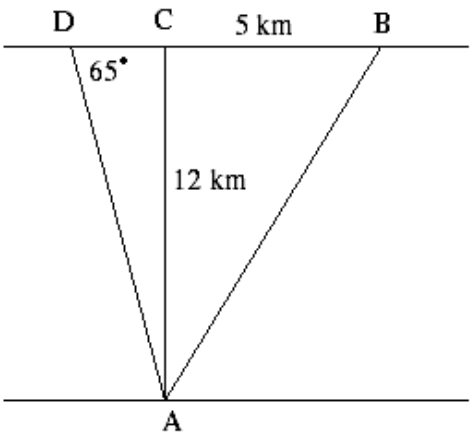
2 marks

c. Volume of cylinder = volume of pyramid
Hence $(22 / 7) *25 * 30 = (1/3) *a^2 *15$
(a denotes the pyramids base length)
a = 21.71

2 marks

Question 2

a.



Using Pythogoras Theorem
$AC^2 + BC^2 = AB^2$
$5^2 + 12^2 = AB^2$ Hence AB = 13 km

2 marks

b. We can say sin value of the angle ABC = 12 / 13
Using the sine rule to triangle ADB

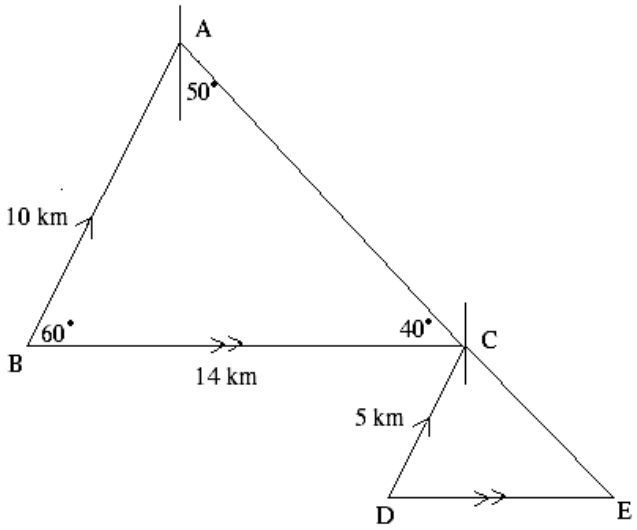
$$\frac{\sin 65}{13} = \frac{\sin(12/13)}{AD}$$

Hence AD = 13.33

3 marks

Question 3

a.



Using sine rule to ABD triangle

$$\frac{\sin 60^0}{AC} = \frac{\sin 40^0}{10}$$

AC = 13.47

2 marks

b. Using similarity triangulation formula. Triangle ABC and CDE are similar triangles. (Since AB // CD and BC // DE hence <ABC = <CDE and <BAC = <DCE)

Hence $\frac{10}{DC} = \frac{AC}{CE}$

$\frac{10}{5} = \frac{13.47}{CE} \Longrightarrow$ CE = 6.735

1 mark

d. W can calculate <BCD as 60^0. (Since AB // CD and BC // DE)
Using the cosine rule
$BD^2 = BC^2 + DC^2 - 2*BC*DC \cdot \cos 30'$
$= 99.75$
$BD = 9.99 = 10$

2 marks

Total 15 marks

Module 3: Graphs and Relations

Question 1

a. Commission is = 25,000*(1.5 /100) = 375 $

1 mark

b. Commission is = 25,000*(1.5 /100)+ 25,000* (1.2/100) + 20,000*(1/100)
= 950 $

2 marks

c. Commission = 30,000 * (1.5/100) + 25,000 * (1.2/100) +
(sales price- 55,000)*(1/100)

1 mark

d. If he got $ 1200
The commission for first 30,000 = 30,000*(1.5 /100) = 450 $
The commission for second 25,000 = 25,000*(1.2 /100) = 300 $
Remind of commission = 1200 – (450 + 300) = 450 $
This commission he gets in interest rate 1%
Hence value = 450 * 100 = 45,000
Hence value of the car = 30,000 + 25,000 + 45,000
= 100,000 $

1 mark

Question 2

a. Cost = 230 + 27 * 15 = 635 $
Revenue = 300 * 15 – 250 = 4250 $

2 marks

b. 635 = 230 + 27n
n = 15
Hence number of computers 15

1 mark

c. Profit = Revenue – Cost
13,170 = 300 n- 250 – (230 + 27 n)
13, 170 = 273n – 480
n = 50

2 marks

Question 3

a. Let x = number of automobiles shipped to Victoria from Factory 1
Let y = number of automobiles shipped to New South Wales from Factory 1
Hence $x + y \leq 130$

1 mark

b. Now we can say (150 – x) = number of automobiles shipped to Victoria from Factory 2
(90 - y) = number of automobiles shipped to New South Wales from Factory 2
Hence $(150-x) + (90-y) \leq 175$
$240 - x - y \leq 175$
$65 \leq x + y$

1 mark

c. Factory 1 shipped x automobiles to Victoria at 120 $ per car.
Factory 1 shipped y automobiles to New South Wales at 350 $ per car.
Factory 2 shipped (150-x) automobiles to Victoria at 400 $ per car.
Factory 2 shipped (90-y) automobiles to New South Wales at 150 $ per car.
Hence Total cost = x*150 + 350*y + (150-x)* 400 + (90-y)*150
= 73,500 – 280x + 200y

3 marks

Total 15 marks

Module 4: Business Related Mathematics

Question 1

a. Deposit value = 700 * (10 /100) = 70 $

1 mark

b. Customer payment = 950 $ - 70 $ = 880 $
Hence quarters instalment = 880 / 8 = 110 $

2 marks

c. Flat rate interest $I = \frac{\Pr t}{100}$

$$110 = \frac{880 * r * 8}{100}$$

$$r = 1.5625$$

1 mark

d. Effective interest $= \frac{2n}{n+1} * \textit{flat rate}$

(n – number of instalments)

$= \frac{2*8}{8+1} * 1.5625$

$= 2.778$

1 mark

Question 2

a. We know $A = PR^T$, $R = 1 + (r/100)$

A = current value

P = Principal value

R = Interest rate

Hence $6047.92 = 4500 * R^{10}$

$R^{10} = 1.3439$

$R = 1.0299 = 1.03$

$R = 1 + (r/100)$

$1.03 = 1 + r / 100$

$3 = r$

Hence inflation rate 3

2 marks

b. $A = PR^T$, $R = 1 + (r/100)$

$13500 = 4500 (1.03)^T$

$3 = (1.03)^T$

$\ln 3 = T \ln 1.03$

$T = 37.16$

$T = 37$ years

2 marks

c. $A = PR^T$, $R = 1 + (r/100)$

$5216.74 = 4500 (1.03)^T$

$1.16 = (1.03)^T$

$\ln 1.16 = T \ln 1.03$

$T = 5$ years

Hence 10 years ago.

2 marks

Question 3

a. Rare of depreciation $= \frac{total\ \ depreciation}{number\ \ of\ \ years}$

But we want to calculate depreciation for a month.

Hence Rare of depreciation $= \frac{total\ \ depreciation}{number\ \ of\ \ years * 12}$

$= 7000 / 8 * 12$

$= 72.92$

2 marks

b. We know BV = P – d T

BV = Book value after T years

P = Cost price

T = Time in years

D = depreciation rate

Hence BV = 25000 – 875 T (since we have to use depreciation rate for year. Hence use d as 875. (7000/8))

2 marks

c. Using part (b)

BV = P – d T

33750 = 25000 – 875 T

T = 10 years

1 mark

Total 15 marks

Module 5: Network and Decision Mathematics

Question 1

a. AB→ BC→ CD→ DB→ BE→ED→ DF→ FE→ EA→ AH→ HF→ FG→ HG → HI→ IG→ GJ→ JI

Here we have to use one edge at once as sub Euler path.

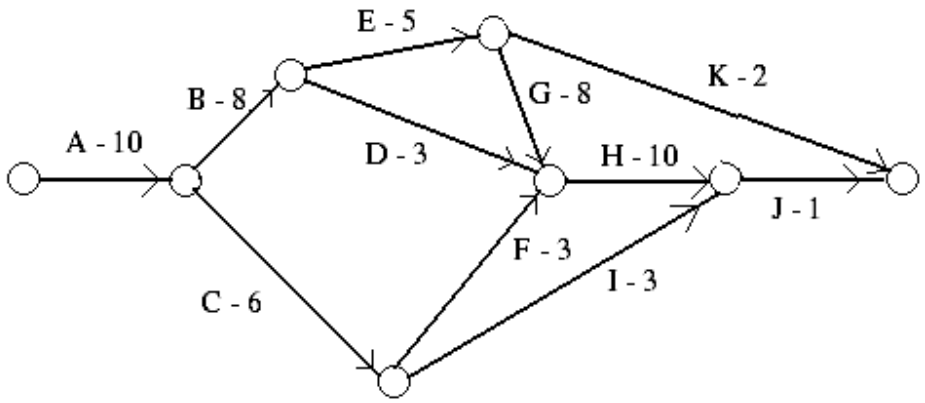
2 marks

b. AB→ BC→ CD→ DB→ BE→ ED→ DF→ FG→ GJ→ JI→ IG→ GH→ HI→ IA
Here we have to use one edge at once as sub Euler circuit with same start and ending point.

3 marks

Question 2

a.



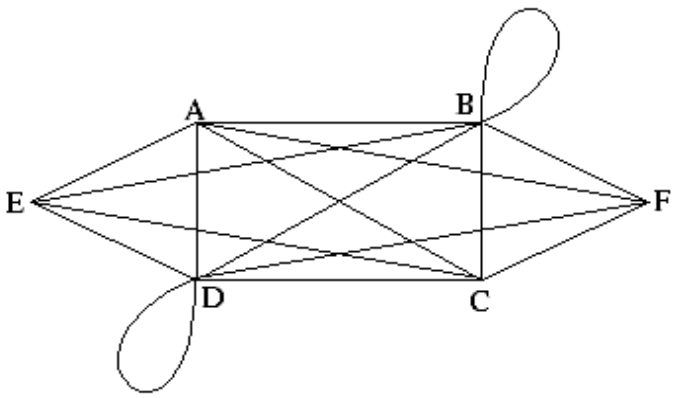
3 marks

b. Earliest completion path is A→ B→ E→ G→ H→ J.
Hence 42 days.

2 marks

Question 3

a.



3 marks

b. EA→ AB→ BF→ FC→ CD→ DE.
Here we have to use one edge at once in Euler circuit with same start and ending point.

1 marks

c. Here we have to find points which have loops.
Hence B and D.

1 mark

Total 15 marks